Flow of Couple Stress Fluid Between Two Parallel Porous Plates

D. Srinivasacharya, N. Srinivasacharyulu, and O. Odelu

***Abstract*—The steady flow of incompressible couple stress fluid flow between parallel porous plates maintained at constant but different temperatures with the assumption that there is a constant suction at upper plate and a constant injection at the lower plate is studied. The governing non-linear ordinary differential equations are solved numerically using quasi-linearization technique. The effects of couple stress fluid parameter, suction Reynolds number and suction-injection ratio on the velocity and temperature are discussed.**

***Index Terms*—parallel plates, suction, injection, couple stress fluid, temperature.**

I. Introduction

The flow through porous boundaries is of great importance both in technological as well as biophysical flows. Examples of this are found in soil mechanics, transpiration cooling, food preservation, cosmetic industry, blood flow and artificial dialysis. A large number of theoretical investigations dealing with steady incompressible laminar flow with either injection or suction at the boundaries have appeared during the last few decades. Several authors ([1] -[4]) have studied the steady laminar flow of an incompressible viscous fluid in a two-dimensional channel with parallel porous walls.

It is known that many of the industrially and technologically important fluids behave like a non-Newtonian fluid. The importance of fluid flow and heat transfer between parallel plates is well known due to the occurrence of such flows in a wide host of industrial applications like the thermal design of industrial equipment dealing with molten plastics, polymeric liquids, foodstuffs, or slurries. Several investigators have extended many of the available heat transfer problems to include the non Newtonian effects. Ariel [5] provided an exact solution for the flow problems of a second grade fluid through two parallel porous plates in two-dimensional and axially symmetric cases. Kamsili [6] solved the problems of power-law fluid flow between parallel porous plates, when the upper plate is stationary and the lower plate is subjected to sudden acceleration. He obtained analytical solution for a Newtonian fluid case and numerical solution for various values of power-law index. The laminar flow of a second grade viscoelastic fluid between two parallel plates, one of which is externally heated and cooled by coolant injection through the other plate, is considered by Kurtcebe and Erim [7]. A two-phase fluid-particle model based on the continuum approach was formulated by Chamkha [8] for the problem of flow and heat transfer in a porous channel with uniform suction and injection applied at the lower and upper plates of the channel, respectively. Sran [9] investigated the Couette and Poiseuille flows of dipolar fluid between parallel plates maintained at constant but different temperatures in addition to being subjected to uniform injection and suction. Yong-Li Chen and Ke-Qin Zhua [10] have derived Analytical solutions of Couette-Poiseuille flow of Bingham fluids between two porous parallel plates. Srinivasacharya *et al* [11] have considered an incompressible laminar flow of a couple stress fluid in a porous channel with expanding or contracting walls. Ansari *et al* [12] have obtained analytical as well numerical solutions for the problem of flow through a porous channel where the flow entry profiles are taken to be Poiseuille Couette combinations using Optimal Homotopy Asymptotic Method (OHAM). Sanchita Ghosh *et al* [13] solved an initial value problem for the motion of an incompressible conducting viscous fluid with embedded small inert spherical particles in a channel bounded by two infinite rigid non-conducting plates.

The couple stress fluid theory developed by Stokes [14] represents the simplest generalization of the classical viscous fluid theory that sustains couple stresses and the body couples. The important feature of these fluids is that the stress tensor is not symmetric and their accurate flow behavior cannot be predicted by the classical Newtonian theory. The fluids consisting of rigid, randomly oriented particles suspended in a viscous medium, such as blood, lubricants containing small amount of polymer additive, electro-rheological fluids and synthetic fluids are examples of these fluids.

The flow of a couple stress fluid between two parallel horizontal stationary plates due to fluid injection through the lower porous plate is considered by Kabadi [15]. The aim of this paper is to consider the flow and heat transfer of an incompressible couple stress fluid between two parallel porous plates assuming constant suction at the upper plate and constant injection at the lower plate. Quasilinearization technique is used to solve the governing nonlinear equations.

II. Formulation of The Problem

Consider a fully developed steady flow of an incompressible couple stress fluid flowing between two parallel porous flat plates distance h apart. Let x and y axes be chosen along and perpendicular to the walls respectively. Assume that there is a suction of velocity V_0 at the upper plate and an injection of velocity V_1 at the lower plate. Without loss of generality, it is assumed that $|V_1| \geq |V_0|$. Let the lower plate is maintained at constant temperature T_1 while the upper plate at constant temperature T_2.

The governing equations of the flow of an incompressible couple stress fluid in the absence of body force and body

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D. Srinivasacharya is with the Department of Mathematics, National Institute of Technology, Warangal – 506004, A.P., India (corresponding author Tel No : +91-870-2462821; Fax No : +91-870-2459547; e-mail: dsc@nitw.ac.in, dsrinivasacharya@yahoo.com).

N. Srinivasacharyulu, and O. Odelu are with the Department of Mathematics, National Institute of Technology, Warangal – 506004, A.P., India

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couple and the energy equations are [10]

$$\nabla \cdot \bar{q} = 0, \tag{1}$$

$$\rho\left[\frac{\partial \bar{q}}{\partial t} + (\bar{q}\cdot\nabla)\bar{q}\right] = -\nabla P - \mu\nabla\times\nabla\times\bar{q} - \eta\nabla\times\nabla\times\nabla\times\nabla\times\bar{q}, \tag{2}$$

$$\rho c\left[\frac{\partial T}{\partial t} + (\bar{q}\cdot\nabla)T\right] = \mu[(\nabla\bar{q}):(\nabla\bar{q})^T + (\nabla\bar{q}):(\nabla\bar{q})] \tag{3}$$
$$+4\eta[(\nabla\bar{\omega}):(\nabla\bar{\omega})^T] + 4\eta'[(\nabla\bar{\omega}):(\nabla\bar{\omega})] + k\nabla^2 T,$$

where ρ is the density, $\bar{q}$ is the velocity vector, P is the fluid pressure, μ is the fluid viscosity, η and η' are the couple stress fluid parameters, k is the thermal conductivity, c is the specific heat capacity at constant temperature, $\bar{\omega}$ is the rotation vector and T is the temperature.

The force stress tensor τ and the couple stress tensor M that arises in the theory of couple stress fluids are given by

$$\tau = (-p + \lambda_1\nabla\cdot\bar{q})I + \mu[\nabla\bar{q} + \nabla\bar{q}^T] + \frac{1}{2}I\times[\nabla\cdot M + \rho C], \tag{4}$$

and

$$M = mI + 2\eta\nabla(\nabla\times\bar{q}) + 2\eta'(\nabla(\nabla\times\bar{q}))^T, \tag{5}$$

where m is $(\frac{1}{3})^{rd}$ trace of M and ρC is the body couple tensor. The quantity λ_1 is the material constant and η' is the constant associated with couple stresses. The dimensions of the material constant λ_1 is that of viscosity where as the dimensions of η and η' are those of momentum. These material constants satisfy the inequalities,

$$\mu \geq 0, 3\lambda + 2\mu \geq 0, \eta \geq 0, \eta' \leq \eta. \tag{6}$$

Since the flow is along x– direction, the flow variables are assumed to be independent of the co ordinate z. We choose the velocity vector as $\bar{q} = u(x,y)\hat{i} + v(x,y)\hat{j}$ and temperature function as $T(x,y)$. Thus the governing equations (1 = 3) reduce to

$$\frac{\partial u}{\partial x} + \frac{1}{h}\frac{\partial v}{\partial \lambda} = 0, \tag{7}$$

$$\rho\left[u\frac{\partial u}{\partial x} + \frac{v}{h}\frac{\partial u}{\partial \lambda}\right] = -\frac{\partial p}{\partial x} + \mu\nabla_1^2 u - \eta\nabla_1^4 u, \tag{8}$$

$$\rho\left[u\frac{\partial v}{\partial x} + \frac{v}{h}\frac{\partial v}{\partial \lambda}\right] = -\frac{1}{h}\frac{\partial p}{\partial \lambda} + \mu\nabla_1^2 v - \eta\nabla_1^4 v, \tag{9}$$

$$\rho c\left[u\frac{\partial T}{\partial x} + \frac{v}{h}\frac{\partial T}{\partial \lambda}\right] = \mu\left[2\left(\frac{\partial u}{\partial x}\right)^2 + \frac{2}{h^2}\left(\frac{\partial v}{\partial \lambda}\right)^2\right. \tag{10}$$
$$\left.+\left(\frac{1}{h}\frac{\partial u}{\partial \lambda} + \frac{\partial v}{\partial x}\right)^2\right] + \eta\left[\left(\nabla_1^2 v\right)^2 + \left(\nabla_1^2 u\right)^2\right] + k\nabla_1^2 T,$$

where $\lambda = \dfrac{y}{h}$ and $\nabla_1^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{1}{h^2}\dfrac{\partial^2}{\partial \lambda^2}$.

The boundary conditions on the velocity profile and temperature are

$$\left.\begin{array}{ll} u(x,\lambda) = 0, & v(x,\lambda) = V_0, \\ \nabla\times\bar{q} = 0, & T(x,\lambda) = T_1. \end{array}\right\} \text{at } \lambda = 0, \tag{11}$$
$$\left.\begin{array}{ll} u(x,\lambda) = 0, & v(x,\lambda) = V_1, \\ \nabla\times\bar{q} = 0, & T(x,\lambda) = T_2. \end{array}\right\} \text{at } \lambda = 1.$$

Following Terril and Shreshta [4], we take velocity components as

$$u(x,\lambda) = \left(\frac{U_0}{a} - \frac{V_1 x}{h}\right)f'(\lambda) \quad \text{and} \quad v(x,\lambda) = V_1 f(\lambda), \tag{12}$$

where U_0 is the entrance velocity, $a = 1 - \dfrac{V_0}{V_1}$ and $f(\lambda)$ is a functions of λ to be determined.

Eliminating pressure from Eqs. 8 and 9 and substituting Eq.12 in the resulting equation we get the following non dimensional equation.

$$s[ff''' - f'f''] - f^{iv} + \alpha^2 f^{vi} = 0, \tag{13}$$

where prime denotes differentiation with respect to λ, $s = \dfrac{\rho h v_1}{\mu}$ and $\alpha^2 = \dfrac{\eta}{\mu h^2}$

Eq. 10 together with Eq. 11, suggests that the form of temperature may be taken as

$$T(x,\lambda) = T_1 + \frac{\mu V_1}{\rho h c}\left[\phi_1(\lambda) + \left(\frac{U}{aV_1} - \frac{x}{h}\right)^2\phi_2(\lambda)\right]. \tag{14}$$

Using Eq. 14 in Eq. 10, and equating the coefficients of $\left(\dfrac{U_0}{a} - \dfrac{V_1 x}{h}\right)^2$ and the terms without $\left(\dfrac{U_0}{a} - \dfrac{V_1 x}{h}\right)$ on both sides of the equation thus obtained, we get

$$\phi_2'' + SPr[\alpha^2(f''')^2 + (f'')^2 + 2f'\phi_2 - f\phi_2'] = 0, \tag{15}$$

$$\phi_1'' + 2\phi_2 + SPr[\alpha^2(f'')^2 + 4(f')^2 - f\phi_1'] = 0, \tag{16}$$

where $Pr = \dfrac{\mu c}{k}$ is the Prandtl number.

The dimensionless form of temperature from Eq. 14 can be written as

$$T = \frac{T - T_1}{T_2 - T_1} = E(\phi_1 + \zeta^2\phi_2), \tag{17}$$

where $E = \dfrac{\mu V_1}{\rho c(T_2 - T_1)}$ is the Eckert number and $\zeta = \left(\dfrac{U_0}{a} - \dfrac{V_1 x}{h}\right)$ is the dimensionless axial variable.

The boundary conditions Eq. 11 in terms of f, ϕ_1 and ϕ_2 are

$$\left.\begin{array}{ll} f(0) = 1 - a, & f(1) = 1, \\ f'(0) = 0, & f'(1) = 0, \\ f''(0) = 0, & f''(1) = 0, \\ \phi_1(0) = 0, & \phi_1(1) = 0, \\ \phi_2(0) = 0, & \phi_2(1) = 1/E = w(\text{say}). \end{array}\right\} \tag{18}$$

III. Solution of The Problem

The nonlinear equations 13, 15 and 16 are converted into the following system of first order differential equations by the substitution (f, f', f'', f''', f^{iv}, f^v, f^{vi} ,ϕ_1, ϕ_1', ϕ_2 , ϕ_2') = (x_1 , x_2, x_3, x_4, x_5, x_6 , x_7 , x_8 , x_9 , x_{10})

$$
\begin{aligned}
\frac{dx_1}{d\lambda} &= x_2,\\
\frac{dx_2}{d\lambda} &= x_3,\\
\frac{dx_3}{d\lambda} &= x_4,\\
\frac{dx_4}{d\lambda} &= x_5,\\
\frac{dx_5}{d\lambda} &= x_6,\\
\frac{dx_6}{d\lambda} &= \frac{S}{\alpha^2}\left\{-x_4x_1 + x_3x_2 + x_2x_3 - x_1x_4\right\} \qquad (19)\\
&+ \frac{1}{\alpha^2}x_5 + \frac{s}{\alpha^2}\left\{-x_2x_3 + x_1x_4\right\},\\
\frac{dx_7}{d\lambda} &= x_8,\\
\frac{dx_8}{d\lambda} &= -2x_9 - SPr(\alpha^2x_3^2 + 4x_2^2 - x_1x_8),\\
\frac{dx_9}{d\lambda} &= x_{10},\\
\frac{dx_{10}}{d\lambda} &= -SPr(\alpha^2x_4^2 + x_3^2 + 2x_2x_9 - x_1x_{10}).
\end{aligned}
$$

The boundary conditions in terms of x_1 , x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} are

$$
\left.\begin{array}{ll}
x_1(0) = 1 - a, & x_2(0) = 0,\\
x_3(0) = 0, & x_7(0) = 0,\\
x_9(0) = 0, & x_1(1) = 1,\\
x_2(1) = 0, & x_3(1) = 0,\\
x_7(1) = 0, & x_9(1) = w.
\end{array}\right\} \qquad (20)
$$

The system of equations 19 is solved numerically subject to the boundary conditions 20 using quasilinearization method (also known as generalized Newton's method) given by Bellman and Kalaba [16]

Let $(x_i^{(k)}, i = 1, 2, ...10)$ be an approximate current solution and $(x_i^{(k+1)}, i = 1, 2, ...10$ be an improved solution of set of equations in 19. By taking Taylor's series expansion around the current solution and neglecting the second and higher order derivative terms, the coupled first order system Eq. 19 is linearized as:

$$
\begin{aligned}
\frac{dx_1^{(k+1)}}{d\lambda} &= x_2^{(k+1)},\\
\frac{dx_2^{(k+1)}}{d\lambda} &= x_3^{(k+1)},\\
\frac{dx_3^{(k+1)}}{d\lambda} &= x_4^{(k+1)},\\
\frac{dx_4^{(k+1)}}{d\lambda} &= x_5^{(k+1)},\\
\frac{dx_5^{(k+1)}}{d\lambda} &= x_6^{(k+1)},\\
\frac{dx_6^{(k+1)}}{d\lambda} &= \frac{S}{\alpha^2}\left\{-x_4^{(k)}x_1^{(k+1)} + x_3^{(k)}x_2^{(k+1)} + x_2^{(k)}x_3^{(k+1)}\right.\\
&- \left.x_1^{(k)}x_4^{(k+1)}\right\} + \frac{1}{\alpha^2}x_5^{(k+1)} + \frac{s}{\alpha^2}\left\{-x_2^{(k)}x_3^{(k)}\right.\\
&+ \left.x_1^{(k)}x_4^{(k)}\right\},\\
\frac{dx_7^{(k+1)}}{d\lambda} &= x_8^{(k+1)},
\end{aligned}
$$

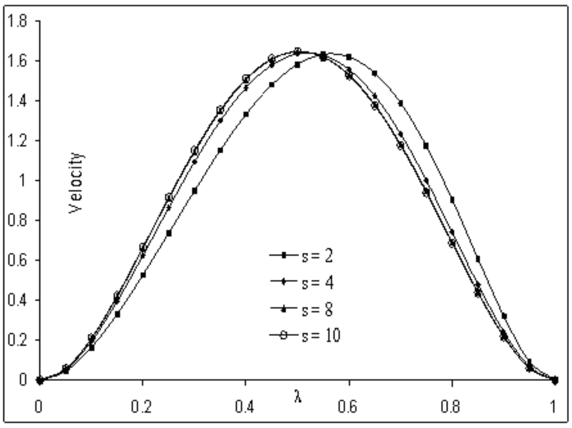

Fig. 1. Effect of Suction Reynolds number (S) on axial velocity (u) for $Pr = 0.5$, $a = 0.2$, $\alpha = 0.5$, $w = 1.0$.

$$
\begin{aligned}
\frac{dx_8^{(k+1)}}{d\lambda} &= SPr\left(x_1^{(k)}x_8^{(k+1)} + x_8^{(k)}x_1^{(k+1)}\right. \qquad (21)\\
&- \left.2\alpha^2x_3^{(k)}x_3^{(k+1)} - 8x_2^{(k)}x_2^{(k+1)}\right) - 2x_9^{(k+1)}\\
&- SPr\left(x_1^{(k)}x_8^{(k)} - \alpha^2x_3^{(k)}x_3^{(k)} - 4x_2^{(k)}x_2^{(k)}\right),\\
\frac{dx_9^{(k+1)}}{d\lambda} &= x_{10}^{(k+1)},\\
\frac{dx_{10}^{(k+1)}}{d\lambda} &= -SPr\left(2\alpha^2x_4^{(k)}x_4^{(k+1)} + 2x_3^{(k)}x_3^{(k+1)}\right.\\
&+ 2x_2^{(k)}x_9^{(k+1)} - x_1^{(k)}x_{10}^{(k+1)} + 2x_9^{(k)}x_2^{(k+1)}\\
&- \left.x_{10}^{(k)}x_1^{(k+1)}\right) + SPr\left(\alpha^2x_4^{(k)}x_4^{(k)} + x_3^{(k)}x_3^{(k)}\right.\\
&+ \left.2x_2^{(k)}x_9^{(k)} - x_1^{(k)}x_{10}^{(k)}\right).
\end{aligned}
$$

To solve for $(x_i^{(k+1)}, i = 1, 2, ...10)$, the solution to five separate initial value problems, denoted by $x_i^{h1}(\lambda), x_i^{h2}(\lambda), x_i^{h3}(\lambda), x_i^{h4}(\lambda), x_i^{h5}(\lambda)$ (which are the solutions of the homogeneous system corresponding to (21)) and $x_i^p(\lambda)$ (which is the particular solution of (21)), with the following initial conditions are obtained by using a Runge-Kutta method.

$$
\left.\begin{array}{l}
x_4^{h1}(0) = 1, \quad x_i^{h1}(0) = 0 \quad \text{for} \quad i \neq 4,\\
x_5^{h2}(0) = 1, \quad x_i^{h2}(0) = 0 \quad \text{for} \quad i \neq 5,\\
x_6^{h3}(0) = 1, \quad x_i^{h3}(0) = 0 \quad \text{for} \quad i \neq 6,\\
x_8^{h4}(0) = 1, \quad x_i^{h4}(0) = 0 \quad \text{for} \quad i \neq 8,\\
x_{10}^{h1}(0) = 1, \quad x_i^{h5}(0) = 0 \quad \text{for} \quad i \neq 10,\\
x_1^{p1}(0) = 1 - a, \quad x_9^{p1}(1) = w,\\
x_2^{p1}(0) = x_3^{p1}(0) = x_4^{p1}(0) = x_5^{p1}(0)\\
= x_6^{p1}(0) = x_7^{p1}(0) = x_8^{p1}(0) = x_{10}^{p1}(0) = 0.
\end{array}\right\} \qquad (22)
$$

Since the differential equations are linear, the principle of superposition holds and the general solution may be written as,

$$
\begin{aligned}
x_i^{(k+1)}(\lambda) &= C_1x_i^{h1}(\lambda) + C_2x_i^{h2}(\lambda) + C_3x_i^{h3}(\lambda) \qquad (23)\\
&+ C_4x_i^{h4}(\lambda) + C_5x_i^{h5}(\lambda) + x_i^p(\lambda),
\end{aligned}
$$

where C_1, C_2, C_3, C_4 and C_5 are the unknown constants and are determine by considering the boundary condition at $\lambda = 1$. This solution $(x_i^{(k+1)}, i = 1, 2, ...10)$ is then compared with solution at the previous step $(x_i^{(k)}, i = 1, 2, ...10)$ and

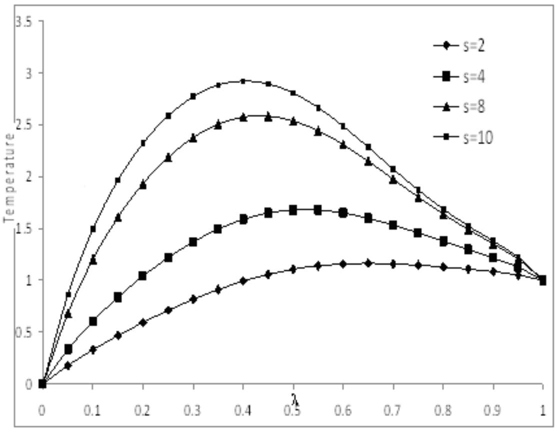

Fig. 2. Effect of Suction Reynolds number(S) on Temperature (T) for $Pr = 0.5$, $a = 0.2$, $\alpha = 0.5$, $w = 1.0$.

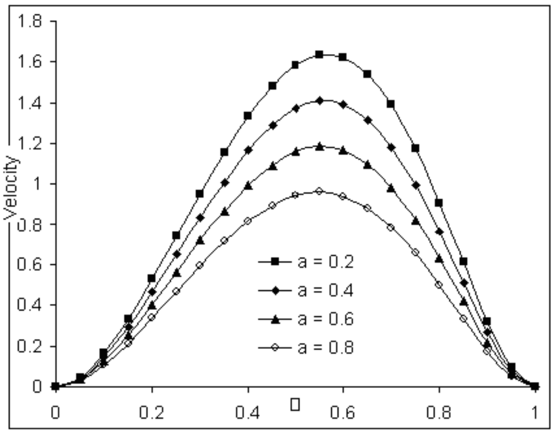

Fig. 3. Effect of a on axial velocity for $Pr = 0.5, S = 10, \alpha = 0.2, w = 1.0$.

further iteration is performed if the convergence has not been achieved or greater accuracy is desired.

IV. Results and Discussion

The velocity component u and the temperature distribution $T$are calculated correct to six places of decimal for various values of S, a, and α.

Fig.1. shows the variation velocity component u with λ for different values of the suction Reynolds number S for $a = 0.2$ and $\alpha = 0.5$. It can be observed from Fig. 1 that as S (i.e. the suction at the upper plate $\lambda = 1$) increases, the axial velocity of the fluid near to the lower plate $\lambda = 0$ decreases and the maximum of the velocity shifts toward the plate $\lambda = 1$. The effect of suction Reynolds number S on the temperature distribution is shown in Fig. 2. The temperature is increasing as the values of suction Reynolds number increases.

Fig. 3 analyzes the effect of a on axial velocity u for the values of $S = 10$ and $\alpha = 0.5$. The velocity u is decreasing as the injection suction ratio increases (as a decreases). Fig.4 depicts the variation of temperature distribution for different values of a. The temperature is increasing as the value of suction injection ratio decreases.

Fig. 5 represents the effect of α on axial velocity (u) for the values of $S = 10$ and $a = 0.2$. It can be observed

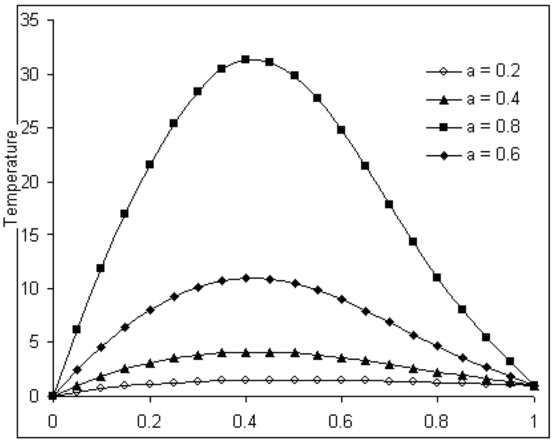

Fig. 4. Effect of a on Temperature (T) for $Pr = 0.5, S = 10, \alpha = 0.2, w = 1.0$.

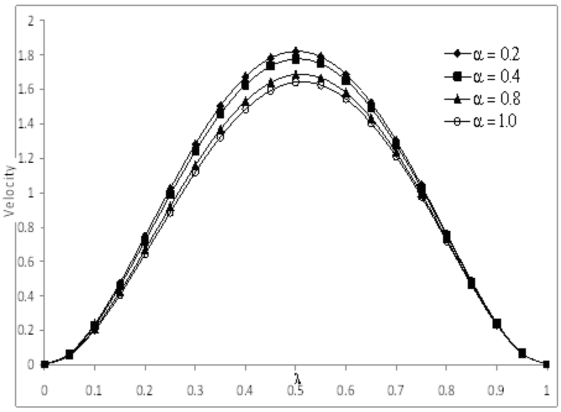

Fig. 5. Effect of α on axial velocity for $Pr = 0.5, S = 10, a = 0.2, w = 1.0$.

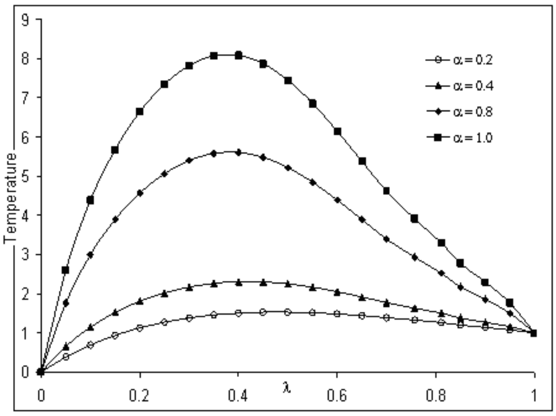

Fig. 6. Effect of α on Temperature (T) for $Pr = 0.5, S = 10, a = 0.2, w = 1.0$.

from Fig. 5 that the velocity decreases as the couple stress fluid parameter α increases. Fig.6 reveals the effect of α on temperature distribution. The temperature increases as the value of the couple stress parameter increases. The effect of Prandtl number on the temperature distribution is presented in Fig.7. It can be observed that the temperature is increasing as the Prandtl number is increasing.

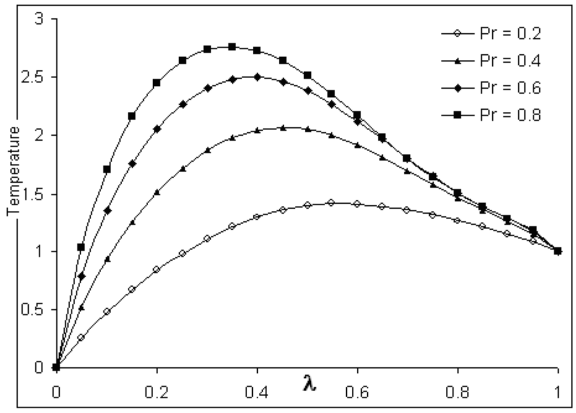

Fig. 7. Effect of Pr on Temperature (T) for $\alpha = 0.4, S = 10, a = 0.2, w = 1.0$.

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