IAENG International Journal of Applied Mathematics, 37:1, IJAM_37_1_6

PID design for improved disturbance attenuation: min max Sensitivity matching approach

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***Abstract*— This paper presents an approach to PID controller tuning based on a simple plant model description; First Order plus Time Delay (FOPTD). The approach is based on the formulation of an optimal approximation problem in the frequency domain for the Sensitivity transfer function of the closed loop. The inclusion of the Sensitivity function allows for a disturbance attenuation specification. The solution to the approximation problem provides a set of tuning rules that constitute a parameterized set that is formulated in the same terms as in [1] and include a third parameter that determines the operating mode of the controller. This factor allows to determine a tuning either for step response or disturbance attenuation. The approach can be seen as an implicit two-degree-of-freedom controller because by using one single parameter the operating mode (servo/regulation) of the control system is determined as well as the appropriate tuning of the controller.**

***Index Terms*— Process Control, PID Tuning, Optimization**

I. INTRODUCTION

Proportional-Integrative-Derivative (PID) controllers are with no doubt the most extensive option that can be found on industrial control applications. Their success is mainly due to its simple structure and meaning of the corresponding three parameters. This fact makes PID control easier to understand by the control engineer than other most advanced control techniques.

Because of the widespread use of PID controllers it is interesting to have simple but efficient methods for tuning the controller. In fact, since Ziegler-Nichols proposed their first tuning rules [2], an intensive research has been done. From modifications of the original tuning rules [3], [4], [5] to a variety of new techniques: analytical tuning [6], [7]; optimization methods [8], [9]; gain and phase margin optimization [8], [10], just to mention a few.

Recently, tuning methods based on optimization approaches with the aim of ensuring good stability robustness have received attention in the literature [11], [12]. However these methods, although effective, use to rely on somewhat complex optimization procedures and do not provide tuning rules. Instead, the tuning of the controller is defined as the solution of the optimization problem. However, from an end-user point of view, it is acknowledged if a precise meaning is given to the tuning parameters instead of just taking the output of the numerical algorithm as the tuning values.

In [13] an approach to PID tuning is presented, based on a combination of a simple model description; First Order plus Time Delay (FOPTD); and closed loop specifications with robustness considerations. The tuning rules are given parameterized form in terms of desired time constant and robustness level and, secondly, a completely automatic tuning determined by the process parameters [1]. The problem with this approach is that the design problem is stated completely in terms of a step response specification. Therefore the resulting tuning provide low disturbance attenuation performance.

The purpose of this paper is to extend this approach in order to include disturbance attenuation specifications. The design problem is stated in similar terms but considering the closed loop Sensitivity function instead of the reference to output relation. The design problem is formulated as an optimal approximation problem in such a way the resulting PID tuning rules include, as a special case, the tuning guidelines provided in [13]. The new tuning rules constitute a parameterized set that is formulated in the same terms as in [1] and include a third parameter that determines the operating mode of the controller. This factor allows to determine a tuning either for step response or disturbance attenuation.

The paper is organized as follows. Section 2 presents the problem formulation: process model, PID structure and the optimization problem based on a min-max optimal approximation problem. Section 3 reviews the solution to the min-max optimization problem and provides the structure of the optimal controller. Starting from the optimal controller structure, Section 4 presents the tuning rules that originate from a reference to output step response specification. Along the same lines, Section 5 extends the results to the case of a Sensitivity based approximation problem in order to include a disturbance attenuation specification. An example

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This work has received financial support from the Spanish CICYT program under grant DPI2004-06393. The financial support from the University of Costa Rica and from the MICIT and CONICIT of the Government of the Republic of Costa Rica for the O.Arrieta's PhD studies is greatly appreciated.

(Advance online publication: 15 August 2007)

is presented in section 6. Section 7 presents a procedure for automatically select the appropriate sensitivity disturbance problem according to the minimization of an ISE criterion. Final conclusions and considerations for further extensions are conducted in section 8.

II. PROBLEM FORMULATION

In this section the controller equations are presented as well as the assumed process model structure and the optimization problem that is posed in order to tune the PID controller.

A. PID Controller

There exists different ways to express the PID control law [14]. In this paper we concentrate on the ISA PID control law [8]:

$$\begin{aligned} u(s) \quad = \quad & K_p \left[br(s) - y(s) + \frac{1}{sT_i}(r(s) - y(s)) \right. \\ & \left. + \frac{sT_d}{1 + sT_d/N}(cr(s) - y(s)) \right] \end{aligned} \quad (1)$$

where $r(s)$, $y(s)$ and $u(s)$ are the Laplace transforms of the reference, process output and control signal respectively. K_p is the PID gain, T_i and T_d are the integral an derivative time constants, finally N is the ratio between T_d and the time constant of an additional pole introduced to assure the properness of the controller. Parameters b and c are called set-point weights and constitute a simple way to obtain a 2-DOF controller. As their choice does not affect the feedback properties of the resulting controlled system, with no loss of generality here we will assume $b = c = 1$. This way, the PID transfer function we will work with can be written as:

$$K(s) = K_p \frac{1 + s(T_i + \frac{T_d}{N}) + s^2 \frac{T_i T_d (N+1)}{N}}{sT_i(1 + s\frac{T_d}{N})} \quad (2)$$

B. Process Model

An important category of industrial processes can be represented by a First Order Plus Dead Time (FOPDT) model as:

$$G_n(s) = \frac{Ke^{-Ls}}{1 + Ts} \quad (3)$$

where K is the process gain, T the time constant and L the time delay. This kind of models are easy to determine by means of a simple step response experiment to get the process reaction curve. In order to deal with the delay term is usual to use a rational approximation. The following simple first order Taylor expansion of the e^{-Ls} term will be used.

$$e^{-Ls} \approx 1 - Ls \quad (4)$$

C. Design Problem Formulation

The approach presented in this paper is based on Sensitivity function optimization. Roughly speaking the goal is to tune the PID controller to match a desired target Sensitivity function. This problem can be formulated as a weighted model matching problem between a specified desired Sensitivity, $S^d(s)$, and the achieved Sensitivity, $S(s) = (1 + G_n(s)K(s))^{-1}$, as:

$$\min_{K(s)} \parallel W(s)(S^d(s) - S(s)) \parallel_\infty \quad (5)$$

The weighting function, $W(s)$, allows to formulate the model matching problem as a frequency dependent approximation problem.

In a previous work [1] a similar design approach was presented where the model matching problem is stated in terms of the Complementary Sensitivity transfer function $T(s) = G_n(s)K(s)(1 + G_n(s)K(s))^{-1}$. To optimize for $T(s)$ constitute a step response design problem. On the other hand, $S(s)$ determines the disturbance attenuation properties of the feedback control system. Here we will show that problem (5) can be stated in such a way that the Complementary Sensitivity optimization results to be a special case of the former.

In order to formulate problem (5) in a more suitable way the controller design is recast on the Internal Model Control framework. This will allow the design problem to be expressed in terms of the IMC parameter. The Internal Model Control (IMC) [15] , [16] is based on the introduction of a model of the plant running in parallel with the actual plant. Comparison with the usual feedback configuration leads to the following relation between the IMC and classical feedback controller:

$$C(s) = \frac{K(s)}{1 + K(s)G_n(s)} \quad (6)$$

$$K(s) = \frac{C(s)}{1 - C(s)G_n(s)} \quad (7)$$

On the basis of the introduced IMC parameter $C(s)$, the closed loop transfer function relations Sensitivity, $S(s)$, and Complementar Sensitivity, $T(s)$, read as follows:

$$T(s) = C(s)G_n(s) \qquad S(s) = 1 - C(s)G_n(s) \quad (8)$$

Therefore, the following min-max problems can be formulated:

$$C_S^o(s) = arg \min_{C(s)} \parallel W(s)(S^d(s) - (1 - G_n(s)C(s))) \parallel_\infty \quad (9)$$

$$C_T^o(s) = arg \min_{C(s)} \parallel W(s)(T^d(s) - G_n(s)C(s)) \parallel_\infty \quad (10)$$

where $C_S^o(s)$ is the IMC solution to the Sensitivity optimization problem, whereas $C_T^o(s)$ is the solution to the Complementary Sensitivity optimization problem. This second controller is introduced just for comparison purposes.

Next section will solve problem (9) and derive the corresponding tuning relation for the ISA-PID controller (2) by using (7).

III. SOLUTION TO THE OPTIMAL APPROXIMATION PROBLEM

The design problem has been formulated in (9) and (10) as an approximation problem in the frequency domain. Both problems are special cases of:

$$\min_{C(s)} \|E(s)\|_\infty = \min_{C(s)} \|W(s)(M(s) - N(s)C(s))\|_\infty \quad (11)$$

Effectively, (10) results from $M(s) = T^d(s)$ and $N(s) = G_n(s)$ and, (9) from $M(s) = 1 - S^d(s)$ and $N(s) = G_n(s)$. Several approaches exists to solve this $\mathcal{H}_\infty$ problem. See [17], [18] among others. Here we will follow a particularization of the solution presented in [19] and also used in [1] where a polynomial approach was taken. This has the advantage of providing the structure of the optimal controller. Therefore, as we will do here, the problem statement can be constrained in order to provide a solution that leads to a PID controller.

First of all $N(s)$, $M(s)$ and $W(s)$ are factored as:

$$N(s) = \frac{n_N(s)}{d_N(s)} \quad M(s) = \frac{n_M(s)}{d_M(s)} \quad W(s) = \frac{n_W(s)}{d_W(s)}$$

The solution to the minimization of the cost function (11) lies in optimal interpolation theory. First, factorize the numerator $n_N(s)$ as:

$$n_N(s) = n_N^+(s) n_N^-(s)$$

where the polynomial $n_N^+(s)$ only has stable roots and $n_N^-(s)$ is the remaining part. In order to obtain a unique factorisation the polynomial $n_N^+(s)$ is assumed to be monic. Let ν =deg($n_N^-(s)$) and $\{z_1, z_2, ..., z_\nu\}$ be the distinct zeros of $n_N^-(s)$. From equation (11) it results that the error function $E(s)$ is subjected to the following interpolation constraints:

$$E(z_i) = W(z_i)M(z_i) \qquad i = 1 \dots \nu \quad (12)$$

If z_i is a zero with multiplicity ν_i, then additional differential interpolation constraints should be imposed.

A well established theory [20] , [21], [17] that solves this problem exists and a closed form solution can be obtained from the following lemma [17]:

Lemma 3.1: The optimal $E^o(s)$ which minimizes $\|E(s)\|_\infty$ is of an all-pass form:

$$E^o(s) = \begin{cases} \rho \frac{q(s)^*}{q(s)} & \text{if } \nu \geq 1 \\ 0 & \text{if } \nu = 0 \end{cases} \quad (13)$$

where $q(s) = 1 + q_1 s + q_2 s^2 + \ldots + q_{\nu-1} s^{\nu-1}$ is a strictly hurwitz polynomial and $q^*(s) = q(-s)$.

Furthermore, the constants ρ and $\{q_i\}_{i=1}^{\nu-1}$ are real and are uniquely determined by the interpolation constraints (12).

Now we will proceed with the application of this lemma in order to compute the optimal $C(s) = C^o(s)$. Note first that in our case $\nu = 1$ and $z_1 = 1/L$. Therefore the interpolation constraints give the following value for the optimal cost ρ:

$$\rho = W(1/L)M(1/L) \quad (14)$$

Application of the above lemma gives the following equation for the optimal parameter $C^o(s)$:

$$W(s)M(s) - W(s)N(s)C^o(s) = \rho \frac{q^*(s)}{q(s)}$$

then,

$$\begin{aligned} C^o(s) &= (W(s)N(s))^{-1} \left(W(s)M(s) - \rho \frac{q^*(s)}{q(s)} \right) \\ &= \frac{d_W(s) d_N(s)}{n_W(s) n_N^+(s) n_N^-(s)} \\ &\quad \left(\frac{n_W(s) n_M(s) q(s) - \rho q^*(s) d_W(s) d_M(s)}{d_W(s) d_M(s) q(s)} \right) \end{aligned} \quad (15)$$

In order for $C^o(s)$ to be a stable transfer function, $n_N^-(s)$ must be a factor of the numerator. That is to say, there must exist a polynomial $\chi(s)$ such that:

$$\begin{aligned} n_N^-(s)\chi(s) &= n_W(s) n_M(s) q(s) \\ &\quad - \rho q^*(s) d_W(s) d_M(s) \end{aligned} \quad (16)$$

It follows that, to determine the optimal controller $C^o(s)$, the $\chi(s)$ polynomial must be known. In any case, the optimal $C^o(s)$ will obey to the following structure:

$$C^o(s) = \frac{d_N(s)\chi(s)}{n_W(s) n_N^+(s) d_M(s) q(s)} \quad (17)$$

Expression (17) provides the structure of the IMC parameter $C(s)$ that solves the optimal approximation problem (11). In the next two sections, this structure will be applied to the case where the approximation problem arises from a Sensitivity and Complementary Sensitivity matching problems.

IV. STEP RESPONSE TUNING

This section reviews the main result of [13] and [1] providing the tuning relations that arise from the application of the solution to the optimal approximation problem to solve the design problem (10). The specification of a target $T^d(s)$ corresponds to a step response specification: the controller is chosen in order to achieve a desired reference to output behavior.

The approximation problem is formulated, according to (10), as:

$$C_T^o(s) = arg \min_{C(s)} \parallel W(s)(T^d(s) - G_n(s)C(s)) \parallel_\infty \quad (18)$$

We will use $T^d(s)$ to specify the desired closed loop time constant, T_M. Therefore it will take the form:

$$M(s) = \frac{n_M(s)}{d_M(s)} = \frac{1}{1 + T_M s} \quad (19)$$

With respect to the weighting function, $W(s)$, in order to automatically include integral action and keep it as simple as possible, we will assume the following form:

$$W(s) = \frac{n_W(s)}{d_W(s)} = \frac{1 + zs}{s} \quad (20)$$

By using this settings, the minimum cost, ρ_T is given by:

$$\min \|E(s)\|_\infty = |\rho_T| = L\frac{(L+z)}{(L+T_M)} \quad (21)$$

and the solution for the optimal, $C_T^o(s)$, is:

$$C_T^o(s) = \frac{1}{K}\frac{(1+Ts)(1+\chi_T^1 s)}{(1+T_M s)(1+zs)} \quad (22)$$

with,

$$\chi_T^1 = z + L - \rho_T \quad (23)$$

.

The resulting feedback controller is:

$$K_T^o(s) = \frac{1}{K(\rho_T + T_M)}\frac{(1+Ts)(1+\chi_T^1 s)}{s(1+T_M\frac{(\rho_T+z)}{(\rho_T+T_M)}s)} \quad (24)$$

can be identified to (2) with the following expressions for the controller parameters:

$$\begin{aligned} K_p^T &= \frac{T_i^T}{K(\rho_T + T_M)} \\ T_i^T &= T + \chi_T^1 - T_M\frac{(\rho_T+z)}{(\rho_T+T_M)} \\ \frac{T_d^T}{N^T} &= T_M\frac{(\rho_T+z)}{(\rho_T+T_M)} \\ N^T + 1 &= \frac{T}{T_i^T}\frac{\rho_T}{L}\frac{(\rho_T+T_M)}{(\rho_T+z)} \end{aligned} \quad (25)$$

These tuning relations provide the four ISA-PID parameters parameterized in terms of the desired T_M and z as determining the frequency range where the solution to (11) is to provide a better match.

It is worth to note that a choice for T_M and z is provided in [1]. If we choose $T_M = \sqrt{2}L$ and $z = \sqrt{2}T_M = 2L$, (25) provides the following simple tuning rule:

$$\begin{aligned} K_p^T &= \frac{T_i^T}{KL2.65} \\ T_i^T &= T + 0.03L \\ \frac{T_d^T}{N^T} &= 1.72L \\ N^T + 1 &= \frac{T}{T_i^T} \end{aligned} \quad (26)$$

V. DISTURBANCE ATTENUATION TUNING

In this section the approximation problem is posed in terms of the Sensitivity function. Therefore specifying a desired disturbance to output target function, $S^d(s)$. Following similar steps as in the previous case, we will get tuning relations that are to be considered for a Disturbance attenuation problem.

The approximation problem is formulated, according to (9), as:

$$C_S^o(s) = arg \min_{C(s)} \parallel W(s)(S^d(s) - (1 - G_n(s)C(s))) \parallel_\infty \quad (27)$$

The target Sensitivity function, $S^d(s)$ is given the following form:

$$S^d(s) = \frac{\gamma s}{T_M s + 1} \quad (28)$$

Therefore, the resulting reference model to be considered in the approximation problem (11) results to be

$$M(s) = 1 - S^d(s) = \frac{n_M(s)}{d_M(s)} = \frac{(T_M - \gamma)s + 1}{1 + T_M s} \quad (29)$$

With respect to the weighting function, $W(s)$, in order to automatically include integral action and keep it as simple as possible, we will assume the following form:

$$W(s) = \frac{n_W(s)}{d_W(s)} = \frac{1 + zs}{s} \quad (30)$$

By using this settings, the minimum cost, ρ_S is given by:

$$\min \|E(s)\|_\infty = |\rho_S| = \frac{(L+z)}{(L+T_M)}(T_M + L - \gamma) \quad (31)$$

and the solution for the optimal, $C_S^o(s)$, is:

$$C_S^o(s) = \frac{1}{K}\frac{(1+Ts)(1+\chi_S^1 s)}{(1+T_M s)(1+zs)} \quad (32)$$

with,

$$\chi_S^1 = z + L - \rho_S + T_M - \gamma \quad (33)$$

.

The resulting feedback controller is:

$$K_S^o(s) = \frac{1}{K(\rho_S + \gamma)}\frac{(1+Ts)(1+\chi_S^1 s)}{s(1+\frac{(\rho_S T_M + \gamma z)}{(\rho_S+\gamma)}s)} \quad (34)$$

can be identified to (2) with the following expressions for the controller parameters:

$$\begin{aligned} K_p^S &= \frac{T_i^S}{K(\rho_S+\gamma)} \\ T_i^S &= T + \chi_S^1 - \frac{(\rho_S T_M + \gamma z)}{(\rho_S+\gamma)} \\ \frac{T_d^S}{N^S} &= \frac{(\rho_S T_M + \gamma z)}{(\rho_S+\gamma)} \\ N^S+1 &= \frac{T}{T_i^S}\chi_S^1 \frac{(\rho_S+\gamma)}{(\rho_S T_M + \gamma z)} \end{aligned} \quad (35)$$

This new set of tuning rules also provide the four ISA-PID. However, this time they are parameterized in terms of the desired T_M and z and a new parameter γ.

It is straightforward to verify that with $\gamma = T_M$, we get $\rho_S = \rho_T$, $\chi_S^1 = \chi_T^1$. Therefore both problems provide the same tuning. The tuning rules (35) can be seen as a parameterized set in terms of γ. Determining the value of γ if the tuning we are using is for step response ($\gamma = T_M$) or for disturbance attenuation ($\gamma \neq T_M$).

This way, the values of T_M and z are first selected in order to determine the desired closed loop time constant. Secondly the value of γ can be determined in terms of the operation mode of the control system. When a reference change is to be applied the controller is to be set to $\gamma = T_M$ and when turning to regulation mode a previously selected $\gamma \neq T_M$ is fixed.

The advantage of this parametrization is that of having tuning for both operating modes under the same tuning rule. One common possibility is the use of a two-degree-of freedom version of the PID controller and to try to handle separately both situations. However, this implies an increase of the tuning parameters.

VI. Example

The purpose of this section is to provide an example of the performance of the parameterized tuning rule and how the performance changes from step response to disturbance attenuation as γ varies. Let us consider the following plant and First Order plus Time Delay approximation:

$$\begin{aligned} G(s) &= \frac{1}{(1+s)(1+0.1s)(1+0.01s)(1+0.001s)} \\ &\approx \frac{e^{-0.073s}}{1.073s+1} \end{aligned} \quad (36)$$

From the First Order pus Time Delay approximation we identify $K_n = 1$, $L_n = 0.073$ and $T_n = 1.073$. These plant parameters give us, by application of the simple tuning rule (26) the PID controller that generates the output shown in figure (1). As it can be seen, the step response is quite acceptable but the load disturbance attenuation is sluggish.

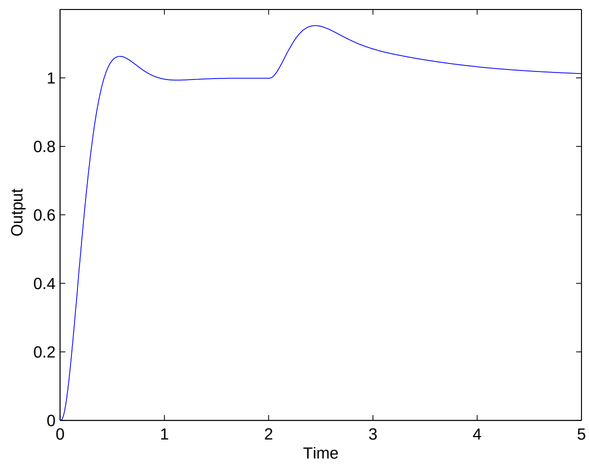

Fig. 1. Output signal generated by application of the step response based tuning.

Application of the disturbance attenuation based tuning, provides an alternative to improve this disturbance attenuation. By using different values for γ and the same values of z and T_M figure (2) clearly shows the performance can be readily improved. Values of γ start with $\gamma = T_M$, providing the same tuning as the step response based, and increasing till $\gamma = 0.9$.

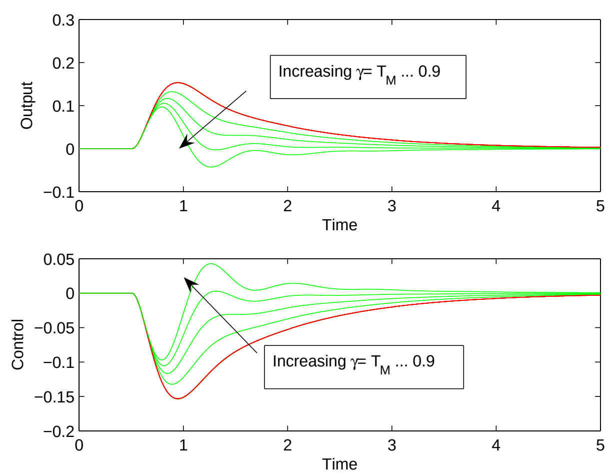

Fig. 2. Output and control signal to an input load disturbance generated by using the Disturbance attenuation tuning and different values of γ

VII. Measuring Performance Improvement

The introduction of the γ parameter allows the search for a new controller that achieves better performance from the disturbance attenuation point of view. However, an immediate question comes into consideration: How much do the performance increase? How to select such γ? A guideline is presented in this section.

Problem (5) defines the disturbance attenuation features of the resulting design by a suitable definition of the target sensitivity function $S^d(s)$. Previous developments have used a γ dependent $S^d(s)$ function as (28). However, by only changing the problem definition we do not have a direct information of the achieved performance improvement. The measure we would like to introduce here comes from a direct interpretation of the ∞-norm as the (system) norm induced by the (signal) 2-norm. effectively, it is well known that (assuming zero reference signal):

$$\|y\|_2 = \|S(s)d\|_2 \leq \|S(s)\|_\infty \|d\|_2 \tag{37}$$

As for each value of γ we will have a different $C_S^o(s)$, we can accordingly write $C_S^o(s;\gamma)$. Each one of these optimal controllers will generate the corresponding Sensitivity function and exhibit a given performance level for the disturbance attenuation measured as the corresponding 2-norm of the output signal. If we concentrate on step disturbance signals, it is possible to compute the associated Integral Squared Error value as a function of γ and get the minimum of such function. This will suggest an automated procedure for selecting γ. Therefore:

$$ISE(\gamma) = \int_0^\infty (y(t))^2 dt = \|y\|_2^2 \tag{38}$$

that can be computed, after Parseval, as:

$$\begin{aligned} ISE(\gamma) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(jw)Y(-jw)dw && (39)\\ &= \frac{1}{2\pi}\oint Y(s)Y(-s)ds && (40)\end{aligned}$$

This last integral is a contour integral up the imaginary axis, then an infinite semicircle in the left half plane. The contribution from this semicircle is zero because $Y(s)$ is strictly proper. By the residue theorem this integral equals the sum of the residues of $Y(s)(-s)$ at its poles in the left half-plane. Straightforward computations leads to:

$$ISE(\gamma) = \frac{zT_M(\rho_S+\gamma)^2 + (zT_M + \chi_S^1 L)^2}{2zT_M(T_M+z)} \tag{41}$$

bearing in mind that $\rho_S = \rho_S(\gamma)$ and $\chi_S^1 = \chi_S^1(\gamma)$. By taking the derivative with respect to γ we can obtain the optimal, γ^o, that minimizes the ISE value (41) as:

$$\frac{\partial ISE(\gamma)}{\partial\gamma} = 0 \Rightarrow \tag{42}$$

$$\gamma^o = \frac{L+T_M}{z-T_M}\left(L+z-\frac{zT_M L + L^2(L+z+T_M)}{zT_M+L^2}\right) \tag{43}$$

If we use the simple rule suggested above, where $T_M = \sqrt{2}L$ and $z = \sqrt{2}T_M$, it turns out that $\gamma^o = \gamma^o(L)$. Therefore once known the time delay, the value of γ can be automatically selected as well as the rest of the PID parameters.

As an example, figure (3) shows the ISE performance corresponding to the system of the previous example. The index is plotted against γ and the situation of the step response tuning is shown as the one corresponding to $\gamma = T_M$.

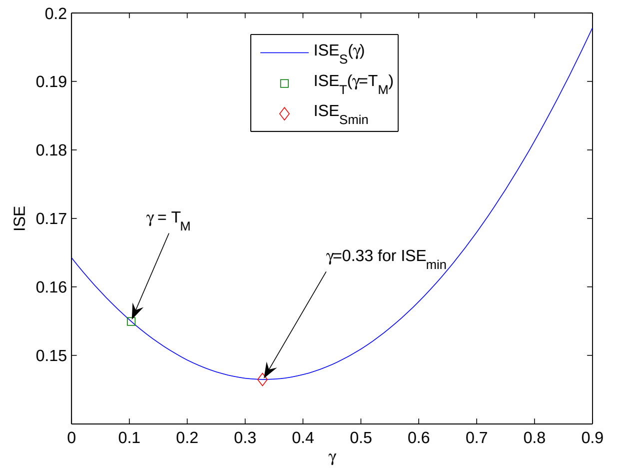

Fig. 3. ISE index for the sensitivity function using disturbance attenuation tuning and different values of γ

An important point is raised after figure (3) concerning the selection of γ. It is seen that a complete automated and guided selection of all the PID parameters can be done once the delay, L, of the system model is known. A consideration has to be done concerning the time domain results of this selection. Even the selected tuning corresponds to the controller $C_S^o(s;\gamma^o)$ it may turn out that the time domain response will not seem to be the *best* one. Regarding figure (2), for example, it is seen that *better* time domain responses are obtained for values $\gamma \approx 0.9$. Therefore, even for small variation of the performance index (see y-axis scale in figure (3)), there can be large variations in the corresponding time domain response. This subject raises the question of selection of the performance index and his correlation with the *shape* of the time domain response it generates. This is a subject of current research.

VIII. CONCLUSIONS

This paper has presented an approach to PID tuning based on an optimal approximation problem. The approximation problem is stated in terms of the Sensitivity function of the closed loop system. An appropriate formulation of the target Sensitivity function generates the tuning of the controller as a parameterized set of tuning rules. This set provides tuning rules for each operating mode of the controller. The overall

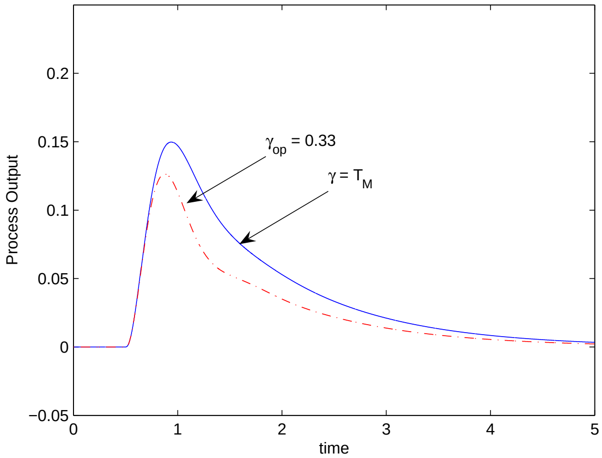

Fig. 4. Output signal to a load-disturbance input by application of step response based tuning ($\gamma = T_M$) and disturbance attenuation tuning with $\gamma = 0.33$

tuning needs three parameters. Two parameters along the lines of previously presented tuning rules and a new third parameter that determines the level of regulation mode of the controller.

An ISE criterion is associated to this third parameter in order to evaluate the performance improvement with respect to the original step response tuning. The maximization of this performance improvement provides a systematic method of determining this third parameter.

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