A New Approach for Numerical Simulation of Pulse Propagation in Non-Uniform Fiber Bragg Grating

F. Emami, *Member, IAENG,* A. H. Jafari, and M. Hatami

***Abstract*—The nonlinear equations of Gaussian apodized and cosine shapes of fiber Bragg gratings (FBGs) are solved for quasi-pulses by using a simple numerical approach. We showed that there is a switching action for these structures related to input pulse power, and the required switching power for cosine shape FBG is 30% less than that needed for apodized FBG. Also, for two configurations, attenuation of the pulse amplitude at the fiber ends is less respect to the uniform gratings. In fact, we have almost constant pulse amplitude and there is attenuation just at the fiber centre where the grating coupling coefficient is high. Our numerical method is applicable to non-solitary pulses too.**

***Index Terms*—Fiber Bragg gratings, Gaussian apodized gratings, nonuniform gratings, soliton**

I. Introduction

Fiber Bragg grating (FBGs) have grate applications in optical communication such as optical multiplexers and de-multiplexers, wavelength division multiplexing (WDM), dense WDM (DWDM), null couplers [1] and so on. The main usage of these structures is filtering action of them; for example negative coefficient and multi tab filters [2]. Tuneable FBGs are used in designing the fiber optical code-division multi-access. The elastic and thermal sensors are other groups of the FBG applications [3].

But, the most important FBG application is the optical switching. It was reported that we can use a self induced nonlinear switch to form an AND gate [4]. Polymeric switches are made by an array of FBGs [5]. Depending to the relationships of the wavelengths of the quasi-continuous waves and the Bragg wavelengths, the resonance effects or a decaying behaviour can exist. In the former case, the light wavelength is in the outer region of the forbidden gap forms a gap soliton [6], and in the latter case when the input light wavelength is near the Bragg's wavelength, we have an attenuated wave. Simulation of the second case is done by Schrödinger's equation [7-8]. But, in general case it is very difficult to consider all of the nonlinear effects on FBG analysis.

In this article, we impose a heuristic method to solve the mentioned equations and simulate the quasi-pulse propagation through a non-uniform FBG such as Gaussian apodized and cosine shapes. We studied the effects of nonlinearities and switching in these structures.

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II. Theory

To simulate (quasi) pulse propagation in a FBG we have to solve the related equations of a FBG considering the *boundary conditions*. Due to the nonlinear Kerr effects, the fiber index varies along the fiber length and it can be written as:

$$n(\omega,z) = \bar{n}(\omega) + n_2 \,|\, E \,|^2 + \delta n_g(z) \tag{1}$$

where δn_g is the periodic index change of the fiber and n_2 is a power dependent nonlinear term. According to the coupled mode theory, there is an energy exchange between the forward and backward waves and the following relations can interpret these variations [9]:

$$\frac{\partial A_f}{\partial z} + \beta_1 \frac{\partial A_f}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_f}{\partial t^2} = i\delta A + i\kappa A_b + i\gamma\left(|\, A_f \,|^2 + 2\,|\, A_b \,|^2\right) \tag{2}$$

$$-i\frac{\partial A_b}{\partial z} + \beta_1 \frac{\partial A_b}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_b}{\partial t^2} = i\delta A_b + i\kappa A_f + i\gamma\left(|\, A_f \,|^2 + 2\,|\, A_b \,|^2\right)A \tag{3}$$

where the detuning δ is the amount of deviation from the Bragg's propagation constant, and γ is a nonlinear Kerr term defined as:

$$\gamma = \frac{\lambda_2 W_0}{cAeft} \tag{4}$$

κ is the coupling coefficient of the grating and gives the extent of the forward, A_f, and backward, A_b, wave amplitudes energy exchange. Coupling coefficient is varied because of the index changes along the fiber length. This factor is a function of the material used in, and the shape of the corrugation (such as the grating period and height).

It is possible to derive a relation for the coupling coefficient in the form of:

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$$\kappa = \frac{\pi}{\lambda}\, \delta n(z)\; g(z) \qquad (5)$$

where *g(z)* is the envelop shape function of the index changes. If we want to survey the non-uniform FBG, the envelop function *g(z)* can be varied and different shapes with different bandwidths generates since the grating shape or equivalently the coupling strength, can be used to set the bandwidth. To do this, we use two special cases of the Gaussian apodized and cosine shapes envelop functions. The term 'apodization' is used for the type of the gratings in a tapered form in the refractive index; a maximum at the middle which approaches to zero at the grating ends. Apodized gratings have large improvement side-lobe suppression and hence narrower bandwidth.

In general, for the envelop shape function *g(z)* we have the form of:

$$g(z) = \exp(-\alpha_0 \frac{z^2}{L^2}) \qquad (6)$$

in the case of Gaussian apodized FBG, and:

$$g(z) = \frac{1}{2}(1 + \cos(-\pi \frac{z}{L})\ ) \qquad (7)$$

for cosine grating, where α_0 and α_1 are the proper defined factors.

III. Numerical Method

There are many methods (in general numerically) to solve the coupled equations (2) and (3) with a predefined *boundary conditions*. All of them have a related complexity, such as the finite element method. But with a heuristic method we can solve these nonlinear equations for quasi-pulse propagation in FBGs, which is fast and exact. Our procedure is based on the Jacobi iterative method and forth-fifth orders Runge-Kutta (R-K) method with non-uniform step size. The predefined data are the *boundary conditions* at the input and output of the structure. At first, we guess some values of the field amplitudes. This is done by considering the equations without any nonlinearity ($\gamma=0$ in (2) and (3)). Using the R-K method, we solve (2) and (3) with these *initial conditions* at the forward direction and for a quasi-pulse at the input. After that, there are initial guess for the field amplitudes at the *boundaries*. Whit these values the coupled equations are solved in the backward directions and hence we will find better estimations. By using the Jacobi iteration method and the mentioned procedure we will have the converged forward and backward field amplitudes A_f and A_b (typically in four or five iterations).

For various FBG structures, we can choose R-K method with a variable step size to guarantee the required accuracy of our results (in some structures this procedure needs a significant amount of computation; we can resolve this complexity by using the Runge-Kutta Fehlberg method). This method is very fast since any numerical method used to solve (2) and (3), considering all the nonlinear conditions, must decompose the equations with the related *boundary conditions*, whereas in our method these *boundary conditions* are reformed to the *initial conditions* which can solve using a simple procedure.

The flowchart of our algorithm is shown in Fig. 1.

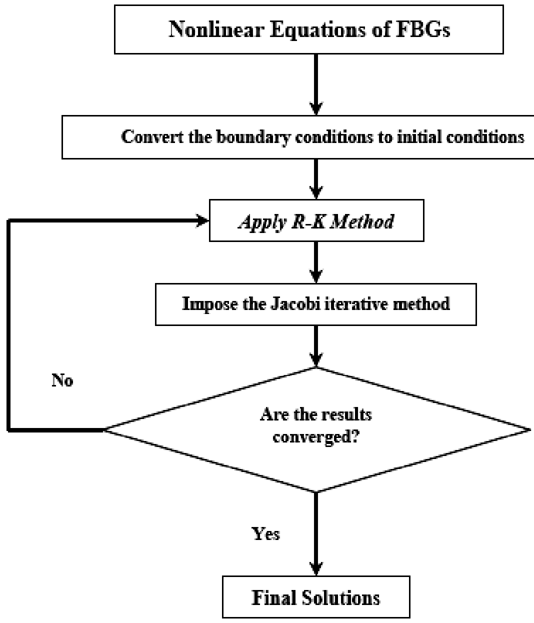

Fig. 1: Solution algorithm to simulate quasi-pulse propagation in an arbitrary non-uniform FBG

IV. Simulation Results

In this section we impose the above algorithm on (2) and (3), and verify the (quasi) pulse propagation in non-uniform FBGs whit the Gaussian apodized and cosine envelop functions. The results are shown in Fig. 2.

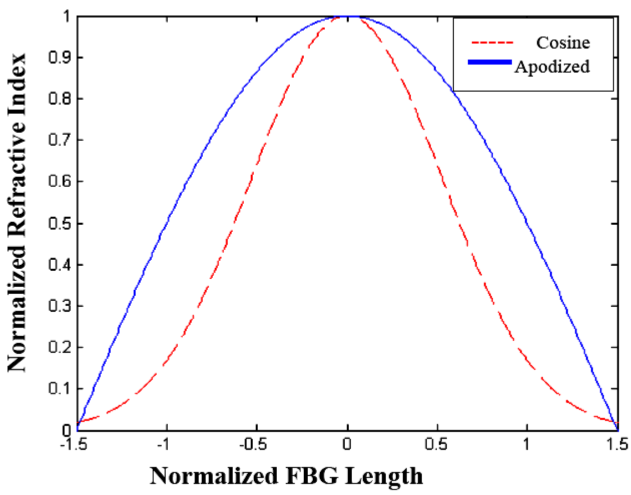

Fig. 2: Normalized profile functions of the refractive index variations for Gaussian apodized and cosine gratings

In these simulations we assumed that λ_b*=1550nm* and all the nonlinear effects are included too. The coupling and detuning parameters are two important parameters for which any variations of them can generate different cases, considering the wavelength variations.

When the Bragg wavelength lies at the forbidden gap of a FBG (or when the detuning is less than the coupling strength;

$\delta < \kappa$), the quasi-pulse propagation is in the form of Fig. 3, for a Gaussian apodized FBG. All the amplitudes are normalized and as seen the forward and backward wave amplitudes are decaying.

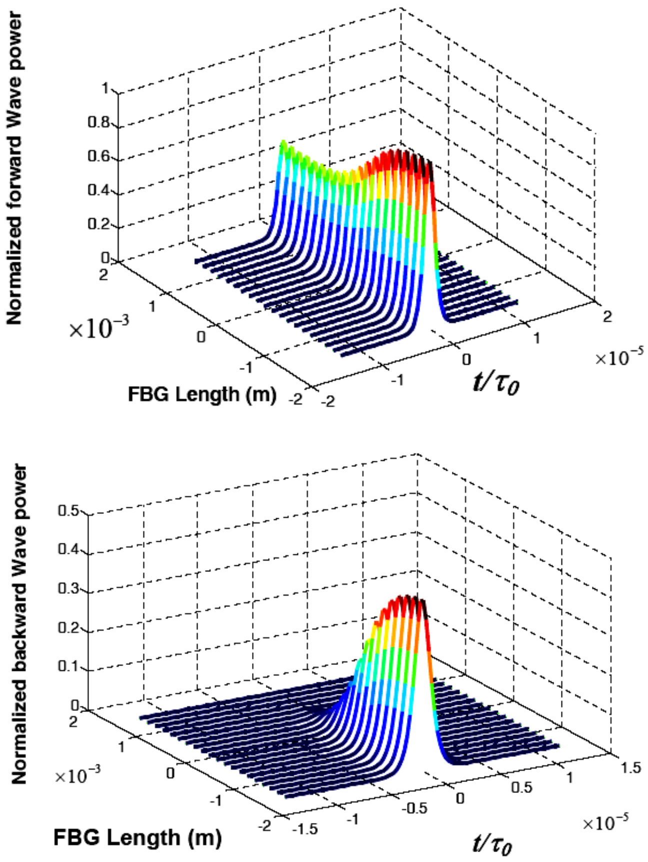

Fig. 3: Simulation of pulse propagation in an apodized FBG ($\delta < \kappa$)

It is evident the FBG acts like a filter; the incoming light is reflected and we don't have illustrious field at the output. In fact, in a Gaussian apodized structure the coupling strength at the either edges of the FBG is low, so there is a minimum energy exchange between the forward and backward field amplitudes. On the other hand, this factor is maximized at the grating middle and the grating effect (or coupling strength) can dominate in this region. The input field attenuates and some of the pulse energy reflects back, hence due to lower coupling at the end of FBG we will have approximately constant pulse amplitude along the fiber.

This is comparable to a uniform FBG where there is a continuous and exponential decay for the field amplitudes; while in a Gaussian apodized FBG there is a negligible interaction between the field and the grating at both ends. The numerical results based on the proposed method are shown in Fig. 4. With a constant total energy at the fiber input, there is an exchange between the forward and backward wave amplitudes during the propagation in a FBG. In fact, this exchange is due to the coupling amount, where measure the wave feedback capability. In each section of the fiber the total energy is constant but the forward and backward wave amplitudes have variable values. This is independent from the coupling amount, since the coupling strength is large at the middle of the fiber and it is very low at both ends in an apodized FBG. Our simulation results for energy exchanges are shown in Fig. 5.

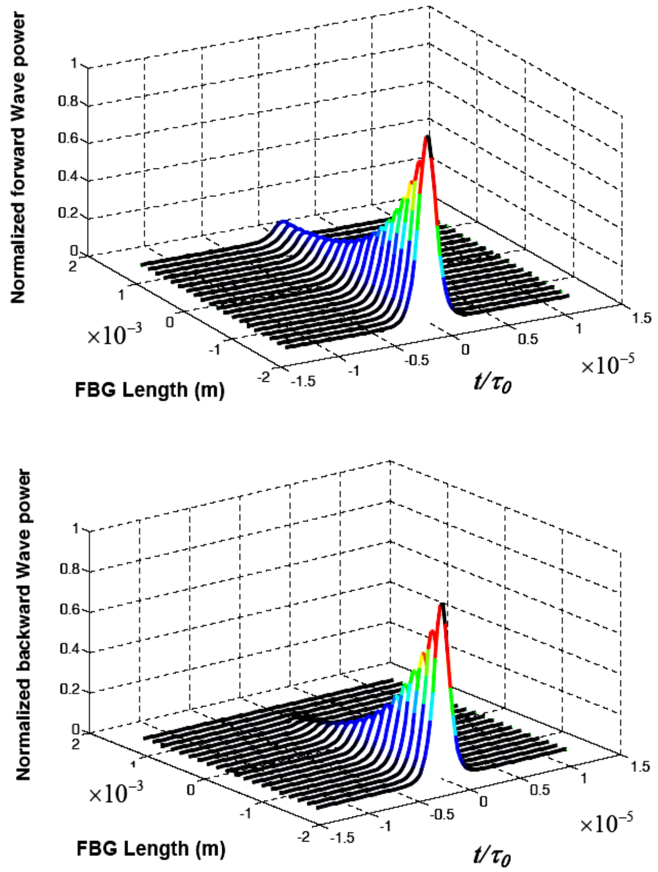

Fig. 4: Simulation of pulse propagation in a uniform FBG ($\delta < \kappa$)

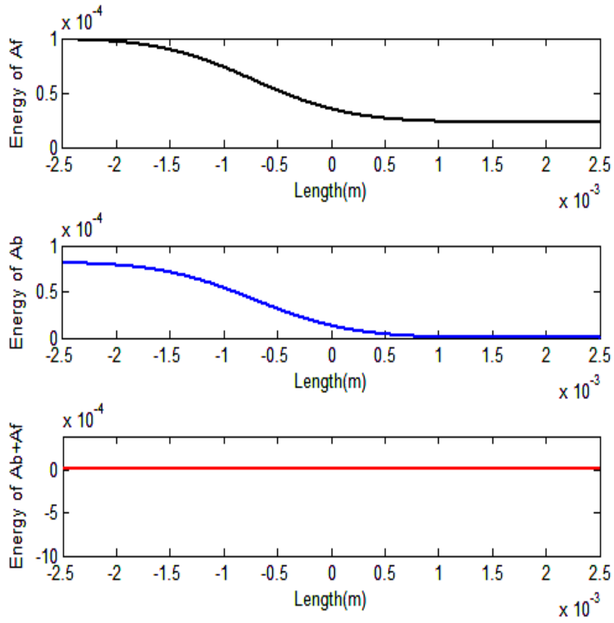

Fig. 5: Simulation of pulse propagation energy for forward and backward wave amplitudes in an apodized FBG ($\delta < \kappa$)

Note that, the constant total energy confirms the mentioned discussions and accuracy of our numerical method. Another non-uniform FBG is cosine envelop structure which defined in (7). Because of higher coupling strength for cosine FBGs, there is a larger energy exchange between the forward and backward field amplitudes and hence higher challenge between them, so the reflected field amplitude will be more. This is simulated and the results are shown in Fig. 6.

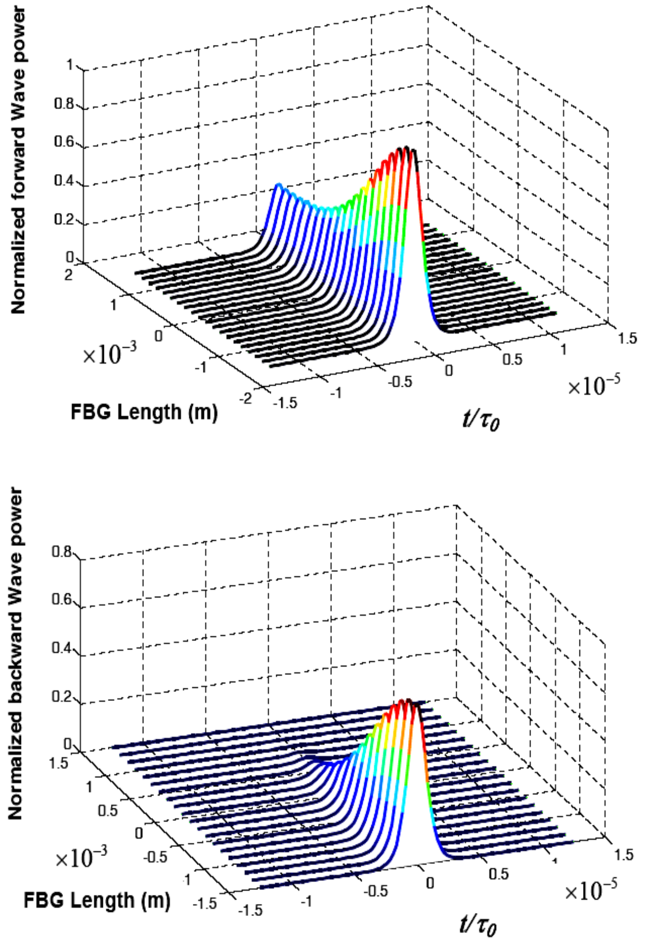

Fig. 6: Simulation of pulse propagation in a cosine envelop FBG ($\delta<\kappa$)

For low amplitude pulses the nonlinear terms in (2) and (3) can be neglected while for high power input pulse the Kerr effect is important and we should consider the nonlinearities in the structure.

We imposed these conditions to an apodized FBG to see the switching effects by application of our proposed method. The results are plotted in Fig. 7. As seen, increasing the input pulse amplitude causes a left frequency shift and the central frequency of the input pulse is not in the Bragg wavelength region, so the input pulse exits from the other side of FBG. This is a switching operation of FBG and FBG acts as an optical switch in this case. As shown in Fig. 7c, the ratio of the output to input pulse power increased for high power inputs. There is a critical power, $P_{Critical}$, so that for input powers more than this value the output power increases. For pulse with low input power, the output to input power ratio is less than the critical power, $P_{Critical}$, (the grating switch off) and for pulses with input powers more than $P_{Critical}$ this ratio tends to "1", means total transmittance (the grating switch on).

The switching behaviors are seen in cosine envelop FBGs too. The results of our simulation for this type of the grating are reported in Fig. 8. Again, for high power pulses the nonlinear effects of the gratins, cause on-off or switching action for cosine envelops FBG structures.

Comparing the results of Fig. 7 and 8, we find that for the low power pulses there is a more switching effect in a cosine envelops FBG respect to a Gaussian apodized FBG. This is due to higher coupling in cosine envelop FBG structures. It is found that, for a cosine FBG the reflected pulse is more affected by the grating shape, respected to the Gaussian apodized FBG and one can decomposes it to two separated soliton like pulses.

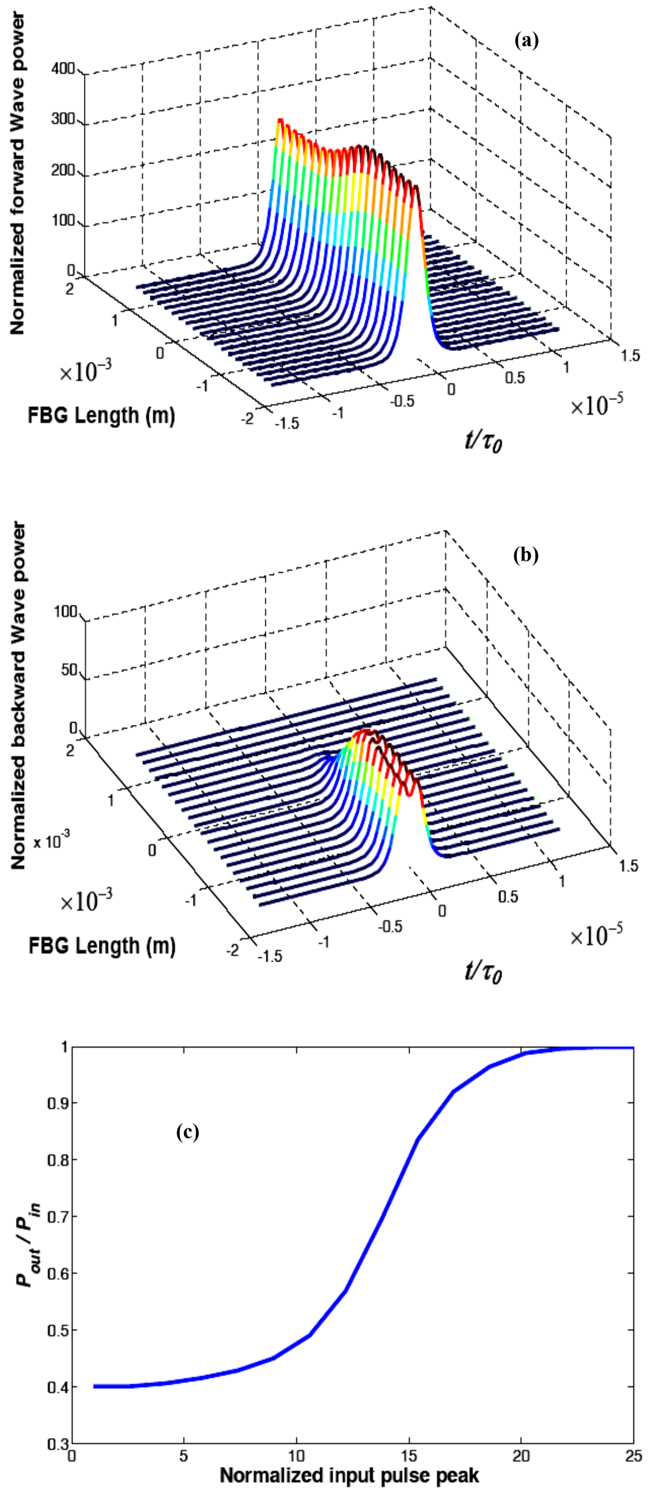

Fig. 7: (a) Forward and (b) backward pulse powers, (c) switching effect due to increasing of the input pulse power in an apodized envelop FBG ($\delta<\kappa$)

Finally, in Fig. 9 we drew the switching behaviors of a uniform FBG. Referring to the figure, it is clear that the switching action is faster respect to the Gaussian apodized and cosine envelops FBGs and this is due to the higher average coupling.

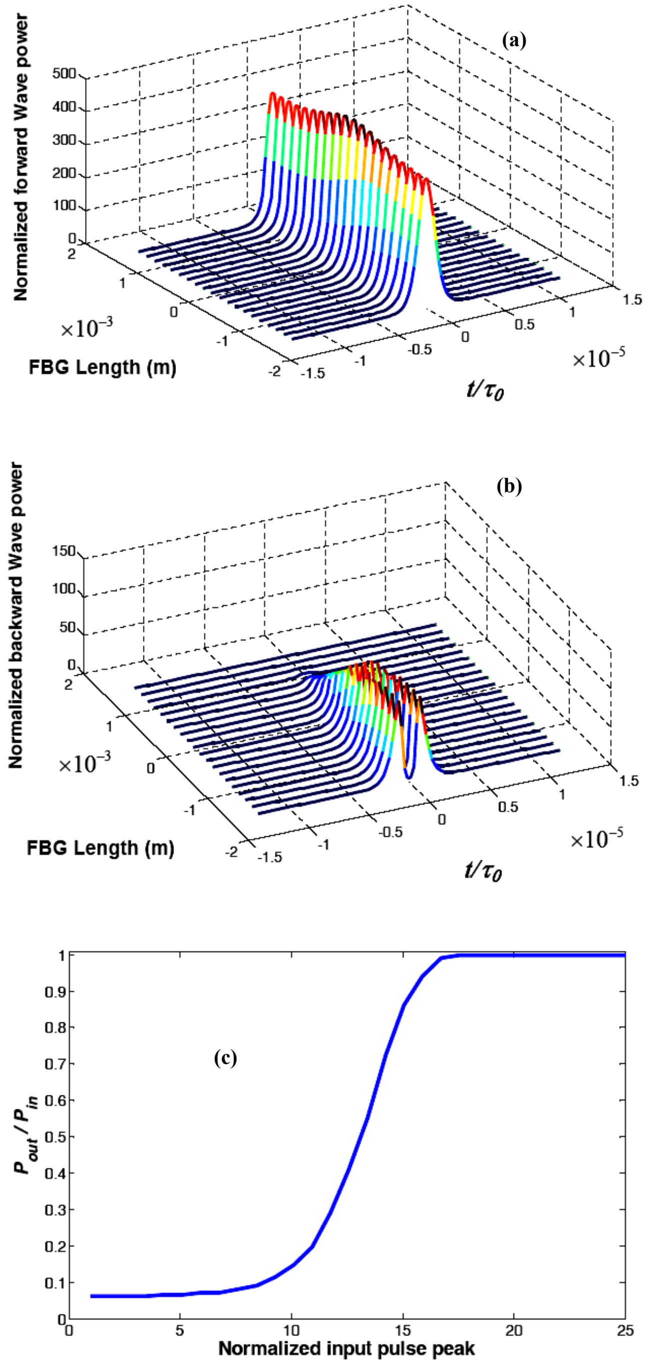

Fig. 8: (a) Forward and (b) backward pulse powers, (c) switching effect due to increasing of the input pulse power in a cosine envelop FBG ($\delta<\kappa$)

V. Conclusions

In this paper we studied the quasi-pulse propagation and the nonlinear effects through the apodized and cosine shapes FBGs using a simple and fast numerical method. We tested our method by imposing it on uniform FBG and then apply the proposed numerical approach to non-uniform structures. This method is useful especially for consideration of nonlinear effects in the FBG governing equations; since these equations are usually complex in general case. Our technique was based on the transforming the *boundary conditions* to some *well-defined initial conditions*, so that the FBG equations can solve fast and simple.

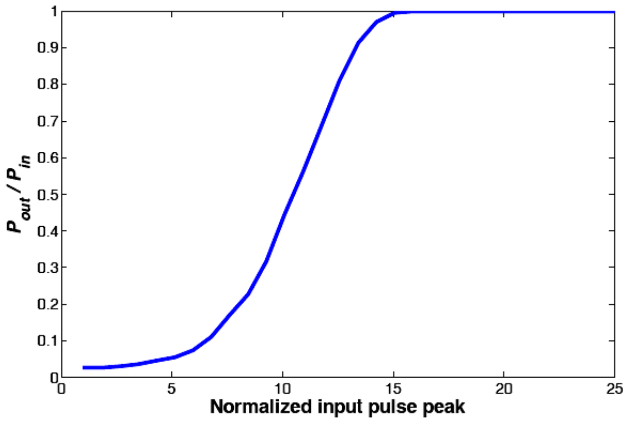

Fig. 9: Switching behavior of a uniform FBG

The *well-defined initial conditions* are derived from uniform structures without any nonlinearity. With this idea, the proposed numerical method would be converged.

After applying the mentioned numerical method, it is found that for lower pulse powers there is a filtering action for two configurations of the chosen gratings, but for cosine shape FBGs more filtering operation is seen. Contrary to non-uniform gratings, which have the exponential attenuation for input pulse amplitude through the fibers, in these types of FBGs the pulse amplitude has almost constant values at both fiber ends during the propagation. This is due to the index lowering at the fiber ends and hence we have the most power transformations at the fiber centre. Indeed, cosine shapes FBGs have more power exchanges between the forward and backward wave amplitudes respect to the Gaussian apodized FBGs.

For pulses with higher powers, the nonlinear effects are seen so the switching effects appear and it is more for apodized FBGs rather than cosine FBGs. In other words, the switching effects are occurred for cosine FBGs in lower powers. In fact, we can make an optical switch using these types of the gratings; with dominant operation for Gaussian apodized FBG structures. The average field coupling for cosine FBGs are grater than the Gaussian apodized FBGs and this is the reason for such switching behaviors.

In general, the proposed method can interpret the mentioned results with a good accuracy. In our proposed procedure, any complicated nonlinearity existing in the FBG equations can be considered in the simulation. It is possible to use this method to simulate the related all optical switches in integrated optic devices. We can explain the non-solitary wave propagation in different fibers using this method with adequate accuracy and convergence.

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