Asymmetric Optimal Hedge Ratio with an Application

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Abstract— **The optimal hedge ratio (OHR) is an important tool for hedging against the price risk. A number of different approaches have been utilized in the literature in order to estimate the OHR, among others, constant parameter and time-varying approaches. One relevant question in this regard that has not been examined, to the best knowledge, is whether the OHR has an asymmetric structure or not. This issue is addressed in the current paper by mathematically proving that the OHR is asymmetric. Furthermore, we offer a method to deal with this asymmetry in the estimation of the underlying OHR. This method is applied to the US equity market using weekly spot and future share prices during the period January 5, 2006 to September 29, 2009. We find empirical evidence that supports the existence of an asymmetric OHR.**

Index Terms— **asymmetry, futures, optimal hedge ratio, US equity market.**

I. Introduction

FINANCIAL risk management and hedging against risk have become more important now due to the recent occurrence of the financial crisis worldwide and the consequent turmoil in financial markets. The optimal hedge ratio (OHR) has therefore important implication for investors in order to hedge against the price risk. Several different approaches have been suggested in the literature in order to estimate the OHR, among others, constant parameter and time-varying approaches have been applied.

The interested reader can refer to the following literature on the optimal hedge ratio: Kroner and Sultan (1993), Kenourgios, Samitas and Drosos (2008), Ghosh and Clayton (1996), Hatemi-J and Roca (2006), Yang and Allen (2004), Baillie and Myers (1991), Sephton (1993), Ahmed (2007).

But, one pertinent issue in this regard, which has not been investigated to our best knowledge, is whether the OHR has an asymmetric character or not. In another word, does a negative price change have the same impact as a positive price change of the same magnitude? This issue is addressed in the current paper by mathematically proving that the OHR is asymmetric.

In addition, we provide a method to deal with this asymmetry in the estimation of the underlying OHR. The asymmetric behaviour of returns and correlations among financial assets have been investigated by among others Longin and Solnik (2001), Ang and Chen (2002), Hong and Zhou (2008), as well as Alvarez-Ramirez, Rodriguez and Echeverria (2009). According to these publications investors seem to respond more to negative shocks than the positive ones. Thus, the question is whether the issue of asymmetry matters in the estimation of the OHR or not. This method suggested in this paper is applied to the US equity market using weekly spot and future share prices during the period January 5, 2006 to September 29, 2009. We find empirical support for an asymmetric OHR. For a conference version of this paper see El-Khatib and Hatemi-J (2011).

The paper is structured as follows. Section 2 makes a brief discussion of the optimal hedge ratio. Section 3 describes the underlying methodology for estimating the asymmetric OHR and it also proves mathematically the asymmetric property of the OHR. Section 4 provides the empirical findings and the last section concludes the paper.

II. Optimal Hedge Ratio

The function of the OHR is to make sure that total value of the hedged portfolio remains unaltered. The hedged portfolio includes the quantities of the spot instrument as well as the hedging instrument and it can be expressed mathematically as the following:

$$V_h = Q_s S - Q_f F \qquad (1)$$

Where V_h represents the value of the hedged portfolio, Q_s and Q_f signify the quantity of spot and futures instrument respectively. S and F are the prices of the underlying variables. Equation (1) can be transformed into changes because the only source of uncertainty is the price. Thus, we can express equation (1) as the following:

$$\Delta V_h = Q_s \Delta S - Q_f \Delta F \qquad (2)$$

Where $\Delta S = S_2 - S_1$ and $\Delta S = F_2 - F_1$. The ultimate goal of the hedging strategy is to achieve $\Delta V_h = 0$, which results in having $\frac{Q_f}{Q_s} = \frac{\Delta S}{\Delta F}$. Now let

$$h = \frac{Q_f}{Q_s} \qquad (3)$$

then we must have

$$h = \frac{\Delta S}{\Delta F} \qquad (4)$$

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Thus, h is the hedge ratio, which can be obtained as the slope parameter in a regression of the price of the spot instrument on the price of the future (hedging) instrument. This can be demonstrated mathematically. Let us substitute equation (3) into equation (2) that results in the following equation:

$$\Delta V_h = Q_s[\Delta S - h\Delta F] \qquad (5)$$

The OHR is the one that minimises the risk of the change of the value of the portfolio that is hedged. This risk is measured by the variance of equation (5), which is given by the following equation:

$$Var[\Delta V_h] = {Q_s}^2[\sigma_s^2 + h^2\sigma_F^2 - 2h\rho\sigma_s\sigma_F] \qquad (6)$$

In this case σ_s^2 represents the variance of ΔS, σ_F^2 signifies the variance of ΔF, and the correlation coefficient between ΔS and ΔF is denoted by ρ. In order to obtain the OHR we need to minimize equation (6) with regard to h. That is,

$$\frac{\partial Var[\Delta V_h]}{\partial h} = {Q_s}^2[2h\sigma_F^2 - 2\rho\sigma_s\sigma_F] = 0, \qquad (7)$$

this gives

$$h^* = \rho\frac{\sigma_s}{\sigma_F}. \qquad (8)$$

The OHR can also be obtained by estimating the following regression model:

$$\Delta S_t = \alpha + h\Delta F_t + u_t. \qquad (9)$$

III. Methodology for Estimating Asymmetric OHR

Assume that we are interested in investigating the relationship between the changes of the spot and future prices when each price index is random walk process. Thus, the changes ΔS_t and ΔF_t can be defined as the following:

$$\Delta S_t = \varepsilon_{1t}, \qquad (10)$$

and

$$\Delta F_t = \varepsilon_{2t}, \qquad (11)$$

where $t = 1,2,\ldots T$, and the variables ε_{1t} and ε_{2t} signify white noise disturbance terms. We define the positive and the negative shocks as the following respectively:

$$\varepsilon_{1t}^+ = \max(\varepsilon_{1t}, 0), \qquad \varepsilon_{1t}^+ = \max(\varepsilon_{2t}, 0),$$

$$\varepsilon_{1t}^- = \min(\varepsilon_{1t}, 0), \quad \text{and} \quad \varepsilon_{1t}^- = \min(\varepsilon_{2t}, 0).$$

It follows that the changes can be defined as

$\Delta S_t^+ = \varepsilon_{1t}^+$, $\Delta S_t^- = \varepsilon_{1t}^-$, $\Delta F_t^+ = \varepsilon_{2t}^+$ and $\Delta F_t^- = \varepsilon_{2t}^-$.

Thus, by using these results we can estimate the following regression models:

$$\Delta S_t^+ = \alpha_1 + h_1\Delta F_t^+ + u_t^+, \qquad (12)$$

$$\Delta S_t^- = \alpha_2 + h_2\Delta F_t^- + u_t^-, \qquad (13)$$

Consequently, h_1 is the OHR for positive price changes and h_2 is the OHR for negative price changes. Per definition we have the following:

$$h_1 = \rho^+\frac{\sigma_{S^+}}{\sigma_{F^+}} = \frac{cov(\Delta S_t^+, \Delta F_t^+)}{\sigma_{F^+}^2} \quad \text{and}$$

$$h_2 = \rho^-\frac{\sigma_{S^-}}{\sigma_{F^-}} = \frac{cov(\Delta S_t^-, \Delta F_t^-)}{\sigma_{F^-}^2}.$$

Given that there are positive as well as negative price changes then h is different from h_1 as well as h_2. The following proposition shows the relationship between h, h_1 and h_2 and proves that h is indeed different from h_1 as well as h_2.

Proposition:

We have

$$h = \begin{bmatrix} h_1\sigma_{F^+}^2 + h_2\sigma_{F^-}^2 + cov(\Delta S_t^+, \Delta F_t^-) \\ +cov(\Delta S_t^-, \Delta F_t^+) \end{bmatrix}\frac{1}{\sigma_F^2} \qquad (14)$$

Thus, we deal with cases that are characterised by both price increases and price decreases during the underlying period.

Proof

The OHR is given by

$$h = \rho\frac{\sigma_S}{\sigma_F} = \frac{cov(\Delta S_t, \Delta F_t)}{\sigma_F^2} \qquad (15)$$

One can observe that

$$\begin{aligned} \Delta S_t &= \Delta S_t^+ + \Delta S_t^- \\ \Delta S_t^+\Delta S_t^- &= 0 \end{aligned} \qquad (16) \quad \text{and}$$

$$\begin{aligned} \Delta F_t &= \Delta F_t^+ + \Delta F_t^- \\ \Delta F_t^+\Delta F_t^- &= 0 \end{aligned} \qquad (17)$$

Using equations (16) and (17), the following is obtained:

$$\begin{aligned} cov(\Delta S_t, \Delta F_t) &= E[(\Delta S_t^+ + \Delta S_t^-)(\Delta F_t^+ + \Delta F_t^-)] \\ &\quad -E[\Delta S_t^+ + \Delta S_t^-]E[\Delta F_t^+ + \Delta F_t^-] \\ &= (E[\Delta S_t^+\Delta F_t^+] - E[\Delta S_t^+]E[\Delta F_t^+]) \\ &\quad +(E[\Delta S_t^+\Delta F_t^-] - E[\Delta S_t^+]E[\Delta F_t^-]) \\ &\quad +(E[\Delta S_t^-\Delta F_t^+] - E[\Delta S_t^-]E[\Delta F_t^+]) \\ &\quad +(E[\Delta S_t^-\Delta F_t^-] - E[\Delta S_t^-]E[\Delta F_t^-]). \end{aligned}$$

This can be expressed as follows:

$$\begin{aligned} cov(\Delta S_t, \Delta F_t) &= cov(\Delta S_t^+, \Delta F_t^+) + cov(\Delta S_t^+, \Delta F_t^-) \\ &\quad +cov(\Delta S_t^-, \Delta F_t^+) + cov(\Delta S_t^-, \Delta F_t^-) \end{aligned} \qquad (18)$$

By replacing equation (18) into equation (15) we obtain the following:

$$h = \begin{bmatrix} cov(\Delta S_t^+, \Delta F_t^+) + cov(\Delta S_t^+, \Delta F_t^-) \\ + cov(\Delta S_t^-, \Delta F_t^+) + cov(\Delta S_t^-, \Delta F_t^-) \end{bmatrix} \frac{1}{\sigma_F^2}.$$

In order to derive equation (14), we use $cov(\Delta S_t^+, \Delta F_t^+) = h_1 \sigma_{F^+}^2$ and $cov(\Delta S_t^-, \Delta F_t^-) = h_2 \sigma_{F^-}^2$.

Equation (14) clarifies the components that are normally used in estimating the OHR. However, if the investor has certain information that would indicate a price change in a given direction then it is better to use certain components of equation (14) not all parts. For example, if a price increase is expected at the maturity date then we expect to have

$\Delta F_t^+ = \Delta F_t$, $\Delta F_t^- = 0$ and $\sigma_F^2 = \sigma_{F^+}^2$.

Therefore, using equation (14) we suggest calculating the following OHR

$$\hat{h} = \left[h_1 \sigma_{F^+}^2 + cov(\Delta S_t^-, \Delta F_t^+)\right] \frac{1}{\sigma_F^2}$$

$$= h_1 + \frac{cov(\Delta S_t^-, \Delta F_t^+)}{\sigma_F^2}.$$

IV. Empirical Findings

The dataset applied in this paper consists of weekly observations of spot and future prices for the US during the period January 5, 2006 to September 29, 2009. The data source is DataStream. The positive and negative shocks of each variable were constructed by the approach outlined in the previous section and by using a program procedure written in Gauss that is available on requested from the authors. The estimation results for the optimal hedge ratios are presented in Table 1. Each value is statistically significant at any conventional significance level. It should be pointed out that the difference between the optimal hedge ratios are not huge in this particular case because the mean values of positive shocks and negative shocks are very close as is indicated in the Table 2.

Table 1. The Estimated Hedge Ratios.

h	h_1	h_2
0.9830 (0.0003)	0.9745 (0.0114)	0.9865 (0.0088)

Notes: The standard errors are presented in the parentheses.

Table 2. The Calculated Mean Values.

$E[\Delta F_t^-]$	$E[\Delta F_t^+]$	$E[\Delta S_t^-]$	$E[\Delta S_t^+]$
-0.010639	0.009668	-0.010451	0.009562

V. Conclusion

The optimal hedge ratio is widely used in financial markets to hedge against the price risk. Different approaches have been suggested for estimating the OHR. This paper is the first attempt, to our best knowledge, to take into account the potential asymmetric character of the underlying OHR. It proves mathematically that the OHR is indeed asymmetric. It also suggests a method to take this asymmetric property in the estimation. The approach is applied to estimating the OHR for the US equity market during the period January 5, 2006 to September 29, 2009. Weekly data are used. The OHR for positive shocks as well negative shocks are estimated separately. Our conjecture is that these separate hedge ratios could be useful to the investor in order to find optimal hedge strategies. This could be achieved by relying more on the OHR for positive cases, (h_1) if the investor expects a price increase at the maturity of the futures contract. On the other hand, it might be safer to rely on the OHR for negative chocks (h_2) in case the investor expects a price decrease at the maturity of the futures. However, if there are no expectations about the direction of any potential price change at the maturity the investor might just rely on the standard OHR (h). It should be mentioned that the *ex post* and *ex ante* problem prevails as for any other empirical calculation.

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