# On Hydromagnetic Channel Flow of a Rotating Two-Phase Fluid Induced by Tooth Pulses

Sanchita Ghosh, A. K. Ghosh and S.Debnath

***Abstract*—An initial value problem is solved for the motion of an incompressible conducting viscous fluid with embedded small inert spherical particles in a channel bounded by two infinite rigid non-conducting plates. Both the plates and the fluid are in a state of solid-body rotation with a constant angular velocity about an axis normal to the plates. An unsteady motion is generated in such a fluid when the upper plate is subjected to velocity tooth pulses in its own plane with the lower plate held fixed. Additionally, an external magnetic field is acting on the particulate suspension in a direction normal to the plates. It is assumed that no electric current exists in the basic state and the magnetic Reynolds number is very small. The method of Fourier analysis is used to derive exact solutions for the fluid and the particle velocities and the skin-friction on the walls. The influence of the particles, the magnetic field and the rotation on the components of the fluid velocity and the wall frictions are examined quantitatively. Some known results are found to emerge as limiting cases of the present analysis.**

***Index Terms*—Hydromagnetic, pulsatile flow, rotating fluid-particle system**

## I. INTRODUCTION

The fluid flow generated by pulsatile motion of the boundary is found to have immense importance in aerospace science, nuclear fusion, astrophysics, atmospheric sciences, cosmical gasdynamics, geophysics and physiological fluid dynamics. The investigation in this direction was initiated by Ghosh[1] who examined the motion of an incompressible viscous fluid in a channel bounded by two rigid coaxial cylinders when the inner cylinder is set in motion by pulses of longitudinal impulses. Subsequently, Chakraborty and Ray[2] studied the unsteady magnetohydrodynamic Couette flow between two parallel plates when one of the plates was subjected to random pulses. Makar[3] presented the solution of magnetohydrodynamic flow between two parallel plates when the upper plate was set in motion by velocity tooth pulses and the induced magnetic field was neglected. Bestman and Njoku[4] constructed the solution of the same problem as that of author[3] without ignoring the effect of induced magnetic field. Regarding the pulsatile motion of a two-phase fluid-particle system, Datta et al.[5,6] examined the heat transfer to pulsatile flows of a dusty fluid in pipes and channels with a view to their applications in the analysis of blood flow. Ghosh and Sarkar[7] considered the hydromagnetic channel flow of a dusty fluid induced by velocity tooth pulses and arrived at the solution by the method of Fourier analysis while the same problem as that of authors[7] was studied by Ghosh and Debnath[8] using the method of Laplace transforms. It was seen that both the methods adopted in [7] and [8] provide the same exact solution of the problem. Most recently, Ghosh and Ghosh[9,10] solved the problems of hydromagnetic flow of a two-phase fluid near a pulsating plate both in a non-rotating and rotating system with a view to their applications in the analysis of suspension boundary layers. On the other hand, several authors including Yang and Healy[11], Nag[12], Nag et al.[13], Mitra and Bhattacharyya[14,15] and Ghosh and Debnath [16,17] discussed various aspects of non-pulsatile flows of a two-phase fluid-particle system both in hydrodynamic and hydromagnetic situations. Finally, it was noticed that the solution of a boundary value problem associated with hydromagnetic Couette flow of an Oldroyd-B fluid in a rotating system was reported by Hayat et al.[18]. In spite of the above works, it is found that the problem of hydromagnetic channel flow of a rotating fluid-particle system caused by pulsatile motion of the boundary has not yet been solved. The objective of the present paper is to study such problem with a view to its applications in hydromagnetic spin-up in a contained fluid[19], the motion of the earth's liquid core[20], the development of sunspot, the solar cycle and the structure of the magnetic stars[21] and in the determination of the effects of the external magnetic field and rotation on the flow of blood in the cardiovascular system[22].

The present problem is concerned with the analysis of unsteady motion developed in an incompressible electrically conducting viscous fluid containing uniformly distributed small inert spherical particles in a channel bounded by two infinite rigid non-conducting plates. Both the plates and the particulate suspension are in a state of sold-body rotation with a constant angular velocity about an axis normal to the plates. An uniform external magnetic field is acting on the system in a direction normal to the plates. Additionally, the upper plate starts moving impulsively from rest relative to the rotating fluid-particle system due to velocity tooth pulses applied on it with the lower plate held fixed. It is assumed that no electric current flows in the basic state and the magnetic Reynolds number is very small. The inquiries are made about the exact solutions for the fluid and the particle velocities and the skin-friction on the walls. The results are computed numerically with a view to disclose the quantitative response of various flow parameters on the components of fluid velocity and the wall frictions. Finally, many known results are recovered as limiting cases of the present analysis.

Manuscript received December 16, 2009.

Sanchita Ghosh, Department of Computer Science, BIT Mesra, Kolkata Extension Centre, 1582 Rajdanga Main Road, Kolkata 7000107, India,(e-mail: $bij_arn@yahoo.com$)

A.K.Ghosh, Retired Professor, Department of Mathematics, Jadavpur University, Kolkata-700032, India (Correspondig author, Phone: +919830155809, e-mail: akg10143@yahoo.co.in)

S.Debnath, Department of Mathematics, Jadavpur University, Kolkata-700032, India, (e-mail:$debnath_swapna@yahoo.co.in$)

**(Advance online publication: 10 February 2011)**

## II. MATHEMATICAL FORMULATION

Following Saffman[23] and Ghosh and Debnath[16], the equations of unsteady motion of an incompressible electrically conducting viscous fluid with embedded identical small inert spherical particles in a rotating coordinate system under an external magnetic field $\mathbf{B}$ are in usual notations:

$$\frac{\partial\, \mathbf{u}}{\partial\, t} + (\mathbf{u}\cdot\nabla)\,\mathbf{u} + 2\,\mathbf{\Omega}\times\mathbf{u} = -\frac{1}{\rho}\,\nabla\,p + \nu\,\nabla^2\,\mathbf{u} + \frac{K\,N}{\rho}\,(\mathbf{v}-\,\mathbf{u}) + \frac{1}{\rho}\,(\mathbf{j}\,\times\,\mathbf{B}) \quad (1)$$

$$m[\frac{\partial\,\mathbf{v}}{\partial\,t} + (\mathbf{v}\cdot\,\nabla)\,\mathbf{v} + 2\,\mathbf{\Omega}\times\mathbf{v}\,] = K(\mathbf{u}-\mathbf{v})] \quad (2)$$

$$\nabla\cdot\mathbf{u} = 0 \;\; and \;\; \frac{\partial\mathbf{N}}{\partial\,t} + \nabla\cdot(N\,\mathbf{v}) = 0 \quad (3)$$

where $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$ represent the velocity of the fluid and the particles respectively, p is the modified fluid pressure including the centrifugal force term, N is the number density of the particles which are distributed uniformly in the fluid of density $\rho$ and kinematic viscosity $\nu$, m is the mass of the particle, K is the Stokes' resistance coefficient which for spherical particles of radius a is $6\,\pi\,\mu\,a$, $\mathbf{j}$ is the current density, $\mathbf{B}$ is the magnetic flux density, and $\mathbf{\Omega}$ is the angular velocity of the coordinate system. The buoyancy force term in (2) is neglected since for most common materials $\frac{\rho}{\rho_p}$ is very very small where $\rho_p$ is the density of the material of the dust. The Maxwell equations with usual MHD approximation are :

$$div\mathbf{B} = 0,\; Curl\mathbf{B} = \mu_0\mathbf{j},\; Curl\;\mathbf{E}^* = -\frac{\partial\,\mathbf{B}}{\partial\,t}, \quad (4)$$

$$\mathbf{j} = \sigma_0(\mathbf{E}^* + \mathbf{u}\times\,\mathbf{B})] \quad (5)$$

where the displacement currents are neglected, $\mu_0 \; and \; \sigma_0$ are constants and $\mathbf{E}^*$ is the electric field.

We now consider the unsteady hydromagnetic flow of an incompressible electrically conducting viscous fluid with uniformly distributed small inert spherical particles in a channel bounded by two infinite rigid non-conducting plates in presence of a constant magnetic field $B_0$ normal to the plates at z=0. Both the two-phase fluid and the plates are in a state of solid-body rotation with constant angular velocity $\Omega$ about the z-axis normal to the plates and in this situation the upper plate sets in motion in its own plane impulsively from rest due to velocity tooth pulses applied periodically on it with the lower plate kept stationary. The x-axis is taken in the direction of length of the lower plate and the y-axis is also fixed in the plate normal to its direction of motion. The flow configuration is shown in figure 1. We assume that no applied or polarization voltage exists i.e., $\mathbf{E}^* = 0$ so that no energy is added or extracted from the fluid by the electric field. We further assume that the magnetic Reynolds number is very small which is plausible for most electrically conducting fluids. This implies that the current is mainly due to induced electric field so that $\mathbf{j} = \sigma_0(\mathbf{u}\times\,\mathbf{B})$ and the applied magnetic field remains essentially unaltered by the electric current flowing through the fluid. We further assume that the induced magnetic field produced by the motion of the fluid is negligible compared to the applied magnetic field so that Lorentz force term in (1) becomes $-\frac{\sigma_0}{\rho}\,B_0^2\,\mathbf{u}$. Moreover, the particles are uniformly distributed in the fluid and the flow field is parallel to the x-direction. This implies that all the physical variables are functions of z and t only and the second equation of (3) is satisfied throughout the flow field when $N = N_0 = constant$. It is also evident from (1) is that $\frac{\partial p}{\partial z} = 0$. Accordingly, $\frac{\partial p}{\partial x}$ and $\frac{\partial p}{\partial y}$ have the same value as in the free stream. We, therefore, assume that both the quantities are zero.

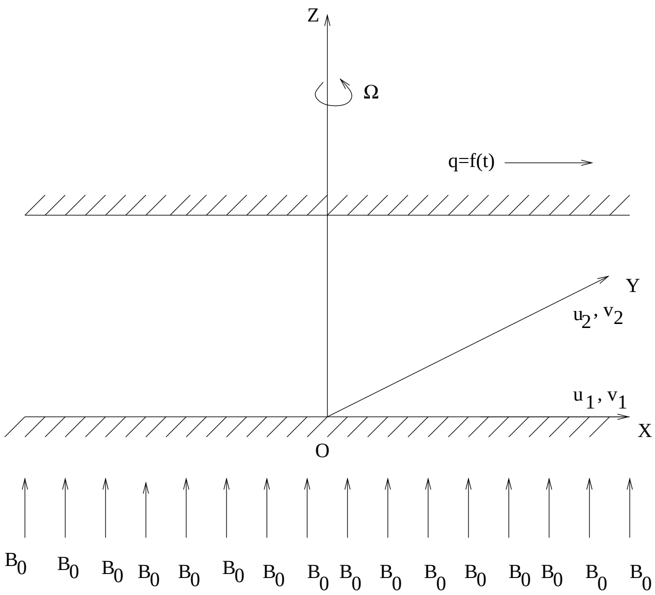


Fig. 1. Geometry of the flow configuration

On the basis of the assumptions made above, the unsteady motion of a two-phase fluid-particle system occupying the space between the plates at a distance h apart is governed by the equations:

$$\frac{\partial q}{\partial t} + 2\,i\,\Omega\,q = \nu\,\frac{\partial^2 q}{\partial z^2} + \frac{k}{\tau}\,(r-q) - \,n\,q \quad (6)$$

$$and \;\; \frac{\partial r}{\partial t} + 2\,i\,\Omega\,r = \frac{1}{\tau}(q-r) \quad (7)$$

where $q = u_1 + iu_2$ is the complex fluid velocity, and $r = v_1 + iv_2$ is the complex particle velocity, $k = \frac{m\;N_0}{\rho}$ is the ratio of the mass density of the particles and the fluid density, usually called, the mass concentration of the particles, $\tau = \frac{m}{K}$ is the relaxation time of the particles and $n = \frac{\sigma_0}{\rho}\,B_0^2$ is the hydromagnetic parameter.

Introducing the non-dimensional variables

$$(q', r') = \frac{(q,r)}{U},\; z' = \frac{z}{h} \;\; and \;\; (t', \lambda) = \frac{\nu(t,\tau)}{h^2} \quad (8)$$

and the non-dimensional flow parameters

$$E = \frac{\Omega\,h^2}{\nu},\quad M^2 = \frac{n\,h^2}{\nu} \quad (9)$$

in equations (6) and (7) and dropping the primes, we get the non-dimensional equations of motion in the form

$$\frac{\partial q}{\partial t} + 2\,i\,E\,q = \frac{\partial^2 q}{\partial z^2} + \frac{k}{\lambda}\,(r-q) - M^2\,q \quad (10)$$

$$and \qquad \frac{\partial r}{\partial t} + 2\,i\,E\,r = \frac{1}{\lambda}\,(q-r) \quad (11)$$

where $M^2 = \frac{\sigma\;h^2\;B_0^2}{\rho\;\nu}$ is the Hartman Number.

The problem now reduces to solving (10) and (11) subject to the boundary and initial conditions given by

$$q(z,t) = f(\ t\ ) \qquad at \quad z=1, \qquad t\ >\ 0\ , \quad (12)$$

$$(q,r) \longrightarrow (0,0) \qquad at\ \ z\ =0, \qquad t\ >\ 0\ , \quad (13)$$

$$(q,r) = (0,0) \qquad at\ t\ \leq\ 0 \qquad for\ \ all\ \ z \quad (14)$$

where f(t) represents the tooth pulses which is an even periodic function of time with period 2T and strength $E_1\ T$.

## III. Solution of the problem

Eliminating r from (10) and (11), we get

$$(1+2iE\lambda+\lambda\frac{\partial}{\partial t})\ \frac{\partial q}{\partial t}\ +\ 2iE(1+2iE\lambda+\lambda\frac{\partial}{\partial t})q$$
$$= (1+2iE\lambda+\lambda\frac{\partial}{\partial t})\frac{\partial^2 q}{\partial z^2} - k(\frac{\partial}{\partial t}+2iE)q$$
$$-M^2(1+2iE\lambda+\lambda\frac{\partial}{\partial t})q. \quad (15)$$

To solve this equation one more initial condition is needed on q in addition to that given in (14). Without loss of generality we assume that

$$\frac{\partial q}{\partial t} = 0 \qquad at \quad t\ \leq\ 0, \qquad 0\ \leq\ z\ \leq\ 1. \quad (16)$$

According to the nature of f(t) mentioned above the mathematical form of q(1,t) may be written as

$$q(1,t) = \frac{E_1}{T}\ \{\ t\ H(t)$$
$$+2\sum_{p=1}^{\infty}\ (-1)^p\ (t-p\ T)\ H(t-p\ T)\ \} \quad (17)$$

where H(t) is the Heaviside step function defined as

$$H(t-T)\ =\ 0,\ t\ <\ T\ and\ H(t-T)\ =\ 1,\ t\ \geq\ T.$$

Using half-range Fourier series the condition (17) may also be expressed as

$$q(1,t) = \frac{E_1}{2} - \frac{4\ E_1}{\pi^2}\ \Sigma_{p=0}^{\infty}\ \ \frac{cos\ [\ (2p\ +1)\ \pi\ t/T\ ]}{(2p\ +\ 1)^2}. \ (18)$$

By virtue of the equation (18), we assume the solution of (15) as

$$q(z,t) = q_s(z) + \frac{1}{2}\ \sum_{p=1}^{\infty}[q_{2p+1}(z)\ exp\ (i\ \frac{(2p+1)\ \pi\ t}{T})$$
$$+\ \bar{q}_{2p+1}(z)\ exp\ (-\ i\ \frac{(2p+1)\ \pi t}{T})]$$
$$+\sum_{n=1}^{\infty} W_n(t) sin\ n\ \pi\ z \ (19)$$

where $\bar{q}$ is the conjugate of q.

Substituting (19) in (15), we have the following equations with appropriate conditions as

$$\frac{d^2 q_s}{dz^2} - L^2 q_s = 0 \quad (20)$$

with $q_s = 0$ on z=0, $q_s = \frac{E_1}{2}$ on z=1,

$$\frac{d^2 q_{2p+1}}{dz^2} - L_p^2 q_{2p+1} = 0 \quad (21)$$

with $q_{2p+1} = 0$ on z = 0,

$q_{2p+1} = \frac{-4E_1}{(2p+1)^2\ \pi^2}$ on z = 1,

$$\lambda\ \frac{d^2 W_n}{dt^2} + [1+k+\lambda(N^2+4iE)]\frac{dW_n}{dt}$$
$$+[(1+2iE\lambda)(N^2+2iE)+2iEk]W_n = 0 \quad (22)$$

$$with \quad W_n(t) = W_n(0) \ \ at \ \ t=0,$$

$$W_n'(t) = W_n'(0) \ \ at \ \ t=0$$

where $W_n(0)$ and $W_n'(0)$ are to be determined.
In the above,

$$N^2 = M^2 + n^2\pi^2,\ \beta_p = \frac{(2p+1)\pi}{T},$$

$$L^2 = M^2 + 2i\ E + \frac{2i\ E\ k}{1+2i\ E\ \lambda},$$

$$L_p^2 = M^2 + i\ (2E+\beta_p)\ [1+\frac{k}{1+i\ \lambda(2\ E+\beta_p)}].$$

The solutions of equations (20) to (22) are

$$q_s(z) = \frac{E_1}{2}\frac{sinh\ Lz}{sinh\ L}, \quad (23)$$

$$q_{2p+1}(z) = -\frac{4\ E_1}{\pi^2}\frac{1}{(2p+1)^2}\ \frac{sinh\ L_p z}{sinh\ L_p}, \quad (24)$$

$$W_n(t) = W_n'(0)\ \frac{exp(m_1 t) - exp(m_2 t)}{m_1 - m_2}$$
$$+W_n(0)\ \frac{m_1\ exp(m_2 t) - m_2\ exp(m_1 t)}{m_1 - m_2} \quad (25)$$

where

$$2\ \lambda\ m_1, 2\lambda\ m_2 = -[\langle 1+k+\lambda(N^2+4iE)\rangle$$
$$\mp\{\langle 1+k+\lambda\ (N^2+4iE)\rangle^2 - 4\lambda$$
$$\langle (1+2iE)(N^2+2iE)+2iEk\rangle\}^{1/2}]. \quad (26)$$

The initial conditions (14) and (16) provide

$$\frac{W_n(0)}{E_1} = (-1)^n\ n\ \pi\ [\frac{1}{L^2+n^2\ \pi^2}$$
$$-\frac{8}{\pi^2} Re\Sigma_{p=0}^{\infty}\frac{1}{(2p+1)^2(L_p^2+n^2\pi^2)}], \quad (27)$$

$$\frac{W_n'(0)}{E_1} = \frac{8\ n\ (-1)^n}{\pi} I_m \Sigma_{p=0}^{\infty}\frac{\beta_p}{(2p+1)^2(L_p^2+n^2\pi^2)} \quad (28)$$

where Re and $I_m$ stand respectively for the real and the imaginary parts of the above expressions.

It is to be noted here that the p-series in (27) and (28) are of orders $\beta_p^{-3}$ and $\beta_p^{-2}$ when $p \to \infty$. The n-series is also convergent since $m_1$ and $m_2$ and $(m_1 - m_2)$ are all of order $-N^2$ as $n \to \infty$.

Finally, the fluid velocity takes the form

$$\frac{q(z,t)}{E_1} = \frac{sinh\ Lz}{2sinhL} - \frac{4}{\pi^2} Re\Sigma_{p=0}^{\infty} \frac{exp(i\beta_p t)}{(2p+1)^2} \frac{sinh\ L_p z}{sinh\ L_p} + \Sigma_{n=1}^{\infty} [\ \frac{W_n'(0)}{E_1}\ \frac{exp(m_1 t) - exp(m_2 t)}{m_1 - m_2} + \frac{W_n(0)}{E_1}\ \frac{m_1\ exp(m_2 t)\ -\ m_2\ exp(m_1 t)}{m_1\ -\ m_2}\ ]\ sin\ n\pi z \quad (29)$$

which is valid for all values of the particle concentration k, the rotation E and the magnetic field M. Since $m_1$ *and* $m_2$ are negative, the steady-state fluid velocity becomes

$$\frac{q(z,t)}{E_1} = \frac{sinhLz}{2sinhL} - \frac{4}{\pi^2} Re\Sigma_{p=0}^{\infty} \frac{exp(i\beta_p t)}{(2p+1)^2} \frac{sinh\ L_p z}{sinhL_p} \quad (30)$$

where both the steady and the harmonic part contain the effect of dust particles which is not the case in absence of rotation. In absence of rotation only the harmonic part of (30) contains the effect of particles due to pulsation when the steady condition is attained.

However, if the fluid is clean $(k \to 0)$, the expression for the fluid velocity (29) reduces to

$$\frac{q(z,t)}{E_1} = \frac{sinh\ L^* z}{2sinh\ L^*} - \frac{4}{\pi^2} Re\Sigma_{p=0}^{\infty} \frac{exp(i\beta_p t)}{(2p+\ 1)^2} \frac{sinh\ L_p^* z}{sinh\ L_p^*} + \Sigma_{n=1}^{\infty} \frac{W_n^*(0)}{E_1} e^{-(N^2+2iE)t} sin\ n\pi z \quad (31)$$

where

$$L^*\ =\ \sqrt{M^2\ +\ 2iE}, \quad L_p^*\ =\ \sqrt{M^2\ +\ i(2E\ +\ \beta_p)}$$

and

$$\frac{W_n^*(0)}{E_1}\ =\ (-1)^n\ n\ \pi\ [\frac{1}{(L^*)^2\ +\ n^2\ \pi^2} - \frac{8}{\pi^2}\ Re\ \Sigma_{p=0}^{\infty} \frac{1}{(2\ p\ +\ 1)^2\ ((L_p^*)^2\ +\ n^2\ \pi^2)}].$$

It is to be noted here that when E=0 the results (29) and (31) coincides exactly with those of authors [7] and the result (31) is identical to that of authors[4].

Further, if $E_1 = 2$ and $T \to 0$, the result (29) provides the solution of hydromagnetic Couette flow of a two-phase fluid in a rotating system. In this case, the solution (29) yields

$$q(z,t) = \frac{sinh\ Lz}{sinh\ L} + \Sigma_{n=1}^{\infty} [W_n'(0) \frac{exp(m_1 t) - exp(m_2 t)}{m_1 - m_2} + W_n(0) \frac{m_1 exp(m_2 t) - m_2 exp(m_1 t)}{m_1 - m_2}] sin\ n\pi z \quad (32)$$

which, for the case of clean fluid $(k \to 0)$, takes the form

$$q(z,t)\ =\ \frac{sinh\sqrt{M^2 + 2iE}\ z}{sinh\sqrt{M^2 + 2iE}} + 2\pi \sum_{n=1}^{\infty} n(-1)^n\ \frac{e^{-(N^2\ +\ 2iE)t}}{N^2\ +\ 2iE}\ sin\ n\pi z. \quad (33)$$

The result (33), when E = 0, agrees with the limiting solution of Hayat et al.[24] and provides the classical hydrodynamic solution when E = 0 and M = 0.

Following [7] and employing the method of Laplace transform, the equivalent form of (29) can be written as

$$\frac{q(z,t)}{E_1} = \frac{sinh\ Lz}{2sinh\ L} - \frac{4}{\pi^2} Re\Sigma_{p=0}^{\infty} \frac{exp(i\beta_p\ t)}{(2p+1)^2} \frac{sinh\ L_p z}{sinh\ L_p} - \frac{2\pi}{T}\ \Sigma_{n=1}^{\infty}\ n(-1)^n\ B\ sin\ n\pi z \quad (34)$$

where $B = B_1 + B_2$,

$$B_j\ =\ \frac{exp(m_j t)}{m_j^2}\ tanh\ (\frac{m_j\ T}{2}) \times [\ 1\ +\ \frac{k\ (1 + iE\lambda)}{1\ +\ \lambda\ (m_j\ +\ 2iE)^2}]^{-1}, \quad j = 1, 2,$$

$$2\,\lambda\, m_1,\ 2\,\lambda\, m_2\ =\ -\,[\langle\, 1 + k + \lambda\,(N^2 + 4\,i\,E)\rangle \mp \{\langle\ 1\ +\ k\ +\ \lambda\ (N^2\ +\ 4\ i\ E)\rangle^2 - 4\ \lambda\ \langle\ (1\ +\ 2\ i\ E)\ (N^2\ +\ 2\ i\ E)\ +\ 2\ i\ E\ k\rangle\}^{1/2}].$$

The result (34) clearly agrees with that of author [3] when E = 0 , k = 0 and the non-oscillatory result of Mitra and Bhattacharyya [14] appears when E = 0, $E_1\ =\ 2V$ and $T\ \to\ 0$.

The particle velocity, in the general case, as obtained from (11) and (29) gives

$$r(z,t) = \frac{1 - exp[-(1+2iE\lambda)t/\lambda]}{1+2iE\lambda} q_s + Re\sum_{p=0}^{\infty} \frac{exp[i\beta_p t] - exp[-(1+2iE\lambda)t/\lambda]}{1 + 2i\lambda(2E+\beta_p)} q_{2p+1} + \sum_{n=1}^{\infty} [\frac{W_n'(0)}{m_1 - m_2} \{ \frac{exp[m_1 t] - exp[-(1+2iE\lambda)t/\lambda]}{1 + m_1\lambda + 2iE\lambda} - \frac{exp[m_2 t] - exp[-(1+2iE\lambda)t/\lambda]}{1 + m_2\lambda + 2iE\lambda} \} + \frac{W_n(0)}{m_1 - m_2} \{ \frac{m_1}{1 + m_2\lambda + 2iE\lambda} (exp[m_2 t] - exp[-(1+2iE\lambda)t/\lambda]) - \frac{m_2}{1 + m_1\lambda + 2iE\lambda} \times (exp[m_1 t] - exp[-(1+2iE\lambda)t/\lambda]) \} ] sin\ n\pi z \quad (35)$$

which in the steady-state (t $\longrightarrow \infty$) yields

$$r(z,t) = \frac{q_s}{1 + 4E^2\lambda^2} e^{-i\phi_1} +\ Re\ \sum_{p=0}^{\infty} \frac{e^{i(\beta_p - \theta_p)}}{1 + 4\ \lambda^2 (2E + \beta_p)^2}\ q_{2p+1} \quad (36)$$

$where\ tan\phi_1 = 2E\lambda,\ tan\theta_p = 2\lambda(2E + \beta_p)$ .

It follows from (30) and (36) that the particles in the steady-state are unable to attain the actual fluid velocity due to the presence of rotation and pulsation. But in the limit $T \to 0,\ E_1 = 2\ and\ E = 0$, we have $u_1 = v_1$. This shows that, in absence of pulsation and rotation,the particles attain the fluid velocity in the steady motion generated by impulsively moved plate. This result is also known from Michael and Miller's[25] analysis.

Finally, the skin-friction on the walls are given by

$$\frac{\tau_0}{E_1} = \frac{L}{2sinhL} - \frac{4}{\pi^2} Re \sum_{p=0}^{\infty} \frac{1}{(2p+1)^2} \frac{L_p}{sinhL_p} e^{i\beta_p t} + \sum_{n=1}^{\infty} n\ \pi\ [\frac{W_n'(0)}{E_1}\ \frac{e^{m_1 t}\ -\ e^{m_2 t}}{m_1\ -\ m_2} + \frac{W_n(0)}{E_1}\ \frac{m_1 e^{m_2 t}\ -\ m_2 e^{m_1 t}}{m_1\ -\ m_2}], \tag{37}$$

$$\frac{\tau_1}{E_1} = \frac{LcoshL}{2sinhL} - \frac{4}{\pi^2} Re \sum_{p=0}^{\infty} \frac{1}{(2p+1)^2} \frac{L_p coshL_p}{sinhL_p} e^{i\beta_p t} + \sum_{n=1}^{\infty} (-1)^n\ n\ \pi\ [\frac{W_n'(0)}{E_1}\ \frac{e^{m_1 t}\ - e^{m_2 t}}{m_1\ -\ m_2} + \frac{W_n(0)}{E_1}\ \frac{m_1 e^{m_2 t}\ -\ m_2 e^{m_1 t}}{m_1\ -\ m_2}]. \tag{38}$$

Many known results can also be retrieved from (37) and (38) including the classical results.

## IV. Numerical Results

The nature of pulses subjected on the upper plate produces developing (increasing) and the retarding (decreasing) flows in the fluid. To investigate the effect of various flow parameters on the fluid velocity corresponding to developing flows at t=1.25 and t=25.0 and the retarding flow at t=3.75, the exact solutions (29) is evaluated for the cases E = 0, E = 0.1 and E = 1.0 when T = 2.0 and $\lambda = 0.1$.The non-zero values of the flow parameters are chosen arbitrarily within their range of validity. The changing nature of the velocity components are incorporated in figures 2 to 21 for different values of the particle concentration k, the magnetic field M and the rotation E. It is observed from figures 2-3, that in absence of rotation (E = 0) and for fixed values of the magnetic field M, the particles decrease the fluid velocity $u_1$ when the flow is developing and increase the same when the flow is retarding. Such a result is expected because of inertia of the particles playing a vital role to resist the fluid motion. Thus a particulate fluid neither grow nor decay as fast as a clean viscous fluid.

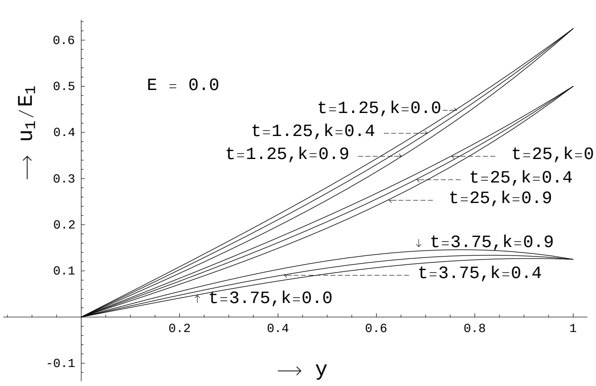


Fig. 2. Fluid velocity profiles in a non-rotating system (E=0.0) when T = 2.0, $\lambda$ = 0.1 and M = 0.0.

It is further noticed that the effect of k on $u_1$ whether decreasing or increasing reduces with the increase of the magnetic field M so that the fluid velocity becomes independent of k when M is very large. On the other hand, when E = 0 and k is fixed, the magnetic field M produces a damping effect on the flow whether it is increasing or decreasing. This effect also reduces with the increase of k and is illustrated in figures 4-5. The above observations are true even when the steady-state is reached where the effect of k persists only in presence of pulsation.

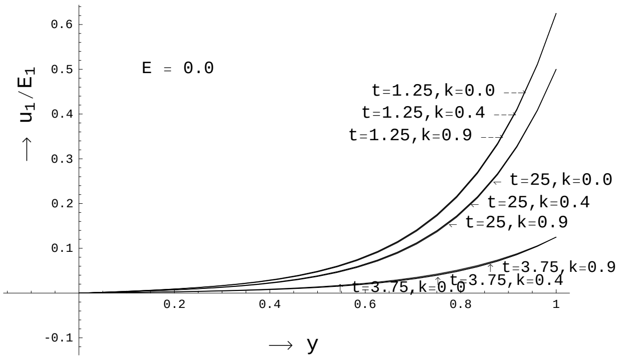


Fig. 3. Fluid velocity profiles in a non-rotating system (E=0.0) when T = 2.0, $\lambda$ = 0.1 and M = 5.0.

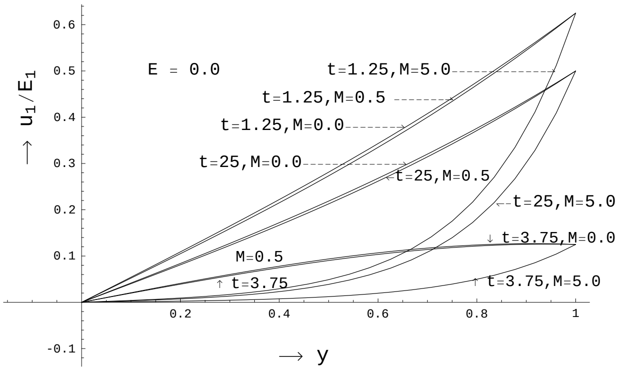


Fig. 4. Fluid velocity profiles in a non-rotating system (E=0.0) when T = 2.0, $\lambda$ = 0.1 and k = 0.0.

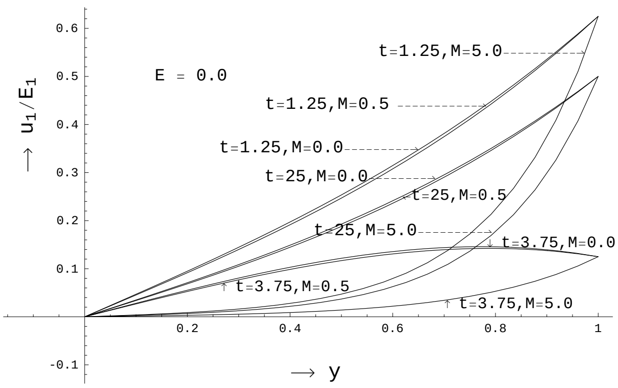


Fig. 5. Fluid velocity profiles in a non-rotating system (E=0.0) when T = 2.0, $\lambda$ = 0.1 and k = 0.9.

However, in presence of rotation, the velocity component $u_1$ varies in a manner similar to that of non-rotating case excepting a significant diminution in its magnitude with the increase of rotation when the flow is developing. For retarding flows, the velocity component $u_1$ increases with rotation E and the particles k but decreases with the magn-

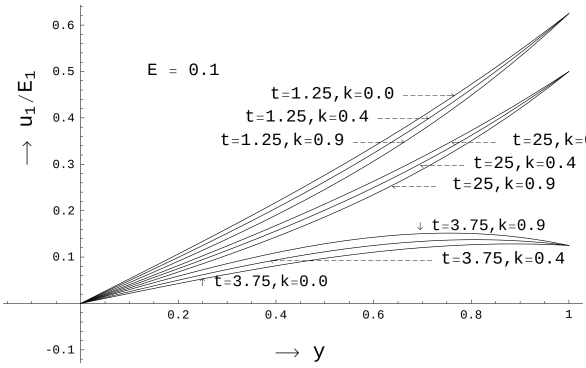


Fig. 6. Distribution of the fluid velocity component $(u_1/E_1)$ in a rotating system (E=0.1) when T = 2.0, $\lambda$ = 0.1 and M = 0.0.

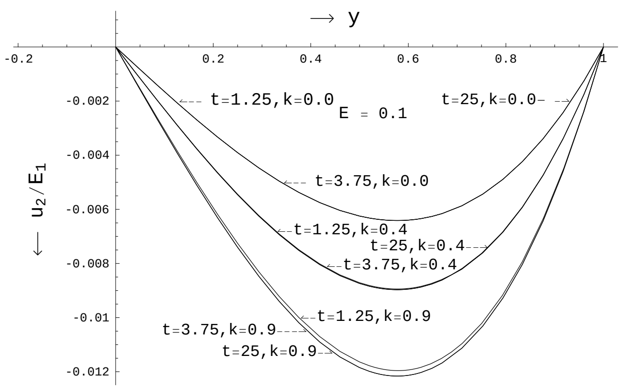


Fig. 7. Distribution of the fluid velocity component $(u_2/E_1)$ in a rotating system (E=0.1) when T = 2.0, $\lambda$ = 0.1 and M = 0.0.

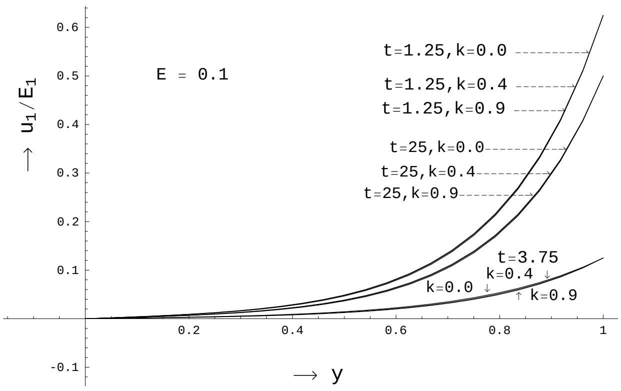


Fig. 8. Distribution of the fluid velocity component $(u_1/E_1)$ in a rotating system (E=0.1) when T = 2.0, $\lambda$ = 0.1 and M = 5.0.

etic field M. In this situation, the increasing effect of k on $u_1$ reduces with the increase of M when E is small and enhances when E is large. Additionally, the damping effect of M on $u_1$ enhances with the increase of k when E is small and reduces when E is large. The observations made above on $u_1$ in presence of rotation, both for the developing and retarding motions, are shown in the figures 6, 8, 10, 12, 14, 16, 18, 20.

It is noticed that the lateral component of fluid velocity $u_2$ appears in a contained fluid only in presence of rotation.

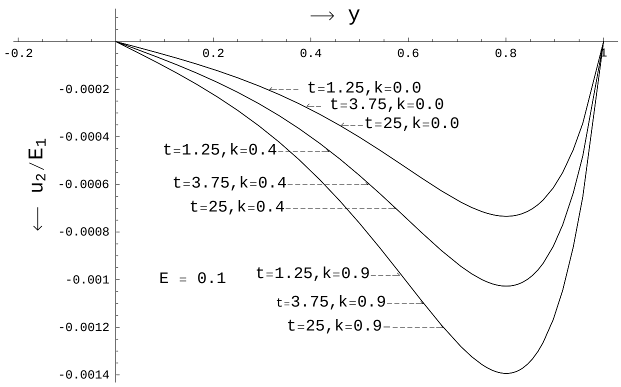


Fig. 9. Distribution of the fluid velocity component $(u_2/E_1)$ in a rotating system (E=0.1) when T = 2.0, $\lambda$ = 0.1 and M = 5.0.

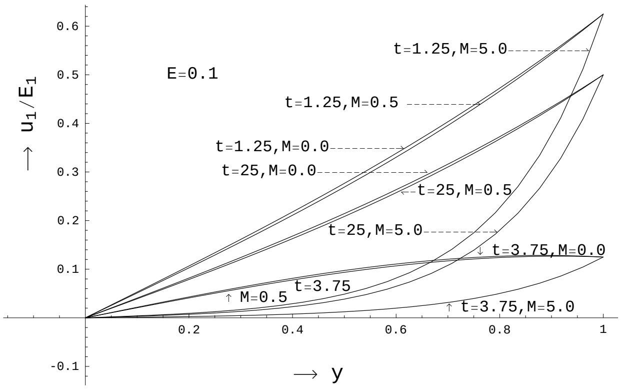


Fig. 10. Distribution of the fluid velocity component $(u_1/E_1)$ in a rotating system (E=0.1) when T = 2.0, $\lambda$ = 0.1 and k = 0.0.

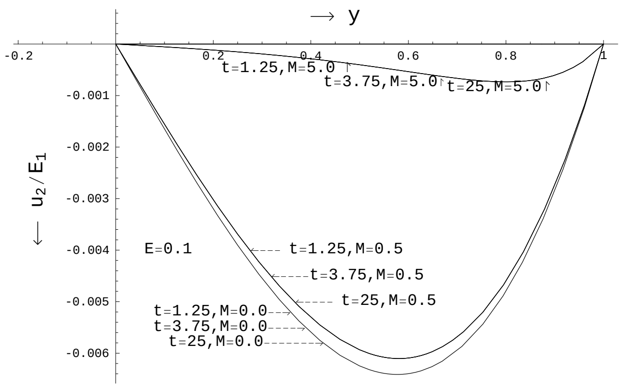


Fig. 11. Distribution of the fluid velocity component $(u_2/E_1)$ in a rotating system (E=0.1) when T = 2.0, $\lambda$ = 0.1 and k = 0.0.

In the developing flow,the magnitude of $u_2$ increases with rotation E and the particles k but decreases with the magnetic field M. Moreover, in this case, for all values of E, the increasing effect of k on $u_2$ enhances with the increase of M and the diminishing effect of M on it reduces with the increase of k. On the other hand, in the retarding motion of the fluid, the magnitude of $u_2$ also increases with E and k but decreases with M. In this situation, the increasing effect of k on the magnitude of $u_2$ enhances with M and the damping effect of M on it reduces with k for all values of rotation.

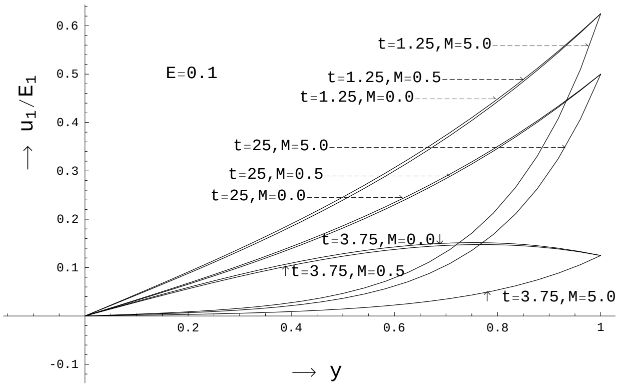


Fig. 12. Distribution of the fluid velocity component $(u_1/E_1)$ in a rotating system (E=0.1) when T = 2.0, $\lambda$ = 0.1 and k = 0.9.

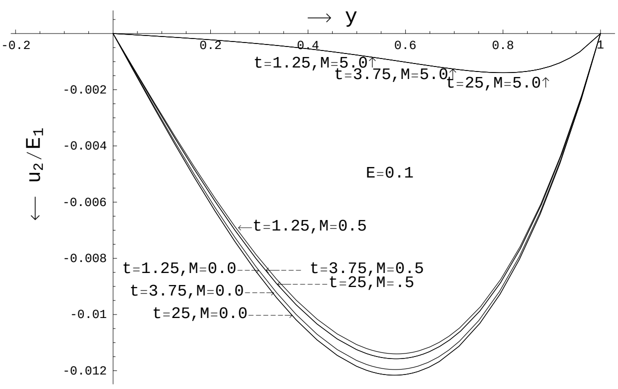


Fig. 13. Distribution of the fluid velocity component $(u_2/E_1)$ in a rotating system (E=0.1) when T = 2.0, $\lambda$ = 0.1 and k = 0.9.

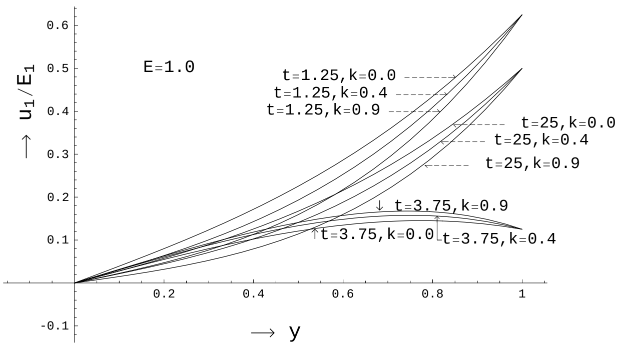


Fig. 14. Distribution of the fluid velocity component $(u_1/E_1)$ in a rotating system (E=1.0) when T = 2.0, $\lambda$ = 0.1 and M = 0.0.

The variation of the fluid velocity component $u_2$ for different values of the flow parameters and time is illustrated graphically in figures 7, 9, 11, 13, 15, 17, 19, 21.

Further enquiry shows that (i) for all values of E,k and M the magnitude of $u_2$ rises sharply near the lower plate, goes to a maximum, then decreases continuously until it becomes zero at the upper plate, (ii) for all E and k, the maximum of $|u_2|$ is shifting more and more towards the upper plate with the increase of M, (iii) for small values of E, the effects of k and M on $u_2$ becomes independent of time t excepting when k large and M small, (iv) for large values of E the effects of

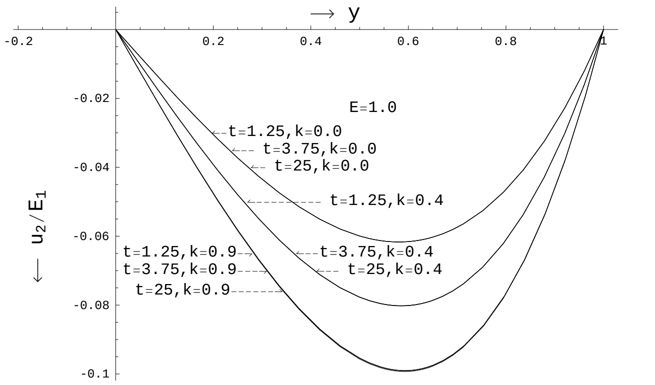


Fig. 15. Distribution of the fluid velocity component $(u_2/E_1)$ in a rotating system (E=1.0) when T = 2.0, $\lambda$ = 0.1 and M = 0.0.

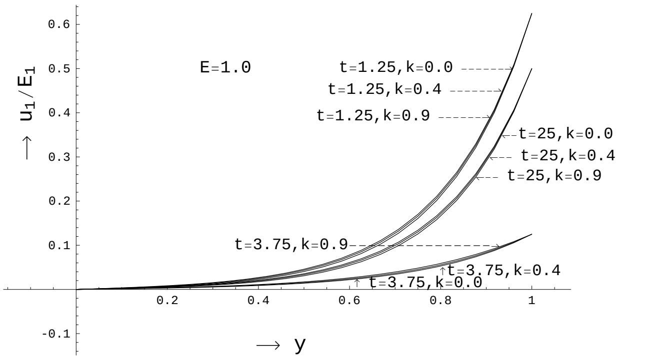


Fig. 16. Distribution of the fluid velocity component $(u_1/E_1)$ in a rotating system (E=1.0) when T = 2.0, $\lambda$ = 0.1 and M = 5.0.

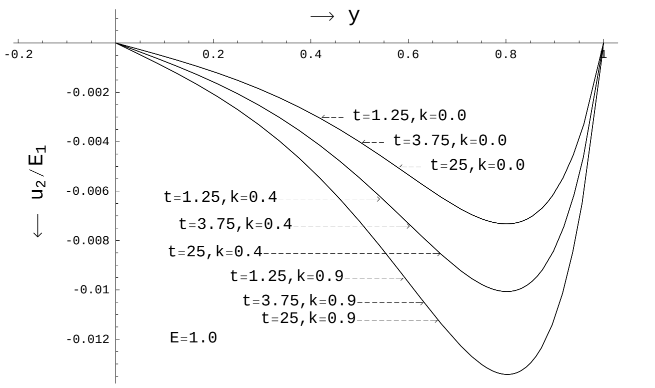


Fig. 17. Distribution of the fluid velocity component $(u_2/E_1)$ in a rotating system (E=1.0) when T = 2.0, $\lambda$ = 0.1 and M = 5.0.

k and M on $u_2$ remain always independent of time t. To investigate the effects of various flow parameters on the components of skin-friction on the plates, the results (37) and (38) are evaluated numerically for the cases E = 0.1 and E = 1.0 when T = 2.0 and $\lambda = 0.1$. These are presented in figures 22, 23 and 24, 25. It is observed that the longitudinal component of skin-friction on both the plates fluctuate in a manner similar to that of pulses imparted in the fluid. On the lower plate, the increase of rotation E and the particles k decrease the magnitude of the longitudinal component of the skin-friction for small values of the magnetic field M when the flow is developing while a reverse effect is found when the flow is retarding. However, for large values of M, the magnitude of the longitudinal component of the skin-friction on the lower plate diminishes greatly and becomes independent of E and k irrespective of the nature of the flow. The behaviour of the longitudinal component of the skin-friction on the upper plate is exactly opposite to that observed at the lower plate. The magnitude of the lateral component of skin-friction on both the walls increase at small rapidly values of time until they maintain a constant value afterwards. It is also significant to notice that, for all values of the flow parameters, the lateral component of skin-friction becomes negative at the lower plate and positive at the upper plate. Such a result is expected in a flow situation described in the present problem.

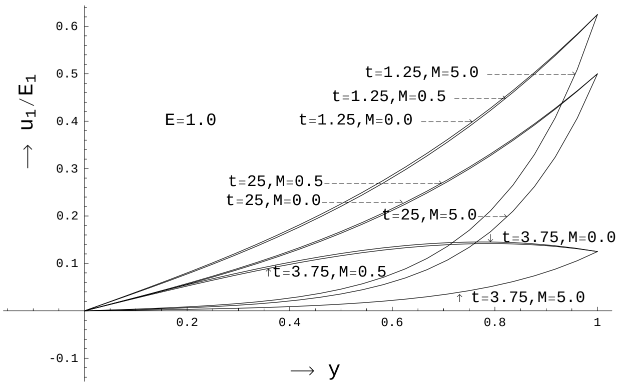


Fig. 18. Distribution of the fluid velocity component $(u_1/E_1)$ in a rotating system (E=1.0) when T = 2.0, $\lambda$ = 0.1 and k = 0.0.

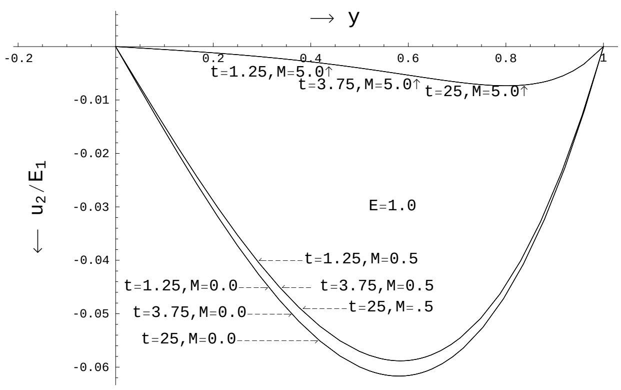


Fig. 19. Distribution of the fluid velocity component $(u_2/E_1)$ in a rotating system (E=1.0) when T = 2.0, $\lambda$ = 0.1 and k = 0.0.

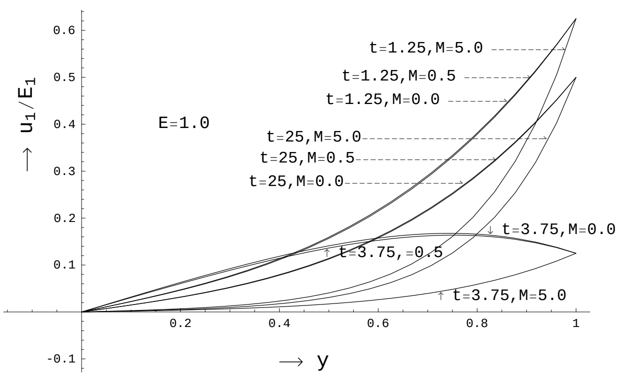


Fig. 20. Distribution of the fluid velocity component $(u_1/E_1)$ in a rotating system (E=1.0) when T = 2.0, $\lambda$ = 0.1 and k = 0.9.

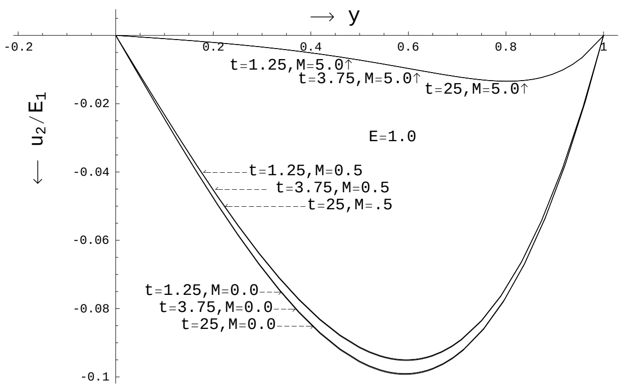


Fig. 21. Distribution of the fluid velocity component $(u_2/E_1)$ in a rotating system (E=1.0) when T = 2.0, $\lambda$ = 0.1 and k = 0.9.

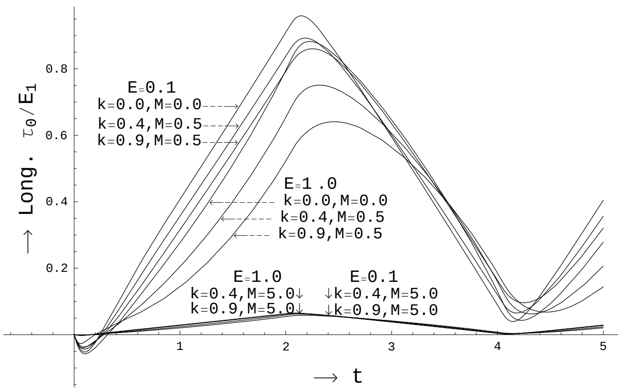


Fig. 22. Distribution of longitudinal component of skin-friction on z = 0 for different values of k, M and E when T = 2.0, $\lambda$ = 0.1

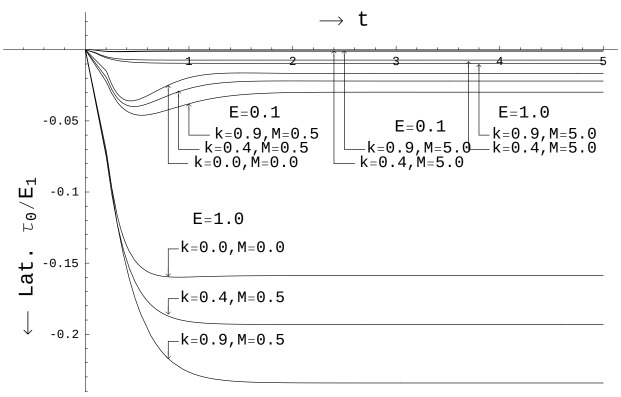


Fig. 23. Distribution of lateral component of skin-friction on z = 0 for different values of k, M and E when T = 2.0, $\lambda$ = 0.1

## V. Conclusion

An exact solution of the problem concerning the motion of a conducting viscous fluid with embedded small inert spherical particles in a channel bounded by two infinite rigid non-conducting parallel plates is found. Both the particulate fluid and the plates are in a state of solid body rotation in presence of an uniform magnetic field. Additionally, an unsteady motion is generated in such a fluid when the upper plate is subjected to velocity tooth pulses and the lower plate is held fixed.

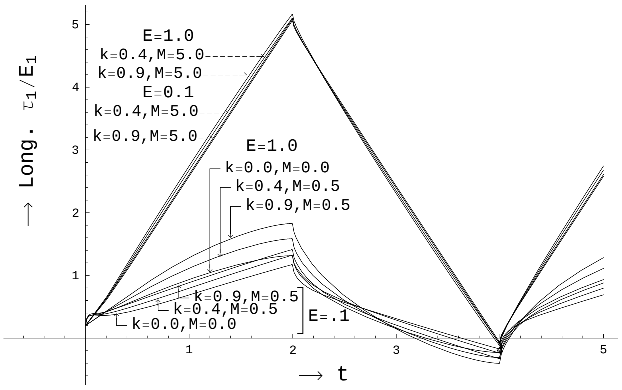


Fig. 24. Distribution of longitudinal component of skin-friction on $z = 1$ for different values of k, M and E when T = 2.0, $\lambda = 0.1$

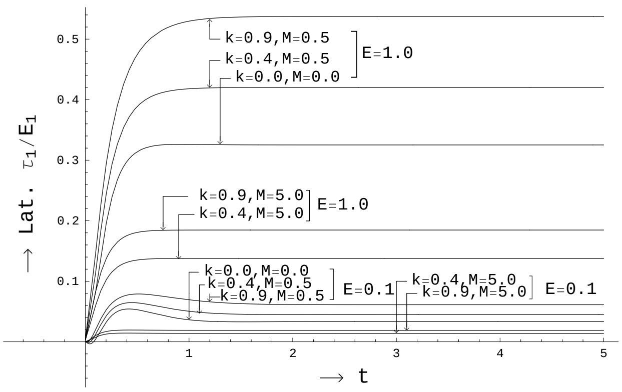


Fig. 25. Distribution of lateral component of skin-friction on $z = 1$ for different values of k, M and E when T = 2.0, $\lambda = 0.1$

The problem, although idealized, bears a good resemblance with the fluid motion contained between two layers of the Earth particularly at the time of earthquake when one of the layers executes periodic motion in the form of tooth pulses. In this situation, the flow field, as evidenced by equations (29) and (34), consists of three distinct parts. Namely, the steady, harmonic and the transient parts. It is found that, at large time, the first two parts represent the steady-state solution for the velocity field, while the third part confirms the existence of the inertial oscillations of frequency $2\,\Omega$ which decay exponentially within the ultimate flow due to the presence of the magnetic field. In other words, the steady state is established in the fluid through inertial oscillations of frequency $2\,\Omega$. Such a phenomenon does not appear in a non-rotating fluid. Finally, we find that the rotation plays an important role to increase the spin-up motion in a contained fluid and modifying the influence of other parameters on the flow and the wall frictions. The present study is more general compared to investigations made by earlier researchers and provides analytical solutions useful for describing many physical flow situations taking place in a rotating system.

## VI. Summary

A closed form solution of the problem of hydromagnetic channel flow of a rotating two-Phase fluid induced by tooth pulses has been obtained by the method of Fourier analysis. The influence of the particles, the magnetic field and the rotation on the components of the fluid velocity and the wall frictions are examined quantitatively.

## Acknowledgment

Authors express their grateful thanks to the referee for suggesting several improvements.

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