Engineering Reliability Analysis in Risk Management Framework: Development and Application in Infrastructure Project

J. Lai, L. Zhang, C.F. Duffield, and L. Aye

Abstract—**Analyzing uncertainty is an essential element of infrastructure project appraisal as critical parameters often exhibit variations that could impact on project feasibility. This paper presents a risk-based, cost-benefit analytical framework to complement decision making. The framework collaborates concepts from life cycle costing, engineering reliability analysis, and autoregression for forecasting. To model uncertainty, the Hasofer-Lind method, or advanced first-order second-moment analysis is applied. The model is tested using a synthetic residential property, with house price as the uncertain variable. Three series of house price are forecasted to simulate varying conditions of the Melbourne, Australia, housing market using historical trend, gross disposable income per capita and consumer price index. From an investor's perspective, it was found that there is less than 1% probability of investment loss if the asset is held for more than 10 years, given a fixed standard deviation of $55,000 in benefit and cost distributions. Furthermore, a market downturn increases the probability of loss to 39% if an asset is held for 5 years. Increasing variations of cost from approximately 5% to 30%, adversely affects probability of loss in all simulated property market conditions. The efficiency of the Hasofer-Lind method was found to be an improved alternative to the more computationally intensive Monte Carlo simulation.**

Index Terms- **AFOSM, property development, house price forecast, reliability analysis, risk management.**

I. Introduction

An important aspect of project appraisal is the accurate collection of reliable input data given that a feasibility decision may be dependent on a few crucial assumptions. However, uncertainty is a considerable concern particularly when appraisals evaluate life cycles over lengthy durations. In infrastructure projects, uncertainty may arise from non-cognitive and cognitive sources [1].

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The risk involved is present when uncertainty impact important project features such as quality, cost and scope [2]. In engineering reliability analysis (ERA), the uncertainty of model outputs is assessed based on the uncertainty within the model system [3]. Although traditionally applied to structural projects regarding resistance and loading [1] [4], ERA has also been utilized in other areas such as composite channels with uncertain runoff [5], water quality involving dissolved oxygen concentrations [6], and reservoir water allocation [3]. The application of ERA in the area of property construction industry is rare to authors' best knowledge. Lai *et al.* (2013) introduced the concept of applying the first-order second-moment (FOSM) method of ERA to financial risk appraisal of infrastructure projects, specifically to a synthesized desalination plant in Victoria, Australia [7]. The risk profile in different project options was presented in a variety of risk metrics including the reliability index (β) and probability of loss (p_f). This paper expands the framework for further application by employing the advanced first-order second-moment (AFOSM), also known as Hasofer-Lind method [1], which has been briefly introduced by Lai *et al.* (2013) [8]. In this study, structural resistance and loading in ERA are taken as monetary benefits (**B**) and costs (**C**). There are numerous advantages of using AFOSM over FOSM in risk analysis. FOSM does not consider distributional information of variables when it is known and is less accurate with non-linear performance functions [1]. An invariance problem exists in FOSM which provides different reliability index when safety margins are mathematically equivalent [1] [4], whereas solutions with AFOSM are independent of the form of performance function. Further, as AFOSM calculation is dimensionless, it has the potential to assess non-monetary social and environmental variables. In addition, although Monte Carlo simulation (MCS) is a popular and commonly used tool for difficult risk management problems [9], ERA is computationally less demanding for complex models with multiple parameters and thus has an advantage over MCS [3] [6].

Uncertainty of financial time series is a popular aspect in risk management theories [10]. In this study, a mathematical model is presented, which is used to assess the risk of investment in property industry under three different housing market scenarios. For the purpose of model illustration, investment periods have been arbitrarily assigned a 30 year period. The relatively long modeling period will allow application of the framework for ease of analysis. The uncertain variable is house price and is

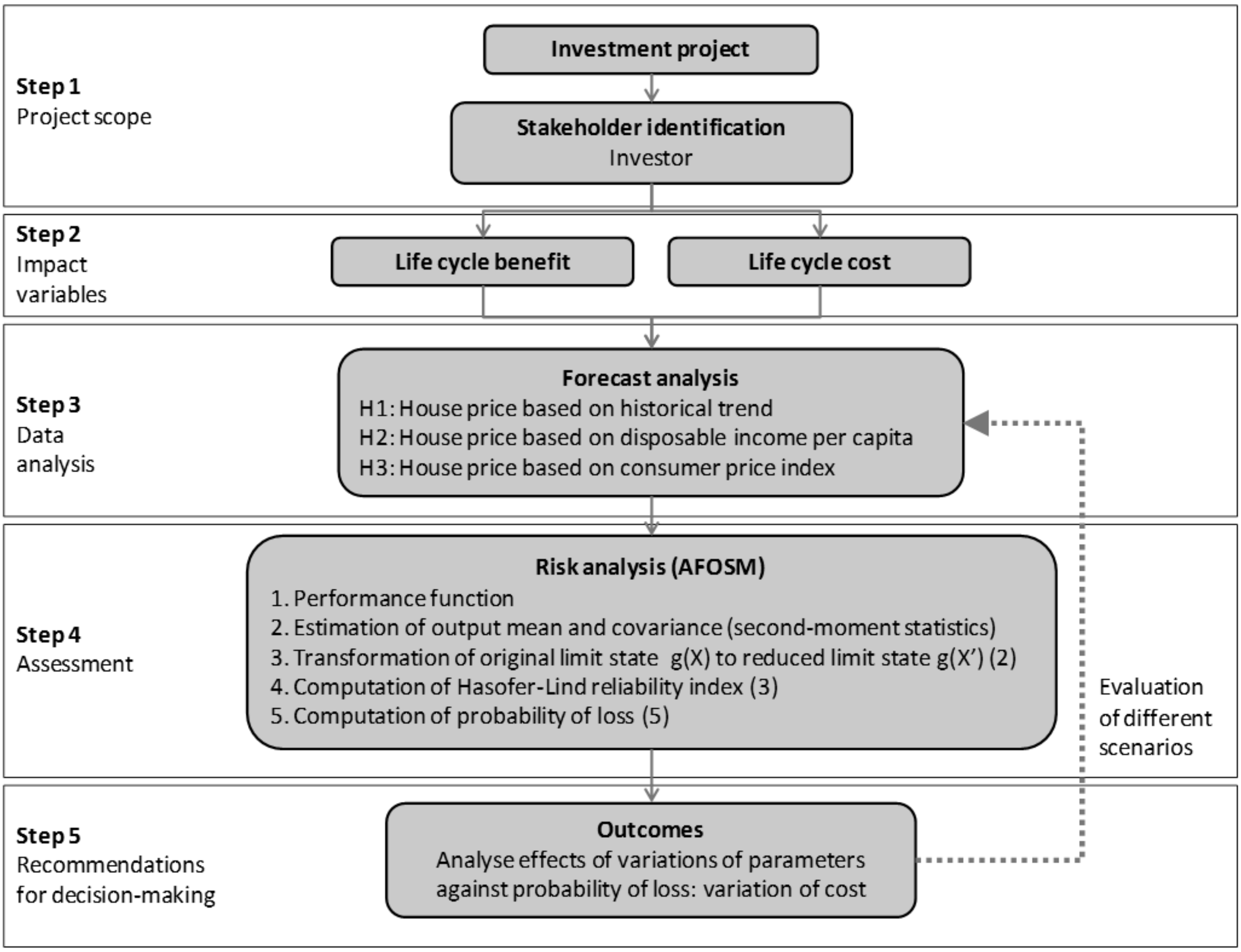


Fig 1. Framework of proposed risk model

modeled using historical trend in house price, gross disposable income per capita (DPI) and consumer price index (CPI). Fig 1 shows the model framework. The objectives of this paper are to apply AFOSM to assess uncertainty of cost and benefit variables in model inputs and outputs, as seen in Step 4 of Fig 1; and to evaluate the effects of changing the degree of uncertainty on project design, focusing mainly on cost uncertainty, as shown in Step 5. Furthermore, the advantages of ERA over MCS will be substantiated. The first part of the paper is the application of the mathematical model AFOSM. The second part is the description of house price modeling and validation of the three forecast of house price. The third part involves the presentation and discussion of results in comparing p_f across different duration and market conditions. The final part verifies the efficiency of ERA over MCS.

II. Research Methodology

A. Reliability analysis

The difference between benefit or cash inflow, **B**, and costs or cash outflows, **C**, is defined by the performance function or safety margin NPV = **B** – **C** = $g(X_1, X_2, ..., X_n)$, where NPV is net present value, and X_n are multiple random variables representing effects of uncertain house prices. A project is considered infeasible when **C** exceeds **B**, and its probability of loss, p_f, is

$$p_f = P(\mathbf{B} < \mathbf{C}) = P(\mathrm{NPV} < 0) \qquad (1)$$

The most probable point of loss is the design point and occurs when the planar of performance function is 0 (i.e. **B** = **C**) along the failure surface or limit state. AFOSM evaluates the failure surface using a reduced system in which variables are standardized to

$$X_i' = \frac{X_i - \mu_{X_n}}{\sigma_{X_n}} \qquad (2)$$

The reduced failure surface is thus $\sigma_{\mathbf{B}}\mathbf{B}' - \sigma_{\mathbf{C}}\mathbf{C}' + \mu_{\mathbf{B}} - \mu_{\mathbf{C}} = 0$ with random variables $g(X_1', X_2', \ldots, X_i')$, where **B**' and **C**' are reduced benefit and cost respectively. The AFOSM system is shown in Fig 2 with intercepts $\left[-(\frac{\mu_B - \mu_C}{\sigma_B}), 0\right]$ and $\left[0, \frac{\mu_B - \mu_C}{\sigma_C}\right]$. The design point marks the minimum distance between the reduced failure surface and origin, and is representative of the point of minimum reliability [5] or most probable failure [1]. The minimum distance is the reliability index as shown in (3), β_{HL}, where * denotes the design point $x_i'^{*} = -\alpha_i \beta_{HL}$, and α is the directional cosines of the reduced coordinate X_i' in (4)

$$\beta_{HL} = \sqrt{(\boldsymbol{x}'^{*})^t(\boldsymbol{x}'^{*})} = -\frac{\sum_{i=1}^{n} x_i'^{*}\left(\frac{\partial g}{\partial X_i'}\right)^{*}}{\left[\sum_{i=1}^{n}(\frac{\partial g}{\partial X_i'})^{2*}\right]^{\frac{1}{2}}} \qquad (3)$$

$$\alpha_i = \frac{\left(\frac{\partial g}{\partial X_i'}\right)^{*}}{\left[\sum_{i=1}^{n}(\frac{\partial g}{\partial X_i'})^{2*}\right]^{\frac{1}{2}}} \qquad (4)$$

p_f, as a measure of functions failing in a system [6], can be rewritten as

$$p_f = \Phi(-\beta_{HL}) = 1 - \Phi(\beta_{HL}) \quad (5)$$

where Φ is the cumulative distribution function of the standard normal variates, and characterizes the probability distribution function (PDF) between $-\infty$ and $-\beta_{HL}$ of the normal standardized variates. In Fig 2, g(**X**')<0 represents the loss region. As the reduced failure surface moves closer to the origin, indicated by an increase in coefficient of variation of cost (CV_C) or an increase in cost uncertainty, the loss region is larger. This study assumes statistically normal and independent random variables. p_f is examined through two different ways: the means; and the standard deviations of benefit and cost distribution.

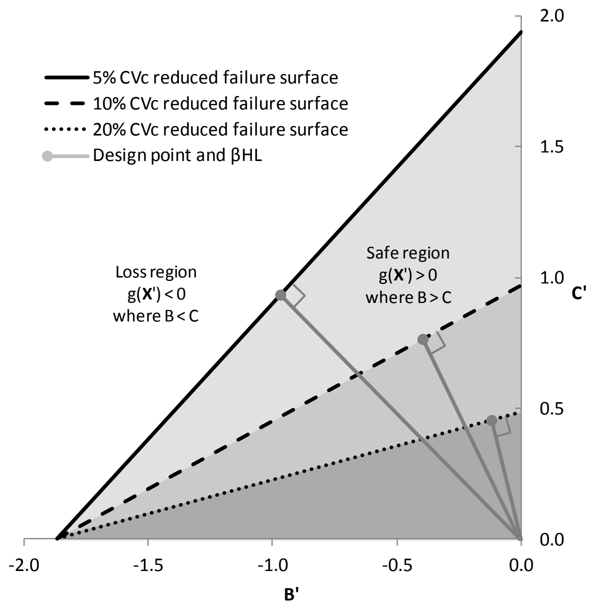


Fig 2. Standardized benefit, **B'**, and cost, **C'** in a reduced coordinate system. The shaded area shows the safe region when benefit exceeds cost. Probability of loss is represented by the reliability index, β_{HL}. The loss region increases with increasing cost coefficient of variation (CV_C). The length of the lines perpendicular to the reduced failure surface represents decreasing β_{HL} for increasing CV_C, indicating that probability of loss is larger as variation in future cost increases.

B. Forecast and validation of house price model

An inherent problem in project appraisals is the uncertainties surrounding variable forecasts. Information received from investors may be ambiguous and unclear [11]. The focus of this paper lies in the variability of Melbourne house price. House price in Melbourne experienced negligible growth in the post-1950s years [12], followed by a gradual increase in the 1960s and subsequently proceeded by significant long-run rise since 1996 [12] [13] [14]. A few substantial price booms have occurred in recent history notably between 1971 and 1974; 1979 and 1981; 1987 and 1989 [13]; and between 1996 and arguably to the present day. The Global Financial Crisis in 2008, compensated by Australia's resource boom in the mid-2000s, had only minor and momentary diminishing impacts on house prices [12]. Multiple studies have explored the driving force behind Melbourne house price [12] [13] [14]. Stapledon (2012) [12] suggested links between land price and house price, as well as income and travel time amongst other variables. Other factors contributing to house price may include demographic [15], policies and housing supply [16], transport infrastructure and zoning policies [17], and immigration rates [18].

Given that Melbourne has experienced substantial booms in the housing industry in the last decade, there are speculations on the timing and impact of a downturn in property price. This is a major concern for investors as downward trends of price fluctuation can adversely impact the profitability of investment and hence the decision to invest. From hereon, house price is in reference to median house price. For simplicity, this study uses three measures to forecast Melbourne house price and its variations to indicate varying market conditions: H1, historical trend in Melbourne house price; H2, relationship between house price and DPI; and H3, relationship between house price and CPI. The three series are described below.

Future prices in series H1 are inferred from historical trend using an autoregressive model. Nagaraja, Brown and Zhao [19] have applied autoregression for house price forecasting. Autoregression assumes an independent variable to be a time-lagged version of dependent variable. House prices usually require 1-2 years to correct itself back to equilibrium [13], and thus market shocks are likely to have an impact on prices that will affect the next period. The duration of the period varies depending on the momentum of the shocks. Autoregression is appropriate to capture such behavior. It has the following general form where x_t is the current year house price, a_m are the autoregression coefficients from least-squares regression, p is the most significant model order, $x_{(t-1)}$ is previous year house price and ε_t are output uncorrelated errors.

$$x_t = \sum_{i=1}^{p} a_m x_{(t-1)} + \varepsilon_t \quad (6)$$

p is determined by the time-lag with the highest order coefficient of less than 0.05 in its *p*-value, rejecting the null hypothesis when the null hypothesis is assumed to be true. This allows a 5% probability of a Type I error to reject the null hypothesis when it is true [20]. The weighting and strength between lagged and current periods are inferred from $a_1, a_2, \ldots, a_m$. The procedures for deducing the p of the model for this paper are briefly described using the following steps: a) identification of scope and appropriate time horizon of historical house price; b) regression of house price employed on different order of lag operators; c) comparison of highest order coefficient *p*-value at 0.05 significance level, orders higher than p are redundant; and d) identification of inconsistencies between regressed values and actual data.

Income has been attributed as an indicator to house price [12], [13], [18]. Series H2 forecasts house price based on changes in Victoria DPI. The change in house price is linked to the fundamental changes of DPI based on a growth rate of 2.5%. The growth rate is taken from the budget forecast of household consumption of 2012-13 in the Australian Government Budget 2013-14 [21]. In the absence of accurate DPI forecast, household consumption is assumed to be an adequate indicator of DPI growth. The relationship between house price and DPI is significant; the estimated long-run elasticity of house price with respect to DPI was found to be 1.7 between 1975 and 2003 at 5 per cent significance level [13]. The house price forecast from DPI is estimated in equation (7), where $E_{h,k}$ is the elasticity of house price, h, with respect to variable k, and $E_{h,k} = \left|\frac{\partial \ln h}{\partial \ln k}\right|$. h_{t+1} and h_t is house price of the next and current period respectively, and k is DPI.

$$h_{t+1} = \left(\frac{E_{h,k} \cdot \partial \ln k}{100} \times h_t\right) + h_t \quad (7)$$

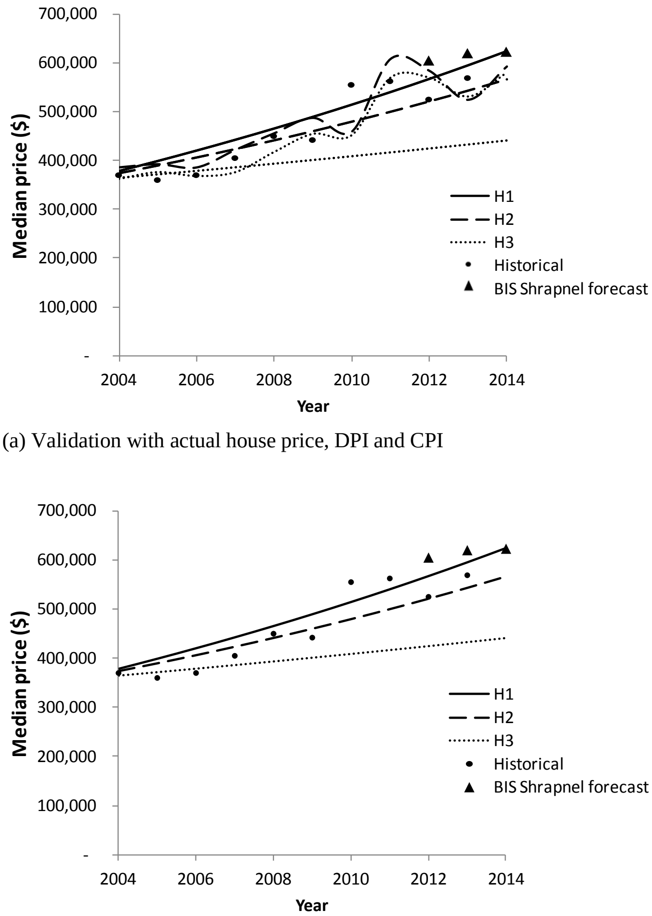
The third series, H3, is modelled based on house price and inflation. The Reserve Bank of Australia (RBA) has an inflation target of 2-3% since 1993; CPI is employed as an indication of inflation [13]. Several reports use CPI of 2.5% as the midpoint of RBA's inflation target range in their forecasts [22] [23] [24], thus the same approach is utilised in this paper. The relationship between house price and CPI is significant; the estimated long-run elasticity of house price with respect to CPI was found to be 0.76 between 1975 and 2003 at 5 per cent significance level [13]. Equation (7) is applied with k as CPI. The deterministic forecast from 2013 for H1, H2 and H3 are reproduced in Appendix 1.

Fig 3(a) and 3(b) show the validation and inference to future market conditions respectively between 2004 and 2014. 2004 onwards were chosen as the validation period due to the elasticity conclusions being drawn from a period ending in 2003 [13]. Fig 3(a) uses actual house price for the current period h_t in equation (7), and actual DPI and CPI values in the H2 and H3 series respectively. Included in both figures are estimates provided from BIS Shrapnel [25], which shows the rationale of the model's forecasts when compared with industry reports. It is clear that H1, H2 and H3 give good indications of the overall long-run trend of house price. Fig 3(b) shows H2 and H3 with the estimated forecast growth rates for DPI and CPI respectively, with h_t in equation (7) estimated from previous years forecasts. H1 is the same in both Fig 3(a) and 3(b). While the series in Fig 3(b) are imperfect, it provides alternative general house prices of different economic scenarios. It does not implicate particular short-term prices. Thus in this paper, H1 is taken to represent a continuing upward trend with current growth rates; H2 as a medium modest price rise; and H3 as a dampening housing market.

While there are many other factors that contribute to explaining house prices, only historical trend, DPI and CPI are chosen for this paper for simplicity. The next section describes the probabilistic component of the three house price series.

C. Probabilistic analysis

ERA requires estimation of variation of variables if not known. The probabilistic variable in this study is house price. For H1, the variation is inferred from the difference in errors between forecast and actual data during the period of available historical information. H2 assumes DPI to grow between 2-3% per year. H3 varies between the inflation target of 2-3% per year.



(a) Validation with actual house price, DPI and CPI

(b) Validation with estimated growth rates for DPI and CPI

Fig 3. (a) Validation of H1, H2 and H3 using actual house price, DPI and CPI data shows all three series provide the overall trend for house price. (b) H2 and H3 use the estimated forecast growth rates for DPI and CPI respectively. The h_t in equation (7) is deduced from forecasts. H1 represents continuing upward rise in market; H2 as medium market conditions; and H3 as dampening housing market. Also included in the figures are historical prices [26], and an industry forecast [25].

D. Data description

The model is applied to a one-storey residential building with a floor area of 205.7 m^2 [27], located in metropolitan Melbourne, Australia. Equations (8) and (9) show the benefit and cost variables considered in this study, where * indicates the variable at year of purchase, and † indicates the variable at year of sale. SP is selling price, determined by the house price and construction cost; and L is loan repayments including principal. Tax is treated as negating benefit. Further assumptions and data sources are outlined in the Appendix.

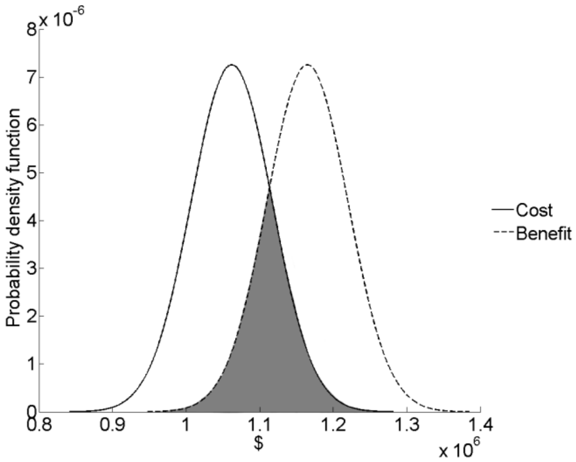
$$B_t = f(Rental\ income, SP^{\dagger}, -tax)_t \quad (8)$$

$$C_t = \begin{Bmatrix} h^*, construction\ cost^*, stamp\ duty^*, \\ land\ tax, L, council\ rates \end{Bmatrix}_t \quad (9)$$

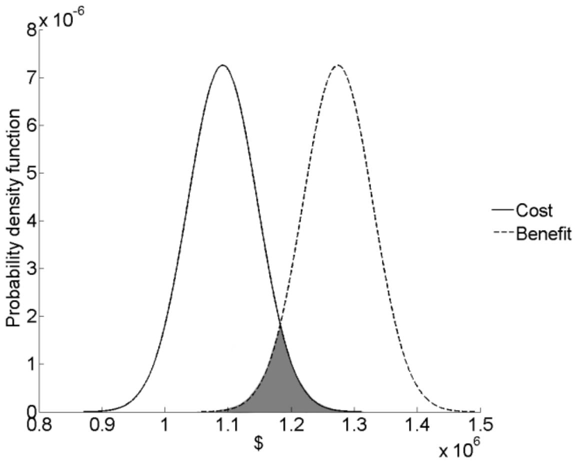
Variation of parameters is subject to change depending on duration. The investment begins in 2013 and is held indefinitely, during which time the investor receives rental income as a function of yield. At the conclusion of investment period, property is sold at a price equal to median house price (H1, H2 or H3) plus construction cost as an indication of a metropolitan house price. Market conditions are assumed to deviate from current trend in 2014. That is, H2 and H3 are employed beyond 2014.

III. Results and discussion

The current house price trend is first employed through series H1 to explore the effect of different distributional means of benefit and cost on p_f. The difference in means is inferred by investment duration. Fig 4(a) and 4(b) show the distribution of benefit and cost when the residential property is held for 5 years and 10 years respectively. The distributional means represent expected benefit and cost. To isolate effects of means, standard deviations are fixed at $55,000. When asset is held for 5 and 10 years, p_f is 9.3% and 1.0% respectively as represented by decreasing area of shaded regions. As series H1 assumes the broad trend of house price to continually rise akin to levels of the late 2000s, it is thus intuitive for rational investors to hold assets for a longer period of time to minimize risks. The results suggest investors have less than 1.0% chance of an unprofitable investment if asset is held beyond 10 years.



(a) Means of cost and benefit at 5 years with series H1



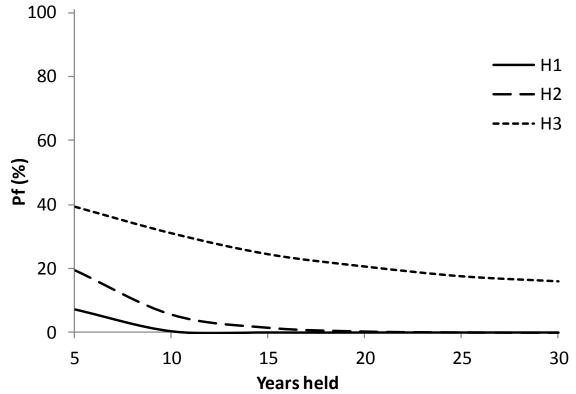
(b) Means of cost and benefit at 10 years with series H1

Fig 4. The shaded area represents probability of loss. (a) Means of cost and benefit at 5 years with probability of loss of 9.3% represented by the shaded area; (b) Means of cost and benefit at 10 years with probability of loss of 1.0%. Standard deviation for cost and benefit of both figures is fixed at $55,000. At 10 years held, probability of loss is decreased by increasing the separation between the two distributional means.

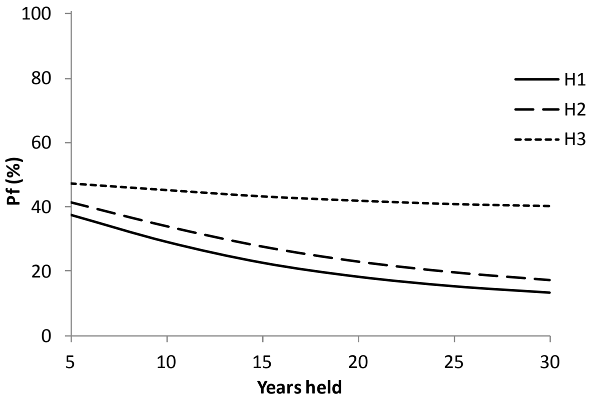
Speculations of Melbourne's current house price rise becoming unsustainable are represented by series H2 and H3. Fig 5(a) illustrates the effect of investment duration on p_f for the three series. The coefficient of variation on cost (CV_C) and benefit (CV_B) is approximately 5%. As H3 indicates the weakest market, it results in the highest chance of investment loss. Investors should take note that if house price moves solely in line with CPI, even for a relatively long term residential investment of 30 years, the risk of investment loss remains high at 16.0%. In a short term of 5 years investment, H3's p_f is 39.1%, whereas H1 and H2 are 7.2% and 19.3% respectively. The difference between H1 and H2 is greatest at short investment duration; however both approach very low levels of p_f at longer durations. For instance at 20 years, both H1 and H2 have p_f of less than 0.5%. These results implicate that while residential property investment remains profitable for continued rise in house prices, a market downturn can adversely affect investors regardless of property being held in the long term.

In addition to house price, life cycle cost is an important factor in making an investment decision. Return from rental income and selling price must cover major costs including construction cost and interest on loan. The effect of high cost uncertainty is subsequently explored. Fig 5(b) shows a high CV_C of 30%. H3 remains the most risky set as it combines both high cost variations and weak market conditions. For a 30 year investment period, p_f of H3 remains above 40%. p_f does not exceed 50% as model forecasts projects expected benefit as always exceeding cost during the 30 year duration. The effect of increased cost variation is also reflected in series H1 and H2 by adverse movements of p_f. While H1 remains the less risky series, in a short term of 5 years H1 and H2 have 37.5% and 41.3% investment loss probability respectively. At 30 years, their p_f remain high at 13.3% and 17.4% respectively. Thus large variations in cost are unfavorable in investment projects, and in all circumstances as represented by this paper, probability of investment loss in residential property is reduced by holding assets in the long run, which is in line with most investors' expectations.

In this section, the efficiency of ERA is compared with the more commonly used MCS. Fig 6 shows the ratio of p_f values obtained through ERA and different number of runs of MCS for series H1 at 5 years held. The p_f for MCS is obtained with equation (10), where N_{loss} is the total instances of loss, and N_{total} is the number of simulations. Simulations were run 10, 100, 1000, and 10000 times. Each set of simulation was repeated 50 times to ensure consistency.



(a) Normal simulation of H1, H2 and H3 (cost coefficient of variation approximately 5%)



(b) H1, H2 and H3 when cost coefficient of variation is 30%

Fig 5. Probability of loss (p_f) is compared with H1, H2 and H3. (a) H3 represents the weakest market with p_f equal to 16.0% at 30 years held. H1 and H2 move at similar levels and at 20 years, p_f is less than 0.5% for both series. (b) When cost coefficient of variation is 30%, representing high cost variations, H3 remains above 40% throughout a 30 year investment period. H1 and H2 have much higher p_f levels than can be seen in (a), for instance p_f are 13.3% and 17.4% respectively at 30 years. In all cases, p_f is lower when residential property is held in the long run.

$$p_{f,MCS} = \frac{N_{loss}}{N_{total}} \quad (10)$$

In order to obtain similar outcomes as ERA, the range of MCS outputs approached satisfactory results at 1000 runs or more. However, running 1000 simulations required 18 minutes. Running 10000 simulations required 3 hours and 46 minutes. MCS is computationally intensive in comparison to ERA, which virtually produces instant results. This has significant implications when uncertainty models involve more complex systems with large number of probabilistic variables. ERA is therefore viewed as a more efficient alternative for modelling p_f.

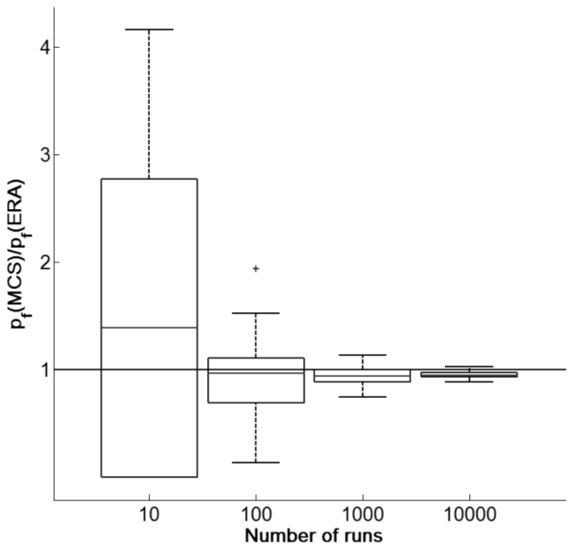


Fig 6. Using the computationally intensive Monte Carlo simulation, more than 1,000 runs are needed to produce a similar range of probability of loss results compared to engineering reliability analysis.

The framework presented provides a means to systematically analyses benefit and cost variables, and summarizes the probabilistic characteristics into β_{HL} and hence p_f. The approach is complementary to traditional financial appraisals as it delves into probabilities which are often neglected in common practice. Further research to advance the proposed model for other application include correlations between variables, skewed variable distributions, combining probabilities and actual loss values as an index, and incorporating non-monetary variables such as environmental, social and wider economic benefits. Subsequent refinements to house price modeling could enhance the accuracy of housing development forecast including factors such as foreign investors demand, residential density, and location premium.

IV. Conclusion

A theoretical framework in risk modeling has been presented in this paper. AFOSM reliability analysis is the focus of the model with house price as the probabilistic variable. Melbourne house price has experienced a series of significant boom periods, but generally fluctuating upwards to the current years. Speculation exists as to if and when house price would fall, posing important implications to property investors. Three series of house price representing varying conditions of property has been employed: H1 based on historical trend; H2 modeled from DPI; and H3 inferred from CPI. H3 represented the weakest market, while H1 is the strongest. A synthetic residential property located in Melbourne, Australia, was evaluated from the perspective of an investor. Benefit and cost of investment were a function of common factors including purchase and selling of property, construction cost, loan repayments, and rental income. p_f was the primary risk metric. It was found that there was a 9.3% and 1.0% chance of investment loss when the property was held for 5 and 10 years respectively, keeping standard deviations fixed at $55,000. Over the long run in a 30 year investment period, p_f remains relatively high at 16.0% for H3, while even at 20 years investment H1 and H2 reports less than 0.5% p_f. It was also found that an overall increase in variation of cost (CV_C of 30%) would adversely affect p_f, with p_f remaining above 40% for the duration of 30 year investment period for H3. At 30 years, H1 and H2 reported 13.3% and 17.4% p_f respectively. AFOSM is computationally less intensive than MCS as it uses substantially less time to generate similar results. It also addresses the invariance problem present in FORM. Moreover, AFOSM has the potential to expand on non-monetary variables to determine β_{HL} due to its unit-less conversions. The model can be further extended to include actual loss and p_f into a single metric. It could also be trialed against other industry sectors to evaluate its validity to aid recommendations for decision making.

Appendix

The house price series H1, H2 and H3 used in this paper are listed in Table I. H1 is constructed from historical median house price from The Real Estate Institute of Victoria [26]. H2 and H3 are based on DPI and CPI respectively. For validation, actual DPI was obtained from World Data Atlas [28] and was cross referenced with the

Australian Bureau of Statistics (ABS), 5204.0 Table 37 series A2421989L and 3101.0 Table 1 series A2133251W for trend consistency. CPI is Melbourne-specific and is extracted from ABS, 6401.0 Table 1 and 2 series A2325811C. For forecasting, DPI growth was inferred from the budget forecast of household consumption of 2012-13 in the Australian Government Budget 2013-14 [21]. In the absence of accurate DPI forecast, household consumption has been assumed to be an adequate indicator for DPI growth. CPI is assumed to be 2.5% [22] [23] [24], taken as the midpoint of RBA's inflation target range of 2-3%.

TABLE I
HOUSE PRICE SERIES H1, H2 AND H3

Year	H1	H2	H3	Construction cost
2013	569,000	569,000	569,000	490,127
2014	623,110	593,325	579,811	508,470
2015	653,335	618,689	590,827	521,441
2016	684,752	645,138	602,053	534,412
2017	717,408	672,718	613,492	547,383
2018	751,352	701,477	625,148	560,355
2019	786,634	731,465	637,026	573,326
2020	823,307	762,735	649,130	586,297
2021	861,426	795,342	661,463	599,269
2022	901,047	829,343	674,031	612,240
2023	942,231	864,797	686,838	625,211
2024	985,039	901,767	699,888	638,182
2025	1,029,535	940,318	713,185	651,154
2026	1,075,784	980,516	726,736	664,125
2027	1,123,858	1,022,433	740,544	677,096
2028	1,173,827	1,066,142	754,614	690,067
2029	1,225,766	1,111,720	768,952	703,039
2030	1,279,753	1,159,246	783,562	716,010
2031	1,335,868	1,208,804	798,450	728,981
2032	1,394,196	1,260,480	813,620	741,952
2033	1,454,823	1,314,366	829,079	754,924
2034	1,517,841	1,370,555	844,832	767,895
2035	1,583,344	1,429,146	860,883	780,866
2036	1,651,429	1,490,242	877,240	793,837
2037	1,722,199	1,553,950	893,908	806,809
2038	1,795,759	1,620,381	910,892	819,780
2039	1,872,219	1,689,653	928,199	832,751
2040	1,951,694	1,761,885	945,835	845,722
2041	2,034,303	1,837,206	963,806	858,694
2042	2,120,168	1,915,746	982,118	871,665
2043	2,209,419	1,997,645	1,000,778	884,636

Construction costs are extracted from the Australian Construction Handbook by Rawlinsons from years 1983 to 2013, items 13.1.2.5, 13.1.3.7, 13.1.3.8. As the focus of this paper is on house price, the forecasting for construction costs was obtained through linear regression for simplicity and is shown in Table I. Stamp duty specifications are from the State Revenue Office Victoria [29], and rental income is the average yield from 1993 to 2009, taken from Westpac Property Outlook. Land tax specifications are also from the State Revenue Office Victoria [30]. City of Melbourne council rates were used [31]. Taxable income includes rental income and capital gain minus interest payment and stamp duty, with the information of the tax specifications extracted from the Australian Taxation Office (ATO) [32]. Discount rate was inferred from the average cash rate between 1993-2013 from the RBA interest rates and yields table [33]. Table II summarizes the assumptions used in the study.

TABLE II
ASSUMPTIONS

Description	Quantity
Currency	All costs and benefits in AUD
Discount rate	5.2%
Base year	2013
Year of house price deviation from trend (applicable for H2 and H3)	2014
Tax rate (%)	Specifications from ATO
Loan for house purchase and construction	80%
Loan interest rate	6%
CPI rate for forecast	2.5%
Disposable income per capita growth rate for forecast	2.5%

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