Optimal Retailer's Replenishment Policy for Seasonal Products with Ramp-type Demand Rate

Hidefumi Kawakatsu *

Abstract—**We discuss the retailer's optimal replenishment policy for seasonal products with a ramp-type demand pattern. In this study, we focus on "Special Display Goods", which are heaped up in end displays or special areas at retail store. They are sold at a fast velocity when their quantity displayed is large, but are sold at a low velocity if the quantity becomes small. We develop the model with a finite time horizon (period of a season) to determine the optimal replenishment policy, which maximizes the retailer's total profit. Numerical examples are also presented to illustrate the theoretical underpinnings of the proposed model.**

Keywords: ***optimal replenishment policy, seasonal product, ramp-type demand rate, special display goods***

1 Introduction

Inventory models with a finite planning horizon and time-varying demand patterns have extensively been studied in the inventory literature[1-7]. Resh et al.[1] and Donaldson[2] established an algorithm to determine both the optimal number of replenishment cycles and the optimal replenishment time for a linearly increasing demand pattern. Barbosa and Friedman[3] and Henery[4] respectively extended the demand pattern to a power demand form and a log-concave function. Hariga and Goyal[5] and Teng[6] extended Donaldson's work by considering various types of shortages. For deteriorating items such as medicine, volatile liquids and blood banks, Dye[7] developed the inventory model under the circumstances where shortages are allowed and backlogging rate linearly depends on the total number of customers in the waiting line during the shortage period. However, there still remain many problems associated with replenishment policies for retailers that should theoretically be solved to provide them with effective indices. We focus on a case where *special display goods*[8, 9, 10] are dealt in. The special display goods are heaped up in the end displays or special areas at retail store. Retailers deal in such special display goods with a view to introducing and/or exposing new products or for the purpose of sales promotions in many cases. They are sold at a fast velocity when their quantity displayed is large, but are sold at a low velocity when their quantity becomes small. Baker and Urban[11] and Urban[12] dealt with a similar problem, but they expressed the demand rate simply as a function of a polynomial form without any practical meaning.

The demand of seasonal merchandise such as clothes, sporting goods, children's toys and electrical home appearances consists of the following three successive periods: in the first phase the demand rate of the product increases with time, and then its demand rate becomes steady. In the final phase the demand rate decreases with time up to the end of the selling season[13]. This type of demand is classified into a time dependant ramp-type demand pattern[13]. The seasonal items have a relatively short selling season (eight to 12 weeks), while they have a relatively long ordering lead-time (three to nine months)[14]. For this reason, the retailers have to commit themselves to a single order to purchase the seasonal items, prior to the start of the season. Recently, Quick Response (QR) system has widely used by manufacturing industries[15]. Quick Response is a vertical strategy where the manufacturer strives to provide products and services to its retail customers in exact quantities on a continuous basis with minimum lead time[16]. Appling the QR system to the manufacture and distribution allows the retailer to re-order the seasonal items during the selling season.

In this study, we develop an inventory model for seasonal products with a ramp-type demand pattern over a finite time horizon (period of a season) to determine the optimal replenishment policy, which maximizes the retailer's total profit. Numerical examples are also presented to illustrate the theoretical underpinnings of the proposed model.

2 Notations and Assumptions

The main notations used in this paper are listed below:

H**:** planning horizon.

n**:** the number of replenishment cycles during the plan-

*Department of Information & Economics, University of Marketing and Distribution Sciences, Kobe, Japan. E-mail: Hidefumi_Kawakatsu@red.umds.ac.jp

(Advance online publication: 23 November 2010)

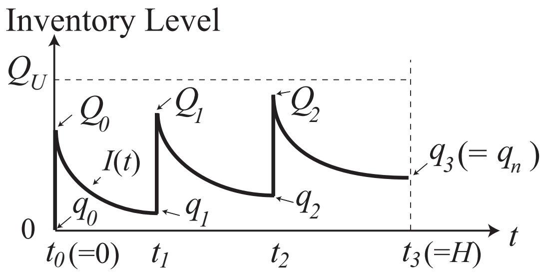

Figure 1: Transition of inventory level ($n = 3$)

ning horizon.

Q_U**:** maximum inventory level.

Q_j, q_j**:** the order-up-to level and the re-order point, respectively, in the jth replenishment cycle($q_0 = 0$, $0 \le q_j < Q_j \le Q_U$, $j = 1, 2, \cdots, n$).

t_j**:** the time of the jth replenishment ($t_{j-1} < t_j$, $t_0 = 0$, $t_n = H$).

p**:** selling price per item.

c**:** acquisition cost per item.

h**:** inventory holding cost per item and unit of time.

K**:** ordering cost per lot.

θ**:** salvage value, per item, of unsold inventory at the end of the planning horizon.

λ**:** a proportional constant of the demand rate.

$\mu(t)$**:** demand rate, at time t, which is independent of the quantity displayed.

The assumptions in this study are as follows:

(1) The finite planning horizon H is divided into n ($n = 1, 2, 3, \cdots$) replenishment cycles.

(2) The retailer deals in the seasonal merchandise. The demand rate first reaches its maximum value, and then becomes a constant or slightly decreases with time. Finally, its rate appreciably decreases with time. The demand rate, $\mu(t)$, which is independent of the quantity displayed is a time dependent ramp-type function and is of the form

$$\mu(t) = \begin{cases} \mu_1(t), & 0 \le t < \gamma_1, \\ \mu_2(t), & \gamma_1 \le t < \gamma_2, \\ \mu_3(t), & \gamma_2 \le t \le H. \end{cases} \quad (1)$$

We assume that $\mu_1'(t) > 0$, $\mu_2'(t) \le 0$, $\mu_3'(t) < 0$, $\lim_{t \to \gamma_1 - 0} \mu_1(t) = \mu_2(\gamma_1)$ and $\lim_{t \to \gamma_2 - 0} \mu_2(t) = \mu_3(\gamma_2)$.

(3) The demand rate is deterministic and significantly depends on the quantity displayed: the items sell well if their quantity displayed is large, but do not when their quantity displayed becomes small. We express such a behavior of special display goods in terms of the following differential equation:

$$\frac{d}{dt} m_j(t) = \lambda \left[Q_{j-1} - m_j(t) \right] + \mu(t), \quad (2)$$

where $m_j(t)$ denotes the cumulative quantity of the objective product sold during $[t_{j-1}, t]$ ($t < t_{j+1}$) and Q_{j-1} signifies the order-up-to level at the beginning of the jth replenishment cycle. A mathematically identical equation has been used to express the behavior of deteriorating items and their optimal ordering policy has been obtained by Abad[17]. Under the model proposed in this study, the demand depends on the quantity heaped and thus depends on time.

(4) The rate of replenishment is infinite and the delivery is instantaneous.

(5) Backlogging and shortage are not allowed.

(6) The retailer orders $(Q_j - q_j)$ units when her/his inventory level reaches q_j. Figure 1 shows the transition of inventory level in the case of $n = 3$.

(7) $v = (p - c - h/\lambda) > 0$. This assumption, $v > 0$, is equivalent to $(p - c)(Q_j - q_j) > h \frac{Q_j - q_j}{\lambda}$. The left-hand-side of the inequality, $(p - c)(Q_j - q_j)$, denotes the cumulative gross profit during $[t_{j-1}, t_j)$, and the right-hand-side of the inequality, $h \frac{Q_j - q_j}{\lambda}$, approximately expresses the cumulative inventory holding cost during $[t_{j-1}, t_j)$. Therefore, $v > 0$ signifies that the gross profit exceeds the inventory holding cost during one replenishment cycle.

3 Total Profit

Since the demand rate consists of three different types components in three successive time periods from assumption (2), the relationship among the demand transfer points $(\gamma_1, \gamma_2)(\gamma_1 < \gamma_2)$ and the cycle times (t_{j-1}, t_j) can be classified into the following six cases:

(a) $t_j < \gamma_1$:

By solving the differential equation in Eq. (2) with the boundary condition $m_j(t_{j-1}) = 0$, the cumulative quantity, $m_j(t)$, of demand for the product at time $t(\ge t_{j-1})$ is given by

$$m_j(t) = Q_{j-1} \left[1 - e^{-\lambda(t - t_{j-1})} \right] + \int_{t_{j-1}}^{t} e^{-\lambda(t-u)} \mu_1(u) du. \quad (3)$$

Since we have $I(t_j) = q_j$, the inventory level of the product at time t becomes

$$\begin{aligned} I(t) &= Q_{j-1} - m_j(t) \\ &= q_j e^{\lambda(t_j - t)} + \int_{t}^{t_j} e^{\lambda(u-t)} \mu_1(u) du. \quad (4) \end{aligned}$$

(b) $t_{j-1} < \gamma_1 \le t_j < \gamma_2$:

In this case, in the same manner as (a), $I(t)$ can be expressed by

$$I(t) = \begin{cases} q_j e^{\lambda(t_j - t)} + \int_{\gamma_1}^{t_j} e^{\lambda(u-t)} \mu_2(u) du \\ \quad + \int_t^{\gamma_1} e^{\lambda(u-t)} \mu_1(u) du, \\ \qquad \text{if } t_{j-1} \le t < \gamma_1, \\ q_j e^{\lambda(t_j - t)} + \int_t^{t_j} e^{\lambda(u-t)} \mu_2(u) du, \\ \qquad \text{if } \gamma_1 \le t \le t_j. \end{cases} \tag{5}$$

(c) $t_{j-1} < \gamma_1$ and $\gamma_2 \le t_j$:

In this case, $I(t)$ can be expressed by

$$I(t) = \begin{cases} q_j e^{\lambda(t_j - t)} + \int_{\gamma_2}^{t_j} e^{\lambda(u-t)} \mu_3(u) du \\ \quad + \int_{\gamma_1}^{\gamma_2} e^{\lambda(u-t)} \mu_2(u) du \\ \quad + \int_t^{\gamma_1} e^{\lambda(u-t)} \mu_1(u) du, \\ \qquad \text{if } t_{j-1} \le t < \gamma_1, \\ q_j e^{\lambda(t_j - t)} + \int_{\gamma_2}^{t_j} e^{\lambda(u-t)} \mu_3(u) du \\ \quad + \int_t^{\gamma_2} e^{\lambda(u-t)} \mu_2(u) du, \\ \qquad \text{if } \gamma_1 \le t < \gamma_2, \\ q_j e^{\lambda(t_j - t)} + \int_t^{t_j} e^{\lambda(u-t)} \mu_3(u) du, \\ \qquad \text{if } \gamma_2 \le t \le t_j. \end{cases} \tag{6}$$

(d) $\gamma_1 \le t_{j-1}$ and $t_j < \gamma_2$:

In this case, $I(t)$ is given by

$$I(t) = q_j e^{\lambda(t_j - t)} + \int_t^{t_j} e^{\lambda(u-t)} \mu_2(u) du. \tag{7}$$

(e) $\gamma_1 \le t_{j-1} < \gamma_2 \le t_j$:

In this case, $I(t)$ can be expressed by

$$I(t) = \begin{cases} q_j e^{\lambda(t_j - t)} + \int_{\gamma_2}^{t_j} e^{\lambda(u-t)} \mu_3(u) du \\ \quad + \int_t^{\gamma_2} e^{\lambda(u-t)} \mu_2(u) du, \\ \qquad \text{if } t_{j-1} \le t < \gamma_2, \\ q_j e^{\lambda(t_j - t)} + \int_t^{t_j} e^{\lambda(u-t)} \mu_3(u) du, \\ \qquad \text{if } \gamma_2 \le t \le t_j. \end{cases} \tag{8}$$

(f) $\gamma_2 \le t_{j-1}$:

In this case, $I(t)$ is given by

$$I(t) = q_j e^{\lambda(t_j - t)} + \int_t^{t_j} e^{\lambda(u-t)} \mu_3(u) du. \tag{9}$$

By using notation $\mu(t)$ in Eq. (1), $m_j(t)$ in Eq. (3) and $I(t)$ in Eqs. from (4) to (9) can respectively be expressed by

$$m_j(t) = Q_{j-1} \left[1 - e^{-\lambda(t - t_{j-1})} \right] + \int_{t_{j-1}}^t e^{-\lambda(t-u)} \mu(u) du, \tag{10}$$

$$I(t) = Q_{j-1} - m_j(t) = q_j e^{\lambda(t_j - t)} + \int_t^{t_j} e^{\lambda(u-t)} \mu(u) du. \tag{11}$$

Therefore, the initial inventory level in jth replenishment cycle is given by

$$Q_{j-1} = I(t_{j-1}) = q_j e^{\lambda(t_j - t_{j-1})} + \int_{t_{j-1}}^{t_j} e^{\lambda(u-t)} \mu(u) du. \tag{12}$$

By letting $Q_{j-1} = I(t_{j-1})$ in Eq. (10), the cumulative quantity of demand during $[t_{j-1}, t_j)$ becomes

$$m(t_{j-1}, t_j) = q_j \left[e^{\lambda(t_j - t_{j-1})} - 1 \right] + \int_{t_{j-1}}^{t_j} e^{\lambda(t_j - t_{j-1})} \mu(u) du. \tag{13}$$

There obviously exists a time $t = t_j^U (> t_{j-1})$ when the inventory level reaches zero, where t_j^U is unique positive solution to

$$\int_{t_{j-1}}^{t_j} e^{\lambda(u - t_{j-1})} \mu(u) du = Q_{j-1}. \tag{14}$$

The left-hand-side of Eq. (14) indicates that the cumulative demand of the product in jth replenishment cycle when the re-order point q_j is set to be zero. The maximum value of t_j can therefore be given by t_j^U.

On the other hand, the cumulative inventory, $A(t_{j-1}, t_j)$, held during $[t_{j-1}, t_j)$ ($t_j \le t_j^U$) is expressed by

$$A(t_{j-1}, t_j) = \int_{t_{j-1}}^{t_j} I(t) dt = \frac{1}{\lambda} \left[m(t_{j-1}, t_j) - \int_{t_{j-1}}^{t_j} \mu(u) du \right]. \tag{15}$$

Hence, the total profit is given by

$$P_n = \sum_{j=1}^{n} \Big[p \cdot m(t_{j-1}, t_j) - c \cdot (Q_{j-1} - q_{j-1})$$

$$\left. -h \cdot A(t_{j-1}, t_j) \right] + \theta q_n - nK$$

$$= v \sum_{j=1}^{n} m(t_{j-1}, t_j) + (\theta - c) q_H + h/\lambda \int_0^H \mu(u) du - nK, \quad (16)$$

where $v = (p - c - h/\lambda)(> 0)$.

4 Optimal Policy

This section analyzes the existence of the optimal policy $(Q_{j-1}, q_j, t_j) = (Q^*_{j-1}, q^*_j, t^*_j)$ for a given n ($j = 1, 2, \cdots, n$), which maximizes P_n in Eq. (16). It is, however, very difficult to conduct analysis under $\theta \neq c$. For this reason, we focus on the case where $\theta = c$.

4.1 Optimal Re-order Point

In this subsection, we examine the existence of (Q^*_j, q^*_j), in case t_{j-1} and t_j are respectively fixed to suitable values.

Let $R(t_{j-1}, t_j)$ be defined by

$$R(t_{j-1}, t_j) \equiv \left[Q_U - \int_{t_{j-1}}^{t_j} e^{\lambda(u - t_{j-1})} \mu(u) du \right] \times e^{-\lambda(t_j - t_{j-1})} \ (> 0). \quad (17)$$

The optimal order-up-to level and the optimal re-order point can be given by

$$(Q^*_j, q^*_j) = (Q_U, R(t_{j-1}, t_j)). \quad (18)$$

The proofs are given in Appendix A.

By letting $(Q_{j-1}, q_j) = (Q_U, R(t_{j-1}, t_j))$ in Eq. (16), the total profit on $(Q_{j-1}, q_j) = (Q_U, R(t_{j-1}, t_j))$ becomes

$$P_n = v \sum_{j=1}^{n} \left\{ Q_U - e^{-\lambda(t_j - t_{j-1})} \left[Q_U - \tilde{m}(t_{j-1}, t_j) \right] \right\} + h/\lambda \int_0^H \mu(u) du - nK, \quad (19)$$

where

$$\tilde{m}(t_{j-1}, t_j) = \int_{t_{j-1}}^{t_j} e^{\lambda(u - t_{j-1})} \mu(u) du. \quad (20)$$

4.2 Optimal Replenishment Time

This subsection makes an analysis of t^*_j that maximizes P_n, for a given (t_{j-1}, t_{j+1}). The analysis with respect to existence of t^*_j becomes considerably complicated under $L'(t_j) > 0$ for $t_j < \gamma_1$. For this reason, when $t_j < \gamma_1$, we focus on the case where $L'(t_j) \leq 0$.

Let us here $\varphi(t^U_j)$ be given by

$$\varphi(t^U_j) \equiv \frac{1}{\lambda} \ln \frac{\lambda Q_U + \mu(t^U_j)}{\mu(t^U_j)} + t^U_j. \quad (21)$$

We here summarize the result of analysis in relation to the optimal replenishment time t^*_j.

The proofs are shown in Appendix B.

We show below that an optimal replenishment time t^*_j exists:

(1) $t_{j+1} < \varphi(t^U_j)$:
In this case, there exists a unique finite t^*_j ($t_{j-1} < t^*_j < \min(t^U_j, t_{j+1})$) that maximizes P_n.

(2) $t_{j+1} \geq \varphi(t^U_j)$:
In this case, P_n is non-decreasing in t_j, and consequently we have $t^*_j = t^U_j$.

If there exists $t^*_j < t^U_j$ for all j ($j = 1, 2, \cdots, n-1$), the total profit is given by

$$P_n = \tilde{v} \left\{ \frac{1}{\lambda} \sum_{j=1}^{n-1} \left[\lambda Q_U + \mu(t^*_j) \right] \left[1 - e^{-\lambda(t^*_{j+1} - t^*_j)} \right] m(t^*_{n-1}, H) \right\} + h/\lambda \int_0^H \mu(u) du - nK. \quad (22)$$

5 Numerical Examples

This section presents numerical examples to illustrate the proposed model.

Suppose that the demand rate which is independent of the quantity displayed is given by

$$\mu(t) = \begin{cases} \alpha e^{\beta t}, & t < \gamma_1, \\ \alpha e^{\beta \gamma_1}, & \gamma_1 \leq t < \gamma_2, \\ \alpha e^{\beta(\gamma_1 + \gamma_2 - t)}, & t \geq \gamma_2, \end{cases} \quad (23)$$

where $\alpha > 0$ and $\beta > 0$[13].

Figure 2 reveals the transition of inventory level along with behavior of (q^*_j, t^*_j) in the case of $(H, Q_U, \lambda, p, c, h, \theta, \alpha, \beta.\gamma_1, \gamma_2) = (100, 350, 0.01, 600, 300, 1, 300, 0.1, 0.15, 30, 70)$ for $K = 2000, 5000, 8000$.

It is observed in Fig. 2 that the number of replenishment cycles decreases with increasing K. This is because when the ordering cost per lot becomes large, the total ordering cost should be slashed by means of increasing the time interval between successive replenishments.

We can also notice in Fig. 2 that q^*_j takes a constant value on the whole in the region of $\gamma_1 < t \leq \gamma_2$. In contrast, in the regions of $t < \gamma_1$ and $t \geq \gamma_2$, the value of q^*_j relatively

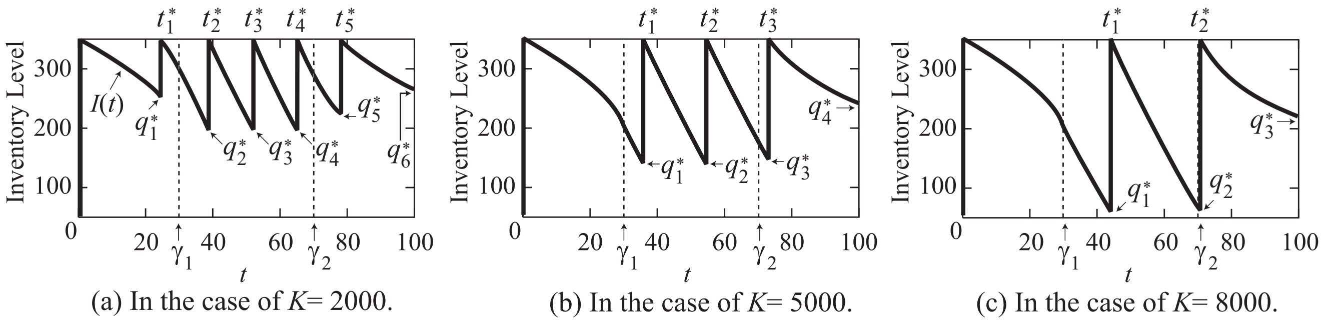

Figure 2: Sensitivity analysis

becomes larger, which signifies that, in these regions, the cumulative quantity displayed increases as the demand rate which is not affected by the inventory level decreases. Heaping up the products to a large quantity reflects the situation where the demand velocity is large. When the demand rate becomes small, the retailer can therefore maintain her/his profit as large as possible by increasing the quantity displayed.

6 Conclusions

In this study, we have proposed an inventory model for seasonal products with the ramp-type demand pattern over a finite time horizon (period of a season) to determine the optimal replenishment policy, which maximizes the retailer's total profit. We particularly focus on the case where the retailer is facing her/his customers' demand by dealing in the special display goods. They are sold at a fast velocity when their quantity displayed is large, but are sold at a low velocity if the quantity becomes small. We have clarified the existence of the optimal order quantity at time t_j, along with the optimal replenishment time which maximize the retailer's total profit. In the real circumstances, retailers frequently place a mirror at their display area, or they display products on a false bottom to increase their quantity displayed in appearance. Taking account of such factors is an interesting extension.

Appendix A

In this appendix, we show the existence of both the optimal order-up-to level and the re-order point $(Q_j, q_j) = (Q_j^*, q_j^*)$ in case n, t_{j-1} and t_j are respectively fixed to suitable values.

At retail stores, they have a maximum value for the inventory level arrowed for some reasons, which is denoted by Q_U. It can easily be shown from Eq. (12) that Q_{j-1} is a function of q_j $(0 \leq q_j < Q_{j-1} \leq Q_U)$, and furthermore, $Q_{j-1} \leq Q_U$ agrees with

$$q_j \leq e^{-\lambda(t_j - t_{j-1})} \times \left[Q_U - \int_{t_{j-1}}^{t_j} e^{\lambda(u - t_{j-1})} \mu(u) du \right]. \quad \text{(A.1)}$$

Let $R(t_{j-1}, t_j)$ express the right-hand-side of Inequality (A.1). We obviously have $R(t_{j-1}, t_j) \geq 0$ for $t_{j-1} \leq t_j < \min(t_j^U, t_{j+1})$.

By differentiating P_n in Eq. (16) with respect to q_j, we have

$$\frac{\partial}{\partial q_j} P_n = v \left[e^{\lambda(t_j - t_{j-1})} - 1 \right] (> 0). \quad \text{(A.2)}$$

Since $(p - c - h/\lambda) > 0$ from assumption (7), we have $\frac{\partial}{\partial q_j} P_n > 0$, and consequently $(Q_{j-1}^*, q_j^*) = (Q_U, R(t_{j-1}, t_j))$.

Appendix B

In this appendix, we show the existence of t_j^* that maximizes P_n for a given (t_{j-1}, t_{j+1}).

By differentiating P_n in Eq. (19) with respect to t_j, we have

$$\begin{aligned} \frac{\partial}{\partial t_j} P_n = v \Big\{ & \lambda e^{-\lambda(t_j - t_{j-1})} \left[Q_U - \tilde{m}(t_{j-1}, t_j) \right] \\ & - \lambda Q_U e^{-\lambda(t_{j+1} - t_j)} \\ & + \mu(t_j) \left[1 - e^{-\lambda(t_{j+1} - t_j)} \right] \Big\} \end{aligned} \quad \text{(B.1)}$$

Let $L(t_j)$ express the terms enclosed in braces { } in the right-hand-side of Eq. (B.1). Since it can easily be proven from assumption (7) that the sign of v is positive, $\frac{\partial}{\partial t_j} P_n \geq 0$ agrees with

$$L(t_j) \geq 0. \quad \text{(B.2)}$$

Furthermore, we have

$$L'(t_j) = -\lambda \Big\{ \lambda e^{-\lambda(t_j - t_{j-1})} \left[Q_U - \tilde{m}(t_{j-1}, t_j) \right]$$

$$
\begin{aligned}
&\quad +\mu(t_j) + e^{-\lambda(t_{j+1}-t_j)}\left[\lambda Q_U + \mu(t_j)\right]\Big\} \\
&\quad +\mu'(t_j)\left[1 - e^{-\lambda(t_{j+1}-t_j)}\right], \qquad \text{(B.3)}
\end{aligned}
$$

$$
\begin{aligned}
L(t_{j-1}) &= [\lambda Q_U + \mu(t_{j-1})] \\
&\quad \times \left[1 - e^{-\lambda(t_{j+1}-t_{j-1})}\right] \ (>0), \qquad \text{(B.4)} \\
L(t_{j+1}) &= -\lambda\Big\{e^{-\lambda(t_{j+1}-t_{j-1})}\tilde{m}(t_{j-1}, t_{j+1}) \\
&\quad +Q_U\left[1 - e^{-\lambda(t_{j+1}-t_{j-1})}\right]\Big\} (<0), \ \text{(B.5)} \\
L(t_j^U) &= \mu(t_j^U)\left[1 - e^{-\lambda(t_{j+1}-t_j^U)}\right] \\
&\quad -\lambda Q_U e^{-\lambda(t_{j+1}-t_j^U)}. \qquad \text{(B.6)}
\end{aligned}
$$

In the case of $t_{j+1} \geq t_j^U$, $L(t_j^U) < 0$ coincides with

$$
t_{j+1} \quad < \quad \frac{1}{\lambda}\ln\frac{\lambda Q_U + \mu(t_j^U)}{\mu(t_j^U)} + t_j^U. \qquad \text{(B.7)}
$$

Let us denote, by $\varphi(t_j^U)$, the right-hand-side of Inequality (B.7).

It can easily be shown from Eq. (B.3) that $L'(t_j) < 0$ in the case of $t_j \geq \gamma_1$ since we have $\mu_2'(t_j) \leq 0$ and $\mu_3'(t_j) < 0$. In the case of $t_j < \gamma_1$, as mentioned in Section 4.2, we focus on the case where $L'(t_j) \leq 0$.

On the basis of the above results, for a given (t_{j-1}, t_{j+1}), we show below that an optimal replenishment time t_j^* exists:

(1) $t_{j+1} < \varphi(t_j^U)$:

In this subcase, the sign of $\frac{\partial}{\partial t_j}P_n$ changes from positive to negative only once, and thus there exists a unique finite t_j^* $(t_{j-1} < t_j^* < \min(t_j^U, t_{j+1}))$ that maximizes P_n.

(2) $t_{j+1} \geq \varphi(t_j^U)$:

In this subcase, P_n is non-decreasing in t_j, and consequently we have $t_j^* = t_j^U$.

References

[1] Resh, M., Friedman, M. and Barbosa, L.C., "On a general solution of the deterministic lot size problem with time-proportional demand", *Operations Reseach*, V24, pp. 718–725, 1976.

[2] Donaldson, W.A., "Inventory replenishment policy for a linear trend in demand: An analytical solution", *Operational Research*, V28, pp. 663–670, 1977.

[3] Barbosa, L.C. and Friedman, M., "Deterministic inventory lot size models.a general root law", *Management Science*, V24, pp. 819–826, 1978.

[4] Henery, R.J., "Inventory replenishment policy for increasing demand", *Journal of the Operational Research Society*, V30, pp. 611–617, 1979.

[5] Hariga M.A. and Goyal, S.K., "An alternative procedure for determining the optimal policy for an inventory item having linear trend", *Journal of the Operational Research Society*, V46, pp. 521–527, 1995.

[6] Teng, J.T., "A deterministic replenishment model with linear trend in demand", *Operations Research Letters*, V19, pp. 33–41, 1996.

[7] Dye, C.Y., Chang, H.J. and Teng, J.T., "A deteriorating inventory model with time-varying demand and shortage-dependent partial backlogging", *European Journal of Operational Research*, V172, N16, pp. 417–429, 2006.

[8] Kawakatsu, H., Sandoh, H. and Hamada T., "An optimal order quantity for special display goods in retailing: Maximization of total profit per unit time (in Japanese)", *Trans. Japan Society for Industrial and Applied Mathematics*, V10, N2, pp. 75–186, 2000.

[9] Kawakatsu, H., Sandoh, H. and Hamada T., "An optimal order quantity for special display goods in retailing: The effect of a mirror and false bottom (in Japanese)", *Trans. Japan Society for Industrial and Applied Mathematics*, V12, N2, pp. 135–154, 2002.

[10] Kawakatsu, H., " Optimal replenishment policy for special display goods with ramp-type demand rate ", *Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering 2010*, WCE 2010, 30 June - 2 July, 2010, London, U.K., pp 1691–1696.

[11] Baker, R.C. and Urban, T.L., "A deterministic inventory system with an inventory-level-dependent demand rate", *The Journal of the Operational Research Society*, V39, pp. 823–831, 1988.

[12] Urban, T.L., "An inventory-theoretic approach to product assortment and shelf-space allocation", *Journal of Retailing*, V74, pp. 15–35, 1998.

[13] Panda, S., Senapati, S. and Basu, M., "Optimal replenishment policy for perishable seasonal products in a season with ramp-type time dependent demand". *Computers & Industrial Engineering*, V54, N2, pp. 301–314, 2008.

[14] Walker, J., "A model for determining price markdowns of seasonal merchandise", *Journal of Product & Brand Management*, V8, N4, pp. 352–361, 1999.

[15] Al-Zubaidi H. and Tyler D., "A simulation model of quick response replenishment of seasonal clothing", *International Journal of Retail & Distribution Management*, V32, N6, pp. 320–327, 2004.

[16] Yan, H., "Retail buyers' perceptions of quick response systems". *International Journal of Retail & Distribution Management*, V26, N6, pp. 237–246, 1998.

[17] Abad, P.L., "Optimal price and order size for a reseller under partial backordering", *Computers & Operations Research*, V28, N1, pp. 53–65, 1/01.