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Abstract—District heating and cooling (DHC) systems have been actively introduced as energy supply systems in urban areas. Since there exist a number of large-size freezers, heat exchangers and boilers in a DHC plant to generate and supply cold water, hot water and steam to a DHC system, the control under an operation plan for these instruments on the basis of the demand of cold water, hot water and steam, called heat load, is important for stable and economical management of DHC systems. In this paper, we formulate an operation planning problem of an actual DHC plant as a nonlinear integer programming problem in consideration of various penalties for violation of contracts. Furthermore, in order to reflect actual decision making situations for DHC plants more appropriately, we formulate a multiobjective operation planning problem to minimize not only the running cost but the amount of primary energy consumption from the viewpoint of saving energy. Then, we propose an interactive fuzzy satisficing method through tabu search for multiobjective operation planning problems to derive a satisficing solution for the decision maker.

Keywords: interactive fuzzy satisficing method, district heating and cooling system, operation planning, contract violation penalty

1 Introduction

District heating and cooling (DHC) systems have been actively introduced as an energy supply system in urban areas for the purpose of saving energy, saving space, inhibiting air-pollution or preventing city disaster. In a typical DHC system, cold water, hot water and steam used for air-conditioning at all facilities in a certain district are made and supplied by a DHC plant, as illustrated in Figure 1.

Since there exist a number of large-size freezers, heat exchangers and boilers in a DHC plant, the control under an operation plan for these instruments on the basis of the amount of cold water, hot water and steam, called heat load, is important for stable and economical management of a DHC system.

In recent years, with the improvement of heat load prediction methods for DHC systems [3, 6], the needs of the formulation of operation planning problems of DHC plants as a mathematical programming one and the development of solution methods to the formulated problems has been increasing [2, 5, 7, 8, 10]. In previous researches [5, 7], the running cost involving the electric power rate and the gas rate based on the meter rate contract and the arrangement cost of instruments was considered as an objective function to be minimized in operation planning problems in DHC plants. However, actual DHC plant op-

Figure 1: An illustration of a district heating and cooling system.
eration companies have other contracts except the meter rate contract with the electric power company and the gas company. Therefore, we should incorporate penalties for violation of contracts into the running cost to estimate the running cost more accurately. In addition, these companies need to minimize not only the running cost but the energy consumption itself from the viewpoint of the energy saving.

Under these circumstances, in this paper, after formulating an operation planning problem of a DHC plant in consideration of contract violation penalties, we formulate a multiobjective operation planning problem to simultaneously minimize the running cost and the amount of primary energy consumption from the viewpoint of saving energy. Then, we propose an interactive fuzzy satisficing method through tabu search to derive a satisficing solution for the decision maker and show its efficiency through numerical experiments using actual data.

2 Operation Planning of DHC Plants

2.1 Structure of a DHC Plant

A DHC plant usually generates cold water, hot water and steam by running many instruments using gas and electricity.

Relations among components in a DHC plant are depicted in Figure 2. From Figure 2, it can be seen that hot water, steam required for heating and cold water required for cooling are generated by running $N_{BW}$ boilers of $p$ types, $N_{DAR}$ absorbing freezers of $q$ types, $N_{ER}$ turbo freezers of $r$ types, $N_{CEx}$ cold water heat exchangers of $s$ types, $N_{IEX}$ ice thermal storage heat exchangers of $u$ types, $N_{HEX}$ hot water heat exchangers of $v$ types and a thermal storage tank using gas and electricity in this DHC plant, where pumps and cooling towers are connected with the corresponding freezers.

For the DHC plant, we need to find an optimal operation plan to minimize the cost of gas and electricity under the condition that the demand for cold water, hot water and steam must be satisfied by running instruments.

2.2 Problem Formulation

Given the (predicted) amount of the demand for cold water $C_{load}(t)$, that for hot water $W_{load}(t)$ and that for steam $S_{load}(t)$ at time $t$, the operation planning problem of the DHC plant can be summarized as follows.

(I) The problem involves $p + q + r + s + u + v + 1$ integer decision variables. Decision variables $(x_1^t, \ldots, x_q^t), (x_{q+1}^t, \ldots, x_{q+r}^t), (x_{q+r+1}^t, \ldots, x_{q+r+s}^t), (x_{q+r+s+1}^t, \ldots, x_{q+r+s+u}^t)$ and $(x_{q+r+s+u+1}^t, \ldots, x_{q+r+s+u+v}^t)$ are corresponding to the number of running instruments of absorbing freezers, that of turbo freezers, that of cold water heat exchangers, that of ice thermal storage tank heat exchangers and that of hot water heat exchangers, while $y_1^t, \ldots, y_p^t$ are that of boilers. In addition, there exists a decision variable $z^t$ which indicates whether some condition holds or not.

(II) The freezer output load rate $P = (C^t_{load} - C^t_{TS})/C^t$, which means the ratio of the difference between the (predicted) amount of the demand for cold water $C_{load}(t)$ and the output of the thermal storage tank which is automatically running, $C_{TS}^t$, to the total output of running freezers $C^t = \sum_{i=1}^{q+r+s+u+v} a_ix^t_i$, must be less than or equal to 1.0, i.e.,

$$C^t \geq C_{load} - C_{TS}$$  (1)

where we denote the rating output of the ith freezer by $a_i$. This constraint means that the total output of running freezers and heat exchangers must exceed the necessary amount of cold water generated in the plant, $C_{load} - C_{TS}$.

(III) The freezer output load rate $P = (C_{load} - C_{TS})/C^t$ must be greater than or equal to 0.2, i.e.,

$$0.2 \cdot C^t \leq C_{load} - C_{TS}.$$  (2)

This constraint means that the total output of running freezers must not exceed five times the difference between the (predicted) amount of the demand for cold water and the output of the thermal storage tank.

(IV) The hot water heat exchanger output load rate $R = W_{load}/W^t$, which means the ratio of the (predicted) amount of the demand for hot water $W_{load}(t)$ to the total output of running heat exchangers $W^t = \sum_{i=q+r+s+u+v+1}^{q+r+s+u+v} b_ix^t_i$, must be less than or equal to 1.0, i.e.,

$$W^t \geq W_{load}$$  (3)

where we denote the rating output of the ith heat exchanger by $w_i$. This constraint means that the total output of running hot water heat exchangers must exceed the (predicted) amount of the demand for hot water.

(V) The boiler output load rate $Q = (S_{DAR}^t + S_{HEX}^t + S_{load}^t - S_{WHS}^t)/S^t$, which means the ratio of the necessary amount of steam generated in the plant to the total output of running boilers $S^t = \sum_{j=1}^{p} f_j y^t_j$, must be less than or equal to 1.0, i.e.,

$$-S_{DAR}^t - S_{HEX}^t + S^t \geq S_{load}^t - S_{WHS}^t$$  (4)

where we denote the rating output of the jth boiler by $f_j$. Furthermore, $S_{DAR}$ and $S_{HEX}$ mean the total amount of steam used by absorbing freezers at time $t$, defined as

$$S_{DAR} = \sum_{i=1}^{q} \Theta(P) \cdot S_{1}^{\max} \cdot x_i$$  (5)

and the total amount of steam used by heat exchangers at time $t$, defined as

$$S_{HEX} = W^t/0.95$$  (6)
Figure 2: Structure of a district heating and cooling plant.
where $S_{i}^{\text{max}}$ is the maximal steam amount used by the $i$th absorbing freezer. In addition, $S_{WHS}$ means the amount of waste heat steam supplied from the outside of this DHC system. $\Theta(P)$ is the rate of use of steam in an absorbing freezer, which is a nonlinear function of the freezer output load rate $P$ in general. For the sake of simplicity, in this paper, we use the following piecewise linear approximation.

$$\Theta(P) = \begin{cases} 
0.8775 \cdot P + 0.0285, & P \leq 0.6 \\
1.125 \cdot P - 0.1125, & P > 0.6 
\end{cases} \quad (7)$$

(VI) The boiler output load rate $Q = (S_{\text{DAR}}' + S_{\text{HEX}}' + S_{\text{load}}' - S_{WHS}')/S'$ must be greater than or equal to 0.2, i.e.,

$$-S_{\text{DAR}}' - S_{\text{HEX}}' + 0.2 \cdot S' \leq S_{\text{load}}' - S_{WHS}'. \quad (8)$$

This constraint means that the total output of running boilers must not exceed five times the necessary amount of steam.

(VII) The minimizing objective function $J(t)$ is the energy cost which is the sum of the gas bill and the electricity bill.

$$J(t) = G_{\text{cost}} \cdot A_{G}^t + E_{\text{cost}} \cdot A_{E}^t \quad (9)$$

where $G_{\text{cost}}$ and $E_{\text{cost}}$ are the unit cost of gas and that of electricity, respectively.

The gas consumption $A_{G}^t$ is defined as the gas amount consumed in the rating running of a boiler $g_j$, $j = 1, 2, \ldots, p$ and the boiler output load rate $Q$.

$$A_{G}^t = \left( \sum_{j=1}^{p} g_j y_j \right) \cdot Q \quad (10)$$

On the other hand, the electricity consumption $A_{E}^t$ is defined as the sum of electricity amount consumed by turbo freezers, accompanying cooling towers and pumps.

$$A_{E}^t = E_{\text{ER}}^t + E_{\text{CT}}^t + E_{\text{DP}}^t + E_{\text{P}}^t$$

$$= \sum_{i=1}^{q+r} \Xi(P) \cdot E_{i}^{\text{max}} \cdot x_i^t + \sum_{i=1}^{q+r} c_{i}^{\text{DP}} \cdot x_i^t$$

$$+ \sum_{i=1}^{q+r} c_{i}^{\text{TP}}' x_i^t + \sum_{i=1}^{q+r} c_{i}^{P} x_i^t \quad (11)$$

where $E_{i}^{\text{max}}$ denotes the maximal electricity amount used by the $i$th turbo freezer, $c_{i}^{\text{DP}}$, $c_{i}^{\text{TP}}'$ and $c_{i}^{P}$ are the electricity amount of cooling tower and those of two kinds of pumps.

In the above equation, $\Xi(P)$ denotes the rate of use of electricity in a turbo freezer, which is a nonlinear function of the freezer output load rate $P$. For the sake of simplicity, in this paper, we use the following piecewise linear approximation.

$$\Xi(P) = \begin{cases} 
0.6 \cdot P + 0.2, & P \leq 0.6 \\
1.1 \cdot P - 0.1, & P > 0.6 
\end{cases} \quad (12)$$

Accordingly, the operation planning problem is formulated as the following nonlinear integer programming problem.

Problem $P(t)$

minimize

$$J(x^t, y^t, z^t) = G_{\text{cost}} \cdot A_{G}^t + E_{\text{cost}} \cdot A_{E}^t \quad (13)$$

subject to

$$- (1 - z^t) \cdot (C^t - (C_{\text{load}}^t - C_{\text{TS}}^t)) \leq 0 \quad (14)$$

$$z^t \cdot (0.2 \cdot C^t) + (1 - z^t) \cdot (0.6 \cdot C^t) \leq C_{\text{load}}^t - C_{\text{TS}}^t \quad (15)$$

$$z^t \cdot (0.6 \cdot C^t - (C_{\text{load}}^t - C_{\text{TS}}^t)) \leq 0 \quad (16)$$

$$-W_{t} \leq -W_{\text{load}} \quad (17)$$

$$z^t \cdot \Theta_1(P) + (1 - z^t) \cdot \Theta_2(P) \leq S_{\text{HEX}}^t - S_{t} \quad (18)$$

$$-z^t \cdot \Theta_1(P) - (1 - z^t) \cdot \Theta_2(P) \leq -S_{\text{load}}^t + S_{WHS} \quad (19)$$

$$x_i^t \in \{0, 1, \ldots, N_{\text{DAR}}\}, \quad i = 1, \ldots, q \quad (20)$$

$$x_i^t \in \{0, 1, \ldots, N_{\text{ER}}\}, \quad i = q + 1, \ldots, q + r \quad (21)$$

$$x_i^t \in \{0, 1, \ldots, N_{\text{CEX}}\}, \quad i = q + r + 1, \ldots, q + r + s \quad (22)$$

$$x_i^t \in \{0, 1, \ldots, N_{\text{HEX}}\}, \quad i = q + r + s + 1, \ldots, q + r + s + u \quad (23)$$

$$y_j^t \in \{0, 1, \ldots, N_{\text{BWJ}}\}, \quad j = 1, \ldots, p \quad (25)$$

$$z^t \in \{0, 1\} \quad (26)$$

where

$$C^t = \sum_{i=1}^{q+r} a_i x_i^t, \quad W^t = \sum_{i=q+r+s+u+1}^{q+r+s+u+v} b_i x_i^t$$

$$S_{t}^t = \sum_{j=1}^{p} f_j y_j^t, \quad P = (C_{\text{load}}^t - C_{\text{TS}}^t)/C^t$$

$$\Theta_1(P) = \sum_{i=1}^{q} (0.8775 \cdot P + 0.0285) \cdot S_{i}^{\text{max}} \cdot x_i^t$$

$$\Theta_2(P) = \sum_{i=1}^{q} (1.125 \cdot P - 0.1125) \cdot S_{i}^{\text{max}} \cdot x_i^t$$

$$\Xi_1(P) = \sum_{i=q+1}^{q+r} (0.6 \cdot P + 0.2) \cdot E_{i}^{\text{max}} \cdot x_i^t$$

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\[
\Xi_3(P) = \sum_{i=q+1}^{q+r} (1.1 \cdot P - 0.1) \cdot E_i^{\max} \cdot x_i^t,
\]
\[
A_0^t = \left( \sum_{j=1}^{p} g_j y_j^t \right) \cdot Q,
\]
\[
Q = (1/S^t) \{ z_0^t \cdot \Theta_1(P) + (1 - z_0^t) \cdot \Theta_2(P) + S_{\text{HEX}}^t + S_{\text{load}}^t - S_{\text{WHIS}}^t \},
\]
\[
A_2^t = \left\{ z_0^t \cdot \Xi_1(P) + (1 - z_0^t) \cdot \Xi_2(P) + \sum_{i=1}^{q+r} c_{i}^{CT} x_i^t + \sum_{i=1}^{q+r} c_{i}^{CP} x_i^t + \sum_{i=1}^{q+r} c_{i}^{CDP} x_i^t \right\}.
\]

In the problem, \( z_0^t = 1, z_0^t = 0 \) mean \( P \leq 0.6, P > 0.6 \), respectively. In the following, let \( \Lambda^t = \{ (x^t)^T, (y^t)^T, z^t \} \) and let \( \Lambda^t \) be the feasible region of \( P(t) \).

Since an operation plan for one day is usually made in the DHC plant operation company every day, we should consider 24-hour operation plans \( \Lambda(0, 24) = ((\Lambda^0)^T, (\Lambda^1)^T, \ldots, (\Lambda^{23})^T) \in \Lambda(0, 24) = \Lambda^0 \times \Lambda^1 \times \cdots \times \Lambda^{23} \). Thus, Sakawa et al. [5, 7] studied multi-period operation planning problems to reflect the practical situation for DHC plants. In multi-period operation plans, we must consider the switching of instruments since instruments running in the previous period may be stopping in the next period, and vice versa. Since the starting and stopping of instruments need more electricity and manpower than the continuous running does, the arrangement cost of instruments should be taken into account in multi-period operation planning.

Thus, they formulated an extended operation planning problem in consideration of the arrangement cost of instruments [5, 7]. To be more specific, we deal with the following problem \( P(0, 24) \) for 24-hour operation planning.

### Extended problem \( P(0, 24) \)

Minimize
\[
J_{0,24}(\Lambda(0, 24)) = J(\Lambda^0) + \sum_{t=1}^{23} \left[ J(\Lambda^t) + \sum_{i=1}^{q+r} + \sum_{j=1}^{p+q+r+s+u+v} \phi_j^t \left| \lambda_j^t - \lambda_j^{t-1} \right| \right] \tag{27}
\]
subject to
\[
\Lambda(0, 24) \in \Lambda(0, 24) \tag{28}
\]

where \( \phi_j \) is the cost of switching of the \( j \)th instrument. In should be noted that \( P(0, 24) \) is a large-scale nonlinear integer programming problem which involves 24 times as many variables as \( P(t) \) does.

### 3 Contract Violation Penalties and Multiobjective Problem

The DHC plant operation company considered in this paper has the following contracts except the meter rate contract with the electric power company and the gas company.

- **Least gas consumption contract:** The DHC plant operation company has a least gas consumption contract with the gas company, where the amount of gas consumption of a year must be greater than or equal to a fixed value \( B_1 \). If the amount of gas consumption of a year is less than \( B_1 \), the DHC plant operation company must pay the penalty \( M_1 \) to the gas company.

- **Greatest electric power contract:** The DHC plant operation company has a greatest electric power contract with the electric power company, where the electric power at any time must be less than or equal to a fixed value \( B_2 \). If the electric power exceeds \( B_2 \) at some time, the DHC plant operation company must pay the penalty \( M_2 \) to the electric power company.

- **Peak cut contract:** The DHC plant operation company has a peak cut contract with the electric power company, where the electric power in the peak period from 13:00 to 16:00 must be less than or equal to a fixed value \( B_3 \). If the electric power exceeds \( B_3 \) in the peak period, the DHC plant operation company must pay the penalty \( M_3 \) to the electric power company.

If any contract is violated, the DHC company has to pay the penalty.

Now, we give mathematical expressions of penalties mentioned above. First, we consider the penalty of the greatest electric power contract, \( PE_2(\cdot) \), and that of the peak cut contract, \( PE_3(\cdot) \). Either the greatest electric power contract or that of the peak cut contract is violated when the electric power exceeds \( B_2 \) or \( B_3 \). Then, the DHC company has to pay \( M_2 \) or \( M_3 \). Thus, we define them as:

\[
PE_2(\lambda^t) = \begin{cases} M_2, & \text{if } A_{E_2}^t > B_2, \\ 0, & \text{otherwise} \end{cases}
\]

\[
PE_3(\lambda^t) = \begin{cases} M_3, & \text{if } A_{E_3}^t > B_3, \\ 0, & \text{otherwise} \end{cases}
\]

Next, we consider the penalty of the least gas consumption contract. Let \( \alpha_m, m = 1, 2, \ldots, 12 \) be the ratio of monthly gas consumption to yearly gas consumption for each month. For a monthly operation plan from the first day (day 1) to the last day (day \( d_m \)) of month

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m, we define monthly thresholds $B_{1,m}$, $m = 1, 2, \ldots, 12$ as $B_{1,m} = B_1 \cdot \alpha_m$. In addition, we define monthly penalties of the least gas consumption contract, $PE_{1,m}$, $m = 1, 2, \ldots, 12$ as $M_{1,m} = M_1 \cdot \alpha_m$. Then, based on the gas consumption from day 1 to day $d$ of month $m$ and the number of remaining days of month $m$, $d_m - (d - 1)$, we define the daily threshold of day $d$ as:

$$B_{1,m}(d) = \frac{B_{1,m} - AG(d - 1)}{d_m - (d - 1)}$$

where

$$AG(d) = \sum_{\tau=1}^{d} \left( \sum_{j=1}^{p} g_j y_j^\tau \right) \cdot Q.$$  

Since the daily threshold $B_{1,m}(d)$ increases as the total gas consumption from day 1 to day $d$ of month $m$ decreases, it can reflect the situation of gas consumption on day $d$.

We also define the daily penalty of day $d$ as:

$$M_{1,m}(d) = \frac{M_{1,m}}{d_m - (d - 1)}.$$  

Then, we can define the penalty of the least gas consumption contract for a 24-hour operation plan $\lambda(0,24)$ as:

$$PE_1(\lambda(0,24)) = \begin{cases} M_{1,m}(d), & \text{if } \sum_{\tau=0}^{23} A_G^\tau < B_{1,m}(d) \\ 0, & \text{otherwise} \end{cases}$$

Introducing these penalties into the objective function of $P(0,24)$, we extend $P(0,24)$ into the following problem with penalties.

**Extended problem with penalties $P'(0,24)$**

minimize 

$$J_{0,24}'(\lambda(0,24)) = J_{0,24}(\lambda(0,24)) + PE_1(\lambda(0,24))$$

$$+ \sum_{\tau=0}^{23} \left[ PE_2(\lambda^\tau) + PE_3(\lambda^\tau) \right]$$

subject to $\lambda(0,24) \in \Lambda(0,24)$

Furthermore, in order to reflect actual decision making situations for DHC plants more appropriately, we formulate a multiobjective extended problem to minimize not only the running cost but the amount of primary energy consumption from the viewpoint of saving energy during $D$ days.

**Multiobjective extended problem $MOP'(0,24,D)$**

minimize 

$$J_l'(\lambda(0,24), \ldots, \lambda(24(D - 1), 24))$$

$$= \sum_{d=0}^{D-1} \left\{ J_{24d,24}(\lambda(24d, 24)) + PE_1(\lambda(24d, 24)) \right\}$$

$$+ \sum_{\tau=0}^{23} \left[ PE_2(\lambda^{24d+\tau}) + PE_3(\lambda^{24d+\tau}) \right]$$

minimize 

$$J_l'(\lambda(0,24), \ldots, \lambda(24(D - 1), 24))$$

$$= \sum_{d=0}^{D-1} \left\{ \sum_{\tau=0}^{23} \alpha_G \cdot A_G^{24d+\tau} + \alpha_E A_E^{24d+\tau} \right\}$$

subject to $\lambda(24d, 24) \in \Lambda(24d, 24)$, $d = 0, 1, \ldots, D - 1$

where $\alpha_G$ and $\alpha_E$ are the coefficient of conversion to primary energy for gas and that for electricity, respectively.

**4 An Interactive Fuzzy Satisficing Method**

In order to consider the imprecise nature of the decision maker’s judgments for each objective function $J_l'(\cdot)$, $l = 1, 2$, if we introduce the fuzzy goals such as “$J_l'(\cdot)$ should be substantially less than or equal to $p_l^\prime$”, the multiobjective extended problem can be transformed as:

maximize $\mu_1(J_l'(\lambda(0,24), \ldots, \lambda(24(D - 1), 24)))$

maximize $\mu_2(J_l'(\lambda(0,24), \ldots, \lambda(24(D - 1), 24)))$

subject to $\lambda(24d, 24) \in \Lambda(24d, 24)$, $d = 0, 1, \ldots, D - 1$

where $\mu_l(J_l'(\cdot))$ are membership functions to quantify the fuzzy goals.

As a reasonable solution concept for the fuzzy multiobjective decision making problem, Sakawa et al. [4, 9] defined M-Pareto optimality on the basis of membership function values and developed an interactive fuzzy satisficing method to derive a satisficing solution guaranteed to be M-Pareto optimal by eliciting the local preference information from the decision maker through interactions. In their method [4, 9], the decision maker interactively updates aspiration levels of achievement for membership values of all fuzzy goals, called reference membership levels, until he is satisfied. To be more specific, for the decision maker’s reference membership levels $\bar{\mu}_l$, the following augmented minimax problem is repeatedly solved:

minimize 

$$\max_{l=1,2} \left\{ \left( \bar{\mu}_l - \mu_l(J_l'(\lambda(0,24), \ldots, \lambda(24(D - 1), 24))) \right) \right\}$$

$$+ \rho \sum_{i=1}^{2} \left( \bar{\mu}_i - \mu_l(J_l'(\lambda(0,24), \ldots, \lambda(24(D - 1), 24))) \right)$$

subject to $\lambda(24d, 24) \in \Lambda(24d, 24)$, $d = 0, 1, \ldots, D - 1$
where $\rho$ is a sufficiently small positive number. If the reference membership levels are not attainable, the optimal solution to (35) is the nearest feasible solution to them in the augmented minimax sense. Otherwise, the optimal solution to (35) could be better than them.

In this paper, we apply and adjust the interactive fuzzy satisficing method to the above multiobjective extended problem (34).

Interactive fuzzy satisficing method for $MOP'(0, 24, D)$

**Step 1** In order to obtain the minimum $J_{l}^{t}$ and (approximate) maximum $J_{l}^{*}$ of $J_{l}^{*}()$, $l = 1, 2$ in $MOP'(0, 24, D)$, solve (36) and (37).

\[
\begin{align*}
\text{minimize} & \quad J_{l}^{t} (\mathbf{\lambda}(0, 24), \ldots, \mathbf{\lambda}(24(D - 1), 24) ) \\
\text{subject to} & \quad \mathbf{\lambda}(24d, 24) \in \mathbf{\Lambda}(24d, 24), \quad d = 0, 1, \ldots, D - 1 \\
\text{maximize} & \quad J_{l}^{*} (\mathbf{\lambda}(0, 24), \ldots, \mathbf{\lambda}(24(D - 1), 24) ) \\
\text{subject to} & \quad \mathbf{\lambda}(24d, 24) \in \mathbf{\Lambda}(24d, 24), \quad d = 0, 1, \ldots, D - 1
\end{align*}
\]

**Step 2** Ask the decision maker to specify membership functions $\mu_{l}(J_{l}^{*}())$, $l = 1, 2$ based on minima and maxima obtained in step 1, and set the initial reference membership levels $\bar{\mu}_{l}$, $l = 1, 2$.

**Step 3** For the current reference membership levels $(\bar{\mu}_{1}, \bar{\mu}_{2})$, solve the corresponding augmented min-max problem (35). It should be noted that the optimal solution to (35) is M-Pareto optimal.

**Step 4** If the decision maker is satisfied with the current solution obtained in step 3, the interaction process is terminated. Otherwise, ask the decision maker to update $\bar{\mu}_{l}$, $l = 1, 2$ in consideration of the current membership function values and objective function values, and go to step 3.

Since problems (36), (37) and (35) are large-scale nonlinear integer programming problems, it is difficult to find an exact optimal solution to it. Thus, we use an approximate solution method using tabu search. To be more specific, we extend the tabu search based on strategic oscillation for multidimensional integer knapsack problems [1] to nonlinear integer programming problems and apply it to solve (36), (37) and (35).

5 Tabu Search Based on Strategic Oscillation

In this paper, we extend tabu search based on strategic oscillation for multidimensional 0-1 knapsack problems [1] to nonlinear integer programming problems. The tabu search proposed in [1] made use of the property of multidimensional 0-1 knapsack problems that the improvement or disimprovement of the objective function value corresponds with the decrease or increase of the degree of feasibility. From the property, it is clear that the optimal solution to multidimensional 0-1 knapsack problems exists in the area near the boundary of the feasible region which is called the promising zone. Thus, the search direction in multidimensional 0-1 knapsack problems can be controlled by checking the change of the objective function value. In the case of nonlinear integer programming problems, the promising zone does not always exist near the boundary of the feasible region since the monotone relation between the objective function value and the degree of feasibility no longer holds. Since the promising zone originally means the area which is considered to include an optimal solution, we define the promising zone for nonlinear integer programming problems as neighborhoods of local optimal solutions. Thus, in order to use not only the change of the objective function value but the degree of feasibility, we introduce the index of surplus of constraints $\delta(\mathbf{\lambda}(t, 24))$ and that of slackness of constraints $\epsilon(\mathbf{\lambda}(t, 24))$.

\[
\begin{align*}
\delta(\mathbf{\lambda}(t, 24)) & \triangleq \sum_{\tau = t}^{t + 23} \sum_{i \in I^{+}} g_{i}^{t}(\mathbf{\lambda}^{\tau}) , \\
\epsilon(\mathbf{\lambda}(t, 24)) & \triangleq \sum_{\tau = t}^{t + 23} \sum_{i \in I^{-}} -g_{i}^{t}(\mathbf{\lambda}^{\tau}) , \\
I^{+} & = \{ i \mid g_{i}^{t}(\mathbf{\lambda}^{\tau}) > 0, \quad i \in \{ 1, \ldots, 8 \} \} , \\
I^{-} & = \{ i \mid g_{i}^{t}(\mathbf{\lambda}^{\tau}) \leq 0, \quad i \in \{ 1, \ldots, 8 \} \}
\end{align*}
\]

where $g_{i}^{t}(\cdot) \leq 0$, $i \in \{ 1, 2, \ldots, 8 \}$ correspond with constraints in $P(t)$.

**Step 0: INITIALIZATION**

Generate an initial solution $\mathbf{\lambda}(t, 24)$ at random, and initialize the tabu list TL, parameters $\mathbf{CN}$, $\mathbf{DN}$ and $\mathbf{AN}$. If $\mathbf{\lambda}(t, 24)$ is feasible, go to step 4. Otherwise, go to step 1.

**Step 1: TS_PROJECT**

The aim of this step is to move the current solution in the infeasible region to the promising zone in the gentlest ascent (disimproving) direction about the objective function with decreasing the surplus of constraints $\delta(\mathbf{\lambda}(t, 24))$.

While $\delta(\mathbf{\lambda}(t, 24))$ is positive, i.e., the current solution is infeasible, repeat finding a non-tabu decision variable which decreases $\delta(\mathbf{\lambda}(t, 24))$ and gives the least disimprovement of the objective function value when its value would be changed, changing the value of the decision variable actually and adding the decision variable to TL. If there does not exist any non-tabu decision variable that decreases $\delta(\mathbf{\lambda}(t, 24))$, select a decision variable randomly, change its value even if $\delta(\mathbf{\lambda}(t, 24))$ increases and add the decision variable to TL. If $\delta(\mathbf{\lambda}(t, 24)) = 0$ and there does not exist any decision variable which improves the objective function value by changing its value, go to step 2.
Step 2: TS_COMPLEMENT
The aim of this step is to search the promising zone intensively.

Let \(\lambda(t,24) := \lambda(t,24)\) and \(\lambda''(t,24) := \lambda'(t,24)\). Then, select several tabu decision variables of \(\lambda''(t,24)\) and change their values. If \(\delta(\lambda''(t,24)) = 0\), then carry out step 4. Otherwise, carry out step 1. If \(J_{l,24}(\lambda''(t,24)) < J_{l,24}(\lambda(t,24))\) for the solution \(\delta(\lambda''(t,24))\) obtained by step 4 or step 1, let \(\lambda(t,24) := \lambda''(t,24)\). This procedure is repeated \(CN\) times. If the previous step of this step is step 1, then go to step 3. If the previous step of this step is step 4, then go to step 5.

Step 3: TS_DROP
The aim of this step is to move the current solution in the promising zone to the inside of the feasible region in the gentlest ascent direction of the objective function with increasing the slackness of constraints \(\epsilon(\lambda(t,24))\)

Repeat finding a non-tabu decision variable which increases \(\epsilon(\lambda(t,24))\) and gives the least disimprovement of the objective function when its value would be changed, changing the value of the decision variable actually and adding the decision variable to TL. If there does not exist any non-tabu decision variable that increases \(\epsilon(\lambda(t,24))\) or the number of repetitions of the above procedure exceeds \(DN\), go to step 4.

Step 4: TS_ADD
The aim of this step is to move the current solution in the feasible region to the promising zone in the steepest descent (improving) direction about the objective function with keeping \(\delta(\lambda(t,24)) = 0\).

While \(\delta(\lambda(t,24)) = 0\), i.e., the current solution is feasible, repeat finding a non-tabu decision variable which keeps \(\delta(\lambda(t,24)) = 0\) and gives the greatest improvement of the objective function value when its value would be changed, changing the value of the decision variable actually and adding the decision variable to TL. If there does not exist such a decision variable, go to step 2.

Step 5: TS_INFEASIBLE_ADD
The aim of this step is to move the current solution in the promising zone to the infeasible region in the steepest descent (if exist) or the gentlest ascent direction about the objective function with decreasing the slackness of constraints \(\epsilon(\lambda(t,24))\) or increasing the surplus of constraints \(\delta(\lambda(t,24))\).

Repeat finding a non-tabu decision variable which decreasing the slackness of constraints \(\epsilon(\lambda(t,24))\) or increasing the surplus of constraints \(\delta(\lambda(t,24))\) and gives the greatest improvement (if exist) or the least disimprovement of the objective function value when its value would be changed, changing the value of the decision variable actually and adding the decision variable to TL. If there does not exist such a decision variable or the number of repetitions of the above procedure exceeds \(AN\), go to step 1.

Table 1: Individual minima \(J_{l,\min}\) and maxima \(J_{l,\max}\), \(l = 1, 2\)

<table>
<thead>
<tr>
<th>Objective function</th>
<th>(J_{l,\min})</th>
<th>(J_{l,\max})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(J_1)</td>
<td>465294344.9</td>
<td>150878375.0</td>
</tr>
<tr>
<td>(J_2)</td>
<td>329818346.2</td>
<td>120242558.1</td>
</tr>
</tbody>
</table>

6 Numerical Experiments
In this section, we deal with an actual DHC plant with boilers of 3 types, absorbing freezers of 4 types, turbo freezers of 4 types, cold water heat exchangers of 2 types, ice thermal storage tank heat exchangers of 2 types and heat exchangers of 3 types. Thus, a daily (24-hour) operation planning problem for a certain day for this plant involves 456 integer decision variables.

Here, we consider monthly operation planning with two objective functions of this DHC plant. After formulating it as a multiobjective extended problem \(MOP(0,24,D)\) which includes \(456 \times 31\) integer decision variables, we apply the proposed interactive fuzzy satisficing method through the tabu search. The problem involves two objective functions: the running cost \(J_l(\cdot)\) and the amount of primary energy consumption \(J_2(\cdot)\).

Numerical experiments are carried out on a personal computer (CPU: Intel Pentium IV Processor 2.40GHz, Memory: 512MB, C_Compiler: Microsoft Visual C++ 6.0) and the number of trials of tabu search is 10.

First, according to step 1 in the interactive fuzzy satisficing method, we calculate the (approximate) minimum of \(J_{l,\min}\) and the (approximate) maximum \(J_{l,\max}\), \(l = 1, 2\). Table 1 shows these values.

Second, according to step 2, the hypothetical decision maker specifies membership functions \(\mu_l(\cdot)\), \(l = 1, 2\) based on the individual minima and maxima of objective functions \(J_l(\cdot)\). In this experiment, we use linear membership functions defined as:

\[
\mu_l(J_l) = \begin{cases} 
1 & \text{if } J_l < J_{l,\min}, \\
\frac{J_l - J_{l,\min}}{J_{l,\max} - J_{l,\min}} & \text{if } J_{l,\min} \leq J_l \leq J_{l,\max}, \\
0 & \text{if } J_l > J_{l,\max}.
\end{cases}
\]

Then, the decision maker sets the initial reference membership levels \((\tilde{\mu}_1, \tilde{\mu}_2) = (1.00, 1.00)\).

Next, according to step 3, the augmented minimax problem (35) for the current reference membership levels is solved by the tabu search. The results are shown in the second column of Table 2.

In step 4, since the decision maker cannot be satisfied with this result and he/she feels that \(J_1\) should be improved even if \(J_2\) becomes worse, the reference membership levels are updated from \((1.00,1.00)\) to \((1.00,0.95)\).
and return to step 3.

Again, the minimax problem (35) is solved for the current reference membership levels, and the results are shown in the third column of Table 2.

Furthermore, the decision maker hopes that $J'_2$ becomes better at the additional expense of $J'_1$ and updates the reference membership levels from $(1.00, 0.95)$ to $(1.00, 0.90)$. The results are shown in the fourth column of Table 2.

In this experiment, since the decision maker is satisfied with this result, this solution is the satisficing solution for the decision maker, and the interaction process stops.

### 7 Conclusion

In this paper, we focused on operation planning of district heating and cooling (DHC) plants considering contracts with the electric power company and the gas company except the meter rate contract. First, we formulated a single period operation planning problem $P(t)$ and a multi-period operation planning problem $P(0, 24)$ as a nonlinear integer programming problem. Second, in consideration of penalties for violation of contracts, we formulated an extended multi-period operation planning problem with the penalties $P'(0, 24)$. Next, in order to reflect actual decision making situations for DHC plants more appropriately, we formulate a multi-objective operation planning problem $MOP'(0, 24, D)$ to minimize not only the running cost but the amount of primary energy consumption from the viewpoint of saving energy during $D$ days. Then, we discussed the application of an interactive fuzzy satificing method through tabu search into multiobjective operation planning problems to derive a satisficing solution for the decision maker. Finally, we show the feasibility and usefulness of the interactive method for multiobjective operation planning problems through a numerical experiment using actual data.

### References


