

# Fast Gait Parameter Estimation for Frontal View Gait Video Data Based on the Model Selection and Parameter Optimization Approach

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**Abstract**—We study the problem of analyzing and classifying frontal view gait video data. In this study, we focus on the human walking speed and amplitude of arm swing and leg swing, we estimate these parameters using the statistical registration and modeling on a video data. To demonstrate the effectiveness of our method, we apply our gait parameter estimation model for the human gait video data. As a result, our model is able to estimate the gait parameters by stably at low calculation cost.

**Index Terms**—Gait analysis, human gait modeling, parameter selection.

## I. INTRODUCTION

WE study the problem of analyzing and classifying frontal view gait video data. A study on the human gait analysis is very important in the fields of the health/sports management, medical research.

Gait analysis is mainly based on motion capture system and video data. The motion capture system can give the precise measurements of trajectories of moving objects, but it requires the laboratory environments and we cannot be used this system in the field study. On the other hand, the video camera is handy to observe the gait motion in the field study.

From the standpoint of health/medical research area. Gage [1] proposed brain paralysis gait analysis using gait video data. Kadaba *et al.* [2] discussed importance of lower limb in the human gait using gait video data too. Many gait analysis have recently analyzing using video analysis software (e.g. Dartfish, Contemplas, Silicon Coach). For example, Borel *et al.* [3] and Grunt *et al.* [4] proposed infantile paralysis gait analysis using lateral view gait video data.

On the other hand, from the standpoint of statistics, Olshen *et al.* [5] proposed the bootstrap estimation for confidence intervals of the functional data with application to the gait cycle data observed by the motion capture system.

However, most studies have not focused on frontal view gait analysis, because such data has many restrictions on analysis based on the filming conditions.

The video data filmed from the frontal view is difficult to analyze, because the subject getting close in to the camera, and data includes the scale-changing parameters [7], [8]. To cope with this, Okusa *et al.* [9] and Okusa & Kamakura [10] proposed a registration for scales of moving object using the method of nonlinear least squares, but Okusa *et al.* [9] and

Okusa & Kamakura [10] did not focus on the human leg swing.

Okusa & Kamakura [12] focus on the gait analysis using arm and leg swing model with estimated parameters and application to the normal/abnormal gait analysis. However, their models have many of parameters, and it raise calculation cost and instability of parameter estimation.

Okusa & Kamakura [13] focus on the calculation cost and parameter estimation stability. The performance of Okusa & Kamakura [13] model is able to speed up the parameter estimation. However, the problem of parameter estimation stability still remains to be solved.

In this study, from the stand point of stability of parameter estimation, we suppose that important gait parameters are walking speed and amplitude of arm swing and leg swing, we redesign the frontal view human gait model. To demonstrate the effectiveness of our method, we apply our gait parameter estimation model for the human gait video data.

As a result, our model is able to estimate the gait parameters stably at low calculation cost.

## II. FRONTAL VIEW GAIT DATA

In this section, we describe an overview of frontal view gait data. Many of gait analysis using lateral view gait data, because lateral view gait is easy to detect the human gait features. However, in a corridor like structure, the subject is approaching a camera. Such case is difficult observe lateral view gait.

In a lateral view gait, at least two cycles or four steps are needed. For more robust estimation of the period of walking, about 8m is recommended. To capture this movement, the camera distance required is about 9m. Practically, having such a wide space is difficult. On the other hand, frontal view gait video is easy to observe 8m (or more) gait steps [8].

Figure 1 is an example of frontal view gait data recorded by Figure 2 situation. Figure 1 illustrates difficulty of frontal view gait analysis. Even if subject do the same motion with the same timing, frontal view gait data includes scale changing components. Figure 3 shows subject's width time-series behavior of frontal view gait data. This figure illustrates frontal view gait data contains many of time-series components.

This research is supported by the Institute of Science and Engineering of Chuo University.

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Fig. 1. Frontal view gait data

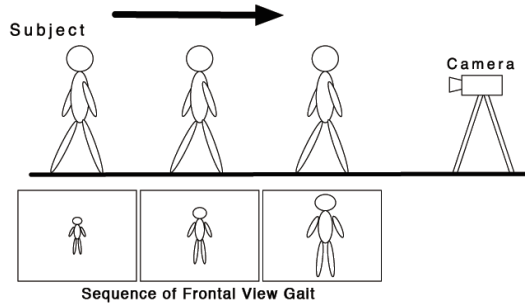


Fig. 2. Filming situation of frontal view gait data

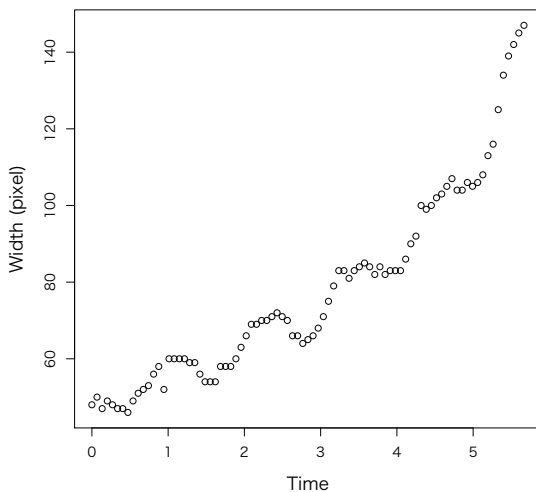


Fig. 3. Time-series behavior of frontal view subject width

### III. MODELING OF FRONTAL VIEW GAIT DATA

#### A. Preprocessing

The raw video data is difficult to observe subject width and height time-series behavior, because data contains background. We separate subject from background using inter-frame subtraction method (Eq. 1).

$$\Delta^{(T)} = |I^{(T+1)} - I^{(T)}|, T = 1, \dots, (n-1),$$

$$\Delta^{(T)}(p, q) = \begin{cases} 1 & (\Delta^{(T)}(p, q) > 0) \\ 0 & (\text{Otherwise}). \end{cases} \quad (1)$$

Here,  $\Delta^{(T)}$  is an inter-frame subtraction image,  $I^{(T)}$  is grey scaled video data image at frame  $T$ ,  $(p, q)$  is the pixel coordinate.

a) *Subject Width/Height Calculation*: Inter-frame subtraction method can separate the subject and background. However, it is difficult to measure the time-series behavior of the subject width and height. In this section, we describe the subject width and height calculation method using inter-frame subtraction data.

Let us suppose that inter-frame subtraction image is binary matrix. We can measure the subject height and width by integration calculation of row and column at each frame. In this study, we focus on the human gait arm and leg swing of the frontal view gait. We assume that subject width and height time-series behavior consist of the arm and leg swing behavior.

#### B. Relationship between camera and subject

Figure 4 shows a relationship between camera and subject. From figure 4, Width and height modeling has same structure. In this section, we describe the subject's width modeling. We can assume simple camera structure. We consider the virtual screen exists between observation point and subject, and we define  $x_i$  as subject width on the virtual screen at  $i$ -th frame ( $i = 1, \dots, n$ ).

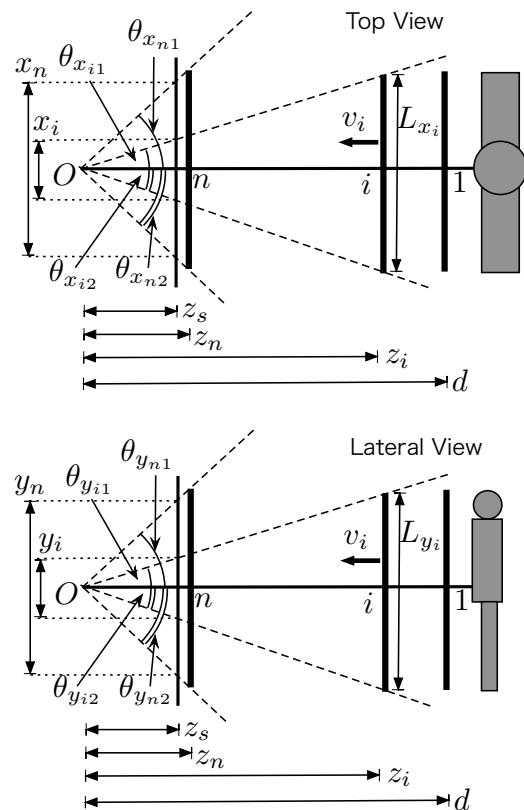


Fig. 4. Relationship between camera and subject

Here we define  $z_i, z_j$  as distance between observation point and subject at  $i$ -th,  $j$ -th frame,  $z_s$  as distance between observation point and virtual screen,  $\theta_{x_{i1}}, \theta_{x_{i2}}$  as subject angle of view from observation point at  $i$ -th frame,  $d$  as distance between observation point and 1st frame,  $v_i$  as subject speed at  $i$ -th frame. Okusa *et al.* [9] defined the subject length  $L$  was constant. We assume that  $L$  has the time-series behavior and we define  $L_i$  is the subject length at  $i$ -th frame.

$x_i$  at  $i$ -th frame depends on  $\theta_{x_{i1}}, \theta_{x_{i2}}$  as shown in Figure 4.

$$x_i = z_s(\tan \theta_{x_{i1}} + \tan \theta_{x_{i2}}). \quad (2)$$

Similarly, the subject length at  $i$ -th frame is

$$L_{x_i} = z_i(\tan \theta_{x_{i1}} + \tan \theta_{x_{i2}}). \quad (3)$$

From Eq.(2), Eq.(3), ratio between  $x_n$  and  $x_i$  is

$$\frac{x_n}{x_i} = \frac{L_{x_n} z_i}{L_{x_i} z_n} \quad (4)$$

Frame interval is equally-spaced (15 fps). Okusa *et al.* [9] assumed the average speed is constant. We can assume that average speed from  $i$ -th frame is  $(n - i) = (z_i - z_n)/\bar{v}$ , therefore  $z_i$  is  $z_i = z_n + \bar{v}(n - i)$ . We substitute  $z_i$  to Eq.(4)

$$x_i = \frac{M_{x_i} \gamma}{\gamma + (n - i)} x_n + \epsilon_i, \quad (5)$$

where  $\gamma$  is  $z_n/\bar{v}$ ,  $M_{x_i}$  is  $L_{x_i}/L_{x_n}$ ,  $\epsilon_i$  is noise. From Eq.(5), predicted value  $\hat{x}_i^{(n)}$  is registration from  $i$ -th frame's scale to  $n$ -th frame's scale

$$\hat{x}_i^{(n)} = \frac{\gamma + (n - i)}{M_{x_i} \gamma} x_i. \quad (6)$$

Similarly, we can define subject height as

$$y_i = \frac{M_{y_i} \gamma}{\gamma + (n - i)} y_n + \epsilon_i, \quad (7)$$

where  $M_{y_i}$  is  $L_{y_i}/L_{y_n}$ .

Next, we discuss the scale changing, human movement, and speed changing parameter estimation model.

### C. Scale changing parameter estimation

From Eq.(5), scale parameter is  $\gamma$ . Solve Eq.(5) for  $\gamma$  shows

$$\gamma = \frac{x_i(n - i)}{x_i - M_{x_i} x_n}. \quad (8)$$

Here  $\gamma$  is the ungaugable parameter, and we estimate it using nonlinear least squares method

$$S(\gamma, M_{x_i}) = \sum_{i=1}^n \left\{ x_i - \frac{M_{x_i} \gamma}{\gamma + (n - i)} x_n \right\}^2. \quad (9)$$

### D. Human movement parameter estimation

$M_{x_i}$  and  $M_{y_i}$  are movement model of the subject. If the subject is the rigid body, movement model  $M_{x_i}$  and  $M_{y_i}$  are constant. Meanwhile, human gait is not a constant.  $M_{x_i}$  and  $M_{y_i}$  needs the movement model because the subject body is moving wildly.

b) *Human gait modeling: arm swing:* Collins *et al.* [14] has reported that arm swing is an very important role in the gait motion. We consider the human gait model based on Collins *et al.* [14] model (see Figure 5).

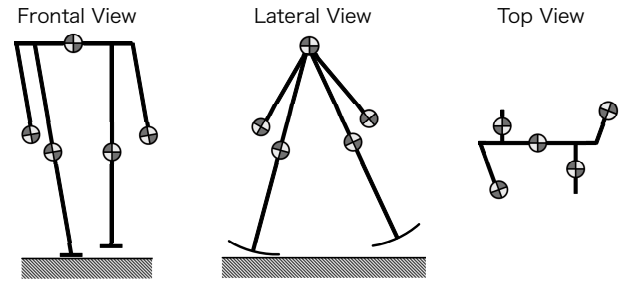


Fig. 5. Gait model

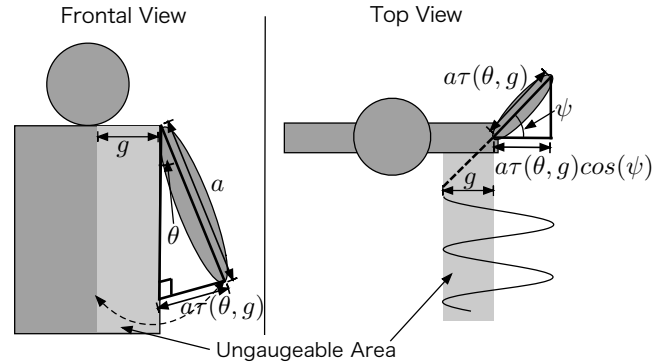


Fig. 6. Arm swing model

It seems reasonable to think that arm swing is single pendulum. Collins *et al.* [14] model assumed that arm swing is move to anteroposterior direction. Our model, on the other hand, can assume that arm swing move to an oblique direction (Figure 6).

Figure 6's model has an ungaugable area. Our method's width/height calculation is based on integration calculation of row and column at each frame. If the arm move to inside the body area, arm length is ungaugable. Arm swing model is

$$x_i = \frac{\left( \frac{W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, i)}{W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, n)} + s \right) \gamma}{\gamma + (n - i)} x_n + \epsilon_i$$

$$W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, i) = P_1 \tau(f i + Q_1, g_1) + P_2 \tau(f i + Q_2, g_2)$$

$$\tau(\theta, g) = \begin{cases} \sin(\theta) + g & (\sin(\theta) + g > 0) \\ 0 & (\text{Otherwise}) \end{cases} \quad (10)$$

where  $P_1 = a_1 \cos(\psi)$  and  $P_2 = a_2 \cos(\psi)$ .  $P_1 \tau(f i + Q_1, g_1)$  and  $P_2 \tau(f i + Q_2, g_2)$  are right and left arm model respectively. From Eq.(10), we estimate each gait parameter using nonlinear least squares method.

$$S(\gamma, P_1, P_2, Q_1, Q_2, g_1, g_2, f, s) = \sum_{i=1}^n \left\{ x_i - \frac{\left( \frac{W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, i)}{W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, n)} + s \right) \gamma}{\gamma + (n - i)} x_n \right\}^2 \quad (11)$$

Here,  $f$  is gait cycle frequency,  $s$  is adjustment parameter,  $P_1, P_2$  are arm swing amplitude parameters,  $Q_1, Q_2$  are arm phase parameters, and  $g_1, g_2$  are ungaugable area parameters.

c) *Human gait modeling: leg swing*: Leg swing modeling is simpler than arm swing model because leg swing model does not have a ungaugable area. Okusa *et al.* [9] and Okusa & Kamakura [10] does not consider the leg swing. It seems reasonable to think like arm swing that leg swing is single pendulum (Figure 7).

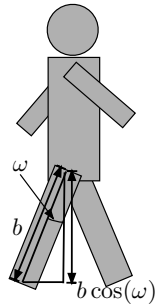


Fig. 7. Leg swing model

Leg swing model is

$$y_i = \frac{\left( \frac{H(b_1, Q_3, f, i)}{H(b_1, Q_3, f, n)} + s \right) \gamma}{\gamma + (n - i)} y_n + \epsilon_i$$

$$H(b_1, Q_3, f, i) = b_1 \cos(fi + Q_3). \quad (12)$$

Here  $b_1$  is leg swing amplitude parameter, and  $Q_3$  is leg phase parameter.

#### E. Speed changing parameter estimation

Frontal view video data is difficult to see the subject's speed. If our gait model is correct, observed value  $x_i$  and  $y_i$  is same as the fitted value of gait model at point  $\ell_i$ . Previous model's  $\ell_i$  assumes equally spaced ( $\ell_i = i = 1, \dots, n$ ). We estimate  $\ell_{x_i}$  and  $\ell_{y_i}$  value for minimize the observed value and model fitted value at  $\ell_i$ . We can define estimated value  $\ell_{x_i}$  and  $\ell_{y_i}$  as a virtual space coordinate at i-th frame (Figure 8).

Eq.5, Eq.7 with the coordinate estimation shows

$$x_i = \frac{M_{x_i} \gamma}{\gamma + (n - \ell_{x_i})} x_n + \epsilon_i$$

$$y_i = \frac{M_{y_i} \gamma}{\gamma + (n - \ell_{y_i})} y_n + \epsilon_i. \quad (13)$$

Here,  $\ell_{x_i}, \dots, \ell_{x_n}$  and  $\ell_{y_i}, \dots, \ell_{y_n}$  are virtual space coordinate parameters of width and height respectively. From Eq.13, arm swing and leg swing model with the coordinate estimation shows Eq.14, Eq.15.

$$x_i = \frac{\left( \frac{W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, \ell_{x_i})}{W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, \ell_{x_n})} + s \right) \gamma}{\gamma + (n - \ell_{x_i})} x_n + \epsilon_i$$

$$W(P_1, P_2, Q_1, Q_2, g_1, g_2, f, \ell_{x_i}) =$$

$$P_1 \tau(f \ell_{x_i} + Q_1, g_1) + P_2 \tau(f \ell_{x_i} + Q_2, g_2)$$

$$\tau(\theta, g) = \begin{cases} \sin(\theta) + g & (\sin(\theta) + g > 0) \\ 0 & (\text{Otherwise}). \end{cases} \quad (14)$$

$$y_i = \frac{\left( \frac{H(b_1, Q_3, f, \ell_{y_i})}{H(b_1, Q_3, f, \ell_{y_n})} + s \right) \gamma}{\gamma + (n - \ell_{y_i})} y_n + \epsilon_i$$

$$H(b_1, Q_3, f, \ell_{y_i}) = b_1 \cos(f \ell_{y_i} + Q_3). \quad (15)$$

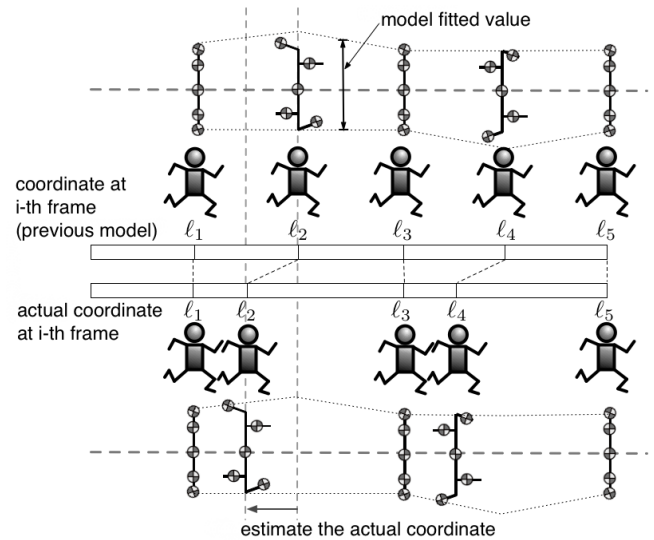


Fig. 8. Virtual space coordinate estimation

We suppose that virtual space coordinate of subject is  $\hat{\ell}_i = (\hat{\ell}_{x_i} + \hat{\ell}_{y_i})/2$ . Then, we can assume that subjects speed is 1st order difference of  $\hat{\ell}_i$ , and acceleration is 2nd order difference of  $\hat{\ell}_i$ .

#### IV. EFFECTIVENESS OF EACH PARAMETERS

In this section, we discuss the effectiveness of each parameters. From the standpoint of gait analysis, we think the subject's parameter stability is most important factor.

Eq.14 and Eq.15 models has many of parameters, we need to estimate  $n + 11$  parameters. It raise calculation cost and instability of parameter estimation.

To cope with this, we confirm the most affected parameters. We choose 30 subjects and calculate the interclass stability index  $C$  for each estimated parameters. Parameter  $k$ 's interclass stability  $C_k$  calculation is

$$C_k = \sum_{p \neq q} |\Theta_{k,p} - \Theta_{k,q}|. \quad (16)$$

This index means most minimum interclass stability index parameter is most stable parameters for gait motion. Where,  $\Theta$  is the set of the estimated parameters from Eq.14 and Eq.15 models, and  $p, q$  are learning and test data's subject ID, respectively. Note, that length of the virtual space coordinate parameters  $\hat{\ell}_i$  are not equal at each subjects, because it depends on moving distance and speed. Therefore, we set  $\hat{\ell} = \frac{1}{n} \sum \hat{\ell}_i$  as a representative value of  $\hat{\ell}_i$ .

For evaluate the  $C_k$  value, we convert  $C_k$  value into  $D_k = 1 - C_k / \sum_{k=1}^n C_k$ . We ascending sorting  $D_k$  and we choose the smaller  $D_k$  value parameters until over 0.9. From this process, finally we can choose parameters  $P, b_1, \gamma, f$ . Parameter  $P, b_1$  and  $\gamma$  are amplitude of arm swing, leg swing and walking speed respectively. We consider this result have relationship to Okusa & Kamakura [12]'s normal/abnormal gait analysis result.

#### A. Modified gait model

From the validation results of effectiveness of each parameters, most effective parameters for the gait analysis are  $P, b_1, \gamma, f$ . Accordingly, we modify the Okusa & Kamakura [13] model for estimate these parameters.

TABLE I  
RSS, AIC, CALCULATION TIME VALUE OF WIDTH AND HEIGHT DATA

Model	Method	Subject ID									
		A	B	C	D	E	F	G	H	I	J
Okusa & Kamakura [11] model	RSS	170.46	67.02	122.28	382.93	193.88	302.20	125.73	55.34	1.17	0.68
	AIC	626.95	558.86	581.82	676.03	663.96	657.80	577.59	510.50	236.67	190.94
	Calc. Time	7.766	6.830	8.231	8.007	6.092	7.090	6.929	8.426	7.462	7.581
Okusa & Kamakura [13] model	RSS	354.50	192.40	229.01	859.82	619.89	580.83	134.61	175.97	1.08	2.83
	AIC	354.79	309.24	311.51	416.31	414.43	386.11	268.71	286.10	-107.87	-26.86
	Calc. Time	0.986	0.669	0.793	0.782	1.093	0.865	0.626	0.912	0.535	0.814
Proposed model	RSS	318.96	168.78	207.15	783.98	545.13	522.80	116.75	160.61	17.10	23.99
	AIC	346.45	298.76	303.89	409.20	403.76	378.00	258.03	279.35	115.87	142.05
	Calc. Time	0.93	0.64	0.76	0.75	1.04	0.77	0.58	0.87	0.50	0.75
Number of frames:		79	80	76	77	83	77	75	74	81	79

Calc. Time: Calculation Time [sec]

Here modified width model is

$$x_i = \frac{\{W(P, Q_1, f, i) + s_1\} \gamma}{\gamma + (n - i)} x_n + \epsilon_i$$

$$W(P, Q_1, f, i) = P \sin(fi + Q_1). \tag{17}$$

Where  $P$  is amplitude of arm swing,  $Q_1$  is phase of human gait,  $f$  is gait cycle frequency and  $s$  is adjustment parameter.

Similarly, modified height model is

$$y_i = \frac{\{H(b_1, Q_1, f, i) + s_2\} \gamma}{\gamma + (n - i)} y_n + \epsilon_i$$

$$H(b_1, Q_1, f) = b_1 \sin(2fi + Q_1). \tag{18}$$

Differences points between Okusa & Kamakura [13] model and our modified model are three points. Firstly, we reduce the model parameters from the validation results of effectiveness of each parameters. Secondly, we standardize the parameters between leg swing and arm swing model. These measures have efficacy for calculation cost and parameter estimation stability. Thirdly, we remove the Okusa & Kamakura [13] model's tuning function like  $1/W(P, Q_1, f, n)$  and  $1/H(P, Q_1, f, n)$ . From the results of our verification, these tuning function is not effective for stability of parameter estimation.

Our modified models are easy and stable to estimate  $P, b_1, \gamma, f$  parameters. In next session, we validate the effectiveness of our model.

## V. EXPERIMENTS AND RESULTS

### A. Gait parameter estimation

To validate the effectiveness of our modified model, we compare Eq.17 and Eq.18 model with Okusa & Kamakura [11], [13] models by Residual Sum of Squares (RSS), Akaike Information Criterion (AIC) [15] and calculation time. We took movie of 10 subjects walking video data from frontal view (10 steps, Male, average height: 176.4cm, sd: 3.07cm) and apply to our proposed method.

Figure 9 is plot of the subject width (pixel) time-series behavior. Here, continuous line represent fitted value of Eq.17. From Figure 9, proposed model is good fitting for time-series behavior of subject's width.

Figure 10 is plot of the subject height (pixel) time-series behavior. Here, continuous line represent fitted value of Eq.18. From Figure 10, proposed model is good fitting for time-series behavior of subject's height like subject's width result.

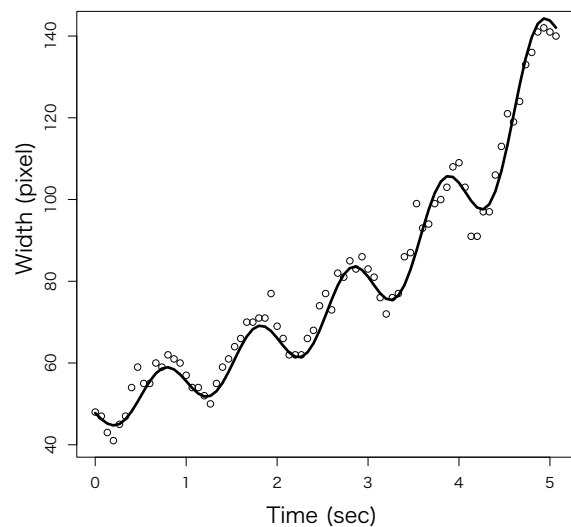


Fig. 9. Fitted Value of subject's width

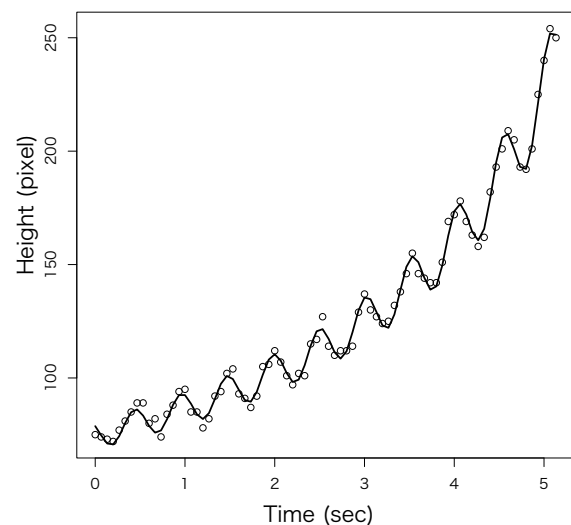


Fig. 10. Fitted Value of subject's height

Table I is RSS, AIC, calculation time value of previous model [11], [13] and proposed model (Eq.17 and Eq.18

model). In Table I, most minimal RSS model is previous model [11]. Meanwhile, most minimal AIC and calculation time model are proposed model (Eq.17, Eq.18 model) except for subject I, J cases. From Table I, our proposed models calculation cost is about 90% faster than previous model.

## VI. CONCLUSION

In this article, we proposed the human gait model for the frontal view human gait analysis. Our model is able to estimate stably human gait feature quantity at low calculation cost. Our model is 90% faster than our previous model.

In next phase, we need to solve the initial value stability of this model. If we adjust initial value appropriately, our model is very stable to estimate gait parameters. We have to seek the initial value setting method for our model.

## REFERENCES

- [1] J. R. Gage, "Gait analysis for decision-making in cerebral palsy." *Bull. Hosp. Jt. Dis. Orthop. Inst.*, vol. 43, no. 2, pp. 147–163, 1982.
- [2] M. P. Kadaba, H. K. Ramakrishnan, and M. E. Wootten, "Measurement of lower extremity kinematics during level walking." *J. Orthop. Res.*, vol. 8, no. 3, pp. 383–392, 1990.
- [3] S. Borel, P. Schneider, and C. J. Newman, "Video analysis software increases the interrater reliability of video gait assessments in children with cerebral palsy," *Gait & posture*, vol. 33, no. 4, pp. 727–729, 2011.
- [4] S. Grunt, P. J. van Kampen, M. M. Krogt, M. A. Brehm, C. A. M. Doorenbosch, and J. G. Becher, "Reproducibility and validity of video screen measurements of gait in children with spastic cerebral palsy." *Gait & posture*, vol. 31, no. 4, pp. 489–494, 2010.
- [5] R. A. Olshen, E. N. Biden, M. P. Wyatt, and D. H. Sutherland, "Gait analysis and the bootstrap," *Ann. Statist.*, pp. 1419–1440, 1989.
- [6] M. Soriano, A. Araullo, and C. Saloma, "Curve spreadsia biometric from front-view gait video," *Pattern Recognit. Lett.*, vol. 25, no. 14, pp. 1595–1602, 2004.
- [7] O. Barnich and M. V. Droogenbroeck, "Frontal-view gait recognition by intra-and inter-frame rectangle size distribution," *Pattern Recognit. Lett.*, vol. 30, pp. 893–901, 2009.
- [8] T. K. M. Lee, M. Belkhatir, and P. A. Lee, "Fronto-normal gait incorporating accurate practical looming compensation," *Pattern Recognit.*, 2008.
- [9] K. Okusa, T. Kamakura, and H. Murakami, "A statistical registration of scales of moving objects with application to walking data. (in Japanese)," *Bull. Jpn. Soc. Comput. Statist.*, vol. 23, no. 2, pp. 94–111, 2011.
- [10] K. Okusa and T. Kamakura, "A statistical registration of scale changing and moving objects with application to the human gait analysis. (in Japanese)," *Bull. Jpn. Soc. Comput. Statist.*, vol. 24, no. 2, 2012.
- [11] —, "Statistical registration and modeling of frontal view gait data with application to the human recognition." in *Int'l. Conf. Comput. Statist. (COMPSTAT 2012)*, 2012, pp. 677–688.
- [12] —, "Gait parameter and speed estimation from the frontal view gait video data based on the gait motion and spatial modeling," *Int'l J. Appl. Math.*, vol. 43, no. 1, pp. 37–44, 2013.
- [13] —, "Fast Frontal View Gait Authentication Based on the Statistical Registration and Human Gait Modeling," *Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering 2013, U.K., 3-5 July, 2013, London.*, pp. 274–279.
- [14] S. H. Collins, P. G. Adamezyk, and A. D. Kuo, "Dynamic arm swinging in human walking," *Proc. R. Soc. B: Biological Sci.*, vol. 276, no. 1673, pp. 3679–3688, 2009.
- [15] H. Akaike, "Information theory and an extension of the maximum likelihood principle," *Int'l. Symp. Inf. Theory*, pp. 267–281, 1973.
- [16] N. Sugiura, "Further analysts of the data by Akaike's information criterion and the finite corrections," *Commun. Statist. - Theory and Methods*, 1978.