The Hamiltonian Alternating Path Problem

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***Abstract*—In this paper, we consider the Hamiltonian alternating path problem for graphs, multigraphs, and digraphs. We describe an approach to solve the problem. This approach is based on constructing logical models for the problem. We use logical models for the Hamiltonian alternating path problem to solve the Hamiltonian path problem and the planning a typical working day for indoor service robots problem. Also, we use these models for Bennett's model of cytogenetics, automatic generation of recognition modules, and algebraic data.**

***Index Terms*—Hamiltonian alternating path, Hamiltonian path, the planning a typical working day for indoor service robots problem, NP-complete, logical models.**

I. Introduction

RECENTLY, a number of Hamiltonian problems for edge-colored graphs were considered (see e.g. [1], [2]). It should be noted that in the last years the concept of alternating trails and the special cases, alternating paths and cycles, appears in various applications (see e.g. [3], [4]). In particular, we can mention that some problems in molecular biology correspond to extracting Hamiltonian or Eulerian paths or cycles colored in specified pattern [5]–[7], transportation and connectivity problems where reload costs are associated to pair of colors at adjacent edges [8], social sciences [9], VLSI optimization [10], etc. Also, there are a number of applications in graph theory and algorithms.

In this paper, we describe an approach to solve the Hamiltonian alternating path problem for graphs, multigraphs, and digraphs. This approach is based on constructing logical models for the problem. Also, we consider an application of this approach to solve the Hamiltonian path problem and the planning a typical working day for indoor service robots problem [11].

II. Preliminaries and problem definitions

Multiple edges are edges that have the same end nodes. A multigraph is a set of nodes connected by edges which is permitted to have multiple edges. Thus, in multigraph, two vertices may be connected by more than one edge. In this paper, we consider only edge-colored multigraphs. We assume that each edge of a multigraph has a color and no two multiple edges have the same color. All multigraphs considered are finite and have no loops. When multigraphs have no multiple edges, we call them graphs, as usual. A digraph is a graph where the edges have a direction associated with them.

An edge of graph with vertices x and y we denote by (x, y). In this case, we assume that $(x, y) = (y, x)$. For multigraphs, an edge with vertices x and y we denote by $z[x, y]$. In this case, we assume that $z_1[x, y] = z_2[x, y]$ if and only if $z_1 = z_2$. For digraphs, we assume that an edge (x, y) is considered to be directed from x to y. In this case, we assume that $(x, y) \neq (y, x)$.

If the number of colors is restricted by an integer c, we speak about c-edge-colored multigraphs. In this paper, we consider only simple paths and cycles. Let G be a c-edge-colored multigraph. A cycle or path in G is called alternating if its successive edges differ in color. An alternating path or cycle is called Hamiltonian if it contains all the vertices of G. If G has a Hamiltonian alternating cycle, G is called Hamiltonian. An alternating path P is called an $(x; y)$-path if x and y are the end vertices of P. The alternating Hamiltonian path problem and the alternating Hamiltonian cycle problem are problems of determining whether an alternating Hamiltonian path or an alternating Hamiltonian cycle exists in a given multigraph.

III. The alternating Hamiltonian path problem for 2-edge-colored multigraphs

The alternating Hamiltonian path problem and the alternating Hamiltonian cycle problem are extensively studied (see e.g. [3], [14]). However, the computational complexity of these problems are not quite clear. In [12], the authors noted that the alternating Hamiltonian cycle problem, even for 2-edge-colored graphs, is trivially **NP**-complete. However, the authors of [12] have not given proof or any other evidence of this fact. In [3], the authors noted that problems on alternating cycles and paths in general 2-edge-colored graphs are at least as difficult as the corresponding ones for directed cycles and paths in digraphs. The authors of [3] gave the following motivation for this fact. "To see that, we consider the following simple transformation attributed to Häggkvist in [13]. Let D be a digraph. Replace each arc xy of D by two (unoriented) edges xz_{xy} and $z_{xy}y$ by adding a new vertex z_{xy} and then colour the edge xz_{xy} red and the edge $z_{xy}y$ blue. Let G be the 2-edge-coloured graph obtained in this way. It is easy to see that each alternating cycle in G corresponds to a directed cycle in D and vice versa. Hence, in particular, the following problems on paths and cycles in 2-edge-coloured graphs are **NP**-complete: the Hamiltonian alternating cycle problem and the problem to find an alternating cycle through a pair of vertices." It should be noted that the motivation is incorrect. For instance, we can consider the digraph

$$D_1 = (\{1, 2, 3\}, \{(1, 2), (2, 3), (1, 3)\}).$$

Using the transformation, we obtain the 2-edge-colored graph

$$G_1 = (\{1, 2, 3, z_{12}, z_{23}, z_{13}\},$$

$$\{(1, z_{12}), (z_{12}, 2), (2, z_{23}),$$

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$$(z_{23},3),(1,z_{13}),(z_{13},3)\})$$

where

$$\{(1,z_{12}),(2,z_{23}),(1,z_{13})\}$$

is the set of red edges and

$$\{(z_{12},2),(z_{23},3),(z_{13},3)\}$$

is the set of blue edges. It is clear that D_1 has a Hamiltonian path. On the other hand, it is easy to check that G_1 has no alternating Hamiltonian path. Similarly, we can consider the digraph

$$D_2 = (\{1,2,3,4\},\{(1,2),(2,3),(3,4),(4,1),(1,3)\}).$$

Using the transformation, we obtain the 2-edge-colored graph

$$G_2 = (\{1,2,3,4,z_{12},z_{23},z_{34},z_{41},z_{13}\},$$

$$\{(1,z_{12}),(z_{12},2),(2,z_{23}),(z_{23},3),$$

$$(3,z_{34}),(z_{34},4),(4,z_{41}),(z_{41},1),$$

$$(1,z_{13}),(z_{13},3)\})$$

where

$$\{(1,z_{12}),(2,z_{23}),(3,z_{34}),(4,z_{41}),(1,z_{13})\}$$

is the set of red edges and

$$\{(z_{12},2),(z_{23},3),(z_{34},4),(z_{41},1),(z_{13},3)\}$$

is the set of blue edges. It is easy to see that that D_2 has a Hamiltonian cycle, but G_2 is not Hamiltonian.

In our investigations, we need an explicit proof of hardness of the alternating Hamiltonian path problem. The proof of the following Proposition 1 gives us an explicit reduction from the Hamiltonian path problem to the alternating Hamiltonian path problem.

Note that the decision version of the alternating Hamiltonian path problem for c-edge-colored multigraphs can be formulated as following.

THE ALTERNATING HAMILTONIAN PATH PROBLEM FOR c-EDGE-COLORED MULTIGRAPHS (c-AHP-M):

INSTANCE: *A c-edge-colored multigraph $G = (V,E)$ where V is the set of vertices and E is the set of edges.*

QUESTION: *Does G have an alternating Hamiltonian path?*

Proposition 1. 2-AHP-M *is* **NP**-*complete.*

Proof. It is clear that c-AHP-M is in **NP**. Therefore, we need to prove only **NP**-hardness of 2-AHP-M. Let us consider the following problem:

HAMILTONIAN PATH PROBLEM FOR GRAPHS (HP-G):

INSTANCE: *A graph $D = (A,B)$.*

QUESTION: *Does D have a Hamiltonian path?*

HP-G is **NP**-complete (cf. [15]). Now, we transform an instance of HP-G into an instance of 2-AHP-M as follows:

$$V = A,$$

$$E = \{r[x,y], b[x,y] \mid (x,y)\in B\},$$

$$G = (V,E),$$

where

$$E_r = \{r[x,y] \mid (x,y)\in B\}$$

is the set of red edges and

$$E_b = \{b[x,y] \mid (x,y)\in B\}$$

is the set of blue edges.

It is clear that if

$$(x_1,x_2),(x_2,x_3),\ldots,(x_{|A|-1},x_{|A|})$$

is a Hamiltonian path in D, then

$$r[x_1,x_2], b[x_2,x_3],\ldots,u[x_{|A|-1},x_{|A|}],$$

where $u = r$ for even $|A|$ and $u = b$ for odd $|A|$, is an alternating Hamiltonian path in G. By definition of G, it is easy to see that

$$u_1[x_1,x_2], u_2[x_2,x_3],\ldots,u_{|A|-1}[x_{|A|-1},x_{|A|}]$$

is an alternating Hamiltonian path in G, then

$$(x_1,x_2),(x_2,x_3),\ldots,(x_{|A|-1},x_{|A|})$$

is a Hamiltonian path in D. Therefore, the multigraph G has an alternating Hamiltonian path if and only if the graph D has a Hamiltonian path.

□

Let us consider the following problem:

THE ALTERNATING HAMILTONIAN CYCLE PROBLEM FOR c-EDGE-COLORED MULTIGRAPHS (c-AHC-M):

INSTANCE: *A c-edge-colored multigraph $G = (V,E)$.*

QUESTION: *Does G have an alternating Hamiltonian cycle?*

Note that Hamiltonian cycle problem for graphs is **NP**-complete (cf. [15]). Using the transformation from the proof of the Proposition 1, it is easy to check that 2-AHC-M is **NP**-complete.

IV. THE ALTERNATING HAMILTONIAN PATH PROBLEM FOR 2-EDGE-COLORED DIGRAPHS

Let us consider the following problem:

THE ALTERNATING HAMILTONIAN PATH PROBLEM FOR c-EDGE-COLORED DIGRAPHS (c-AHP-D):

INSTANCE: *A c-edge-colored digraph $G = (V,E)$.*

QUESTION: *Does G have an alternating Hamiltonian path?*

Proposition 2. 2-AHP-D *is* **NP**-*complete.*

Proof. It is easy to see that c-AHP-D is in **NP**. Therefore, we need to prove only **NP**-hardness of 2-AHP-D. Let us consider the following problem:

HAMILTONIAN PATH PROBLEM FOR DIGRAPHS (HP-D):

INSTANCE: *A digraph* $D = (A, B)$.
QUESTION: *Does* D *have a Hamiltonian path?*
HP-D is **NP**-complete (cf. [15]).

Let D be an instance of HP-D. Let $H = (A_1, B_1)$ be a digraph such that

$$A = \{a_1, a_2, \ldots, a_n\},$$

$$A_1 = A \cup \{a_{n+1}, a_{n+2}\},$$

$$B_1 = B \cup \{(a_{n+1}, a_i), (a_i, a_{n+2}) \mid 1 \le i \le n\}.$$

Let

$$(x_1, x_2), (x_2, x_3), \ldots, (x_{n-1}, x_n)$$

be a Hamiltonian path in D. It is clear that

$$(a_{n+1}, x_1), (x_1, x_2), (x_2, x_3), \ldots, (x_{n-1}, x_n), (x_n, a_{n+2})$$

is a Hamiltonian path in H. Now, let

$$(x_1, x_2), (x_2, x_3), \ldots, (x_{n+1}, x_{n+2})$$

be a Hamiltonian path in H. Since

$$(x, a_{n+1}) \notin B_1$$

for any x, it is clear that

$$x_i \ne a_{n+1}$$

where $2 \le i \le n+2$. Similarly, since

$$(a_{n+2}, x) \notin B_1$$

for any x, it is clear that

$$x_i \ne a_{n+2}$$

where $1 \le i \le n+1$. Therefore, $a_{n+1} = x_1$, $a_{n+2} = x_{n+2}$. Thence,

$$(x_2, x_3), (x_3, x_4), \ldots, (x_n, x_{n+1})$$

is a Hamiltonian path in D. Thus, the digraph H has a Hamiltonian path if and only if the digraph D has a Hamiltonian path.

Now, we transform H into an instance of 2-AHP-D as follows:

$$Z = \{z_{x_i x_j} \mid (x_i, x_j) \in B_1\},$$

$$A_2 = \{a_{n+3}, a_{n+4}, \ldots, a_{|B_1|+2}\},$$

$$V = A_1 \cup Z \cup A_2,$$

$$E = \{(x_i, z_{x_i x_j}), (z_{x_i x_j}, x_j) \mid (x_i, x_j) \in B_1\} \cup$$

$$\{(a_{n+2}, a_{n+3})\} \cup$$

$$\{(a_k, z_{x_i x_j}), (z_{x_i x_j}, a_p) \mid n+3 \le k \le |B_1|+1,$$

$$n+4 \le p \le |B_1|+2, (x_i, x_j) \in B_1\},$$

$$G = (V, E),$$

where

$$E_r = \{(x_i, z_{x_i x_j}) \mid (x_i, x_j) \in B_1\} \cup$$

$$\{(a_{n+2}, a_{n+3})\} \cup$$

$$\{(z_{x_i x_j}, a_p) \mid n+4 \le p \le |B_1|+2, (x_i, x_j) \in B_1\}$$

is the set of red edges and

$$E_b = \{(z_{x_i x_j}, x_j) \mid (x_i, x_j) \in B_1\} \cup$$

$$\{(a_k, z_{x_i x_j}) \mid n+3 \le k \le |B_1|+1, (x_i, x_j) \in B_1\}$$

is the set of blue edges.

Let

$$(a_{n+1}, x_1), (x_1, x_2), (x_2, x_3), \ldots, (x_{n-1}, x_n), (x_n, a_{n+2})$$

be a Hamiltonian path in H. It is easy to check that

$$(a_{n+1}, z_{a_{n+1} x_1}), (z_{a_{n+1} x_1}, x_1),$$

$$(x_1, z_{x_1 x_2}), (z_{x_1 x_2}, x_2),$$

$$(x_2, z_{x_2 x_3}), (z_{x_2 x_3}, x_3),$$

$$\ldots,$$

$$(x_{n-1}, z_{x_{n-1} x_n}), (z_{x_{n-1} x_n}, x_n),$$

$$(x_n, z_{x_n a_{n+2}}), (z_{x_n a_{n+2}}, a_{n+2}),$$

$$(a_{n+2}, a_{n+3}),$$

$$(a_{n+3}, u_1), (u_1, a_{n+4}),$$

$$\ldots,$$

$$(a_{|B_1|+1}, u_{|B_1|-n-1}), (u_{|B_1|-n-1}, a_{|B_1|+2}),$$

$$u_i \in Z \backslash \{z_{a_{n+1} x_1}, z_{x_1 x_2}, z_{x_2 x_3}, \ldots, z_{x_{n-1} x_n}, z_{x_n a_{n+2}}\},$$

$$u_i = u_j \Leftrightarrow i = j,$$

is an alternating Hamiltonian path in G.

Now, let

$$(x_1, x_2), (x_2, x_3), \ldots, (x_{2|B_1|}, x_{2|B_1|+1})$$

be an alternating Hamiltonian path in G. Note that $(y, a_{n+1}) \notin E$ for any y. Therefore, it is clear that $x_1 = a_{n+1}$. So, $x_2 = z_{a_{n+1}a_i}$ where $i \in \{1, 2, \ldots, n\}$. Since $(a_{n+1}, z_{a_{n+1}a_i}) \in E_r$, it is easy to see that $(x_2, x_3) = (z_{a_{n+1}a_i}, a_i)$. Similarly, $(x_j, x_{j+1}) = (a_{n_k}, z_{a_{n_k}a_{n_{k+1}}})$ or $(x_j, x_{j+1}) = (z_{a_{n_k}a_{n_{k+1}}}, a_{n_{k+1}})$ for any $3 \leq j \leq 2n+2$. Clearly,

$$(x_3, x_5), (x_5, x_7), \ldots, (x_{2n+1}, x_{2n+3})$$

is a Hamiltonian path in H. Therefore, the digraph G has an alternating Hamiltonian path if and only if the digraph D has a Hamiltonian path.

□

Let us consider the following problem:

THE ALTERNATING HAMILTONIAN CYCLE PROBLEM FOR c-EDGE-COLORED DIGRAPHS (c-AHC-D):

INSTANCE: *A c-edge-colored digraph $G = (V, E)$.*

QUESTION: *Does G have an alternating Hamiltonian cycle?*

Proposition 3. 2-AHC-D *is* **NP**-*complete.*

Proof. It is easy to see that c-AHC-D is in **NP**. Therefore, we need to prove only **NP**-hardness of 2-AHC-D.

Let D be an instance of HP-G. Let $H = (A_1, B_1)$ be a digraph defined in proof of the Proposition 2. Let

$$Z = \{z_{x_i x_j} \mid (x_i, x_j) \in B_1\},$$

$$A_2 = \{a_{n+3}, a_{n+4}, \ldots, a_{|B_1|+3}\},$$

$$V = A_1 \cup Z \cup A_2,$$

$$E = \{(x_i, z_{x_i x_j}), (z_{x_i x_j}, x_j) \mid (x_i, x_j) \in B_1\} \cup$$

$$\{(a_{n+2}, a_{n+3}), (a_{|B_1|+2}, a_{|B_1|+3})\} \cup$$

$$\{(a_k, z_{x_i x_j}), (z_{x_i x_j}, a_p) \mid n+3 \leq k \leq |B_1|+1,$$

$$n+4 \leq p \leq |B_1|+2, (x_i, x_j) \in B_1\},$$

$$G = (V, E),$$

where

$$E_r = \{(x_i, z_{x_i x_j}) \mid (x_i, x_j) \in B_1\} \cup$$

$$\{(a_{n+2}, a_{n+3})\} \cup$$

$$\{(z_{x_i x_j}, a_p) \mid n+4 \leq p \leq |B_1|+2, (x_i, x_j) \in B_1\}$$

is the set of red edges and

$$E_b = \{(z_{x_i x_j}, x_j) \mid (x_i, x_j) \in B_1\} \cup$$

$$\{(a_{|B_1|+2}, a_{|B_1|+3})\} \cup$$

$$\{(a_k, z_{x_i x_j}) \mid n+3 \leq k \leq |B_1|+1, (x_i, x_j) \in B_1\}$$

is the set of blue edges.

It is not hard to check that the digraph G has an alternating Hamiltonian cycle if and only if the digraph D has a Hamiltonian path.

□

V. LOGICAL MODELS

The satisfiability problem (SAT) was the first known **NP**-complete problem. The problem SAT is the problem of determining if the variables of a given boolean function in conjunctive normal form (CNF) can be assigned in such a way as to make the formula evaluate to true. Different variants of SAT were considered. In particular, the problem 3SAT is the problem of determining if the variables of a given 3-CNF can be assigned in such a way as to make the formula evaluate to true. Encoding problems as Boolean satisfiability (see e.g. [11], [16]–[27]) and solving them with very efficient satisfiability algorithms (see e.g. [28]–[32]) has recently caused considerable interest. In this paper we consider reductions from c-AHP-M and c-AHP-D to SAT and 3SAT.

Let $G = (V, E)$ be a 2-edge-colored multigraph, E_r is the set of red edges of G and E_b is the set of blue edges of G. Let

$$V = \{v_1, v_2, \ldots, v_n\}.$$

Let

$$\varphi[1] = \wedge_{1 \leq i \leq n} \vee_{1 \leq j \leq n} x[i, j],$$

$$\varphi[2] = \wedge_{1 \leq i \leq n} \wedge_{1 \leq j[1] < j[2] \leq n} (\neg x[i, j[1]] \vee \neg x[i, j[2]]),$$

$$\varphi[3] = \wedge_{1 \leq j \leq n} \wedge_{1 \leq i[1] < i[2] \leq n} (\neg x[i[1], j] \vee \neg x[i[2], j]),$$

$$\delta[1] = \wedge_{1 \leq i < n} \wedge_{\substack{1 \leq j[1] \leq n \\ 1 \leq j[2] \leq n \\ (v_{j[1]}, v_{j[2]}) \notin E}} (\neg x[i, j[1]] \vee$$

$$\neg x[i+1, j[2]]),$$

$$\psi[1] = \wedge_{1 \leq i < n, i = 2k-1} \wedge_{\substack{1 \leq j[1] < j[2] \leq n \\ e[v_{j[1]}, v_{j[2]}] \notin E_r}} (\neg y \vee$$

$$\neg x[i, j[1]] \vee \neg x[i+1, j[2]]),$$

$$\psi[2] = \wedge_{1 < i \leq n, i = 2k} \wedge_{\substack{1 \leq j[1] < j[2] \leq n \\ e[v_{j[1]}, v_{j[2]}] \notin E_b}} (\neg y \vee$$

$$\neg x[i, j[1]] \vee \neg x[i+1, j[2]]),$$

$$\psi[3] = \wedge_{1 \le i < n, i=2k-1} \wedge_{\substack{1 \le j[1] < j[2] \le n \\ e[v_{j[1]}, v_{j[2]}] \notin E_b}} (y \vee \neg x[i, j[1]] \vee \neg x[i+1, j[2]]),$$

$$\psi[4] = \wedge_{1 < i \le n, i=2k} \wedge_{\substack{1 \le j[1] < j[2] \le n \\ e[v_{j[1]}, v_{j[2]}] \notin E_r}} (y \vee \neg x[i, j[1]] \vee \neg x[i+1, j[2]]),$$

$$\xi[1] = (\wedge_{i=1}^{3} \varphi[i]) \wedge \delta[1] \wedge (\wedge_{j=1}^{4} \psi[j]).$$

It is clear that $\xi[1]$ is a CNF. It is easy to check that $\xi[1]$ gives us an explicit reduction from 2-AHP-M to SAT.

Now, let $G = (V, E)$ be a c-edge-colored multigraph, E_t is the set of edges of tth color where $1 \le t \le c$. Let

$$\rho[1, m] = \wedge_{1 \le i \le m} \vee_{1 \le j \le c} u[i, j],$$

$$\rho[2, m] = \wedge_{1 \le i \le m} \wedge_{1 \le j[1] < j[2] \le c} (\neg u[i, j[1]] \vee \neg u[i, j[2]]),$$

$$\rho[3, m] = \wedge_{1 \le j \le c} \wedge_{1 \le i < m} (\neg u[i, j] \vee \neg u[i+1, j]),$$

$$\eta[t] = \wedge_{1 \le i < n} \wedge_{\substack{1 \le j[1] < j[2] \le n \\ e[v_{j[1]}, v_{j[2]}] \notin E_t}} (\neg u[i, t] \vee \neg x[i, j[1]] \vee \neg x[i+1, j[2]]),$$

$$\xi[2] = (\wedge_{i=1}^{3} \varphi[i]) \wedge \delta[1] \wedge (\wedge_{j=1}^{3} \rho[j, n-1]) \wedge (\wedge_{t=1}^{c} \eta[t]).$$

Clearly, $\xi[2]$ is a CNF. It is easy to check that $\xi[2]$ gives us an explicit reduction from c-AHP-M to SAT.

Let

$$\delta[2] = \wedge_{\substack{1 \le j[1] \le n \\ 1 \le j[2] \le n \\ (v_{j[1]}, v_{j[2]}) \notin E}} (\neg x[n, j[1]] \vee \neg x[1, j[2]]),$$

$$\psi[5] = \wedge_{\substack{1 \le j[1] < j[2] \le n \\ e[v_{j[1]}, v_{j[2]}] \notin E_r}} (\neg y \vee \neg x[n, j[1]] \vee \neg x[1, j[2]])$$

and

$$\psi[6] = \wedge_{\substack{1 \le j[1] < j[2] \le n \\ e[v_{j[1]}, v_{j[2]}] \notin E_b}} (y \vee \neg x[n, j[1]] \vee \neg x[1, j[2]])$$

for $n = 2k - 1$,

$$\psi[5] = \wedge_{\substack{1 \le j[1] < j[2] \le n \\ e[v_{j[1]}, v_{j[2]}] \notin E_b}} (\neg y \vee \neg x[n, j[1]] \vee \neg x[1, j[2]])$$

and

$$\psi[6] = \wedge_{\substack{1 \le j[1] < j[2] \le n \\ e[v_{j[1]}, v_{j[2]}] \notin E_r}} (y \vee \neg x[n, j[1]] \vee \neg x[1, j[2]])$$

for $n = 2k$,

$$\xi[3] = (\wedge_{i=1}^{3} \varphi[i]) \wedge (\wedge_{p=1}^{2} \delta[p]) \wedge (\wedge_{j=1}^{6} \psi[j]),$$

$$\tau[t] = \wedge_{\substack{1 \le j[1] < j[2] \le n \\ e[v_{j[1]}, v_{j[2]}] \notin E_t}} (\neg u[n, t] \vee \neg x[n, j[1]] \vee \neg x[1, j[2]]),$$

$$\xi[4] = (\wedge_{i=1}^{3} \varphi[i]) \wedge (\wedge_{p=1}^{2} \delta[p]) \wedge (\wedge_{j=1}^{3} \rho[j, n]) \wedge (\wedge_{t=1}^{c} \eta[t]) \wedge (\wedge_{t=1}^{c} \tau[t]).$$

Note that $\xi[3]$ is a CNF and $\xi[4]$ is a CNF. It is easy to check that $\xi[3]$ gives us an explicit reduction from 2-AHC-M to SAT and $\xi[4]$ gives us an explicit reduction from c-AHC-M to SAT. Note that

$$\begin{aligned} \alpha \quad \Leftrightarrow \quad & (\alpha \vee \beta_1 \vee \beta_2) \wedge \\ & (\alpha \vee \neg \beta_1 \vee \beta_2) \wedge \\ & (\alpha \vee \beta_1 \vee \neg \beta_2) \wedge \\ & (\alpha \vee \neg \beta_1 \vee \neg \beta_2), \end{aligned} \tag{1}$$

$$\begin{aligned} \vee_{j=1}^{l} \alpha_j \quad \Leftrightarrow \quad & (\alpha_1 \vee \alpha_2 \vee \beta_1) \wedge \\ & (\wedge_{i=1}^{l-4} (\neg \beta_i \vee \alpha_{i+2} \vee \beta_{i+1})) \wedge \\ & (\neg \beta_{l-3} \vee \alpha_{l-1} \vee \alpha_l), \end{aligned} \tag{2}$$

$$\begin{aligned} \alpha_1 \vee \alpha_2 \quad \Leftrightarrow \quad & (\alpha_1 \vee \alpha_2 \vee \beta) \wedge \\ & (\alpha_1 \vee \alpha_2 \vee \neg \beta), \end{aligned} \tag{3}$$

$$\begin{aligned} \vee_{j=1}^{4} \alpha_j \quad \Leftrightarrow \quad & (\alpha_1 \vee \alpha_2 \vee \beta_1) \wedge \\ & (\neg \beta_1 \vee \alpha_3 \vee \alpha_4) \end{aligned} \tag{4}$$

where $l > 4$. Using relations (1) – (4) we can obtain explicit transformations of $\xi[i]$ into $\gamma[i]$, $1 \le i \le 4$, such that $\xi[i] \Leftrightarrow \gamma[i]$ and $\gamma[i]$ is a 3-CNF. It is not hard to check that we can use $\xi[i]$ for 2-AHC-D and c-AHC-D.

VI. SAT SOLVERS FOR ALTERNATING HAMILTONIAN PROBLEMS

There is a well known site on which solvers for SAT are posted [33]. In addition to the solvers the site also represented a large set of test problems. This set includes a randomly generated problems of 3SAT and SAT. We have designed generators of natural random instances for 2-AHP-M, c-AHP-M, 2-AHC-M, c-AHC-M, 2-AHP-D, c-AHP-D, 2-AHC-D, c-AHC-D. We used the algorithms fgrasp and posit from [33]. Also, we consider our own genetic algorithms OA[1] (see [30]), OA[2] (see [31]), and OA[3] (see [32]) for SAT which based on algorithms from [33]. We used heterogeneous cluster based on three clusters (Cluster USU, umt, um64) [34]. Each test was runned on a cluster of at least 100 nodes. Note that due to restrictions on computation time (20 hours) we used savepoints.

Selected experimental results are given in Tables I – III.

TABLE I
EXPERIMENTAL RESULTS FOR 2-AHP-M

time	average	max	best
1	26 h	214 h	42 min
2	23 h	208 h	38 min
3	32 h	226 h	46 min
4	31.4 h	213 h	43 min
5	23.1 h	236 h	32.8 min
6	20.2 h	227 h	27.3 min
7	27.9 h	249 h	49 min
8	26.8 h	234 h	47.4 min
9	2.3 h	54.2 h	2.4 min
10	1.7 h	53.8 h	1.9 min
11	9.1 h	61.3 h	2.8 min
12	8.6 h	59.7 h	3.2 min
13	8.2 h	57.3 h	27 sec
14	7.9 h	56.9 h	8 sec
15	9.4 h	64.2 h	29 sec
16	9.7 h	63.7 h	19 sec
17	2.6 h	55.9 h	2.9 min
18	1.9 h	56.7 h	1.8 min
19	9.6 h	63.2 h	3.3 min
20	9.1 h	61.5 h	3.4 min
21	8.6 h	60.1 h	36 sec
22	8.4 h	59.2 h	43 sec
23	10.1 h	66.1 h	18 sec
24	9.9 h	65.8 h	24 sec
25	1.2 h	17.2 h	3.9 min
26	47 min	6.8 h	2.7 min
27	4.3 h	13.7 h	1.5 min
28	4.2 h	11.4 h	1.2 min
29	5.6 h	16.2 h	3 sec
30	5.8 h	16.7 h	8 sec
31	6.1 h	17.9 h	11 sec
32	6.7 h	17.8 h	14 sec

In Table I, 1 – fgrasp with 3SAT reduction $\gamma[1]$ and random data from [33]; 2 – fgrasp with 3SAT reduction $\gamma[1]$ and natural random data; 3 – fgrasp with 3SAT reduction $\gamma[2]$ and random data from [33]; 4 – fgrasp with 3SAT reduction $\gamma[2]$ and natural random data; 5 – posit with 3SAT reduction $\gamma[1]$ and random data from [33]; 6 – posit with 3SAT reduction $\gamma[1]$ and natural random data; 7 – posit with 3SAT reduction $\gamma[2]$ and random data from [33]; 8 – posit with 3SAT reduction $\gamma[2]$ and natural random data; 9 – OA[1] with 3SAT reduction $\gamma[1]$ and random data from [33]; 10 – OA[1] with 3SAT reduction $\gamma[1]$ and natural random data; 11 – OA[1] with 3SAT reduction $\gamma[2]$ and random data from [33]; 12 – OA[1] with 3SAT reduction $\gamma[2]$ and natural random data; 13 – OA[1] with SAT reduction $\xi[1]$ and random data from [33]; 14 – OA[1] with SAT reduction $\xi[1]$ and natural random data; 15 – OA[1] with SAT reduction $\xi[2]$ and random data from [33]; 16 – OA[1] with SAT reduction $\xi[2]$ and natural random data; 17 – OA[2] with 3SAT reduction $\gamma[1]$ and random data from [33]; 18 – OA[2] with 3SAT reduction $\gamma[1]$ and natural random data; 19 – OA[2] with 3SAT reduction $\gamma[2]$ and random data from [33]; 20 – OA[2] with 3SAT reduction $\gamma[2]$ and natural random data; 21 – OA[2] with SAT reduction $\xi[1]$ and random data from [33]; 22 – OA[2] with SAT reduction $\xi[1]$ and natural random data; 23 – OA[2] with SAT reduction $\xi[2]$ and random data from [33]; 24 – OA[2] with SAT reduction $\xi[2]$ and natural random data; 25 – OA[3] with 3SAT reduction $\gamma[1]$ and random data from [33]; 26 – OA[3] with 3SAT reduction $\gamma[1]$ and natural random data; 27 – OA[3] with 3SAT reduction $\gamma[2]$ and random data from [33]; 28 – OA[3] with 3SAT reduction $\gamma[2]$ and natural random data; 29 – OA[3] with SAT reduction $\xi[1]$ and random data from [33]; 30 – OA[3] with SAT reduction $\xi[1]$ and natural random data; 31 – OA[3] with SAT reduction $\xi[2]$ and random data from [33]; 32 – OA[3] with SAT reduction $\xi[2]$ and natural random data. It is easy to see that OA[3] demonstrates the best performance. Note that SAT reductions give us slightly better best time, but 3SAT reductions give us significantly better average time. It should be noted that these observations are also valid for c-AHP-M, 2-AHC-M, c-AHC-M, 2-AHP-D, c-AHP-D, 2-AHC-D, c-AHC-D. Therefore, we consider only OA[3] with 3SAT reductions in in Tables II and III.

TABLE II
EXPERIMENTAL RESULTS FOR c-AHP-M, 2-AHC-M, AND c-AHC-M

time	average	max	best
1	4.9 h	16.8 h	2.1 min
2	4.3 h	13.2 h	1.7 min
3	5.1 h	19.2 h	5.7 min
4	4.7 h	14.1 h	4.4 min
5	7.2 h	28.1 h	11 min
6	6.3 h	16.3 h	9.2 min

In Table II, 1 – OA[3] for c-AHP-M with 3SAT reduction $\gamma[2]$ and random data from [33]; 2 – OA[3] for c-AHP-M with 3SAT reduction $\gamma[2]$ and natural random data; 3 – OA[3] for 2-AHC-M with 3SAT reduction $\gamma[3]$ and random data from [33]; 4 – OA[3] for 2-AHC-M with 3SAT reduction $\gamma[3]$ and natural random data; 5 – OA[3] for c-AHC-M with 3SAT reduction $\gamma[4]$ and random data from [33]; 6 – OA[3] for c-AHC-M with 3SAT reduction $\gamma[4]$ and natural random data.

TABLE III
EXPERIMENTAL RESULTS FOR 2-AHP-D, c-AHP-D, 2-AHC-D, AND c-AHC-D

time	average	max	best
1	19 min	2.3 h	57 sec
2	9 min	1.6 h	32 sec
3	1.3 h	4.7 h	1.9 min
4	1.1 h	3.4 h	1.1 min
5	2.2 h	5.3 h	3.8 min
6	1.9 h	3.9 h	2.4 min
7	4.3 h	21.4 h	6.3 min
8	3.8 h	9.7 h	4.8 min

In Table III, 1 – OA[3] for 2-AHP-D with 3SAT reduction $\gamma[1]$ and random data from [33]; 2 – OA[3] for 2-AHP-D with 3SAT reduction $\gamma[1]$ and natural random data;

3 – OA[3] for c-AHP-D with 3SAT reduction $\gamma[2]$ and random data from [33]; 4 – OA[3] for c-AHP-D with 3SAT reduction $\gamma[2]$ and natural random data; 5 – OA[3] for 2-AHC-D with 3SAT reduction $\gamma[3]$ and random data from [33]; 6 – OA[3] for 2-AHC-D with 3SAT reduction $\gamma[3]$ and natural random data; 7 – OA[3] for c-AHC-D with 3SAT reduction $\gamma[4]$ and random data from [33]; 8 – OA[3] for c-AHC-D with 3SAT reduction $\gamma[4]$ and natural random data.

VII. Bennett's model of citogenetics

Regularities in a biological sequence can be used to identify important knowledge about the underlying biological system (see e.g. [35]–[37]). In particular, the study of genome rearrangements has drawn a lot of attention in recent years (see e.g. [38], [39]). According to Bennett's model of cytogenetics (see e.g. [40]) the order in chromosome complements is based on a similarity relation which gives rise to a multigraph G. The multigraph G is the edge disjoint union of two of its subgraphs G_1 and G_2. We can assume that the edges of G_1 is the set of red edges and the edges of G_2 is the set of blue edges. The order of the chromosomes is determined by an alternating Hamiltonian path of G (see e.g. [40]).

We have designed generators of natural instances of 2-AHP-M for Bennett's model of cytogenetics. Selected experimental results are given in the Table IV.

TABLE IV
Experimental results for Bennett's model of cytogenetics

time	average	max	best
1	36 min	5.2 h	3.4 min
2	24.1 min	3.7 h	23 sec

In Table IV, 1 – OA[3] with 3SAT reduction $\gamma[1]$; 2 – OA[3] with 3SAT reduction $\gamma[2]$. It should be noted that in this case $\gamma[2]$ gives us significantly better performance.

VIII. Automatic generation of recognition modules

Usage of specialized modules of recognition gives us a significant advantage in solving concrete robotic and technical vision tasks (see e.g. [41]–[43]). However, in this case, we need a large number of such modules. It is clear that we get a significant advantage if we use an automatic generation of such modules (see e.g. [44]). One of the main problems of automatic generation of recognition modules is a problem of study of new objects. In many self-learning systems, to solve this problem we need a training set which represents a set of configurations of the system and a set of confirmations and contradictions for the object (see e.g. [44], [45]). It is easy to see that such training set can be represented by some 2-edge-colored digraph D. In this case, to setup a self-learning process we need to solve 2-AHP-D for D. We have designed a generator of natural instances for automatic generation of recognition modules. Selected experimental results are given in the Table V.

In Table V, 1 – OA[3] with 3SAT reduction $\gamma[1]$; 2 – OA[3] with 3SAT reduction $\gamma[2]$.

TABLE V
Experimental results for automatic generation of recognition modules

time	average	max	best
1	31.4 min	7.5 h	2.9 min
2	54.6 min	11.1 h	19.3 min

IX. Robot self-awareness

Robot self-awareness is an another area where we need to consider a set of configurations of the system and a set of confirmations and contradictions for the object. In particular, we have considered Occam's razor model to anticipate collisions of the mobile robot with objects from the environment [46].

Using a model of on-line navigation in environments with dynamic obstacles, the robot needs to solve the question "Is it true that an obstacle 1 has continued behind an obstacle 2?" and to construct a local strategy of motion depending on the answer. To solve the question the robot can use some Occam's razor model. Such model can be constructed by some genetic algorithm. Another genetic algorithm can be used to construct a local strategy of motion depending on the answer. Usage of two genetic algorithms requires some solution of the problem of their co-training. We use a sequential approach to learning. First, we train a genetic algorithm for Occam's razor model. After this, we train a genetic algorithm for construction of a local strategy of motion depending on the answer.

It should be noted that quality of learning of a genetic algorithm for construction of a local strategy depends essentially on the distribution of obstacles (see Tables VI and VII).

TABLE VI
Quality of prediction of a genetic algorithm with different probability of answer "yes"

number of generations	$p = 0.5$	$p = 0.3$	$p = 0.1$	$p = 0.7$	$p = 0.9$
0	43 %	37 %	49 %	38 %	47 %
10	44 %	39 %	50 %	39 %	48 %
10^2	46 %	41 %	53 %	40 %	49 %
10^3	55 %	42 %	54 %	41 %	50 %
10^4	71 %	47 %	55 %	45 %	52 %
10^5	94 %	56 %	56 %	55 %	53 %
10^6	97 %	68 %	59 %	63 %	57 %

In Table VI, we consider random distributions where p is the probability of answer "yes".

TABLE VII
Quality of prediction of a genetic algorithm with probability $p = 0.5$ of answer "yes"

number of generations	constant	polynomial	exponential
0	46 %	44 %	41 %
10	48 %	45 %	42 %
10^2	50 %	47 %	43 %
10^3	60 %	53 %	48 %
10^4	77 %	63 %	56 %
10^5	97 %	85 %	74 %
10^6	99 %	91 %	86 %

In Table VII, we consider sequences of answers with constant, polynomial, and exponential subword complexities.

In view of results of Tables VI and VII, it is of interest to consider 2-AHP-D to generate a set of configurations of the system and a set of confirmations and contradictions for the object. Selected experimental results are given in the Table VIII.

TABLE VIII
THE DEPENDENCE OF THE NUMBER OF GENERATIONS AND QUALITY OF PREDICTION OF THE GENETIC ALGORITHM

0	10	10^2	10^3	10^4
44 %	47 %	58 %	83 %	96 %

X. ALGORITHMIC PROBLEMS IN ALGEBRA AND GRAPHS

Many algorithmic problems of algebra are solved by investigation of sequences of applications of defining relations. In particular, we can mention algorithmic problems of rings (see e.g. [47]–[50]), semigroups (see e.g. [51]–[55]), and groups (see e.g. [56]). Such sequences can be represented by different graph models.

There are a number of general connections between graphs and algorithmic problems in algebra. In particular, we can mention Cayley networks (see e.g. [39]), rewrite systems (see e.g. [57]), configuration graph for interpretations of Minsky machines (see e.g. [58]–[65]) and Turing machines (see e.g. [66]), etc. Note that in case of Minsky machines and Turing machines, traditional connections with graphs are slightly inconvenient. In Minsky machines and Turing machines, we consider directed transformations. In algebraic models, we consider undirected transformations. Therefore, it is natural to use Häggkvist transformation [13] and consider 2-edge-colored multigraphs instead of traditional digraphs.

Using this approach we have created a test set based on the algebraic data. Selected experimental results are given in the Table IX.

TABLE IX
EXPERIMENTAL RESULTS FOR THE ALGEBRAIC DATA

time	average	max	best
1	38.2 min	4.9 h	7.2 min
2	1.8 h	16.3 h	43.6 min

In Table IX, 1 – OA[3] with 3SAT reduction $\gamma[1]$; 2 – OA[3] with 3SAT reduction $\gamma[2]$.

XI. THE PLANNING A TYPICAL WORKING DAY FOR INDOOR SERVICE ROBOTS PROBLEM

Different problems of planning and scheduling are among the most rapidly developing areas of modern computer science (see e.g. [67]–[71]). In particular, planning problems for mobile robots are of considerable interest for many years (see e.g. [11], [18], [22], [31], [72]–[81]). In [11], we have obtained an explicit reduction from the planning a typical working day for indoor service robots problem to HP-D. Also, in [11], we have used an explicit reduction [82] from HP-D to 3SAT. Note that we can use $\gamma[1]$ and $\gamma[2]$ to to create a solver for the planning a typical working day for indoor service robots problem and HP-D. We have designed generators of natural instances for the planning a typical working day for indoor service robots problem and HP-D. Selected experimental results are given in the Table X.

TABLE X
EXPERIMENTAL RESULTS FOR THE PLANNING A TYPICAL WORKING DAY FOR INDOOR SERVICE ROBOTS PROBLEM

time	average	max	best
1	35.2 sec	57.3 min	2.3 sec
2	2.7 min	6.1 h	19 sec
3	3.2 min	8.4 h	23 sec
4	1.2 h	11.2 h	22.1 min
5	51.8 min	14.1 h	16.3 min

In Table X, 1 – OA[3] with 3SAT reduction $\gamma[1]$; 2 – OA[3] with 3SAT reduction $\gamma[2]$; 3 – OA[3] with 3SAT reduction [82]; 4 – fgrasp with 3SAT reduction [82]; 5 – posit with 3SAT reduction [82].

XII. THE HAMILTONIAN PATH PROBLEM

It is easy to see that we can use SAT solvers for 2-AHP-M and c-AHP-M to solve HP-D. Selected experimental results are given in the Table XI.

TABLE XI
EXPERIMENTAL RESULTS FOR THE HAMILTONIAN PATH PROBLEM

time	average	max	best
1	15.2 min	2.7 h	43 sec
2	26.1 min	5.4 h	8.4 min

In Table XI, 1 – OA[3] with 3SAT reduction $\gamma[1]$; 2 – OA[3] with 3SAT reduction $\gamma[2]$.

XIII. CONCLUSION

In this paper we have considered an approach to create solvers for alternating Hamiltonian problems. In particular, explicit polynomial reductions from the decision versions of these problems to 3SAT is constructed. Also, we have considered computational experiments for alternating Hamiltonian problems, Bennett's model of cytogenetics, automatic generation of recognition modules, algebraic data, the planning a typical working day for indoor service robots problem, and the Hamiltonian path problem.

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