Terminal Cost Distribution in Discrete-Time Controlled System with Disturbance and Noise-Corrupted State Information

Valery Y. Glizer, Vladimir Turetsky, and Josef Shinar

Abstract—Recursive formula for the terminal cost distribution in a scalar linear discrete-time system with disturbance and noise corrupted measurements is obtained. The system is subject to a linear saturated control strategy. The distributions of the initial state and the estimator error are assumed to be known. The disturbance is independent of the state/control and its distribution is known. The general result is applied to an interception problem with different types of disturbance. An illustrative numerical example confirms that the analytical method can replace extensive Monte Carlo simulations.

Index Terms—linear discrete-time system, robust transferring strategy, noisy measurements, terminal state distribution.

I. INTRODUCTION

VARIOUS real life control problems (including interception and navigation) can be formulated as a problem of transferring a system to a prescribed hyperplane in the state space at a prescribed time by bounded control in the presence of noise corrupted measurements and unknown bounded disturbance [1], [2], [3], [4]. This problem can be reduced to a scalar one, where the new state variable is the distance between a current point on the trajectory of the uncontrolled system motion and the hyperplane. By this scalarization, the problem of transferring to a prescribed hyperplane becomes a problem of robust transferring to zero.

If perfect state information is available, several classes of deterministic feedback strategies that robustly transfer a scalar system from some domain of initial positions to zero, are known. Among such robust transferring strategies are differential game based bang-bang strategies [1], [2], as well as various linear, saturated linear and weakly nonlinear strategies [3], [5], [6], [7], [8], [9].

In real life applications, the state information is corrupted by measurement noise and only part of the state variables can be directly measured. This fact impedes significantly the practical implementation of theoretically robust transferring strategies. Moreover, an estimator, reconstructing and filtering the state variables, becomes an indispensable component of the control loop. Due to the noisy measurements and the uncertain (random) disturbance the control function receives, instead of the exact value of , a random estimator output becomes the estimation error. As the consequence, the terminal value of becomes a random variable with an a-priori unknown distribution. In order to appreciate the extent of performance deterioration of a deterministic robust transferring strategy by using such a stochastic data, the distribution of the terminal value of has been found.

In the current practice, such a distribution is obtained, for any given estimator/control strategy combination and specified disturbance and noise models, by a large set of Monte Carlo simulations [10], [11]. Such a-posteriori test is necessary for validation purpose, but cannot be applied for an insightful control system design. There is an obvious need for an analytical a-priori estimate of the control strategy performance as a part of the integrated control system design.

State estimates in the presence of deterministic information errors were obtained in [12] and [13]. In [13], such estimates are used to construct a robust control of a dynamic system with inexact state information. In interception problems, the scalar state variable is the zero-effort miss distance and its terminal value is the actual miss distance itself. Under some general linear assumptions without taking into account the system dynamics, the miss distance distribution was investigated in [14]. In the case of linear interceptor strategies, the dependence of the miss distance on the measurement noises was analyzed, by means of the adjoint approach in [15] and [16]. Unfortunately, this approach can be applied only in the case of non saturated linear strategies.

In this paper, the system dynamics is modeled by a discrete-time scalar linear equation controlled by a saturated linear transferring strategy. Assuming that the distributions of initial state , the estimation error and disturbance are known, a recurrence formula for the distribution of is obtained. The random variable is the linear combination of two dependent random variables - the state at the previous time step and the control variable (nonlinearly depending on via the saturation function) and an independent random variable . This makes the problem to be mathematically nontrivial. The disturbance free version of the problem was studied in [17].

II. PROBLEM STATEMENT

A. Original Control Problem

Consider the controlled system

\[ \dot{X} = A(t)X + b(t)u + c(t)v + f(t), \]

(1)

where \( X \in \mathbb{R}^n \) is the state vector; \( t \in [t_0, t_f] \), \( X(t_0) = X_0 \), \( t_f \) is a fixed time instant, \( t_0 \in [0, t_f] \); the matrix function

(Advance online publication: 27 February 2012)
A(t) and the vector functions \( b(t), c(t), f(t) \) are differentiable for a sufficient number of times on the interval \([0, t_f]\). The scalar control \( u \) and disturbance \( v \) are assumed to be measurable on \([t_0, t_f]\) and satisfying the constraints
\[
|u(t)| \leq 1, \quad |v(t)| \leq 1, \quad t \in [t_0, t_f].
\]  
(2)
The target set is the hyperplane \( D = \{X \in \mathbb{R}^n \mid d^T X + d_0 = 0\} \), where \( d \in \mathbb{R}^n \) is a prescribed non-zero vector, \( d_0 \) is a prescribed scalar, the superscript \( T \) denotes the transposition. The control objective is to guarantee \( X(t_f) \in D \) against any admissible disturbance function \( v(t) \).

By the transformation of the state variable of (1),
\[
z = z(t, X) = d^T \Phi(t, t) X + \int_t^{t_f} \Phi(t, \tau) f(\tau) d\tau + d_0,
\]  
(3)
the system (1) is reduced to the scalar one
\[
z = h_1(t) u + h_2(t) v, \quad z(t_0) = z_0,
\]  
(4)
where \( h_1(t) = d^T \Phi(t_f, t) b(t), \) \( h_2(t) = d^T \Phi(t_f, t) c(t) \), \( z_0 = z(t_0, X_0) \), \( \Phi(t, t_0) \) is the fundamental matrix of the homogenous system \( X = A(t) X \). The control objective becomes to guarantee \( z(t_f) = 0 \).

It is assumed that the control is given by a saturated linear strategy
\[
u(t, z) = \text{sat}(K(t) z),
\]  
(5)
where
\[
\text{sat}(y) = \begin{cases} 
1, & y > 1, \\
-1, & y < -1, \\
0, & |y| \leq 1,
\end{cases}
\]  
(6)
the gain function \( K(t) \) satisfies the conditions [9], guaranteeing that the linear strategy \( u = K(t) z \) is robust transferring.

B. Discrete-Time Estimation Problem

Define the division of the interval \([0, t_f]\): \( t_0 < t_1 < \ldots < t_N = t_f \), where \( t_{n+1} - t_n = \Delta t, n = 0, \ldots, N - 1 \). The discrete-time version of the system (4) is
\[
z_{n+1} = z_n + b_n u_n + c_n v_n,
\]  
(7)
where for the simplest Euler approximation of the differential equation (4),
\[
b_n = \Delta t h_1(t_n), \quad c_n = \Delta t h_2(t_n).
\]  
(8)
The control is
\[
u_n = \text{sat}(k_n (z_n + \eta_n)),
\]  
(9)
where \( k_n = K(t_n) \) is the control gain and \( \eta_n \) is the estimation error. The probability density functions \( f_{z_n}(x) \) of \( z_0 \) and \( f_{\eta_n}(x) \) of \( \eta_n, n = 0, 1, \ldots, N - 1 \), are assumed to be known. For any \( n \), the random value \( v_n \) is independent of \( z_n \) and \( u_n \). Its probability density function \( f_{v_n}(x) \) is assumed to be known. The problem is to obtain the probability density function \( f_{z_n}(x) \).

Denote
\[
\begin{align*}
  w_{1n} &\triangleq z_n + b_n u_n, \quad \text{and} \\
  w_{2n} &\triangleq c_n v_n.
\end{align*}
\]  
(10)
(11)
Since \( v_n \) is independent of \( z_n \) and \( u_n \), the random values \( w_{1n} \) and \( w_{2n} \) are independent. Thus, due to (7) and (10) – (11), the convolution formula [18] can be applied:
\[
f_{z_{n+1}}(x) = \int_{-\infty}^{\infty} f_{w_{1n}}(x - \xi) f_{w_{2n}}(\xi) d\xi,
\]  
(12)
where \( f_{w_{1n}}(x) \) and \( f_{w_{2n}}(x) \) are the probability density functions of \( w_{1n} \) and \( w_{2n} \), respectively. However, since the random variables \( z_n \) and \( u_n \) are dependent, the distribution function of \( w_{1n} \) cannot be calculated by using the convolution formula.

III. Solution

Since the probability density function \( f_{w_{2n}}(x) \) is assumed to be known, the calculation of \( f_{z_{n+1}}(x) \), due to (12), is reduced to the calculation of \( f_{w_{1n}}(x) \).

A. Calculation of \( f_{w_{1n}}(x) \)

Due to (7) – (9), the distribution function of \( w_{1n} \) is
\[
F_{w_{1n}}(x) = P(w_{1n} < x) = p_1 P(k_n (z_n + \eta_n) > 1) + p_2 P(k_n (z_n + \eta_n) \leq 1) + p_3 P(k_n (z_n + \eta_n) < -1),
\]  
(13)
where \( p_1, p_2 \) and \( p_3 \) are the conditional probabilities
\[
p_1 = P(z_n + b_n < x \mid k_n (z_n + \eta_n) > 1),
\]  
(14)
\[
p_2 = P((1 + b_n k_n) z_n + b_n k_n \eta_n < x \mid k_n (z_n + \eta_n) \leq 1),
\]  
(15)
\[
p_3 = P(z_n - b_n < x \mid k_n (z_n + \eta_n) < -1).
\]  
(16)
Thus, the problem is reduced to calculating the conditional probabilities (14) – (16).

1) Calculation of \( p_1 p_2 \) and \( p_3 \): By using (14) and the formula for the probability of the product of dependent events,
\[
p_1 p_2 = \bar{P}_1 P(z_n < x - b_n) / P(z_n + \eta_n > 1/k_n),
\]  
(17)
where
\[
\bar{P}_1 = P(z_n + \eta_n > 1/k_n \mid z_n < x - b_n).
\]  
(18)
Let calculate the conditional probability \( \bar{P}_1 \).

First, instead of the event \( z_n < x - b_n \), consider the event \( z_n \in (a, x - b_n) \), where \( a \) is a negative number with sufficiently large absolute value:
\[
\bar{P}_{1a} = P(z_n + \eta_n > 1/k_n \mid z_n \in (a, x - b_n)).
\]  
(19)
Note that
\[
\bar{P}_1 = \lim_{a \to -\infty} \bar{P}_{1a}.
\]  
(20)
Let divide the interval \((a, x - b_n)\) into \( M \) subintervals of equal length \( \Delta x = (x - b_n - a)/M \): \( x_j = a + j \Delta x, \) \( j = 0, 1, \ldots, M \). Then, since the events \( z_n \in (x_j, x_{j+1}) \), \( j = 0, \ldots, M - 1 \), are mutually exclusive,
\[
\bar{P}_{1a} \approx \sum_{j=0}^{M-1} P(z_n \in (x_j, x_{j+1}) \mid z_n \in (a, x - b_n)) \times
\]  
(Advance online publication: 27 February 2012)
\begin{equation}
P(z_n + \eta_n > 1/k_n \mid z_n \in (x_j, x_{j+1})). \tag{21}
\end{equation}

Let start with calculating the first conditional probability under the sum in (21). Note that
\begin{equation}
P\left(\left[ z_n \in (x_j, x_{j+1}) \right] \cap \left[ z_n \in (a, x - b_n) \right] \right) = P\left( z_n \in (x_j, x_{j+1}) \right) P(z_n \in (a, x - b_n)). \tag{22}
\end{equation}

At the other hand,
\begin{equation}
P\left( \left[ z_n \in (x_j, x_{j+1}) \right] \cap \left[ z_n \in (a, x - b_n) \right] \right) = P\left( z_n \in (x_j, x_{j+1}) \right) P(z_n \in (a, x - b_n)). \tag{23}
\end{equation}

For sufficiently small \( \Delta x \), the second conditional probability under the sum in (21) can be approximated as
\begin{equation}
P\left( z_n + \eta_n > 1/k_n \mid z_n \in (x_j, x_{j+1}) \right) \approx P\left( \eta_n > 1/k_n - \bar{x}_j \right), \tag{24}
\end{equation}
where \( \bar{x}_j = (x_j + x_{j+1})/2 \).

Due to (21) and (24) – (25),
\begin{equation}
\hat{p}_{1a} \approx \frac{1}{P(z_n \in (a, x - b_n))} \sum_{j=0}^{M-1} P\left( z_n \in (x_j, x_{j+1}) \right) \times
\end{equation}
\begin{equation}
P\left( \eta_n > 1/k_n - \bar{x}_j \right) = \frac{1}{x-b_n} \sum_{j=0}^{M-1} \int_{x_{j+1}}^{x_{j+1}} f_{z_n}(y) dy \int_{1/k_n-x_j}^{\infty} f_{\eta_n}(y) dy, \tag{26}
\end{equation}

where \( f_{z_n}(y) \) and \( f_{\eta_n}(y) \) are the probability density functions of the random variables \( z_n \) and \( \eta_n \), respectively. Since
\begin{equation}
\int_{x_j}^{x_{j+1}} f_{z_n}(y) dy \approx f_{z_n}(\bar{x}_j) \Delta x, \tag{27}
\end{equation}

the equation (26) can be rewritten as
\begin{equation}
\hat{p}_{1a} \approx \frac{1}{x-b_n} \sum_{j=0}^{M-1} \int_{a}^{\infty} f_{z_n}(\bar{x}_j) f_{\eta_n}(y) \Delta y \Delta x. \tag{28}
\end{equation}

Hence,
\begin{equation}
\lim_{M \to \infty} \sum_{j=0}^{M-1} \left[ f_{z_n}(\bar{x}_j) \int_{1/k_n-x_j}^{\infty} f_{\eta_n}(y) dy \right] \Delta x = \int_{a}^{\infty} f_{z_n}(y) dy \int_{a}^{\infty} f_{\eta_n}(y) dy ds. \tag{29}
\end{equation}

By virtue of (20),
\begin{equation}
\hat{p}_1 = \int_{a}^{\infty} f_{z_n}(y) dy \int_{a}^{\infty} f_{\eta_n}(y) dy ds. \tag{30}
\end{equation}

Due to (17) and (30),
\begin{equation}
\hat{p}_1 = \int_{x-b_n}^{\infty} f_{z_n}(s) \int_{1/k_n-s}^{\infty} f_{\eta_n}(y) dy ds. \tag{31}
\end{equation}

The random variables \( z_n \) and \( \eta_n \) are independent. Therefore,
\begin{equation}
f_{z_n+\eta_n}(y) = f_{z_n}(y) * f_{\eta_n}(y) = \int_{-\infty}^{\infty} f_{z_n}(y-s) f_{\eta_n}(s) ds. \tag{32}
\end{equation}

Finally,
\begin{equation}
p_1 = \int_{-\infty}^{\infty} f_{z_n}(s) \int_{1/k_n-s}^{\infty} f_{\eta_n}(y) dy ds. \tag{33}
\end{equation}

The calculation of \( p_3 \) is similar to the calculation of \( p_1 \), resulting in
\begin{equation}
p_3 = \int_{-\infty}^{\infty} f_{z_n}(s) \int_{1/k_n-s}^{\infty} f_{\eta_n}(s) dx ds. \tag{34}
\end{equation}
2) Calculation of $p_2$: Consider the case $b_n \geq 0$. By definition of the conditional probability,

$$p_2 = \frac{P\left((z_n, \eta_n) \in R(x) \land (z_n, \eta_n) \in Q\right)}{P\left((z_n, \eta_n) \in Q\right)} = \frac{P\left((z_n, \eta_n) \in S(x)\right)}{P\left((z_n, \eta_n) \in Q\right)},$$

where (see Fig. 1)

$$R(x) \triangleq \{(z_n, \eta_n): \eta_n < -Az_n + B(x)\},$$

$$A = 1 + \frac{1}{b_n k_n}, \quad B(x) = \frac{x}{b_n k_n},$$

$$Q \triangleq \{(z_n, \eta_n): -z_n - 1/k_n \leq \eta_n \leq -z_n + 1/k_n\},$$

$$S(x) \triangleq R(x) \cap Q.$$

By virtue of (35), (37) and (43) – (44),

$$p_2 = \frac{1}{C_n} \left\{ \int_{-b_n}^{b_n} \int_{-s+1/k_n}^{-s-1/k_n} f_{z_n}(s) f_{\eta_n}(y) dy ds + \int_{x-b_n}^{x+b_n} \int_{-s+1/k_n}^{-s-1/k_n} f_{z_n}(s) f_{\eta_n}(y) dy ds \right\},$$

where

$$C_n = \frac{1/k_n}{1/k_n} \int_{-b_n}^{b_n} \int_{-s+1/k_n}^{-s-1/k_n} f_{z_n}(y-s) f_{\eta_n}(s) dy ds.$$
respectively; are the lateral accelerations of the evader and the pursuer, line-of-sight; the engagement is modeled by the system (1), where $x_n(t)$, $y_n(t)$, $r_n(t)$, $s_n(t)$ are the states of the evader and pursuer, respectively; $w_n(t)$, $b_n(t)$, $d_n(t)$ are the disturbances at times $t_n$, $t_b$, $t_d$, respectively; $\tau_p$, $\tau_e$, $\sigma_p$, $\sigma_e$ are the time constants of the evader and pursuer; $a_p\max$, $a_e\max$ and $v_p\max$, $v_e\max$ are the bounds of the lateral acceleration commands of the objects; $\varphi_p$, $\varphi_e$ are the aspect angles; $F(t)\equiv 0$, $X_0 = (0, X_2, 0, 0)^T$, $X_2 = V_e\varphi_e(0) - V_p\varphi_p(0)$. The control of the pursuer $u$ and the evader $v$ are the normalized lateral acceleration commands, satisfying the constraints (2). The objective of the pursuer is to nullify the miss distance $|X_1(t_f)|$, i.e. in the target hyperplane, $d = (1, 0, 0, 0)^T$, $d_0 = 0$. In the scalarized system (4), $z_0 = t_f X_2$. Finally, the probability function $f(x)$ is obtained by applying the recurrence formulae (50) and (12) $N$ times. Finally, the probability function $f(x)$ is

$$f(x) = \int_{-\infty}^{\infty} f_N(y)dy + \int_{-\infty}^{x+b_n} \frac{1}{b_n k_n} \int_{x-b_n}^{x+b_n} \left[ f_{2n}(s)f_{n}(s - As + B(x))\right] ds.$$ \hfill (50)

\section*{IV. INTERCEPTION PROBLEM}

\subsection*{A. Problem Outline}

As an example, a planar engagement between two point-mass objects (pursuer and evader) is considered. It is assumed that the dynamics of each object is expressed by a first-order transfer function with the time constants $\tau_p$ and $\tau_e$, respectively. The velocities $v_p$ and $v_e$ and the bounds of the lateral acceleration commands $a_p\max$ and $a_e\max$ of the objects are constant. The geometry of such a planar engagement is presented in Fig. 2.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{interception_geometry.png}
\caption{Interception geometry}
\end{figure}

Assuming that the aspect angles $\varphi_p$ and $\varphi_e$ are small, the engagement is modeled by the system (1), where $X_1$ is the relative separation between the objects, normal to the initial line-of-sight; $X_2$ is the relative normal velocity; $X_3$ and $X_4$ are the lateral accelerations of the evader and the pursuer, respectively; $t_f = \tau_0/(v_p + v_e)$, where $\tau_0$ is the initial range between the objects;

$$A(t) \equiv \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & -1 \\
0 & 0 & -1/\tau_e & 0 \\
0 & 0 & 0 & -1/\tau_p
\end{bmatrix},$$ \hfill (52)

$$b(t) \equiv (0, 0, a_p\max/\tau_p)^T, \quad c(t) \equiv (0, 0, a_e\max/\tau_e, 0)^T.$$ \hfill (53)

The controls of the pursuer $u$ and the evader $v$ are the normalized lateral acceleration commands, satisfying the constraints (2). The objective of the pursuer is to nullify the miss distance $|X_1(t_f)|$, i.e. in the target hyperplane, $d = (1, 0, 0, 0)^T$, $d_0 = 0$. In the scalarized system (4), $z_0 = t_f X_2$. Finally, the probability function $f(x)$ is obtained by applying the recurrence formulae (50) and (12) $N$ times. Finally, the probability function $f(x)$ is

$$f(x) = \int_{-\infty}^{x} f_N(\xi) d\xi.$$ \hfill (51)
The equations (64) – (65) yield

\[ f_{w_{2n}}(x) = \begin{cases} 
\delta(x - c_n), & n \leq n_{sw}, \\
\delta(x + c_n), & n > n_{sw}.
\end{cases} \]  

(66)

Consequently, by using (12),

\[ f_{w_{n+1}}(x) = \begin{cases} 
f_{w_{1n}}(x - c_n), & n \leq n_{sw}, \\
f_{w_{1n}}(x + c_n), & n > n_{sw}.
\end{cases} \]  

(67)

3) Random switch bang-bang disturbance: In this case, the evader also employs the bang-bang strategy (62), but the switch time \( t_{sw} \) is random, uniformly distributed over the interval \([0, t_f] \). In the discrete model (7), it is assumed that \( t_{sw} = \Delta t n_{sw} \), where \( n_{sw} \) can accept any value from the set \( \{0, 1, \ldots, N - 1\} \) with the probability \( p = \frac{1}{N} \).

Let calculate the probability

\[ p_n^+ = P(w_{2n} = c_n). \]  

(68)

Due to (62),

\[ p_n^+ = P(n \leq n_{sw}) = 1 - F_{n_{sw}}(n), \]  

(69)

where \( F_{n_{sw}}(x) \) is the probability function of \( n_{sw} \):

\[ F_{n_{sw}}(x) = \begin{cases} 
0, & x \leq 0, \\
\frac{1}{N}, & 0 < x \leq 1, \\
\frac{2}{N}, & 1 < x \leq 2, \\
\ldots \\
1, & x > N - 1,
\end{cases} \]  

(70)

yielding

\[ F_{n_{sw}}(n) = \frac{n}{N}, \quad n = 0, 1, \ldots, N - 1, \]  

(71)

and, by (69),

\[ p_n^+ = 1 - \frac{n}{N}. \]  

(72)

Therefore, the disturbance term \( w_{2n} \) has a random value

\[ w_{2n} = \begin{cases} 
c_n, & p = p_n^+, \\
-c_n, & p = 1 - p_n^+.
\end{cases} \]  

(73)

where \( p \) is the probability.

Thus, the probability function of \( w_{2n} \) is

\[ F_{w_{2n}}(x) = \begin{cases} 
0, & x \leq -c_n, \\
\frac{n}{N}, & -c_n < x \leq c_n, \\
1, & x > c_n.
\end{cases} \]  

(74)

By differentiating (74), the probability density function is

\[ f_{w_{2n}}(x) = \frac{n}{N} \delta(x + c_n) + \left(1 - \frac{n}{N}\right) \delta(x - c_n). \]  

(75)

Equation (12) along with (75) yields

\[ f_{z_{n+1}}(x) = \frac{n}{N} f_{w_{1n}}(x + c_n) + \left(1 - \frac{n}{N}\right) f_{w_{1n}}(x - c_n). \]  

(76)

4) Random value disturbance: In this case, it is assumed that for any \( t \in [0, t_f] \), the disturbance \( v(t) \) has a random value, uniformly distributed on the interval \([-1, 1]\). Thus, in the discrete model (7), the random variable \( w_{2n} \) is uniformly distributed on the interval \([-c_n, c_n]\), yielding the probability density function

\[ f_{w_{2n}}(x) = \begin{cases} 
\frac{1}{2c_n}, & x \in (-c_n, c_n], \\
0, & x \notin (-c_n, c_n].
\end{cases} \]  

(77)

The latter, along with (12), leads to

\[ f_{z_{n+1}}(x) = \int_{-c_n}^{c_n} f_{w_{1n}}(x - \xi) d\xi. \]  

(78)

C. Numerical Illustration

In this subsection the analytical results are compared to the outcome of extensive Monte Carlo simulations. It is assumed that the initial value \( z_0 \) and the estimation errors \( \eta_n \) are gaussian: \( z_0 \sim \mathcal{N}(0, 0.1), \eta_n \sim \mathcal{N}(\mu_n, \sigma_n) \), \( n = 0, \ldots, N - 1 \). The set of such values of \( \mu_n \) and \( \sigma_n \) were extracted from a realistic Monte Carlo simulation with noisy line-of-site measurements and an estimator in the control loop. For the sake of comparison, the same set of values were chosen for all the four types of disturbance. The data for the comparisons are \( t_f = 1 \) s, \( N = 10, \Delta t = 0.1 \) s, \( \tau_p = 0.2 \) s, \( \sigma_p^{max} = 30 \) m/s², \( \tau_e = 0.2 \) s, \( \sigma_e^{max} = 15 \) m/s².

The cumulative distribution function of the miss distance \( |z_N| \) is calculated as

\[ F_{|z_N|}(x) = F_{z_N}(x) - F_{z_N}(-x), \]  

(79)

where \( F_{z_n}(x) \) is given by (51).

![Fig. 3. Simulative and theoretical distribution functions of \( |z_N| \) for constant disturbance](image-url)

In Figs. 3 – 6, the cumulative distributions of \( |z_N| \), obtained by Monte Carlo simulation of (4) and by using (50), (51) and (79), are depicted and compared to the results of Monte Carlo simulation of (4) for the four different types of disturbance.
In this problem, it is assumed that the state information is corrupted by an error with known distribution and the initial state distribution is also known. Moreover, the system is subject to an additive random disturbance with known distribution. The control is realized by a saturated linear control strategy. The formulation is motivated by various real-life control problems, such as the interception problem, where validating robust transferring deterministic strategies in realistic stochastic environment is of a high practical importance.

The problem is mathematically nontrivial, because the evaluation of the sum of two dependent random variables is required. The solution is based on proper discretization of some conditional probabilities. The resulting formula allows to obtain the final state distribution without carrying out a great amount of Monte Carlo simulation runs.

The general result is used to compare the miss distance distribution in an interception problem with four different types of disturbance using a given set of estimation errors with the outcomes of a large set of Monte Carlo simulations. The numerical examples confirm that the large number of Monte Carlo runs can be replaced by using analytic formulae.

V. CONCLUSIONS

The problem of evaluating the probability distribution of the final state of a scalar discrete-time system is solved.

For all cases 2000 runs of Monte Carlo simulations were performed. In these simulations there was no estimator in the loop and the estimation errors were the same as in the analytical expressions. It is seen that in all cases the two curves are very close.

REFERENCES

