An Optimal Quantity Discount Policy for Deteriorating Items with a Single Wholesaler and Two Retailers

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Abstract—In Japanese large-scale super markets, the same items are sold at different departments in the same store. In many cases, managers of these departments independently order the items since they are under competition and are separately evaluated by their superiors. They can possibly reduce their costs if either of them purchases the items for two departments in cooperation with each other. This study considers the quantity discount problem between a single wholesaler and two buyers (retailers). The wholesaler attempts to increase her/his profit by controlling the buyer’s order quantity through a quantity discount strategy. The buyers try to maximize their profits by considering both whether to cooperate with each other and whether to accept the seller’s offer. We formulate the above problem as a Stackelberg game between a single wholesaler and two buyers to analyze the existence of the seller’s optimal quantity discount pricing policy, which maximizes her total profit per unit of time. The same problem is also formulated as a cooperative game. Numerical examples are presented to illustrate the theoretical underpinnings of the proposed model.

Index Terms—quantity discounts, deteriorating items, total profit, Stackelberg game.

I. INTRODUCTION

In Japanese large-scale super-markets, the same items are sold at different departments in the same store. For instance, assorted sushi boxes are sold both at the ready-made dish department (sozai corner) and the fish department. Managers of these departments independently order the fish as ingredients for sushi since they are under competition and are separately evaluated by their superiors. They can possibly reduce their costs if either of them purchases the items for two departments in cooperation with each other. Several super-markets have recently applied this strategy.

This study discusses the quantity discount problem[1], [2], [3], [4], [5], [6] between a single wholesaler (wholesaler) and two buyers (retailers). The wholesaler purchases items from upper-leveled supplier and sells them to two retailers. The wholesaler attempts to increase her/his profit by controlling the retailers’ order quantities through a quantity discount strategy. The retailers try to maximize their profits by considering both whether to cooperate with each other and whether to accept the wholesaler’s offer. We consider the case where the retailers deal in perishable items such as fresh fruits, fishes, sushi boxes and vegetables, and where both the wholesaler’s and the retailers’ inventory levels are continuously depleted due to the combined effects of its demand and deterioration. Yang[7] and Kawakatsu[8] have developed the model to determine an optimal pricing and a ordering policy for deteriorating items with quantity discounts. However, they focused on the quantity discount problem between a single seller and a single buyer.

Our previous work[9] has formulated the above problem as a Stackelberg game between a single wholesaler and two buyers to analyze the existence of the wholesale’s optimal quantity discount pricing policy, which maximizes her/his total profit per unit of time. In this study, we also formulate the same problem as a cooperative game. Numerical examples are presented to illustrate the theoretical underpinnings of the proposed model.

II. NOTATION AND ASSUMPTIONS

The wholesaler uses a quantity discount strategy in order to improve her/his profit. The wholesaler proposes, for the retailers, an order quantity per lot along with the corresponding discounted wholesale price, which induces the retailers to alter their replenishment policies. We consider two options throughout the present study as follows:

Option $V_1$: The retailer $i$ ($i = 1, 2$) does not adopt the quantity discount proposed by the wholesaler. When the retailer $i$ chooses this option, she/he purchases the products from the wholesaler at an initial price in the absence of the discount, and she/he determines her/himself an optimal order quantity which maximizes her/his own total profit per unit of time.

Option $V_2$: The retailer $i$ accepts the quantity discount proposed by the wholesaler.

The main notations used in this paper are listed below:

- $Q^j_i$: the order quantity per lot for the retailer $i$ under Option $V^j_i$ ($i = 1, 2$, $j = 1, 2$). $T^j_i$: the length of the order cycle for the retailer $i$ under Option $V^j_i$.
- $h_i$: the inventory holding cost for the retailer $i$ per item and unit of time.
- $a_i$: the ordering costs per lot for the retailer $i$.
- $\theta_i$: the deterioration rate of the retailer $i$’s inventory.
- $p_i$: the retailers’ unit selling price, i.e., unit purchasing price for their customers.
- $\mu_i$: the constant demand rate of the product for the retailer $i$.
- $c_i$: the wholesaler’s unit acquisition cost (unit purchasing cost from the upper-leveled manufacturer).

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Fig. 1. Transition of Retailers’ Inventory Level

\[ \begin{align*}
Q_{t}^{(0)}(j) & = 0 \\
T_{t}^{(0)}(j) & = T_{0}^{(0)} \\
Q_{t}(j) & = T_{2}(j) - T_{1}(j) \\
T_{t}(j) & = T_{2}(j)
\end{align*} \]

A. Under Option V1

If the retailer \( i \) chooses Option V1, her/his order quantity per lot and her/his unit acquisition cost are respectively given by \( Q_{t}^{(1)} = Q \left( T_{t}^{(1)} \right) \) and \( p_{s} \), where \( p_{s} \) is the unit initial price in the absence of the discount. In this case, she/he determines her/himself the optimal order quantity \( Q_{t}^{(1)} = Q_{t}^{*} \) which maximizes her/his total profit per unit of time.

Under assumption 1), Kawakatsu et al.[9] have formulated the retailer \( i \)'s total profit per unit of time, which is given by

\[ \pi_{t}^{(1)}(T_{t}^{(1)}) = p_{g} \int_{0}^{T_{t}^{(1)}} \mu dt - p_{s} Q \left( T_{t}^{(1)} \right) - h_{1} \int_{0}^{T_{t}^{(1)}} I_{B}(t) dt - a_{i} \]

where \( \mu = \mu_{i}/\theta_{i}. \)

We have also shown that there exists a unique finite \( T_{t}^{(1)} = T_{t}^{(1)*} (> 0) \) which maximizes \( \pi_{t}^{(1)}(T_{t}^{(1)}) \) in Eq. (3). The optimal order quantity is therefore given by

\[ Q_{t}^{(1)*} = Q \left( T_{t}^{(1)*} \right) = \rho_{i} \left[ e^{\theta_{i}} e^{\theta_{t}} - 1 \right]. \]

The total profit per unit of time becomes

\[ \pi_{t}^{(1)*} = \rho_{i} \left[ (p_{g} \theta_{i} + h_{i}) - \theta_{i} \left( p_{s} + h_{i} \right) e^{\theta_{t}} e^{\theta_{t}} \right]. \]

B. Under Option V2

If the retailer \( i \) chooses Option V2, the order quantity and unit discounted wholesale price are respectively given by \( Q_{t}^{(2)} = Q \left( T_{t}^{(2)} \right) = \rho_{i} \left[ e^{\theta_{t}} e^{\theta_{t}} - 1 \right] \) and \( (1 - y) p_{s} \). The retailer \( i \)'s total profit per unit of time can therefore be expressed by

\[ \pi_{t}^{(2)}(T_{t}^{(2)}, y) = \rho_{i} \left( p_{g} \theta_{i} + h_{i} \right) - \left( 1 - y \right) p_{s} - \frac{h_{i}}{\theta_{i}} Q \left( T_{t}^{(2)} \right) + a_{i}. \]

Let \( p^{(1)} \) and \( p^{(2)} \) be define by \( p^{(1)} = p_{s} \) and \( p^{(2)} = (1 - y) p_{s} \), respectively, then \( \pi_{t}^{(1)}(T_{t}^{(1)}) \) in Eq. (3) and \( \pi_{t}^{(2)}(T_{t}^{(2)}, y) \) in Eq. (6) can be rewritten as follows:

\[ \pi_{t}^{(j)} = \rho_{i} \left( p_{g} \theta_{i} + h_{i} \right) - \frac{h_{i}}{\theta_{i}} Q \left( T_{t}^{(j)} \right) + a_{i}. \]

IV. RETAILERS’ OPTIMAL POLICY UNDER THE COOPERATIVE GAME

This section discusses a cooperative game between two retailers. In this study, we focus on the situation where there are two departments in the same store, and therefore we assume that the transportation cost of the product from one retailer to the other is zero. This signifies that the retailers can possibly reduce their costs by adopting the strategy that either of the retailers purchases the products from the wholesaler and stocks them, and then she/he distributes the products to the other retailer.

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Based on these observations, the joint profit function per unit of time can therefore be expressed by

\[
J \left( T_i^{(j)} \right) = \frac{\mu_1 + \mu_2}{\theta_i} \left( p_s \theta_i + h_i \right) - \left( p_s + \frac{h_s}{\theta_i} \right) Q \frac{T_i^{(j)}}{T_i^{(j)}} + a_i.
\]

(8)

A. Under Option \( V_1 \)

Under Option \( V_1 \), we can prove that there exist a unique finite positive \( T_i^{(1)} = T_i^{(1)*} \), which maximizes \( J \left( T_i^{(j)} \right) \) in Eq. (8), and the maximum joint profit becomes

\[
J^{(1)*} = \max_{i=1,2} J_i^{(1)},
\]

(9)

where \( J_i^{(1)} = \frac{\mu_1 + \mu_2}{\theta_i} \times \left[ (p_s \theta_i + h_i) - (p_s \theta_i + h_i) e^{\theta_i T_i^{(1)*}} \right] \).

(10)

Equation (10) signifies a local maximum value of the joint profit when the retailer \( i \) is in charge of ordering and inventory management.

Let \( R \) denote the retailer who is in charge of ordering and inventory control and bargains with the wholesaler on behalf of two retailers, and then \( R \) is given by

\[
R = \begin{cases} 
1, & \text{if } J_1^{(1)} \geq J_2^{(1)}, \\
2, & \text{if } J_1^{(1)} < J_2^{(1)}. 
\end{cases}
\]

(11)

The analysis with respect to comparing \( J_1^{(1)} \) with \( J_2^{(1)} \) becomes considerably complicated since Eq. (10) includes the term \( T_i^{(1)*} \) which is determined by a nonlinear equation solution. Neglecting higher order terms of \( \theta_i \) in the expansion of \( e^{\theta_i T_i^{(1)}} \), we have \( e^{\theta_i T_i^{(1)}} \approx 1 + \theta_i T_i^{(1)} + [\theta_i T_i^{(1)}]^2 / 2 \).

In this case, \( J_1^{(1)} \) in Eq. (9) can be expressed as

\[
J_1^{(1)*} = \begin{cases} 
J_1^{(1)}, & \text{if } a_1(p_s \theta_i + h_i) \leq a_2(p_s \theta_i + h_i), \\
J_2^{(1)}, & \text{if } a_1(p_s \theta_i + h_i) > a_2(p_s \theta_i + h_i).
\end{cases}
\]

(12)

It can also be shown in this case that \( J_{R}^{(1)} > \sum_{i=1}^{2} \pi_i^{(1)*} \).

We therefore focus on the case where \( J_{R}^{(1)} > \sum_{i=1}^{2} \pi_i^{(1)*} \) in the following sections.

B. Under Option \( V_2 \)

The wholesaler offers the quantity discount to the retailer \( R (R = 1, 2) \) which is defined in Eq. (11).

Under Option \( V_2 \), the retailer \( R \)'s joint profit per unit of time can be expressed by

\[
J_2^{(2)} \left( T_R^{(2)}, y \right) = \frac{\mu_1 + \mu_2}{\theta_R} \left( p_s \theta_R + h_R \right) - \left[ (1 - y)p_s + \frac{h_s}{\theta_R} \right] Q \frac{T_R^{(2)}}{T_R^{(2)}} + a_R.
\]

(13)

V. RETAILERS’ OPTIMAL RESPONSE AND SHAPLEY VALUE IMPUTATION

A. Retailers’ optimal response

This subsection discusses the retailer \( R \)'s optimal response. The retailer \( R \) prefers Option \( V_1 \) over Option \( V_2 \) if \( J_1^{(1)*} > J_2^{(2)} \), but when \( J_1^{(1)*} < J_2^{(2)} \), she/he prefers \( V_2 \) to \( V_1 \). The retailer \( R \) is indifferent between the two options if \( J_1^{(1)*} = J_2^{(2)} \left( T_R^{(2)}, y \right) \), which is equivalent to

\[
y = \frac{1}{p_s Q} T_R^{(2)} \left( 1 - \rho e^{\theta_R T_R^{(2)} e^{\theta_R T_R^{(2)*}}} \right) \times \left( p_s + \frac{h_R}{\theta_R} + a_R \right).
\]

(14)

Let us denote, by \( \psi \left( T_R^{(2)} \right) \), the right-hand-side of Eq. (14). It can easily be shown from Eq. (14) that \( \psi(T_2) \) is increasing in \( T_R^{(2)} \), which maximizes the joint profit is given by

\[
J_{\pi}^{(1)*} = \begin{cases} 
J_1^{(1)*}, & \text{if } J_1^{(1)*} \geq J_2^{(2)} \left( T_R^{(2)}, y \right), \\
J_2^{(2)} \left( T_R^{(2)}, y \right), & \text{if } J_1^{(1)*} < J_2^{(2)} \left( T_R^{(2)}, y \right).
\end{cases}
\]

(15)

B. Shapley value imputation

We focus on the case where two retailers maximize their joint profit and share their cooperative profit according to the Shapley value \([10], [11]\). In this subsection, we determine the retailers’ allocation of profit based on the concept of Shapley value. The Shapley value is one of the commonly used sharing mechanisms in static cooperation games with transferable payoff\([10], [11]\).

Let \( x_i \) denote the retailer \( i \)'s allocation of the cooperative profit \( (i = 1, 2) \). In this study, \( x = (x_1, x_2) \) can be called an imputation \([9]\), and then \( x_1 \) and \( x_2 \) are respectively given by

\[
x_1 = \frac{\pi_1^{(1)*} + J_{\pi}^{(1)*} - \pi_2^{(1)*} \cdot 2}{2},
\]

(16)

\[
x_2 = \frac{J_{\pi}^{(1)*} - \pi_1^{(1)*} + \pi_2^{(1)*} \cdot 2}{2}.
\]

(17)

VI. WHOLESALER’S TOTAL PROFIT AND OPTIMAL POLICY

The retailers adopt the cooperative strategy to increase their profit as mentioned in Section IV. The wholesaler can therefore regard the retailers as a single retailer since either of the retailers is in charge of ordering and inventory management. In this case, the wholesaler’s total profit per unit of time can be formulated in the same manner as our previous formulation\([8]\). For this reason, in the following we briefly summarize the results associated with the wholesaler’s profit under Option \( V_1 \) and \( V_2 \) and her/his optimal policy.

The length of the wholesaler’s order cycle is given by \( N^{(j)} T_R^{(j)} \) under Option \( V_j \ (j = 1, 2) \), where \( N^{(j)} \) is a positive integer. This is because the wholesaler can possibly improve her/his total profit by increasing the length of her/his order cycle from \( T_R^{(j)} \) to \( N^{(j)} T_R^{(j)} \).
The wholesaler’s inventory is only depleted by deterioration except when the wholesaler ships the products to the retailer \( R \), as in assumption (1). The wholesaler’s inventory level, \( I_S(t) \), at time \( t \) can therefore be expressed by the following differential equation:

\[
dI_S(t)/dt = -\theta_s I_S(t),
\]

with a boundary condition \( I_S(jT_R^{(2)}) = z_k(T_R^{(2)}) \) under Option \( V_j \), where \( z_k(T_R^{(2)}) \) denotes the remaining inventory at the end of the \( k \)th shipping cycle.

In this case, the wholesaler’s total profit per unit of time under Option \( V_1 \) is given by

\[
P^{(1)}\left(N^{(1)}, T_R^{(1)*}\right) = \left(p_s + \frac{h}{\theta_R} \right) Q(T_R^{(1)*}) - \left(c_s + \frac{h}{\theta_R} \right) S\left(N^{(1)}, T_R^{(1)*}\right) + a_s,
\]

In contrast, under Option \( V_2 \), the wholesaler’s total profit per unit of time becomes

\[
P^{(2)}\left(N^{(2)}, T_R^{(2)}, y\right) = \left(1 - y\right)p_s + \frac{h}{\theta_R} Q(T_R^{(2)}) - c_s + \frac{h}{\theta_R} S\left(N^{(2)}, T_R^{(2)}\right) + a_s,
\]

where

\[
Q\left(T_R^{(2)}\right) = \frac{\mu_1 + \mu_2}{\theta_R} \left(\theta_a T_R^{(2)} - 1\right),
\]

\[
S\left(N^{(2)}, T_R^{(2)}\right) = \frac{Q\left(T_R^{(2)}\right)^\frac{\gamma}{\theta_R}}{e^\theta T_R^{(2)}/\gamma - 1} - 1.
\]

The wholesaler’s optimal values for \( T_R^{(2)} \) and \( y \) can be obtained by maximizing her/his total profit per unit of time considering the retailer \( R \)’s optimal response which was discussed in Subsection V-A. Henceforth, let \( \Omega_j \) \((j = 1, 2) \) be defined by

\[
\Omega_1 = \left\{ \left(T_R^{(2)}, y\right) \mid y \leq \psi\left(T_R^{(2)}\right) \right\},
\]

\[
\Omega_2 = \left\{ \left(T_R^{(2)}, y\right) \mid y \geq \psi\left(T_R^{(2)}\right) \right\}.
\]

Figure 2 depicts the region of \( \Omega_j \) \((j = 1, 2) \) on the \((T_R^{(2)}, y)\) plane.

### A. Optimal Policy under Option \( V_1 \)

If \( \left(T_R^{(2)}, y\right) \in \Omega_1 \setminus \Omega_2 \) in Fig. 2, the retailer \( R \) will naturally select Option \( V_1 \). In this case, the wholesaler can maximize her/his total profit per unit of time independently of \( T_2 \) and \( y \) on the condition of \( \left(T_R^{(2)}, y\right) \in \Omega_1 \setminus \Omega_2 \). Hence, the wholesaler’s locally maximum total profit per unit of time in \( \Omega_1 \setminus \Omega_2 \) becomes

\[
P^{(1)*} = \max_{N^{(1)} \in N} P^{(1)}\left(N^{(1)}, T_R^{(1)*}\right),
\]

where \( N \) signifies the set of positive integers.

### B. Optimal Policy under Option \( V_2 \)

On the other hand, if \( \left(T_R^{(2)}, y\right) \in \Omega_2 \setminus \Omega_1 \), the retailer \( R \)'s optimal response is to choose Option \( V_2 \). Then the wholesaler’s locally maximum total profit per unit of time in \( \Omega_2 \setminus \Omega_1 \) is given by

\[
P^{(2)*} = \max_{N^{(2)} \in N} \hat{P}^{(2)}\left(N^{(2)}\right),
\]

where

\[
\hat{P}^{(2)}\left(N^{(2)}\right) = \max_{\left(T_R^{(2)}, y\right) \in \Omega_2 \setminus \Omega_1} P^{(2)}\left(N^{(2)}, T_R^{(2)}, y\right).
\]

More precisely, we should use ”sup” instead of ”max” in Eq. (25).

For a given \( N^{(2)} \), we show below the existence of the wholesaler’s optimal quantity discount pricing policy \( \left(T_R^{(2)}, y\right) = \left(T_R^{(2)}, y^*\right) \) which attains Eq. (25). It can easily be proven that \( P^{(2)}\left(N^{(2)}, T_R^{(2)}, y\right) \) in Eq. (20) is strictly decreasing in \( y \), and consequently the wholesaler can attain \( \hat{P}^{(2)}\left(N^{(2)}\right) \) in Eq. (25) by letting \( y = \psi\left(T_R^{(2)}\right) \). By letting \( y = \psi\left(T_R^{(2)}\right) \) in Eq. (20), the total profit per unit of time on \( y = \psi\left(T_R^{(2)}\right) \) becomes

\[
P^{(2)}\left(N^{(2)}, T_R^{(2)}\right) = \frac{\mu_1 + \mu_2}{\theta_R} \left(p_s \theta_a + h_R\right) e^{\theta_a T_R^{(2)}} - 1.
\]

\[
\hat{P}^{(2)}\left(N^{(2)}\right) = \max_{\left(T_R^{(2)}, y\right) \in \Omega_2 \setminus \Omega_1} P^{(2)}\left(N^{(2)}, T_R^{(2)}, y\right).
\]

Let us now define \( L\left(N^{(2)}\right) \) as follows:

\[
L\left(N^{(2)}\right) = C \theta_T Q \left(T_R^{(2)}\right) \left(e^{\theta T_2} - 1\right)^2 - \frac{1}{N^{(2)} T_R^{(2)}} \left[C \cdot S\left(N^{(2)}, T_R^{(2)}\right) - H\left(N^{(2)}\right) Q\left(T_R^{(2)}\right) + a_s N^{(2)} + a_s\right],
\]

where

\[
C = \left(c_s + h_s/\theta_R\right),
\]

\[
H\left(N^{(2)}\right) = \left(h_s/\theta_s - h_R/\theta_R\right) N^{(2)}.
\]
\[
\begin{align*}
\left[ \rho \theta_R e^{\rho \eta T_R^{(2)}} - Q \left( T_R^{(2)} \right) \right] \\
\times \left[ \frac{e^{N(2) \theta_R T_R^{(2)}} - 1}{\theta_R T_R^{(2)} - 1} - H \left( N(2) \right) \right].
\end{align*}
\]  

(29)

We here summarize the results of analysis in relation to the optimal quantity discount policy which attains \( p^{(2)} \) \((N(2))\) in Eq. (25) when \( N(2) \) is fitted to a suitable value.

1) \( N(2) = 1 \):

In this case, there exists a unique finite \( T_R^{(2)} \) \((> T_R^{(1)*})\) which maximizes \( p^{(2)} \) \((N(2), T_R^{(2)})\) in Eq. (26), and therefore \( T_R^{(2)*}, y^* \) is given by

\[
\left( T_R^{(2)*}, y^* \right) \rightarrow \left( \tilde{T}_R, \tilde{y} \right),
\]

(30)

where \( \tilde{y} = \psi \left( \tilde{T}_R^2 \right) \).

The wholesaler’s total profit then becomes

\[
P^{(2)} \left(N(2)\right) = \frac{\mu_1 + \mu_2}{\theta_R} \left[ (p_0 \theta_R + h_R) e^{\rho \eta T_R^{(1)*}} + (\epsilon_s \theta_R + h_R) e^{\rho \eta T_R^{(2)*}} \right].
\]

(31)

2) \( N(2) \geq 2 \):

Let us define \( \tilde{T}_R^{(2)} \) \((> T_R^{(1)*})\) as the unique solution (if it exists) to

\[
L(T_R^{(2)}) = a_R N(2) + a_s.
\]

(32)

In this case, the optimal quantity discount pricing policy is given by Eq. (30).

C. Optimal Policy under Option \( V_1 \) and \( V_2 \)

In the case of \( \left( T_R^{(2)}, y \right) \in \Omega_1 \cap \Omega_2 \), the retailer is indifferent between Option \( V_1 \) and \( V_2 \). For this reason, this study confines itself to a situation where the wholesaler does not use a quantity discount policy \( \left( T_R^{(2)}, y \right) \in \Omega_1 \cap \Omega_2 \).

VII. WHOLESALER’S OPTIMAL POLICY UNDER THE COOPERATIVE GAME

This section discusses a cooperative game between the wholesaler and the retailer \( R \). We focus on the case where the wholesaler and the retailer \( R \) attempt to maximize their joint profit per unit of time. We here introduce some additional notations \( N(3) \) and \( T(3) \), which correspond to \( N(2) \) and \( T(2) \) respectively, under Option \( V_2 \) in the previous section.

Let \( J_P \left(N(3), T(3), y \right) \) express the joint profit function per unit of time for the wholesaler and the retailer \( R \), i.e., let

\[
J_P \left(N(3), T(3), y \right) = p^{(2)} \left(N(3), T(3), y \right) + J^{(2)} \left(T(3), y \right),
\]

where

\[
J_P \left(N(3), T(3), y \right) = \frac{\mu_1 + \mu_2}{\theta_R} \left[ (p_0 \theta_R + h_R) e^{\rho \eta T_R^{(1)*}} + (\epsilon_s \theta_R + h_R) e^{\rho \eta T_R^{(2)*}} \right].
\]

(33)

It can easily be proven from Eq. (33) that \( J_P \left(N(3), T(3), y \right) \) is independent of \( y \) and we have

\[
J_P \left(N(3), T(3), y \right) = p^{(2)} \left(N(3), T(3) \right) + J^{(2)} \left(T(3), y \right).
\]

VIII. NUMERICAL EXAMPLES

Table I reveals the results of sensitivity analysis in reference to \( x_1, x_2, S^{(1)} = S(N(3) + T_R^{(1)}) \) and \( P^{(1)} \) \((= P^{(N(3) + T_R^{(1)})})\) under Option \( V_1 \) \((j = 1, 2)\) for \( (p_0, \rho_0, \epsilon_s, h_s, \theta_s, a_s) = (600, 300, 100, 1, 0.01, 1000), (b_1, \theta_1, \mu_1) = (1.1, 0.013, 1200, 6) \) and \( (b_2, \theta_2, \mu_2) = (1.5, 0.015, 1300, 5) \) when \( a_s = 500, 1000, 2000 \) and 3000. In this case, we obtain \( \pi_1 = 1526.521 \) and \( \pi_2 = 1214.124 \), which are independent of \( a_s \).

In Table I(a) indicates that both \( S^{(1)*} \) and \( N^{(1)*} \) are non-decreasing in \( a_s \). As mentioned in Section II, under Option \( V_1 \), the retailer \( R \) does not adopt the quantity discount offered by the wholesaler, which signifies that the wholesaler cannot control the retailer \( R \)’s ordering schedule. In this case, the wholesaler’s cost associated with ordering should be reduced by increasing her/his own length of order cycle and lot size by means of increasing \( N^{(1)} \). Table I(a) also implies \( x_1 = x_2 \) \((i = 1, 2)\).

Table I(b) shows that, under Option \( V_2 \), \( S^{(2)*} \) increases with \( a_s \), in contrast, \( N^{(2)*} \) takes a constant value, i.e., \( N^{(2)*} = 1. \) Under Option \( V_2 \), the retailer accepts the quantity discount proposed by the wholesaler. The wholesaler’s lot size can therefore be increased by stimulating the retailer to alter her/his order quantity per lot through the quantity discount strategy. If the wholesaler increases \( N^{(2)} \) one step, her/his lot size also significantly jumps up since \( N^{(2)} \) takes a positive integer. Under this option, the wholesaler should increase her/his lot size using the quantity discount rather than increasing \( N^{(2)} \) when \( a_s \) takes larger values. Table I reveals that we have \( P^{(1)*} > P^{(2)*} \). This indicates that using the quantity discount strategy can increase the wholesaler’s total profit per unit of time. We can notice in Table I that \( x_1 = x_2 \) \((i = 1, 2)\) for each value of \( a_s \). This signifies that the retailers’ profit do not increase even if they accept the quantity discount proposed by the wholesaler.
IX. CONCLUSION

In this study, we have discussed a quantity discount problem between a single wholesaler and two retailers under circumstances where both the wholesaler’s and the retailers’ inventory levels of the product are depleted not only by demand but also by deterioration. In Japanese large-scale super markets, the same items are sold at different departments at the same store. In many cases, managers of these departments independently order the items since they are under competition and are separately evaluated by their superiors. They can possibly reduce their costs if either of them purchases the items for two departments in cooperation with each other.

The wholesaler is interested in increasing her/his profit by controlling the retailers’ order quantities through the quantity discount strategy. The retailers attempt to maximize their profit by considering both whether to cooperate with each other and whether to accept the wholesaler’s proposal. The analysis with respect to comparing the cooperative solution with non-cooperative one becomes considerably complicated since the local maximum values of the players’ total profit per unit of time cannot be expressed as closed form expressions. For this reason, we have shown that the retailers can increase their profit by means of adopting the cooperative strategy in the case where higher order terms of the deterioration rate in the expansion of the exponential can be ignored. Focusing on such a situation, the wholesaler can regard the retailers as a single retailer since either of the retailers is in charge of ordering and inventory management. In this case, we can formulate the above problem as a Stackelberg game between the wholesaler and the retailers in the same manner as our previous formulation[8], [9]. In this study, we have also formulated the same problem as a cooperative game between the wholesaler and the retailer $\tilde{R}$. The result of our analysis reveals that the wholesaler is indifferent between the cooperative and non-cooperative options. This is because the inventory holding cost is assumed to be independent of the value of the item. The relaxation of such a restriction is an interesting extension.

REFERENCES


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