A Multi-stage Financial Hedging Strategy for a Risk-averse Firm with Contingent Payment

Qiang Li, Lap Keung Chu

***Abstract*—This paper aims at addressing the contingent sales price risk mitigation problem of a risk-averse firm which procures some kind of commodity from the spot market as the major input for production. The downstream buyer pays the firm following a contingent payment rule by which the exact amount depends on the input commodity spot price when the product is physically delivered. In order to reduce the volatility originating from the contingent payment, a multi-stage financial hedging strategy using commodity futures contracts is proposed. This approach allows the firm to adjust the position in commodity futures market dynamically. The close-form optimal hedging strategies are presented when the firm adopts the exponential or mean-variance utility to characterize the risk-averse attitude.**

***Index Terms*—commodity, financial hedging, risk aversion, volatile raw material price**

I. Introduction

UNDER today's fast-changing business environment, small-and-medium-sized firms are facing considerable challenges in maintaining a healthy financial status, due to the pressures from both the selling side and the sourcing side. On the selling side, the firm's profit margin is squeezed seriously by powerful buyers who are seeking for a low procurement payment. For instance, in the American automobile industry, the powerful automakers put great pressure on numerous small suppliers. The profit margin is so low that many suppliers routinely lose money and even go bankrupt in the past decade [1-2]. On the sourcing side, the firms suffer from the fluctuating raw material prices of the input commodity as they rely more on the spot market for the acquisition of raw materials [3]. According to a report released by the Efficio Consulting, over 55% of the interviewees, who are procurement professionals at various levels, considered the commodity price instability as their single biggest challenge [4]. To help reduce these pressures and enhance the competitiveness of the supply chain, various forms of contracts have been designed to effectively cope with the volatile commodity risks (to name a few, [5-8], etc.).

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Rather than designing contracts, this study chooses to explore the uses of financial instruments to mitigate commodity price risk exposure. Due to the high liquidity and accessibility, commodity futures contracts traded in the commodities exchange markets are selected to develop the hedging strategy. There are plenty of commodities exchange markets around the world where various input commodities futures are available. For example, the Shanghai Futures Exchange (SHFE), the New York Mercantile Exchange (NYMEX) and the London Metal Exchange (LME). The type of risk to deal with here is incurred from the contingent payment required by the powerful downstream buyer. Under this context, the future payment received by the firm depends on the future spot price of the commodity, which is the major input for production. Therefore, this payment is uncertain when the contract is signed at the beginning. Such payment is found to be common in real practices. For instance, large contract manufacturer such as Foxconn adopts such contingent payment to deal with its component suppliers. Therefore, the component supplier will be paid based on the raw material price on the delivery date instead of the contract signing date [8]. This kind of contingent payment belongs to the so-called flexible contract, dynamic contract, or index-linked payment contract in [8], [9], and [10], respectively. Details on the contingent payment are introduced in the next section.

This paper addresses the risk management problem of a small-and-medium-sized firm suffering from the contingent payment linked with the input commodity spot price. In the model, the demand is known at the beginning, i.e., the firm satisfies the customer demand in a make-to-order fashion. To simplify the analysis, we assume that the firm procures the input commodity from the spot market only. Specifically, Li and Chu [11] considered a similar situation where a two-stage problem is studied. In a similar study, Ni et al. [12] proposed a multi-stage hedging strategy to mitigate the volatility of procurement cost arising from erratic commodity spot price under quadratic utility criterion. While they studied a problem with long planning horizon with unknown demand we consider a situation where the make-to-order firm receives uncertain contingent payment from the buyer.

To characterize the risk-averse attitude of the firm, mean-variance and exponential utility functions are employed for this analysis. Since the seminal work of Markowitz [13], mean-variance analysis has been popular in many research areas such as finance and operation management [14-17]. Exponential utility is another popular approach to characterize the risk behavior which is also supported by many researches (e.g., [18]). Following the finance literature, terminal wealth is defined to represent the sum of the given initial monetary wealth and the revenue received from operational and hedging activities over the entire planning horizon. The goal of the firm is then to maximize the expected utility with respect to the terminal wealth. Following the approach of Anderson and Danthine [19], this study proposes an effective financial hedging strategy for the firm to mitigate the terminal wealth volatility.

II. Model Formulation

This section describes a multi-stage futures hedging model with $N+1$ trading time points. In this context, the planning horizon could be divided into N stages arbitrarily depending on the need of the firm management. t is used to index the trading points, $t \in \{1, 2, \cdots, N\}$. At the beginning of the planning horizon ($t=1$), given the initial wealth W_0, the firm procures the commodity from the spot market at the price of S_1 for production to satisfy the customer order Q. At the same time, the firm initializes the position, θ_1, in the commodity futures market. After the initialization, the futures position will be adjusted dynamically in the subsequent stages. Specifically, for any intermediate trading point $2 \le t \le N$, the profit or loss of the futures contracts entered at $t-1$ is realized. This implies that the future contracts are assumed to be "marked to market", i.e., all profit or loss of a futures position are realized at the end of each trading time point. There is no operation decisions needed to be made at these time points. At the end of the final stage, the futures contracts will be settled in cash. That is, at $t=N+1$ the firm will not hold any futures contract ($\theta_{T+1}=0$). At the moment, the spot market price of the commodity, S_{T+1}, is observed. The production process is completed and the processed product is then delivered to the buyer at the price of $S_{N+1}+p$, where p represents the exogenous unit markup. The aforementioned delivery price is the realization of the predetermined uncertain contingent payment at the beginning of the planning horizon. In other words, the contingent payment received by the firm in this model is the sum of a unit markup and the market value of one unit input commodity at the time when products are transferred. Note that the real material acquisition cost is determined at the beginning of the production horizon, which is S_1. This type of contingent payment is a special case in [8]. Without loss of generality, one unit processed product requires one unit of input commodity and the firm's production cost is zero. The result also holds true for a more general case with nonzero production cost as the term of production cost can be eliminated by adjusting the term of markup.

For model tractability, the financial market is assumed to be complete. Moreover, to preclude risk-free arbitrage opportunities, the expectation and variance of the stochastic commodity prices are taken under the risk-neutral probability measure, which is standard in finance. The complete financial market assumption guarantees that such probability measure exists uniquely. In our setting, there exists a futures contract written on the price of the same commodity used for production and its maturity date is the same as the end of the planning horizon, i.e., $t=N+1$. The complete financial market assumption and the existence of the futures contract with perfectly matched maturity date imply that no basis risk exists in the hedge. In this paper, the terminal wealth, W_{N+1}, denotes the sum of the firm's initial wealth and the profit or loss from both operation and trading in commodity futures during the planning horizon. The objective of the firm is to maximize the expected utility of the terminal wealth. This assumption implies that the firm is only concerned with the terminal wealth at the end of the planning horizon. This is true in the sense that firms in practice care more about the profit of each quarter (or half year or year). In addition, we assume that the firm has sufficient working capital to maintain the position of the futures contracts during the entire planning horizon.

The mathematical notations for the model are listed as follows.

Q the demand of the product, which is known at the beginning of the production horizon, i.e., $t=1$.

p the unit fixed markup which is exogenous and deterministic

r the constant risk-free interest rate

S_t the spot price of the input commodity at t

F_t the time-t price of the futures contracts maturing at the end of the planning horizon

θ_t the position of the futures contracts at t (a long position is represented as $\theta_t < 0$)

W_t the wealth of the firm at t, W_0 is the initial wealth

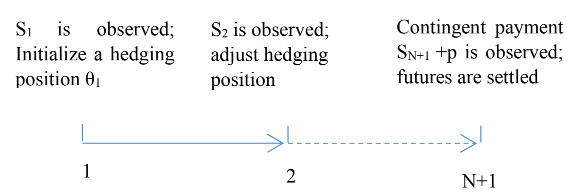

Fig. 1 Sequence of events

Following the sequence of events and using the notations defined above, the uncertain terminal wealth of the firm can be obtained as follows. The wealth at $t=1$ is equal to the firm's initial wealth subtracting the ordering cost from the spot market, i.e.,

$$W_1 = W_0 - S_1 Q \quad (1)$$

The firm's wealth at $t=2$ (immediately after θ_1 is executed but before θ_2 is initiated) is

$$W_2 = (1+r)W_1 + (F_1 - F_2)\theta_1 \tag{2}$$

where θ_1 is the size of the firm's position in this futures contract from $t=1$ to $t=2$. Note that $\theta_2 - \theta_1$ is the amount of futures the firm sells at $t=2$.

Similarly, the firm's wealth at the subsequent trading time points can be derived. Specifically, the firm's terminal wealth at $t = N+1$ can be obtained

$$W_{N+1} = (1+r)W_N + (F_N - F_{N+1})\theta_T + (p + S_{N+1})Q \tag{3}$$

Notice that S_{N+1} is equal to F_{N+1} in our model due to the absence of basis risk.

To summarize, the wealth at each trading points during the planning horizon can be described as:

$$W_t = \begin{cases} W_0 - S_1 Q, & t=1 \\ (1+r)W_{t-1} + (F_{t-1} - F_t)\theta_{t-1}, & 2 \le t \le N \\ (1+r)W_N + (F_N - F_{N+1})\theta_N & \\ \qquad + (p + S_{N+1})Q, & t = N+1 \end{cases} \tag{4}$$

The optimal hedging strategy can thus be obtained by solving the following multi-stage hedging problem:

$$\max_{\{\theta_t\}_{t=1}^N} E_1\left[U\left(W_{N+1}\right)\right] \tag{5}$$

where $E_1[\cdot]$ represents that the expectation is taken conditional on the σ-algebra $\mathscr{F}_1$, which represents the information available at the beginning of horizon. The price information evolution process is then a filtration, which is an increasing family of σ-algebra $\mathscr{F}_t, t = 1, 2, \cdots. N+1$ [20]. Similarly, in the ensuing analysis, $E_t[\cdot] = E[\cdot \mid \mathscr{F}_t]$ and $Var_t(\cdot) = Var(\cdot \mid \mathscr{F}_t)$, where $\mathscr{F}_t$ denotes the information available at the beginning of stage t.

In the following sections, the specific objectives of the firm under both utility functions and the corresponding optimal hedging strategies are presented, respectively.

III. Mean-Variance Utility Criterion

This section describes the case in which the firm employs a mean-variance utility function of the following form

$$E_1\left[U\left(W_{N+1}\right)\right] = E_1\left[W_{N+1}\right] - \frac{1}{2}\lambda Var_1\left(W_{N+1}\right) \tag{6}$$

where λ is a strictly positive constant representing risk aversion.

The following theorem determines the optimal position of the futures contracts for the firm to hold at each stage and the corresponding optimal utility of the terminal wealth in the presence of financial hedging.

Theorem 1: The optimal hedging strategy with respect to the mean-variance criterion is:

$$\theta_t^* = \frac{Q}{\beta_t}, t = 1, 2, \cdots, N. \tag{7}$$

where $\beta_t = (1+r)^{N-t}$

The maximal utility of the terminal wealth is given as

$$\begin{aligned} &\max_{\{\theta_t\}_{t=1}^N} E_1\left[U\left(W_{N+1}\right)\right] \\ &= (1+r)^N W_0 + \left(p + F_1 - (1+r)^N S_1\right)Q \end{aligned} \tag{8}$$

This expression shows that the volatility of the firm's utility with respect to the terminal wealth is perfectly mitigated by the proposed hedging strategy because all the terms are deterministic now.

Proof: The optimal hedging position in the futures market can be derived by induction.

To start with, we prove that the statement in the theory holds for the last stage.

At $t = N$, the beginning of the final stage, the firm chooses the best θ_N by taking $\{\theta_t\}_{t=1}^{N-1}$ as given. The optimal policy can be obtained by solving the following problem:

$$\max_{\theta_N}\left\{E_N\left[W_{N+1}\right] - \frac{1}{2}\lambda Var_N\left(W_{N+1}\right)\right\} \tag{9}$$

The above conditional expectation of the utility of the terminal wealth at $t = N$ is

$$\begin{aligned} E_N\left[W_{N+1}\right] &= V_N\left(Q, W_N\right) + \left(F_N - E_N\left[F_{N+1}\right]\right)\theta_N \\ &\quad + E_N\left[F_{N+1}\right]Q \end{aligned} \tag{10}$$

where $V_N\left(Q, W_N\right) = (1+r)W_N + pQ$.

The corresponding conditional variance at $t = N$ is

$$\begin{aligned} Var_N\left(W_{N+1}\right) &= E_N\left[\left(W_{N+1} - E_N\left[W_{N+1}\right]\right)^2\right] \\ &= ({\theta_N}^2 - 2\theta_N Q + Q^2) Var_N\left(F_{N+1}\right) \end{aligned} \tag{11}$$

Taking the first derivation with respect to θ_N, the necessary and sufficient conditions for the optimal θ_N are given by the following equation

$$\left(F_N - E_N\left[F_{N+1}\right]\right) - \lambda\left(\theta_N Var_N\left(F_{N+1}\right) - Q Var_N\left(F_{N+1}\right)\right) = 0$$

Since there is no arbitrage opportunity, i.e., $F_N = E_N\left[F_{N+1}\right]$, we have

$$\theta_N^* = Q.$$

Note that $\beta_N = (1+r)^{N-N} = 1$. Thus, the statement holds for the last stage.

Next we suppose that the statement in the theory holds for any stage $1 < t \le N$, i.e., $\theta_t^* = \dfrac{Q}{\beta_t}$. The proof will be completed if we can prove that the statement holds for stage

$t-1$.

At the beginning of stage $t-1$, the optimal hedging position in the futures market are known based on the assumption, i.e.,

$$\theta_s^* = \frac{Q}{\beta_s}, s = t, t+1, \cdots, N.$$

Specifically, the optimal hedging position in futures market for stage $t-1$ can be derived by solving the following problem:

$$\max_{\theta_{t-1}} \left\{ E_{t-1}\left[W_{N+1}\right] - \frac{1}{2}\lambda Var_{t-1}\left(W_{N+1}\right) \right\} \tag{12}$$

Following (4), the terminal wealth can be written as:

$$\begin{aligned} W_{N+1} &= \beta_{t-1} W_{t-1} + \sum_{s=t-1}^{N} \beta_s (F_s - F_{s+1}) \theta_s \\ &\quad + (p + S_{N+1}) Q \end{aligned} \tag{13}$$

Substituting $\{\theta_s^*\}_{s=t}^N$ into (13), the terminal wealth can be rewritten as

$$W_{N+1} = \beta_{t-1}[(1+r)W_{t-1} + (F_{t-1} - F_t)\theta_{t-1}] + (p + F_t)Q \tag{14}$$

Following the procedure in the last stage, the expectation and variance of utility of the terminal wealth at $t-1$ can be derived:

$$E_{t-1}[W_{N+1}] = (1+r)\beta_{t-1}W_{t-1} + (p + E_{t-1}[F_t])Q \tag{15}$$

$$Var_{t-1}(W_{N+1}) = (\beta_{t-1}^2 \theta_{t-1}^2 - 2\beta_{t-1}\theta_{t-1}Q + Q^2) Var_{t-1}(F_t) \tag{16}$$

By taking the first derivative of the objective function in (12), i.e., $E_{t-1}\left[W_{N+1}\right] - \frac{1}{2}\lambda Var_{t-1}\left(W_{N+1}\right)$, the necessary and sufficient conditions for the optimal θ_{t-1} can be obtained:

$$\theta_{t-1}^* = \frac{Q}{\beta_{t-1}}.$$

By far, the proof of the optimal hedging position in the futures market is completed.

Substituting $\{\theta_t^*\}_{t=1}^N$ into (6), the corresponding utility of the terminal wealth with the optimal financial hedging strategy is

$$\begin{aligned} & E_1\left[W_{N+1}\right] - \frac{1}{2}\lambda Var_1\left(W_{N+1}\right)\Big|_{\{\theta_t^*\}_{t=1}^N} \\ &= E_1\left[W_{N+1}\right]\Big|_{\{\theta_t^*\}_{t=1}^N} \\ &= (1+r)^N W_0 + \left(p + F_1 - (1+r)^N S_1\right) Q \end{aligned} \tag{17}$$

IV. Exponential Utility Criterion

This section describes the case in which the firm employs an exponential utility function on the terminal wealth, i.e.,

$$U\left(W_{N+1}\right) = -\exp\left(-\rho W_{N+1}\right) \tag{18}$$

where $\rho > 0$ represents the firm's risk sensitivity. A large ρ implies the firm has a more risk-averse attitude. Note that $U'\left(W_{N+1}\right) > 0$ and $U''\left(W_{N+1}\right) < 0$. Therefore, the firm's objective is to maximize the expected value of a strictly concave utility function of the terminal wealth, i.e.,

$$\max_{\{\theta_t\}_{t=1}^N} E_1\left[-\exp\left(-\rho W_{N+1}\right)\right] \tag{19}$$

Theorem 2: The optimal hedging strategy with the exponential utility is

$$\theta_t^* = \frac{Q}{\beta_t}, t = 1, 2, \cdots, N. \tag{20}$$

where $\beta_t = (1+r)^{N-t}$

The corresponding maximal utility of the terminal wealth is given as

$$\max_{\{\theta_t\}_{t=1}^N} E_1\left[-\exp\left(-\rho W_{N+1}\right)\right] = -\exp\left(-\rho V_0\left(Q\right)\right) \tag{21}$$

where

$$V_0\left(Q\right) = (1+r)^N W_0 + \left(p + F_1 - (1+r)^N S_1\right) Q \tag{22}$$

The results indicate that the volatility of the utility of the terminal wealth can be perfectly hedged by the proposed strategy as all the terms in the optimal utility are deterministic.

Proof: Similar to the case with mean-variance utility, this problem can be solved by induction. To start with, we prove that the statement in the theory holds for the last stage.

At $t = N$, the beginning of the final stage, the firm should choose the best θ_N given $\{\theta_t\}_{t=1}^{N-1}$. The optimal policy can be obtained by solving the following problem:

$$\max_{\theta_N} E_1\left[-\exp\left(-\rho W_{N+1}\right)\right] \tag{23}$$

where

$$W_{N+1} = (1+r)W_N + \left(F_N - F_{N+1}\right)\theta_N + \left(p + S_{N+1}\right)Q$$

By taking the first derivative with respect to θ_N, the condition for the optimal solution can be derived:

$$E_N\left[\rho \exp\left(-\rho W_{N+1}\right)\left(F_N - F_{N+1}\right)\right] = 0 \tag{24}$$

Or, equivalently

$$F_N E_N\left[\exp\left(-\rho W_{N+1}\right)\right] = E_N\left[\exp\left(-\rho W_{N+1}\right) F_{N+1}\right] \tag{25}$$

It can be shown that $\theta_N = \frac{Q}{\beta_N} = Q$ is the solution of the above equation. Moreover, this is the only solution for the problem because of the concavity of the utility function.

Next we suppose that the statement in the theory holds for any stage $1 < t \le N$, i.e., $\theta_t^* = \frac{Q}{\beta_t}$. The proof will be completed if we can prove that the statement holds for stage $t-1$.

Following (4), the terminal wealth can be written as:

$$\begin{aligned} W_{N+1} = \beta_{t-1} W_{t-1} + \sum_{s=t-1}^{N} \beta_s (F_s - F_{s+1}) \theta_s \\ +(p + S_{N+1}) Q \end{aligned} \tag{26}$$

At the beginning of stage $t-1$, the optimal hedging position in the futures market are known based on the assumption, i.e.,

$$\theta_s^* = \frac{Q}{\beta_s}, s = t, t+1, \cdots, N.$$

Substituting $\{\theta_s^*\}_{s=t}^{N}$ into (26), the terminal wealth can be rewritten as

$$W_{N+1} = \beta_{t-1} \cdot [(1+r) W_{t-1} + (F_{t-1} - F_t)\theta_{t-1}] + (p + F_t) Q$$

Specifically, the optimal hedging position in futures market for stage $t-1$ can be derived by solving the following problem:

$$\max_{\theta_{t-1}} \left\{ E_{t-1}[-\exp(-\rho W_{N+1})] \right\} \tag{27}$$

Again, by taking the first derivative with respect to θ_{t-1}, the condition for the optimal solution can be derived:

$$E_{t-1}\left[\rho \exp(-\rho W_{N+1})(F_{t-1} - F_t)\right] = 0 \tag{28}$$

Or, equivalently

$$F_{t-1} E_{t-1}\left[\exp(-\rho W_{N+1})\right] = E_{t-1}\left[\exp(-\rho W_{N+1}) F_t\right] \tag{29}$$

Substituting $\theta_{t-1} = \frac{Q}{\beta_{t-1}}$ into (29), it is easy to check that the equality holds as $\exp(-\rho W_{N+1})$ does not dependent on F_t. Moreover, there exist one unique solution for the problem due to the concavity. Thus, we can conclude that $\theta_{t-1}^* = \frac{Q}{\beta_{t-1}}$.

By far, the proof of the optimal hedging position in the futures market is completed.

At last, substituting $\{\theta_t^*\}_{t=1}^{N}$ into the objective function, the corresponding utility of the terminal wealth with the optimal financial hedging strategy can be derived.

V. Concluding Remarks

In this study, a simple yet effective multi-stage hedging strategy is proposed for a risk-averse small-and-medium-sized firm to minimize the volatility of the utility with respect to the terminal wealth. The volatility arises from the contingent payment received from the buyer, which depends on the commodity spot price on the day when the order is satisfied. It is worth noting that the strategy is quite general because it does not involve any specific models for price evolution of the futures contracts and the underlying commodity. The optimal positions in the commodity futures market are the same for the firm with respect to exponential utility and mean-variance utility. In other words, the firm's optimal hedging strategy is independent of these two risk preferences. This is because that there is only one uncertainty source in this paper, i.e., the volatility of the input commodity price. In this simple case without demand uncertainty, we can see that all the variability in the terminal wealth could be fully hedged. In a general case with demand uncertainty, financial hedging strategy might depend on the utility function chosen to represent the risk attitude. It is expected that when demand uncertainty is considered, only partial hedge could be achieved. It will be a challenge and interesting issue for future study. For other further research directions, realistic issues such as transaction cost and basis risk might be taken into account.

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