Asymmetric Liquidity Risk Premia in Intraday High Frequency Trading

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Abstract—**The traditional Value at Risk (VaR) is a very popular tool for measuring market risk, but it does not incorporate liquidity risk. This paper proposes an extended VaR model to integrate liquidity risk in intraday trading strategies when analyzing high frequency order book data. We estimate the one step ahead liquidity adjusted intraday VaR (LAIVaR) for both bid and ask positions, considering several threshold trading sizes. We also quantify the liquidity risk premium by comparing our result with the standard VaR approach, applying the approach in 3 UK bank stocks. The liquidity risk premia of different volumes for the Northern Rock stock are larger on the bid side in 5 minutes and 10 minutes trading intervals. In contrast, in the case for the Royal Bank of Scotland, the liquidity risk premium on the ask side is larger than on the bid side when the volume is high. For HSBC, the liquidity risk premium is roughly the same on both sides.**

Keywords: Liquidity adjusted intraday VaR, liquidity risk premium, asymmetric market behaviour.

1 Introduction

The growth of the risk management industry can be traced directly to the increased volatility of financial market since the early 1970s. Liquidity risk is a key factor of the cause of many serious market crises. The infamous disaster from the Long Term Capital Management (LTCM) in late 1998, Russian financial crisis in 1998 and the collapse of credit market in 2008 evidence the dangers of ignoring the effects of liquidity. In September 2007,the British retail bank Northern Rock could not refinance itself in the credit market and faced bankruptcy due to the lack of liquidity. These big lessons teach us that the liquidity plays a very important role in financial markets, in particular when it comes to trading. Therefore, a good risk measurement has to take liquidity risk in to account. However, the definition of liquidity is ambiguous and has many different interpretations. "*A liquid market is a market in which a bid-ask price is always quoted, its spread is small enough and small trades can be immediately executed with minimal effect on price* (Black (1971))".

A concept that is even more difficult to predict and measure is *liquidity risk*. In a real "friction market", investors hardly get the mid-price that is used in many risk applications and a more rigorous approach of risk management is needed. Bangia, Diebold, Schuermann, and Stroughair (1999) argue that the liquidity risk is an important component in order to capture the overall risk. Lawrence and Robinson (1997) stress that the failure to consider liquidity may lead to an underestimation of the VaR by 30%.

Although more and more market practitioners have recognized that liquidity risk is a very serious concern for firms, plenty studies have separately analyzed the VaR and liquidity. Only a few studies incorporate liquidity into VaR, not to speak of VaR at intraday level (see, for example, Beltratti and Morana (1999), Dionne, Duchesne, and Pacurar (2006) or Colletaz, Hurlin, and Tokpavi (2007)). The literature include only a few former studies where researchers have incorporated liquidity risk with conventional VaR. In general, there are two different methods: the first one is the stochastic horizon method. Lawrence and Robinson (1997) determine the holding period of VaR according to the size of position and the characteristics of liquidity market. The second method models market price changes induced by the selling the underlying asset within a fixed time horizon. For example, Glosten, Jagannathan, and Runkle (1997) use this method to derive the optimal strategy of liquidation that maximize the value over a pre-specified period. Therefore, they consider the impact of the size of the position and the period of execution on the value under liquidation of the position. Bertsimas and Lo (1998) use a similar method to determine the dynamic optimal strategy for minimizing the cost of execution.

The motivation for our paper is as follows: Firstly, liquidity risk is a key factor for the health of the financial system. The conventional VaR models do not take the liquidity risk in to account. The conventional VaR models heavily rely on the implied assumption that an asset can be traded at a certain price at any quantity within a fixed period of time. This assumption is not realistic un-

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†Wing Lon Ng is grateful to the German Research Foundation (DFG) for financial support through research grant 531283. Both authors would like to thank seminar participants at the University of Essex for their helpful comments and Jian Jiang who provided the order book data.

der real market conditions, especially in intraday trading, as execution is not always guaranteed, i.e. the conventional VaR models do not capture the liquidity risk that traders and investors are exposed to. This paper therefore attempts to measure additional risk due to liquidity in the VaR using intraday data and extends the existing literature in the following way: We consider the endogenous liquidity risk, taking into account the volume effect to model the liquidity adjusted intraday VaR (LAIVaR), which measured on the basis of the size of investors' positions.

Secondly, the result shows that there is an asymmetry in up and down movement in the equity market. Downward movements typically have a higher magnitude than upward movements. Xiangli L (2008) prove the upside risk is important by studying upside VaR and extreme upside risk spillover in Chinese copper futures market and spot market. Our paper investigates both upside and downside VaR process. In particular, we are interested in differentiating between both bid and ask sides since different market sides have to face different price movements as well. We estimate the one step ahead LAIVaR of both market sides in order to quantify their real risk position.

The outline of the paper is as follows. Section 2 provides a literature review. Section 3 describes the methodology and Section 4 presents the data and the empirical results. Section 5 concludes.

2 Literature Review

2.1 Liquidity and Liquidity Risk

Liquidity plays a very important role in the financial market. However, the definition of liquidity is ambiguous and has several versions. Generally speaking, the liquidity is a ability for participants to execute large trades rapidly at with a small impact on prices (CGFS (2000)).

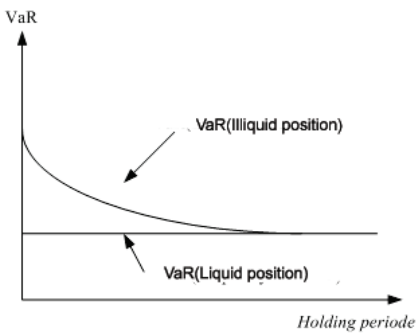

Figure 1: The relationship between liquid and illiquid position of VaR and holding period (Source Dowd (1998)

Market liquidity risk can be summarized as the risk arising from the higher cost and difficulty to execute the trade which is caused by illiquidity market. According to Dowd (1998) "*a market can be very liquid most of the time, but lose its liquidity in a major crisis*". In general, we could differentiate the liquidity risk between normal liquidity risk and crisis liquidity risk. During the crisis period, the liquidity risk should be taken more serious in to account because market lose its liquidity. Dowd (1998) also point out the relationship between the liquid position of VaR and the holding period (see Figure 1). In a highly liquid position the investors can settle the position quickly to get the market price without any significant liquidation cost. But in a illiquid position, one must pay liquidity cost to close his position. The longer the investor wait, the lower of liquidity cost. Moreover, during the waiting period, the asset price can change in a worse way.

Bangia, Diebold, Schuermann, and Stroughair (1999) divide the liquidity risk into two components. Exogenous liquidity risk is influenced by the character of market. On the other hand, endogenous liquidity risk is specific affected by participant's action and position. For example, the larger the trade size, the bigger the ask-bid spread, and the higher the endogenous liquidity risk will become. Figure 2 shows the relationship between the endogenous risk and the the investors' position.

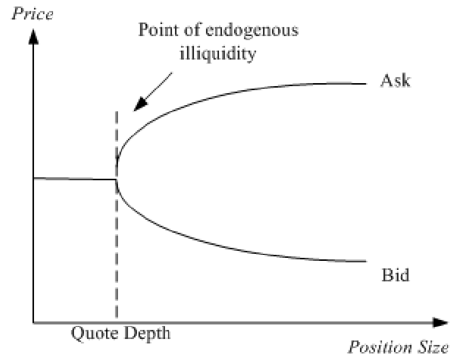

Figure 2: The effect of position size on liquidation value. Source: Bangia, Diebold, Schuermann, and Stroughair (1999)

2.2 Models of Liquidity Adjusted VaR

Conventional VaR models have an implied assumption which is that an asset can be traded at a fixed price within a fixed period of time. This assumption is not realistic under real market conditions. Obviously, the conventional VaR models do not capture the liquidity risk.

Former studies reported in the literature incorporated liquidity risk with conventional VaR by using optimal execution strategy. More specifically, there are two general methods: one is Stochastic Horizon methods; another is modeling the changing of market price induced by the selling off with fixed time horizon. Lawrence and Robinson (1997) determine the holding period of VaR according to the size of position and the characteristics of liquidity market. The authors use the second kind of method which is deriving the optimal strategy of liquidation that will maximize the value over a fixed time horizon. Glosten, Jagannathan, and Runkle (1997) therefore consider the impact of the size of the position and the period of execution on the value under liquidation of the position. Bertsimas and Lo (1998) use the similar method to derive the dynamic optimal strategy with the aim of minimizing the expected cost.

Bangia, Diebold, Schuermann, and Stroughair (1999) develop a liquidity adjusted VaR (LAVaR) model (named as the BDSS model after the name of the authors) which is a fundamental framework for integrating liquidity risk into the standard VaR. The BDSS model mainly focuses on exogenous liquidity risk which take the bid-ask spread into account. The LAVaR simply represents the sum of conventional VaR (computed by mid-price) and the liquidity risk adjusted part (computed by ask-bid spread).

$$LAVaR = Mid_t[(1 - e^{\mu - \alpha\sigma}) + \frac{1}{2}(\overline{S} + \alpha'\tilde{\sigma})] \quad (1)$$

where Mid_t is the mid-price of the asset at time t, $\overline{S} = (Ask - Bid)/Mid$ is the the average relative spread, $\tilde{\sigma}$ is the volatility of relative spread and α' is the quantle of the relative spread distribution.

However, the BDSS model has several drawbacks: Firstly, the model is based on the normal distribution which differs from reality. Secondly, the method ignores the endogenous liquidity risk which is also important. Thirdly, the assumption of perfect correlation between liquidity risk and VaR would lead to an overestimation of the LAVaR. Erwan (2001) extends the BDSS model by using the weighted average spread which incorporates the endogenous risk effect instead of the ask-bid spread. He also points out that for illiquid stocks, the endogenous liquidity risk represents half of the total market risk and must not be neglected.

Hisata and Yamai (2000) propose a framework for the quantification of the LAVaR model that considers the market impact induced by the trader's own liquidation. They derive the optimal execution strategy according to level of market liquidity and the scale of the investor's position. They choose the holding period as an endogenous variable and provide both a discrete time model and continuous time model for LAVaR measurement.

Further, Agnelidis and Benos (2006) investigate intraday LAVaR in Athens Stock Exchange and extend the model from Madhavan, Richardson, and Roomans (1997) by incorporating trading volume and take both endogenous and exogenous liquidity risk into account. Their result also shows that the liquidity risk must not be neglected. Moreover, the LAVaR exhibits a U-shaped pattern throughout the day. In contrast, Giot and Gramming (2006) introduce a GARCH model to derive the LAVaR in an automated auction market. Their empirical model is based on the BDSS framework and model the liquidity risk by calculating the weighted average bid price from the real order book data. They incorporate the endogenous risk impact by corresponding the trader's position. Their result shows that liquidity in VaR accounts significantly and the liquidity risk exhibits an L-shape pattern throughout the day.

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As showed in Figure 3, the conventional VaR model like RiskMetrics (Morgan (1996)) measures the uncertainty of assets returns and do not include the liquidity risk. So a improved approach of modeling VaR should take the whole market risk into account.

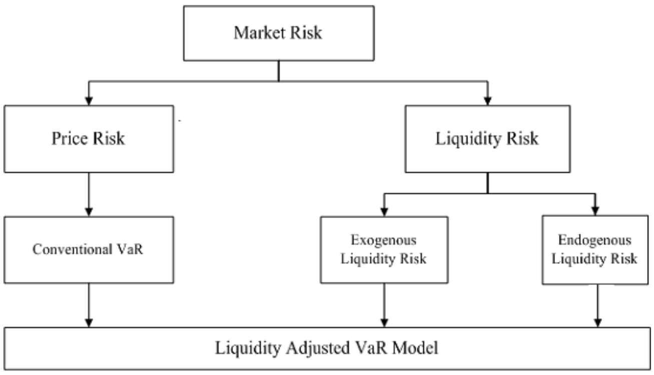

Figure 3: Taxonomy of Market Risk and the VaR Models (Source: Adapted from Bangia, Diebold, Schuermann, and Stroughair (1999).)

2.3 Multivariate GARCH Models

Multivariate GARCH models were initially developed in the late 1980s. Basically the models study the moving process both of variance and covariances which is different with univariate GARCH models. There are three important classes of multivariate models, namely (a) the VECH model, (b) the diagonal VECH model Bollerslev, Engle, and Wooldridge (1998) and (c) the BEKK model Engle and Kroner (1995).

The VECH model is proposed by Bollerslev, Engle, and Wooldridge (1998) which is the original version of the multivariate GARCH model:

$$Y_t = \mu_t + \varepsilon_t \quad (2)$$

with $\varepsilon_t \mid \Psi_{t-1} \sim N(0, H_t)$, and

$$Vech(H_t) = C + \sum_{i=1}^{q} A_i Vech(\varepsilon_{t-i}\varepsilon'_{t-i}) + \sum_{j=1}^{p} B_j Vech(H_{t-j}) \tag{3}$$

where Y_t is an $N \times 1$ vector which denotes the return at time t, μ_t is the conditional mean of Y_t, ε_t is the innovation vector, Ψ_{t-1} is the set of information available time $t-1$, C is a $N(N+1)/2 \times 1$ vector, A_i and B_j are $N(N+1)/2 \times N(N+1)/2$ matrices, and $Vech(.)$ denotes the column-stacking operator applied to the lower portion of an $N \times N$ symmetric matrix.

The number of parameters in VECH model equals: $\frac{2N(N+1)+N^2(N+1)^2(p+q)}{4}$. For example, if we assume N=2, and the simple GARCH(1,1) model, then there are 21 parameters need to be estimated; if N=3, then there are 78 parameters need to be estimated. Thus, the estimation of VECH model is very complex.

Therefore Bollerslev, Engle, and Wooldridge (1998) develop the diagonal VECH model in order to reduce the parameters to estimate in VECH model, which is written as:

$$h_{ij,t} = \omega_{ij} + \alpha_{ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1} + b_{ij}h_{ij,t-1} \tag{4}$$

where ω_{ij}, α_{ij} and b_{ij} are parameters.

Later, Engle and Kroner (1995) present the BEKK model which imposes positive definiteness restrictions to ensure the H matrix being positive. The general format of conditional covariance matrix can be represented as:

$$\begin{aligned} H_t = CC' &+ \Sigma_{k=1}^{K}\Sigma_{i=1}^{q} A_{ik}\varepsilon_{t-i}\varepsilon'_{t-i}A'_{ik} \\ &+ \Sigma_{k=1}^{K}\Sigma_{i=1}^{p} B_{ik}H_{t-i}B'_{ik} \end{aligned} \tag{5}$$

where C is a lower triangular parameter matrix, A_{ik} and B_{ik} are $N \times N$ matrices. As long as C is positive definite, the conditional covariance matrix is also positive definite because the other terms in (4) are expressed in quadratic form.

For example, we assume $K = 1$ and apply GARCH(1,1) model:

$$H_t = CC' + A_{11}\varepsilon_{t-1}\varepsilon'_{t-1}A'_{11} + B_{11}H_{t-1}B'_{11} \tag{6}$$

In the bivariate case, the BEKK becomes

$$\begin{aligned} H_t = CC' &+ A \begin{bmatrix} \varepsilon_{1t-1}^2 & \varepsilon_{1t-1}\varepsilon_{2t-1} \\ \varepsilon_{2t-1}\varepsilon_{1t-1} & \varepsilon_{2t-1}^2 \end{bmatrix} A' \\ &+ B \begin{bmatrix} h_{11t-1} & h_{12t-1} \\ h_{21t-1} & h_{22t-1} \end{bmatrix} B' \end{aligned} \tag{7}$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}.$$

The commonly method to estimate a multivariate GARCH model is conditional log likelihood function which has the following form:

$$L(\theta) = -\frac{TN}{2}ln2\pi - \frac{1}{2}\sum_{t=1}^{T}(ln|H_t| + \varepsilon_t H_t^{-1}\varepsilon_t) \tag{8}$$

where θ denotes the vector of all the unknown parameters, and $H_t = (\sigma_{ijt})_{N\times N}$. Numerical maximization yields the maximum likelihood estimates with asymptotic normal standard errors.

3 Methodology

Different positions face different risks. Developing the discussed models further, we estimate the liquidity adjusted intraday VaR (LAIVaR) model for the bid side, which is for the investor who wants to buy, as well as for the ask side, which is for the investor who wants to sell. Researchers normally use daily data to analysis financial problems. Compared with the low frequency data, the shorter time horizon of high frequency data can present more detailed information about the market behavior. Let $v_{i,t}$ denote the corresponding volumes of orders queuing in the book at time t at positions $i = 1, ..., n$. Similar to Giot (2005), we first define for both bid (B) and ask (A) sides the volume-weighted average prices (VWAP) $B_t(v)$ and $A_t(v)$ to trade a certain volume v in the next short time interval, based on the individual bid and ask prices $B_{i,t}(v)$ and $A_{i,t}(v)$, i.e.

$$B_t(v) = \frac{\sum_j B_{i,t}v_{i,t}^{BID}}{\sum_j v_{i,t}^{BID}}$$

$$A_t(v) = \frac{\sum_j A_{i,t}v_{i,t}^{ASK}}{\sum_j v_{i,t}^{ASK}}$$

where v is the pre-specified threshold volume to be traded at time t when executing at least the first j queuing orders on the bid or ask side, such that $v \leq \sum_{min(n)} v_{i,t}$.

This variable is an ex-ante measure of liquidity which indicates an immediate execution trading cost. With a given volume v (inside the depth), we can compute the price impact by using the information of the full limited order book data. In order to capture the liquidity risk we adopt the model from Giot (2005) and define two log ratio return processes as

$$r_t^{BID}(v) = \ln\frac{B_t(v)}{B_{t-1}(v)}$$

$$r_t^{ASK}(v) = \ln\frac{A_t(v)}{A_{t-1}(v)}$$

representing the VWAP returns.

It is reported in former studies that financial intraday data have a consistent diurnal pattern of trading activities over the course of a trading day, due to certain institutional characteristics of organized financial markets, such as opening and closing hours or lunch time. Since it is necessary to take the daily *deterministic* seasonality into account (Andersen and Bollerslev (1999)), smoothing techniques are required to obtain deseasonalized observations. To remove the seasonality property of high frequency data, Giot and Gramming (2006) assumed a deterministic seasonality in the intraday volatility, and defined the deseasonalized return as

$$D_t^{BID} = \frac{r_t^{BID}}{\sqrt{\phi_t^{BID}}}$$
$$D_t^{ASK} = \frac{r_t^{ASK}}{\sqrt{\phi_t^{ASK}}}$$

where r_t denotes the raw log VWAP-returns and ϕ_t the deterministic seasonality pattern of intraday volatility. Following their approach, we first chose 30 minutes interval raw return as nodes for the whole trading day and then use cubic splines to smooth the average squared sample returns in order to get the intraday seasonal volatility component ϕ_t (see also Giot (2000) and Giot (2005)).

Having computed the deseasonalized VWAP return process, we apply a GARCH(1,1) model

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \tag{9}$$

for both market sides with h_t as the conditional variance for the (deseasonalized) VWAP-returns and ε_t as normally distributed innovations. The LAIVaR at time t for the two return process given confidence level α can be modelled as

$$\text{LAIVaR}_t = \mu_t + Z_\alpha \sigma_t \tag{10}$$

with σ_t as the volatility component. Based on the estimated conditional variance, the standard deviation of the raw return at time t is $\sigma_t = \sqrt{h_t \phi_t}$. From (10), we can estimate the LAIVaR for both bid and ask sides which can be displayed as LAIVaR_t^{BID} and LAIVaR_t^{ASK} respectively.

In the "frictionless" market, the frictionless VaR is computed by the mid-price. In order to quantify the liquidity risk premium, we also need to compute the intraday VaR ($IVaR^{MID}$) based on the mid-price as a benchmark and compare it with the LAIVaR. We define the log ratio return of mid price $r_{mid,t}$ as

$$r_t^{MID} = \ln\left(\frac{P_t^{MID}}{P_{t-1}^{MID}}\right) \tag{11}$$

where P_t^{MID} is the mid-price at time t and model the mid-price return process using a GARCH(1,1) volatility process. Similarly, the IVaR of mid-price returns at time $t-1$ is given by:

$$IVaR_t^{MID} = \mu_t^{MID} + Z_\alpha \sigma_t^{MID} \tag{12}$$

To compare the difference of the liquidity risk, we translate our results back to price intraday VaR which means the worst $\alpha\%$ predicted price if one execute his asset at time t. Most studies in the literature ignore upside risk and only focus on the downside risk, however in our paper the upside risk is a measure for traders who have a long position of his asset. A higher upside risk also means a higher cost. We define the liquidity risk premium λ_t as the difference between mid-price IVaR and LAIVaR

$$\lambda_t = \begin{cases} \frac{1}{T}\sum_{t=1}^{T}(PVaR_{m(t)} - LaIVaR_{(t)}) & (D) \\ \frac{1}{T}\sum_{t=1}^{T}(LaIVaR_{(t)} - PVaR_{m(t)}) & (U) \end{cases} \tag{13}$$

where U and D denote upside risk and downside risk respectively.

Finally, we are also interested in the relative cost of liquidity risk and the difference of the LAIVaR between the bid and ask side. To capture the LAIVaR of VWAP-prices for different levels on both bid and ask side of the order book *jointly*, we apply the dynamic conditional correlation (DCC) multivariate GARCH model proposed by Engle (2002). Consider the bivariate filtrated normally distributed return process

$$r_t \mid \Psi_{t-1} \sim N(0, H_t) \tag{14}$$

with the covariance matrix

$$H_t = D_t R_t Dt \tag{15}$$

where R_t represents the correlation matrix of the returns on both market sides. Further, Engle (2002) assumes that

$$\begin{aligned} D_t &= diag(\sqrt{h_t}) & (16)\\ Q &= (1-a-b)\overline{Q} + a\varepsilon_{t-1}\varepsilon_{t-1}' + bQ_{t-1} & (17)\\ R_t &= (diag(Q_t))^{-\frac{1}{2}} Q_t (diag(Q_t))^{-\frac{1}{2}} & (18) \end{aligned}$$

where

$$\overline{Q} = T^{-1}\sum_{t=1}^{T} \varepsilon_t \varepsilon_t' \quad . \tag{19}$$

The residuals are assumed to be

$$\varepsilon_{it} = r_{it}/\sqrt{h_{it}} \tag{20}$$

with $h_{i,t} = \alpha_0 + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}$ where i stand for the ith asset. Following Engle (2002), the log-likelihood function can be written as

$$\begin{aligned} L(\theta,\varphi) &= \sum_{t=1}^{T} L_t(\theta,\varphi) \\ &= -\frac{1}{2}\sum_{t=1}^{T}(log|D_t R_t D_t| + r_t' D^{-1} R_t^{-1} D^{-1} r_t) \\ &= -\frac{1}{2}\sum_{t=1}^{T}(\underbrace{2log|D_t| + r_t' D^{-1} r_t}_{L_v(\theta)} \\ &\quad - \underbrace{\varepsilon_t'\varepsilon_t + log|R_t| + \varepsilon_t' R_t \varepsilon_t}_{L_c(\theta,\varphi)}) \end{aligned}$$

allowing a two step estimation approach as it can be decomposed into a volatility part

$$L_v(\theta) = -\frac{1}{2}\sum_{t=1}^{T}(2log|D_t| + r_t' D^{-2} r_t) \quad (21)$$

$$= \frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{n}(log(h_{i,t} + \frac{r_{i,t}^2}{h_{i,t}})) \quad (22)$$

and a correlation part

$$L_c(\theta, \varphi) = -\frac{1}{2}\sum_{t=1}^{T}(log|R_t| + \varepsilon_t' R_t \varepsilon_t - \varepsilon_t' \varepsilon_t) \quad . \quad (23)$$

Hence, we first estimate the parameters $\theta = (\alpha_0, \alpha_i, \beta)$ in (22) in the univariate GARCH models, and then substitute θ into (23) to estimate the parameter $\varphi = (a, b)$.

4 The Empirical Analysis

The historical order book using empirical data is extracted from the SETS (Stock Exchange Trading System) that is operated by the London Stock Exchange. The SETS is a powerful platform providing a electronic market for the trading of the constituents of the FTSE. This study only considers the continuous trading phase, where the order book is open and visible for all registered market participants. It starts after the opening auction at 8 am, where the opening price is determined as the price which maximizes the volume that can be traded, and ends at 4.30 pm with the launch of the daily closing auction. The sample period of our data ranges from 1^{st} March 2007 to 31^{st} March 2007. The data set contains full order book information including all events recorded in the order book (limit orders, market orders, iceberg orders, cancelations, changes, full/partial executions) and their matching outcomes.

Different volume sizes executed have different liquidity risk effect. We present two liquidity executions in this paper which are based on a big and a small size of volume. Executing big volume orders has bigger liquidity risk than small volume. We measure the investor's risk on both downside and upside risk which depend on investors' trading strategy (short or long position). In this paper, we choose three different liquidity stocks from the SETS limit order book which are Northern Rock (NR), Royal Bank of Scotland (RBS) and Hongkong and Shanghai Banking Corporation (HSBC). Table 1 gives a list of several average volumes which reflect the liquidity activity for the three selected stocks and shows that HSBC have the largest trade size in every situation. If we compare the average cumulated volume of total ask and bid, NR has the smallest size. According to these facts we choose several different representative threshold volume sizes to reflect different liquidity positions for each stock indicated.

Table 1: Data description

Average volume of ...	NR	RBS	HSBC
Best ask	2979	2038	28386
Best bid	2802	2039	18450
Best three ask orders	7504	8762	57074
Best three bid orders	6654	9015	41743
Total ask side	345420	1077160	3939042
Total bid side	346030	1116740	4348526
Threshold Size (small)	2000	10000	50000
Threshold Size (medium)	10000	50000	100000
Threshold Size (large)	20000	100000	200000

We filter every 5 minute and 10 minute snapshots of the order book to get an equally spaced time series data. Table 2 presents the GARCH model parameter estimates (with the standard errors in brackets) based on the VWAP returns for the three stocks with different threshold volume values. For stock NR and HSBC, all α parameters are as expected smaller than β which means that the updated variance is mainly based on the past variance and less effected by "news". However for stock RBS bid side, the past variance is mainly depend on the "innovation" part.

Figure 4 and Figure 5 display both upside and downside the LAIVaR (with α=5%) of prices and compares this with the frictionless IVaR, based on a 5 minutes and 10 minutes sampling frequency. The graphs compare of the conventional VaR result with our approach. The volume choice can make a big difference of the estimation of VaR. For huge size of the volume execution of all three assets, the LAIVaR is always above the conventional VaR for upside risk and lower for downside risk, and the difference is obvious. The LAIVaR also displays asymmetric between upside and downside position. For algorithmic trader who always adjust their position in short time period, it is important to take liquidity risk in to account. The upside and downside LAIVaR allow traders know exactly how large the risk of long and short position. As shown in the Figure 4 and Figure 5, the huge volume gain more liquidity risk and higher cost. Hence, the conventional method which use mid-price to measure IVaR is underestimating the risk.

Figure 6 and 7 show the dynamic conditional correlation and the conditional variance for bid and ask position of three assets. For each asset, there are results for two different volumes. In 5 minute case, the most fluctuant correlation is the sample volume equal to 2000 of Northern Rock, which ranges from -0.7 to 1.

We examined the effect of our liquidity risk by the liquidity risk premium λ. Figure 8 and 9 displays the forecasted risk premium λ of different volume for both ask and bid side. Liquidity risk is higher when volume size are bigger for all three assets. For larger volume size there are more

Table 2: Estimated Parameters at 5 minutes frequency

NR v=2000	5 minute		10 minute	
	Ask	Bid	Ask	Bid
a_0	$2.8047e-7$	$2.6218e-7$	$1.8026e-6$	$2.2207e-6$
	$(3.6838e-8)$	$(3.5863e-8)$	$(7.8993e-5)$	$(5.7645e-5)$
a_1	0.2367	0.2013	0.1817	0.1136
	(0.0121)	(0.0100)	(0.0218)	(0.0143)
β_1	0.7202	0.7570	0.5255	0.5499
	(0.0193)	(0.0169)	(0.0088)	(0.0087)
v=10000	Ask	Bid	Ask	Bid
a_0	$3.6017e-7$	$4.7074e-7$	$2.1940e-6$	$2.3968e-6$
	$(4.5469e-8)$	$(7.5653e-8)$	$(7.3669e-5)$	$(5.8352e-5)$
a_1	0.2609	0.1585	0.0961	0.1867
	(0.0105)	(0.0131)	(0.0230)	(0.0526)
β_1	0.6924	0.73557	0.5601	0.5200
	(0.0187)	(0.0298)	(0.0940)	(0.0701)
v=20000	Ask	Bid	Ask	Bid
a_0	$2.6298e-7$	$4.3733e-7$	$2.2782e-6$	$3.9412e-6$
	$(2.6819e-8)$	$(5.0698e-8)$	$(5.5326e-5)$	$(4.1364e-5)$
a_1	0.2631	0.1564	0.1641	0.1836
	(0.0087)	(0.0103)	(0.0103)	(0.0204)
β_1	0.7148	0.7630	0.4838	0.4796
	(0.0128)	(0.0189)	(0.0082)	(0.0068)
RBS v=10000	Ask	Bid	Ask	Bid
a_0	$7.6274e-7$	$1.2803e-6$	$4.739e-6$	$4.4760e-6$
	$(3.6187e-8)$	$(4.715e-8)$	$(4.818e-6)$	$(4.3153e-6)$
a_1	0.2706	0.4580	0.3543	0.5556
	(0.0102)	(0.0264)	(0.0016)	(0.0163)
β_1	0.5710	0.3041	0.4001	0.0012
	(0.0168)	(0.0204)	(0.0214)	(0.0211)
v=50000	Ask	Bid	Ask	Bid
a_0	$1.9233e-7$	$1.4687e-6$	$5.1494e-6$	$4.4429e-6$
	$(1.4278e-8)$	$(4.6034e-8)$	$(1.8548e-7)$	$(1.6625e-7)$
a_1	0.3472	0.6126	0.5928	0.77175
	(0.0164)	(0.0226)	(0.0125)	(0.0258)
β_1	0.5736	0.3041	0.0231	0.064
	(0.0164)	(0.0204)	(0.0168)	(0.0243)
v=100000	Ask	Bid	Ask	Bid
a_0	$1.376e-6$	$1.5055e-6$	$4.8786e-6$	$5.9844e-6$
	$(1.9968e-5)$	$(4.094e-8)$	$(4.898e-6)$	$(5.094e-6)$
a_1	0.4953	0.2300	0.5622	0.5763
	(0.0142)	(0.0201)	(0.0142)	(0.0136)
β_1	0.5046	0.2318	0.3017	0.012
	(0.0106)	(0.0132)	(0.0213)	(0.0332)
HSBC v=50000	Ask	Bid	Ask	Bid
a_0	$1.3154e-7$	$1.5111e-7$	$2.3042e-7$	$2.3374e-7$
	$(7.0776e-9)$	$(7.2394e-9)$	$(5.0498e-7)$	$(5.2187e-7)$
a_1	0.3012	0.2579	0.4032	0.4255
	(0.0168)	(0.0157)	(0.0213)	(0.0221)
β_1	0.6371	0.6429	0.5344	0.5526
	(0.0124)	(0.0171)	(0.0246)	(0.0165)
v=100000	Ask	Bid	Ask	Bid
a_0	$1.0400e-7$	$1.6348e-7$	$5.1758e-7$	$3.7381e-7$
	$(7.0012e-9)$	$(7.7124e-9)$	$(3.0834e-8)$	$(2.0726e-8)$
a_1	0.2383	0.2953	0.5839	0.4229
	(0.0146)	(0.0152)	(0.0323)	(0.0234)
β_1	0.7131	0.6201	0.4090	0.5630
	(0.0182)	(0.0121)	(0.0190)	(0.0273)
v=200000	Ask	Bid	Ask	Bid
a_0	$1.3685e-7$	$1.4553e-7$	$2.3844e-7$	$2.4243e-7$
	$(7.5151e-9)$	$(5.9531e-9)$	$(5.8665e-7)$	$(5.9865e-7)$
a_1	0.2781	0.2932	0.4871	0.4332
	(0.0164)	(0.0126)	(0.0244)	(0.0216)
β_1	0.6429	0.6381	0.5002	0.5582
	(0.0161)	(0.0126)	(0.0192)	(0.0298)

big jumps of risk premium which can effect the traders who plan to execute large volumes in short time. The risk premium also shows different with same volume but different trading positions.

Table 3 reports the liquidity risk premium of price LAIVaR for the three stocks with two different frequencies (5 minutes and 10 minutes). The values in brackets are the mean liquidity risk premium in percentage. The BDSS model based on the bid-ask spread only considers price impact. For improvement, we propose LAIVaR model to adjust conventional VaR by incorporating simultaneously the exogenous liquidity risk and the endogenous liquidity risk. The results show how the conventional VaR methods heavily underestimate the risk. Especially for the large volume size case, liquidity risk premium indicate a significant impact in the entire risk profile. In contract to Giot and Gramming (2006) who investigate the bid side liquidity risk premium, we are interested in the asymmetric effect of liquidity risk in both

Table 3: LAIVaR Risk Premia (λ).

		NR	
5 minutes:	v=2000	v=10000	v=20000
Ask	0.3370	1.0938	1.6755
(%)	(0.0005)	(0.0010)	(0.0014)
Bid	0.8781	1.3210	2.9364
(%)	(0.0008)	(0.0012)	(0.0025)
		RBS	
	v=10000	v=50000	v=100000
Ask	1.9241	3.9213	9.4878
(%)	(0.0011)	(0.0029)	(0.0048)
Bid	1.3305	6.7812	11.9636
(%)	(0.0007)	(0.0032)	(0.0054)
		HSBC	
	v=50000	v=100000	v=200000
Ask	0.9624	1.0362	1.4567
(%)	(0.0010)	(0.0013)	(0.0017)
Bid	0.7133	0.07982	1.4827
(%)	(0.0008)	(0.0009)	(0.0017)
		NR	
10 minutes:	v=2000	v=10000	v=20000
Ask	0.4063	1.2894	1.8695
(%)	(0.0004)	(0.0011)	(0.0016)
Bid	0.6982	1.1995	2.4876
(%)	(0.0007)	(0.0010)	(0.0021)
		RBS	
	v=10000	v=50000	v=100000
Ask	1.1095	1.6382	7.012
(%)	(0.0008)	(0.0009)	(0.0048)
Bid	0.8963	3.7678	9.5969
(%)	(0.0006)	(0.0019)	(0.0048)
		HSBC	
	v=50000	v=100000	v=200000
Ask	0.2229	0.4780	0.6150
(%)	(0.0003)	(0.0005)	(0.0007)
Bid	0.3769	0.5975	0.6471
(%)	(0.0004)	(0.0007)	(0.0007)

ask and bid side. Because the asymmetric information of two sides and ask side risk is also important especially for investors who are in short position. For example, the liquidity risk premia of different volumes for the NR stock are larger on bid side in both 5 minutes and 10 minutes cases. However in the case of RBS, the liquidity risk premium of ask side is larger than bid side when the volume is high (v = 10000). For HSBC, the liquidity risk premium is roughly the same on both sides. The results also show that the trend of liquidity risk premium is similar in both 5 minutes and 10 minutes frequency.

General speaking, by examining the liquidity risk premium, one can reveal the liquidity risk component when measuring the VaR model. An investor, especially for the one who have to execute large size volume of asset, must take into account the effect of liquidity in order to trade more rationally.

5 Conclusion

This paper extends the conventional VaR measurement methodology by incorporating the liquidity risk of trading asset and trade positions of market participators. We use the information of limited order book data to study the asymmetric risk effect for bid and ask side.

Our method provides an new practical empirical technique which can help the algorithmic traders to quantify their risk depending on their market position. We establish the liquidity risk premium to quantify the liquidity risk between different volume sizes which provides a specified structure of liquidity risk. This approach improves the BDSS model by incorporating the endogenous liquidity risk effect to instead of the ask-bid spread. Compared with Giot and Gramming (2006), we use different real return process which can reflect the real market information to measure liquidity adjusted intraday VaR (LAIVAR). Furthermore, we also proposed the asymmetric behaviors of both upside and downside LAIVaR and a liquidity risk premium in our analysis.

Our results show that the liquidity risk is a crucial factor in estimating VaR. Negligence of liquidity cost will lead to underestimation of risk as the conventional VaR model. We further contribute by studying and contrasting the patterns of LAIVaR and liquidity risk premium between bid side and ask side of an order drive stock market. We provide significant and specific information for investors who want to go long or short. Therefore, the modeling of the LAIVaR allows investors to adjust positions with a benchmark for the optimal order scheduling.

References

[1] T. Agnelidis and A. Benos. Liquidity adjusted value at risk based on the components of bid-ask spread. *Applied Financial Economics*, 16(11):835–851, 2006.

[2] T. G. Andersen and T. Bollerslev. Intraday periodicity and volatility persistence in financial markets. *Journal of Empirical Finance*, 4(2), 1999.

[3] A. Bangia, F. Diebold, T. Schuermann, and J. Stroughair. Modeling liquidity risk with implications for traditional market risk measurement and management. The Wharton Financial Institutions Center, 1999.

[4] A. Beltratti and c. Morana. Computing value at risk with high frequency data. *Journal of Empirical Finance*, 6(5):431–455, 1999.

[5] D. Bertsimas and A. W. Lo. Optimal control of execution costs. *Journal of Financial Markets*, 1(1):1–50, 1998.

[6] F Black. Towards a fully automated exchange : Part 1. *Financial Analyst Journal*, 27(1):29–34, 1971.

[7] T. Bollerslev, R. Engle, and J. Wooldridge. A capital asset pricing model with time varying covariances. *Journal of Political Economy*, 96(1):116–131, 1998.

[8] CGFS. Asymmetric dynamics in the correlations of global equity and bond returns. Market liquidity: research findings and selected policy implications, Basel, 2000.

[9] G. Colletaz, C. Hurlin, and S. Tokpavi. Irregularly spaced intraday value at risk (isivar) models - forecasting and predictive abilities. Working paper, University of Orléans, 2007.

[10] G. Dionne, P. Duchesne, and M. Pacurar. Intraday value at risk (ivar) using tick-by-tick data with application to the toronto stock exchange. Working paper, HEC Montreal, 2006.

[11] K. Dowd. *Beyond Value at Risk : the New Science of Risk Management*. John Wiley and Sons., Chichester and New York, 1998.

[12] R. Engle and K. Kroner. Multivariate simultaneous generalized arch. *Journal of Political Economy*, 11(6):122–150, 1995.

[13] R. F. Engle. Dynamic conditional correlation: A simple class of generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics*, 20(3):339–350, 2002.

[14] L. S. Erwan. Incorporating liquidity risk in var models. Working Paper, University of Rene, 2001.

[15] P. Giot. Time transformations, intraday data, and volatility models. *Journal of Computational Finance*, 4(2):31–62, 2000.

[16] P. Giot. Market risk models for intraday data. *The European of Journal of Finance*, 11(4):309–324, 2005.

[17] P. Giot and J. Gramming. How large is liquidity risk in an automated auction market? *Empirical Economics*, 30(9):867–887, 2006.

[18] L. R. Glosten, R. Jagannathan, and D. E. Runkle. Mopping up liquidity. *Risk*, 10(12):170–173, 1997.

[19] Y. Hisata and Y. Yamai. Research toward the practical application of liquidity risk evaluation methods. Discussion Paper, Institute for Monetary and Economic Studies, Bank of Japan, 2000.

[20] C. Lawrence and G. Robinson. Liquidity, dynamic hedging and value at risk. *Risk Management for Financial Institutions*, 1(9):63–72, 1997.

[21] A. Madhavan, M. Richardson, and M. Roomans. Why do security prices change? a transaction-level analysis of nyse stocks. *Review of Financial Studies*, 10(4):1035–1064, 1997.

[22] J.P. Morgan. Morgan technical document. Fourth Edition, 1996.

[23] Shouyang W Yiongmiao H Yi L Xiangli L, Siwei C. An empirical study on information spillover effects between the chinese copper futures market and spot market. *Physica A: Statistical Mechanics and its Applications*, 387(4):899–914, 2008.

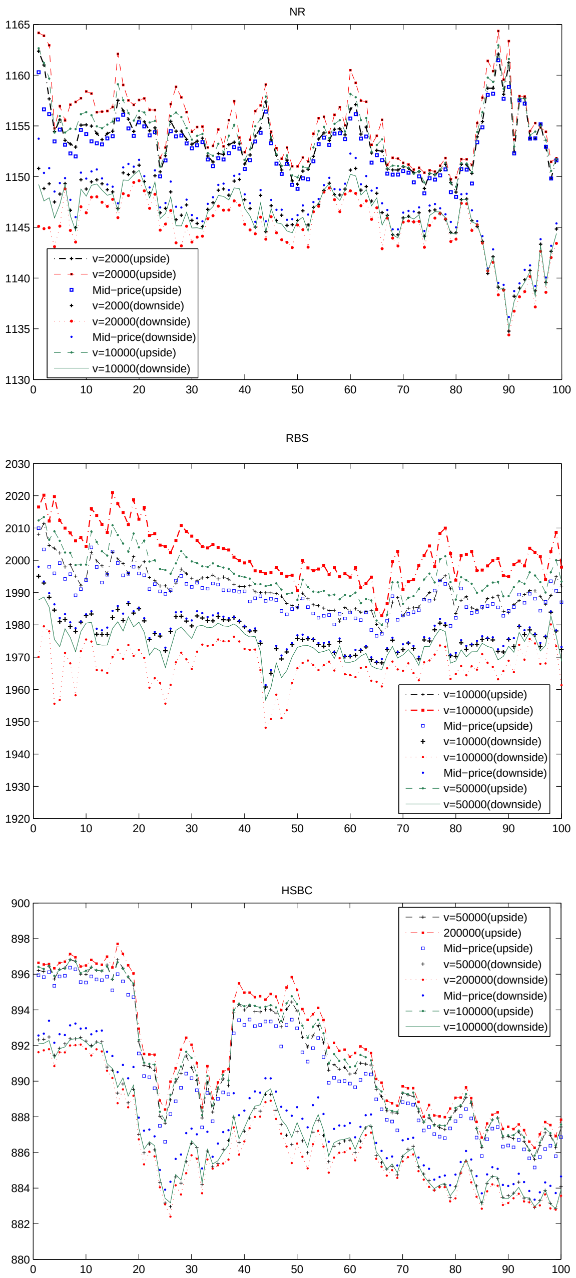

Figure 4: Price IVaR (α=5%) of three companies with 5 minutes sampling frequency

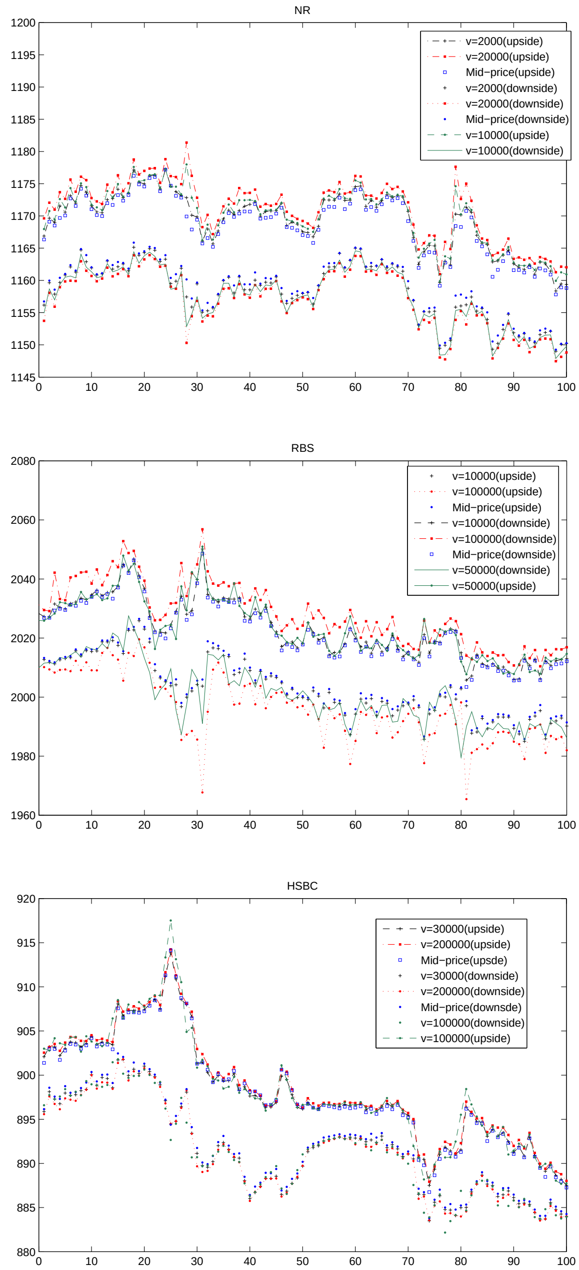

Figure 5: Price IVaR (α=5%) of three companies with 10 minutes sampling frequency

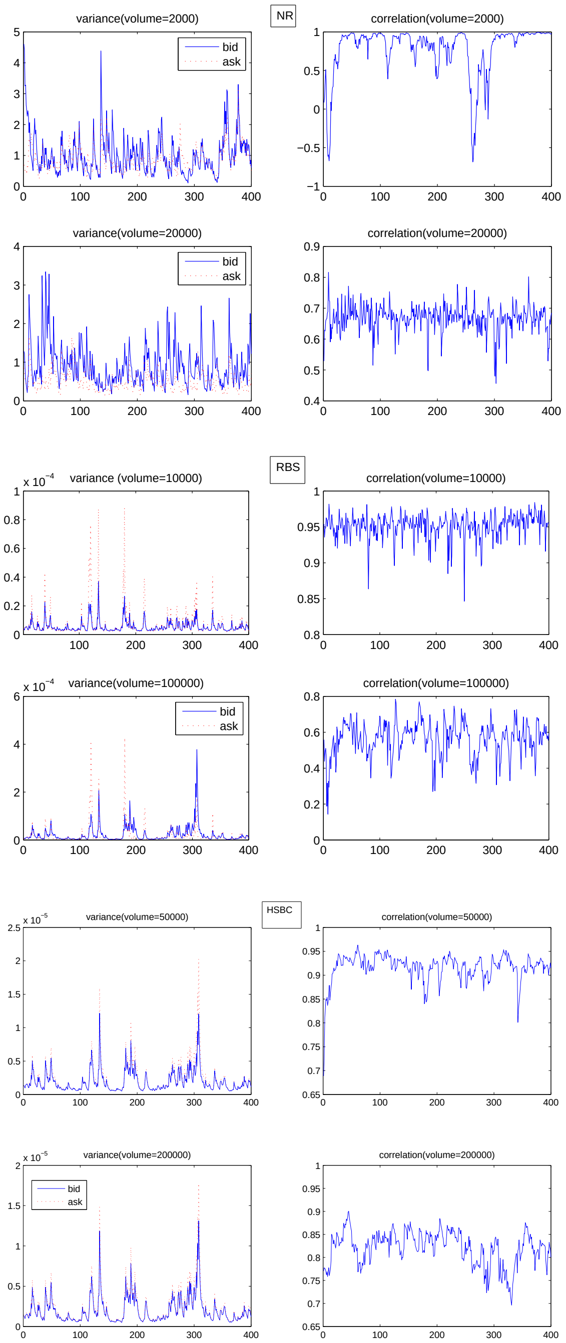

Figure 6: Variance and correlation with different volume sizes for 5 minutes sampling frequency

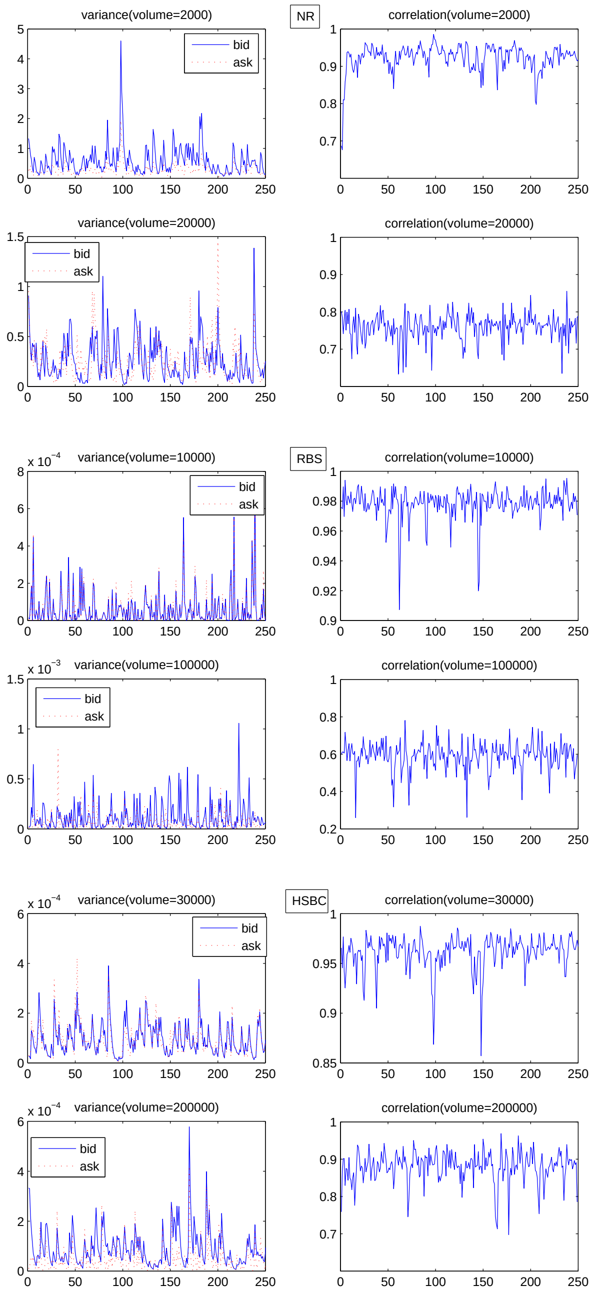

Figure 7: Variance and correlation with different volume sizes for 10 minutes sampling frequency

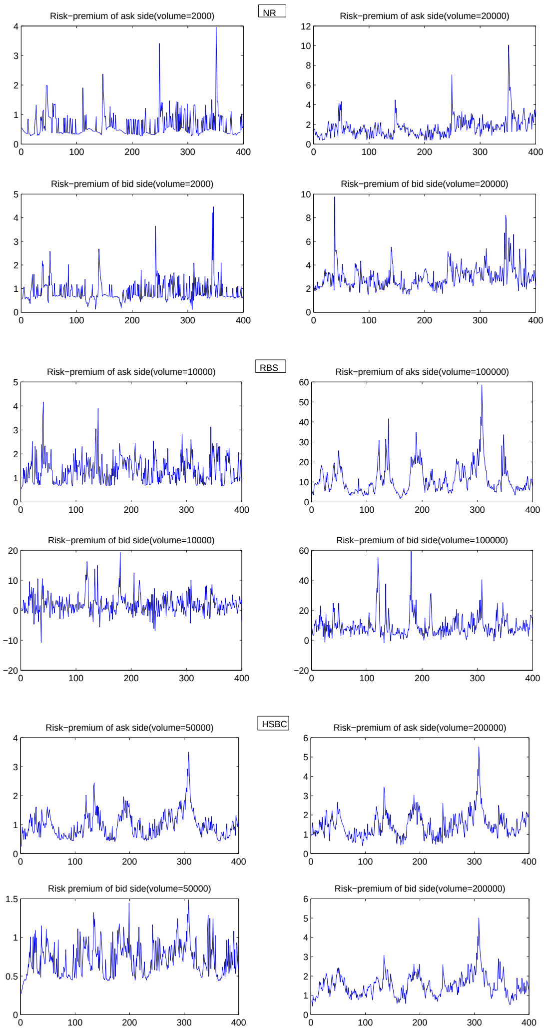

Figure 8: Risk-premium with 5 minutes sampling frequency and different volume sizes

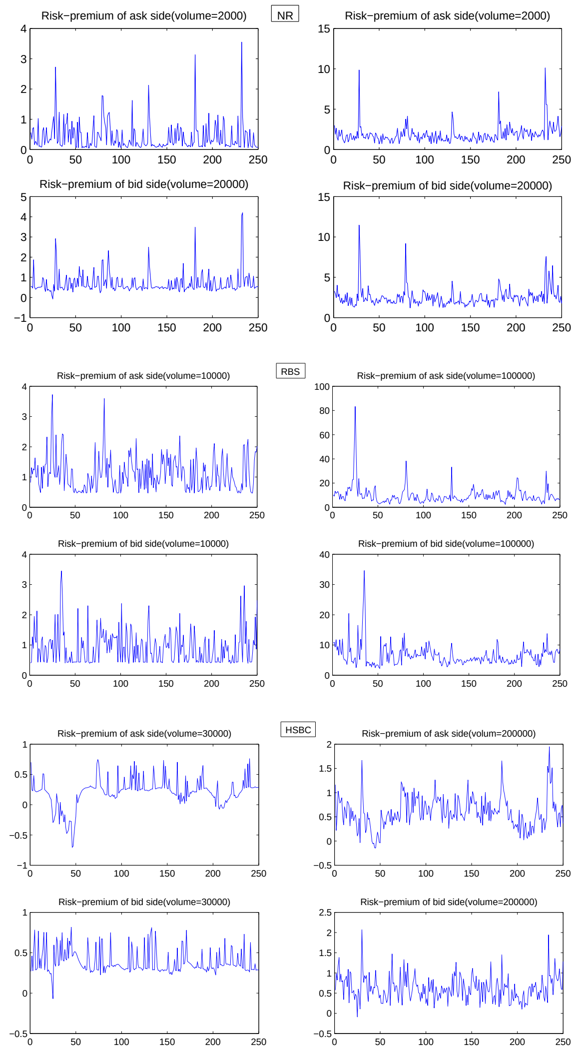

Figure 9: Risk-premium with 10 minutes sampling frequency and different volume size