Boundedness and Exponential Stability of Positive Solutions for Nicholson-type Delay System

Changjin Xu, and Maoxin Liao

Abstract—**In this paper, we study an Nicholson-type delay system with delays. New criteria for the boundedness and exponential stability of positive solutions of Nicholson-type system with time-varying delays are established by applying the fundamental solution matrix, inequality techniques and Lyapunov method. Two examples with their computer simulations are presented to illustrate the effectiveness of the theoretical findings. Our results are new and supplement some previously known ones.**

Index Terms—**Nicholson-type delay system, positive solution, exponential stability, delay, Lyapunov method.**

I. Introduction

THE classical Nicholsons blowflies model

$$\dot{x}(t) = -ax(t) + bx(t-\tau)e^{-cx(t-\tau)} \tag{1}$$

was introduced by Nicholson [1] to model laboratory fly population. Here $x(t)$ denotes the size of the population at time t, b denotes the maximum per capita daily egg production, $\frac{1}{c}$ denotes the size at which the population reproduces at its maximum rate, a denotes the per capita daily adult dath rate, and τ denotes the generation rate. The dynamical behavior has been investigated by Gurney et al. [2] and Nisbet and Gurney [3]. Recently, considerable effort has been devoted to studying the various Nicholsons blowflies models and their modifications. For example, supposing that a harvesting function is the delayed estimate of the true population, Berezansky et al. [4] introduced the following Nicholsons blowflies model with a linear harvesting term:

$$\begin{cases} \dot{x}(t) = -ax(t) + bx(t-\tau_1)e^{-cx(t-\tau_1)} \\ \qquad\quad - hx(t-\tau_2), \\ a, b, c, h, \tau_1, \tau_2 \in (0, +\infty), \end{cases} \tag{2}$$

and gave an open problem: How about the dynamics of (2). Considering that the parameters in the model are pseudo almost periodic functions, Duan and Huang discussed the the existence and convergence dynamics of positive pseudo almost periodic solutions of the following Nicholsons blowflies model with varying coefficients and a linear harvesting term:

$$\begin{cases} \dot{x}(t) = -a(t)x(t) + b(t)x(t-\tau_1(t))e^{-c(t)x(t-\tau_1(t))} \\ \qquad\quad - h(t)x(t-\tau_2(t)), \end{cases} \tag{3}$$

where $a(t), b(t), c(t), h(t) \in (0, +\infty)$, $\tau_1(t), \tau_2(t) \in [0, +\infty)$ are continuous functions. Noticing that in real natural word, the change of the environment and impulsive effect play an important role in numerous biological and ecological dynamical systems [5], Alzabut [5] focused on the positive almost periodic solution of the following delay Nicholson's blowflies model with impulsive effect which is a generalized form of model (1)

$$\begin{cases} \dot{x}(t) = -\alpha(t)x(t) + \sum_{i=1}^{n}\beta_i(t)x(t-\tau) \\ \qquad\quad \times e^{-\lambda_i(t)x(t-\tau)} + h(t), t \neq \theta_k, \\ \Delta x(\theta_k) = \gamma_k x(\theta_k) + \delta_k, k \in \mathbb{N}, \end{cases} \tag{4}$$

where $\alpha(t), \beta_i(t), \lambda_i(t), h(t) \in [\mathbb{R}^+, \mathbb{R}^+], \tau > 0$ and $\gamma_k, \delta_k \in \mathbb{R}, k \in \mathbb{N}$, $h(t)$ is a harvesting function, $\Delta x(t)$ represents the difference $x(t^+) - x(t^-)$, where $x(t^+)$ and $x(t^-)$ define the limits from right and left, respectively, θ_k denotes the instants at which size of the population suffers an increment of δ_k units. By applying the contraction mapping principle and Gronwall-Bellman's inequality, Alzabut [5] obtained some sufficient conditions which guarantee the existence and exponential stability of positive almost periodic solution for the model (4). For more details on Nicholson's blowflies models, we refer the reader to [6-28].

In 2011, to describe the models of Marine Protected Areas and B-cell Chronic Lymphocytic Leukemia dynamics [29], Berezansky [30] have investigated the global dynamics of the following Nicholson-type delay system

$$\begin{cases} \dot{x}_1(t) = -a_1x_1(t) + b_1x_2(t) \\ \qquad\quad + c_1x_1(t-\tau)e^{-x_1(t-\tau)}, \\ \dot{x}_2(t) = -a_2x_2(t) + b_2x_1(t) \\ \qquad\quad + c_2x_2(t-\tau)e^{-x_2(t-\tau)}, \end{cases} \tag{5}$$

with initial conditions:

$$x_i(s) = \varphi_i(s), s \in [-\tau, 0], \varphi_i(0) > 0, \tag{6}$$

where $\varphi_i \in C([-\tau, 0], [0, +\infty)), a_i, b_i, c_i$ and τ are nonnegative constants, $i = 1, 2$.

Here shall point out that the existence of positive solutions of Nicholson-type delay systems plays an important role in characterizing their dynamical behavior. Then the research on the positive solutions of Nicholson-type delay systems has important theoretical value and tremendous potential for application. Thus it is worth while to investigate the existence and stability of positive solutions for Nicholson-type delay system. To the best of our knowledge, there is no paper published on the existence and exponentially stability of positive solutions for Nicholson-type delay systems.

Motivated by the discussions above, we will investigate the existence and exponential stability of positive solutions of

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the following Nicholson-type delay system

$$\begin{cases} \dot{x}_1(t) = -a_1x_1(t) + b_1x_2(t) \\ \qquad\qquad + c_1(t)x_1(t-\tau)e^{-x_1(t-\tau)}, \\ \dot{x}_2(t) = -a_2x_2(t) + b_2x_1(t) \\ \qquad\qquad + c_2(t)x_2(t-\tau)e^{-x_2(t-\tau)}, \end{cases} \tag{7}$$

which is more general than (5).

The purpose of this paper is to present sufficient conditions which ensure the existence and exponential stability of positive solutions of system (7). Applying the fundamental solution matrix, Lyapunov function and constructing fundamental function sequences based on the solution of Nicholson-type delay models, we establish some sufficient conditions which guarantee the existence and global exponential stability of positive solutions of (7). In addition, two examples are presented to illustrate the effectiveness of our main results. Our results are essentially new and complement some previously known ones.

The rest of this paper is organized as follows. In Section 2, we give some notations and preliminary results. In Section 3, we present our main results on the existence and global exponential stability of positive solutions of the Nicholson-type delay system. In Section 4, we support our main theoretical finding by two examples with their computer simulations. A brief conclusion is drawn in Section 5.

II. Preliminary results

In this section, we shall present some notations and introduce some lemmas which are used in the following sections. Denote

$$\bar{c}_1 = \sup_{t\in R}|c_1(t)|, \bar{c}_2 = \sup_{t\in R}|c_2(t)|,$$

For any vector $V = (v_1, v_2)^T$ and matrix $D = (d_{ij})_{2\times 2}$, we define the norm as

$$||v|| = \left(v_1^2 + v_2^2\right)^{\frac{1}{2}}, ||D|| = \left(d_{11}^2 + d_{12}^2 + d_{21}^2 + d_{22}^2\right)^{\frac{1}{2}},$$

respectively. Let $\varphi(s) = (\varphi_1(s), \varphi_2(s))^T$, where $\varphi_i(s) \in C([-\tau, 0], R), i = 1, 2$. Define

$$||\varphi|| = \sup_{-\tau\le s\le 0}\left(\varphi_1(s)|^2 + \varphi_2(s)|^2\right)^{\frac{1}{2}}.$$

We assume that system (7) always satisfies the following initial conditions:

$$\varphi_{i0}(s) = \varphi_i(s), -\tau \le s \le 0, i = 1, 2. \tag{8}$$

In order to obtain our main results in this paper, we make the assumptions as follows.

(H1) $a_1 + a_2 > 0, a_1a_2 > b_1b_2$.

(H2)

$$\begin{cases} -2a_1 + b_1 + \dfrac{\bar{c}_1}{e^2} + b_2 + \dfrac{\bar{c}_1}{e^2} < 0, \\ -2a_2 + b_2 + \dfrac{\bar{c}_2}{e^2} + b_1 + \dfrac{\bar{c}_2}{e^2} < 0. \end{cases}$$

Definition 2.2. *The solution $x^*(t) = (x_1^*(t), x_2^*(t))^T$ of system (7) is said to globally exponentially stable if there exist constants $\beta > 0$ and $M > 1$ such that*

$$\sum_{i=1}^{n}|x_i(t) - x_i^*(t)| \le Me^{-\beta t}||\varphi - \varphi^*||^2$$

for each solution $x(t) = (x_1(t), x_2(t))^T$ of system (7).

Next, we present three important lemmas which are used for proving our main results in Section 3.

Lemma 2.1. *Let*

$$A = \begin{bmatrix} -a_1 & b_1 \\ b_2 & -a_2 \end{bmatrix}.$$

If (H1) *holds, then we have*

$$||\exp At|| \le e^{-\alpha t}$$

for all $t \ge 0$.

Proof Let λ be the characteristic exponent of the matrix A, then we have

$$\det\begin{bmatrix} \lambda + a_1 & -b_1 \\ -b_2 & \lambda + a_2 \end{bmatrix} = 0$$

which leads to

$$\lambda^2 + (a_1 + a_2)\lambda + a_1a_2 - b_1b_2 = 0.$$

Thus we obtain the characteristic exponents of the matrix A are

$$\lambda_{1,2} = \frac{-(a_1 + a_2) \pm \sqrt{(a_1 + a_2)^2 - 4(a_1a_2 - b_1b_2)}}{2}.$$

By (H1), we can conclude that λ_1 and λ_2 have negative real parts. In view of [31] and the definition of matrix norm, we get

$$||\exp At|| \le \exp\left(\max\{\text{Re}(\lambda_1), \text{Re}(\lambda_2)\}t\right) \le e^{-\alpha t},$$

where

$$\alpha = \min\{-\text{Re}(\lambda_1), -\text{Re}(\lambda_2)\}.$$

Lemma 2.2. *If* (H2) *holds, then there exists $\beta > 0$ such that*

$$\begin{cases} \beta - 2a_1 + b_1 + \dfrac{\bar{c}_1}{e^2} + b_2 + \dfrac{\bar{c}_1}{e^2}e^{\beta\tau} \le 0, \\ \beta - 2a_2 + b_2 + \dfrac{\bar{c}_2}{e^2} + b_1 + \dfrac{\bar{c}_2}{e^2}e^{\beta\tau} \le 0. \end{cases}$$

Proof Let

$$\varrho_1(\beta) = \beta - 2a_1 + b_1 + \frac{\bar{c}_1}{e^2} + b_2 + \frac{\bar{c}_1}{e^2}e^{\beta\tau},$$
$$\varrho_2(\beta) = \beta - 2a_2 + b_2 + \frac{\bar{c}_2}{e^2} + b_1 + \frac{\bar{c}_2}{e^2}e^{\beta\tau}.$$

Obviously, $\varrho_1(\beta)$ and $\varrho_2(\beta)$ are continuously differential functions with respect to β. We can easily check that

$$\frac{d\varrho_1(\beta)}{d\beta} = 1 + \beta\frac{\bar{c}_1}{e^2}e^{\beta\tau} > 0,$$
$$\lim_{\beta\to+\infty}\varrho_1(\beta) = +\infty, \varrho_1(0) = 0,$$
$$\frac{d\varrho_2(\beta)}{d\beta} = 1 + \beta\frac{\bar{c}_2}{e^2}e^{\beta\tau} > 0,$$
$$\lim_{\beta\to+\infty}\varrho_2(\beta) = +\infty, \varrho_2(0) = 0.$$

By using the intermediate value theorem, there exist constants $\beta_l^* > 0 (l = 1, 2)$ such that

$$\varrho_l(\beta_l^*) = 0, l = 1, 2.$$

Let $\beta_0 = \min\{\beta_1^*, \beta_2^*\}$, then it follows that $\beta_0 > 0$ and

$$\varrho_l(\beta_0) \le 0, l = 1, 2.$$

This completes the proof of Lemma 2.2.

III. Main results

In this section, we present our main results on the existence and exponentially stability of positive solution for (7).

Theorem 3.1. *Assume that* (H1) *holds. Then for any solution* $(x_1(t), x_2(t))^T$ *of system (7) there exists a constant*

$$\Theta = ||\varphi||^2 + \frac{2}{e\alpha}$$

such that

$$|x_1(t)| \leq \Theta, |x_2(t)| \leq \Theta$$

for all $t > 0$.

Proof Let

$$z(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, A = \begin{bmatrix} -a_1 & b_1 \\ b_2 & -a_2 \end{bmatrix},$$

$$F(x_1(t), x_2(t)) = \begin{bmatrix} c_1(t)x_1(t-\tau)e^{-x_1(t-\tau)} \\ c_2(t)x_2(t-\tau)e^{-x_2(t-\tau)} \end{bmatrix},$$

then system (7) can be written as the following equivalent form

$$\dot{z}(t) \leq Az(t) + F(x_1(t), x_2(t)). \tag{9}$$

Solving the inequality (9), we have

$$z(t) \leq e^{At}z(0) + \int_0^t e^{A(t-s)}[F(x_1(s), x_2(s))]ds.$$

It follows from Lemma 2.1 that

$$\begin{aligned} ||z(t)|| &\leq e^{-\alpha t}||z(0)|| + \int_0^t e^{\alpha(t-s)} \\ &\quad \times ||F(x_1(s), x_2(s))||ds \\ &\leq ||\varphi||^2 + \frac{1}{\alpha}\left(1 - e^{-\alpha t}\right)\frac{2}{e} \\ &\leq ||\varphi||^2 + \frac{2}{e\alpha}. \end{aligned} \tag{10}$$

Let

$$\Theta = ||\varphi||^2 + \frac{2}{e\alpha}. \tag{11}$$

Then it follows that

$$|x_1(t)| \leq \Theta, |x_2(t)| \leq \Theta$$

for all $t > 0$. This completes the proof of Theorem 3.1.

Theorem 3.2. *Assume that* (H1) *and* (H2) *are satisfied. Then any solution* $x^*(t) = (x_1^*(t), x_2^*(t))^T$ *of system (7) is globally exponentially stable.*

Proof Let

$$y_1(t) = x_1(t) - x_1^*(t), y_2(t) = x_2(t) - x_2^*(t). \tag{12}$$

It follows from system (7) that

$$\begin{cases} \dot{y}_1(t) = -a_1y_1(t) + b_1y_2(t) + c_1(t) \\ \qquad \times \left[x_1(t-\tau)e^{-x_1(t-\tau)} \right. \\ \qquad \left. -x_1^*(t-\tau)e^{-x_1^*(t-\tau)}\right], \\ \dot{y}_2(t) = -a_2y_2(t) + b_2y_1(t) + c_2(t) \\ \qquad \times \left[x_2(t-\tau)e^{-x_2(t-\tau)} \right. \\ \qquad \left. -x_2^*(t-\tau)e^{-x_2^*(t-\tau)}\right]. \end{cases} \tag{13}$$

By direct computation, we have

$$\begin{cases} \frac{1}{2}\frac{dy_1^2(t)}{dt} = -a_1y_1^2(t) + b_1y_1(t)y_2(t) \\ \qquad + c_1(t)y_1(t)\left[x_1(t-\tau)e^{-x_1(t-\tau)} \right. \\ \qquad \left. -x_1^*(t-\tau)e^{-x_1^*(t-\tau)}\right], \\ \frac{1}{2}\frac{dy_2^2(t)}{dt} = -a_2y_2^2(t) + b_2y_1(t)y_2(t) \\ \qquad + c_2(t)y_2(t)\left[x_2(t-\tau)e^{-x_2(t-\tau)} \right. \\ \qquad \left. -x_2^*(t-\tau)e^{-x_2^*(t-\tau)}\right]. \end{cases} \tag{14}$$

In view of the fact that $\sup_{v\geq 0}\left|\frac{1-v}{e^v}\right| = \frac{1}{e^2}$, we get

$$\begin{cases} \frac{dy_1^2(t)}{dt} \leq -2a_1y_1^2(t) + b_1(y_1^2(t) + y_2^2(t)) \\ \qquad + \bar{c}_1\frac{1}{e^2}(y_1^2(t) + y_1^2(t-\tau)) \\ \frac{dy_2^2(t)}{dt} \leq -2a_2y_2^2(t) + b_2(y_1^2(t) + y_2^2(t)) \\ \qquad + \bar{c}_2\frac{1}{e^2}(y_2^2(t) + y_2^2(t-\tau)). \end{cases} \tag{15}$$

Now we consider the following Lyapunov function

$$\begin{aligned} V(t) &= e^{\beta t}\left[y_1^2(t) + y_2^2(t)\right] \\ &\quad + \frac{\bar{c}_1}{e^2}\int_{t-\tau}^t e^{\beta(s+\tau)}y_1^2(s)ds \\ &\quad + \frac{\bar{c}_1}{e^2}\int_{t-\tau}^t e^{\beta(s+\tau)}y_2^2(s)ds, \end{aligned} \tag{16}$$

where β is given by Lemma 2.2. Differentiating $V(t)$ along solutions to system (7), together with (15), we have

$$\begin{aligned} \frac{dV(t)}{dt} &\leq \beta e^{\beta t}\left[y_1^2(t) + y_2^2(t)\right] \\ &\quad + e^{\beta t}\left[-2a_1y_1^2(t) + b_1(y_1^2(t) + y_2^2(t)) \right. \\ &\quad \left. + \bar{c}_1\frac{1}{e^2}(y_1^2(t) + y_1^2(t-\tau))\right] \\ &\quad + e^{\beta t}\left[-2a_2y_2^2(t) + b_2(y_1^2(t) + y_2^2(t)) \right. \\ &\quad \left. + \bar{c}_2\frac{1}{e^2}(y_2^2(t) + y_2^2(t-\tau))\right] \\ &\quad + \frac{\bar{c}_1}{e^2}\left[e^{\beta(t+\tau)}y_1^2(t) - e^{\beta t}y_1^2(t-\tau)\right] \\ &\quad + \frac{\bar{c}_2}{e^2}\left[e^{\beta(t+\tau)}y_2^2(t) - e^{\beta t}y_2^2(t-\tau)\right] \\ &= e^{\beta t}\left[\beta - 2a_1 + b_1 + \frac{\bar{c}_1}{e^2} + b_2 \right. \\ &\quad \left. + \frac{\bar{c}_1}{e^2}e^{\beta\tau}\right]y_1^2(t) \\ &\quad + e^{\beta t}\left[\beta - 2a_2 + b_2 + \frac{\bar{c}_2}{e^2} + b_1 \right. \\ &\quad \left. + \frac{\bar{c}_2}{e^2}e^{\beta\tau}\right]y_2^2(t). \end{aligned} \tag{17}$$

It follows from Lemma 2.2 that $\frac{dV(t)}{dt} \leq 0$ which implies that $V(t) \le V(0)$ for all $t > 0$. Thus

$$\begin{aligned} & e^{\beta t}\left[y_1^2(t)+y_2^2(t)\right] \\ \le & y_1^2(0)+y_2^2(0) \\ & +\frac{\bar{c}_1}{e^2}\int_{-\tau}^{0} e^{\beta(s+\tau)}y_1^2(s)ds \\ & +\frac{\bar{c}_1}{e^2}\int_{-\tau}^{0} e^{\beta(s+\tau)}y_2^2(s)ds \\ \le & ||\varphi-\varphi^*||^2+\frac{\bar{c}_1}{e^2}\frac{1}{\beta}e^{\beta\tau}||\varphi-\varphi^*||^2 \\ & +\frac{\bar{c}_2}{e^2}\frac{1}{\beta}e^{\beta\tau}||\varphi-\varphi^*||^2 \\ = & \left[1+\frac{\bar{c}_1}{e^2}\frac{1}{\beta}e^{\beta\tau}+\frac{\bar{c}_2}{e^2}\frac{1}{\beta}e^{\beta\tau}\right]||\varphi-\varphi^*||^2. \end{aligned} \quad (18)$$

Let

$$M = 1+\frac{\bar{c}_1}{e^2}\frac{1}{\beta}e^{\beta\tau}+\frac{\bar{c}_2}{e^2}\frac{1}{\beta}e^{\beta\tau} > 1. \quad (19)$$

Then Eq.(18) can be rewritten as

$$y_1^2(t)+y_2^2(t) \le Me^{-\beta t}||\varphi-\varphi^*||^2 \quad (20)$$

for all $t > 0$. Thus

$$\begin{aligned} & (x_1(t)-x_1^*(t))^2(t)+y(x_2(t)-x_1^*(t))^2(t) \\ \le & Me^{-\beta t}||\varphi-\varphi^*||^2 \end{aligned} \quad (21)$$

for all $t > 0$. Thus the solution $x(t) = (x_1(t), x_2(t))^T$ of system (7) is globally exponentially stable.

Remark 3.1. *In [4], Berezansky et al. established the sufficient conditions for the existence, positiveness and permanence of solutions of system (6). In [23], Berezansky et al. obtained the explicit conditions on the existence of positive global solutions of Nicholson-type delay system. In this paper, we consider the bounded and exponential stability of system (7) with varying coefficients by the fundamental solution matrix, Lyapunov function and constructing fundamental function sequences based on the solution of models. (7) is more general than system (6) and the results in [4,23] cannot be applicable to system (7) to obtain the boundedness and exponential stability of positive solutions. This implies that the results of this paper are essentially new.*

IV. EXAMPLES

In this section, we give two examples to illustrate our main results obtained in previous sections.

Example 4.1. Consider the following Nicholson-type system with time-varying delays

$$\begin{cases} \dot{x}_1(t) = -a_1x_1(t)+b_1x_2(t) \\ \qquad\quad +c_1(t)x_1(t-\tau)e^{-x_1(t-\tau)}, \\ \dot{x}_2(t) = -a_2x_2(t)+b_2x_1(t) \\ \qquad\quad +c_2(t)x_2(t-\tau)e^{-x_2(t-\tau)}, \end{cases} \quad (22)$$

where $a_1 = 5, a_2 = 4, b_1 = -2, b_2 = -2, c_1(t) = e^2(0.5 + 0.5\sin t), c_2(t) = e^2(0.4 + 0.6\cos t), \tau = 0.5$. It is easy to check that all the conditions (H1) and (H2) are satisfied. Thus system (22) has exactly one positive solution which is globally exponentially stable. The results are illustrated in Fig.1.

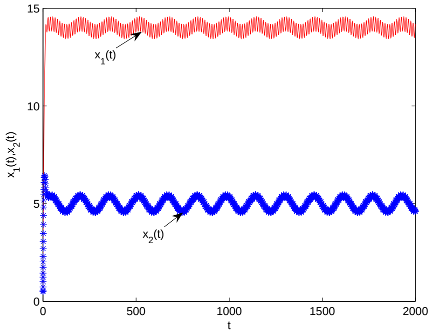

Fig. 1. Transient response of state variables $x_1(t)$ and $x_2(t)$.

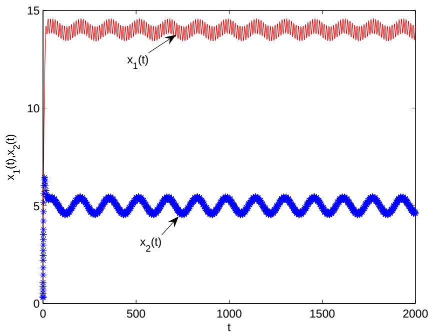

Fig. 2. Transient response of state variables $x_1(t)$ and $x_2(t)$.

Example 4.2. Consider the following Nicholson-type system with time-varying delays

$$\begin{cases} \dot{x}_1(t) = -a_1x_1(t)+b_1x_2(t) \\ \qquad\quad +c_1(t)x_1(t-\tau)e^{-x_1(t-\tau)}, \\ \dot{x}_2(t) = -a_2x_2(t)+b_2x_1(t) \\ \qquad\quad +c_2(t)x_2(t-\tau)e^{-x_2(t-\tau)}, \end{cases} \quad (23)$$

where $a_1 = 6, a_2 = 5, b_1 = -3, b_2 = -3, c_1(t) = e^3(0.6 + 0.6\cos t), c_2(t) = e^3(0.6 + 0.4\sin t), \tau = 0.3$. It is easy to check that all the conditions (H1) and (H2) are satisfied. Thus system (23) has exactly one positive solution which is globally exponentially stable. The results are illustrated in Fig. 2.

Example 4.3. Consider the following Nicholson-type system with time-varying delays

$$\begin{cases} \dot{x}_1(t) = -a_1x_1(t)+b_1x_2(t) \\ \qquad\quad +c_1(t)x_1(t-\tau)e^{-x_1(t-\tau)}, \\ \dot{x}_2(t) = -a_2x_2(t)+b_2x_1(t) \\ \qquad\quad +c_2(t)x_2(t-\tau)e^{-x_2(t-\tau)}, \end{cases} \quad (24)$$

where $a_1 = 7.2, a_2 = 6.1, b_1 = -3.8, b_2 = -2.5, c_1(t) = e^3(0.16+0.16\sin t), c_2(t) = e^4(0.16+0.12\sin t), \tau = 0.12$. It is easy to check that all the conditions (H1) and (H2) are satisfied. Thus system (24) has exactly one positive solution which is globally exponentially stable. The results are illustrated in Fig. 3.

Example 4.4. Consider the following Nicholson-type system

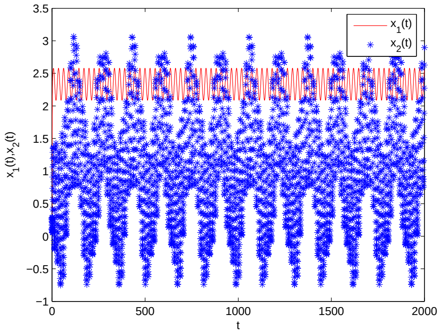

Fig. 3. Transient response of state variables $x_1(t)$ and $x_2(t)$.

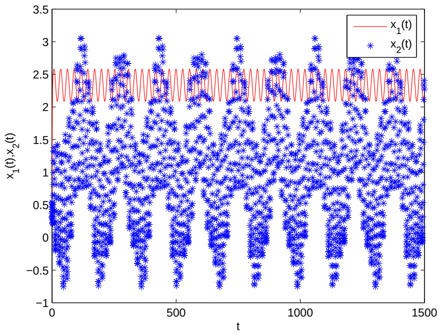

Fig. 4. Transient response of state variables $x_1(t)$ and $x_2(t)$.

with time-varying delays

$$\begin{cases} \dot{x}_1(t) = -a_1x_1(t) + b_1x_2(t) \\ \qquad\quad + c_1(t)x_1(t-\tau)e^{-x_1(t-\tau)}, \\ \dot{x}_2(t) = -a_2x_2(t) + b_2x_1(t) \\ \qquad\quad + c_2(t)x_2(t-\tau)e^{-x_2(t-\tau)}, \end{cases} \tag{25}$$

where $a_1 = 6.23, a_2 = 5.78, b_1 = -2.45, b_2 = -3, c_1(t) = e^5(0.77+0.77\sin t), c_2(t) = e^7(0.77+0.62\cos t), \tau = 0.72$. It is easy to check that all the conditions (H1) and (H2) are satisfied. Thus system (25) has exactly one positive solution which is globally exponentially stable. The results are illustrated in Fig. 4.

Example 4.5. Consider the following Nicholson-type system with time-varying delays

$$\begin{cases} \dot{x}_1(t) = -a_1x_1(t) + b_1x_2(t) \\ \qquad\quad + c_1(t)x_1(t-\tau)e^{-x_1(t-\tau)}, \\ \dot{x}_2(t) = -a_2x_2(t) + b_2x_1(t) \\ \qquad\quad + c_2(t)x_2(t-\tau)e^{-x_2(t-\tau)}, \end{cases} \tag{26}$$

where $a_1 = 8, a_2 = 3, b_1 = -2, b_2 = -4, c_1(t) = e^5(0.38 + 0.38\sin t), c_2(t) = e^5(0.38+0.44\cos t), \tau = 0.52$. It is easy to check that all the conditions (H1) and (H2) are satisfied. Thus system (26) has exactly one positive solution which is globally exponentially stable. The results are illustrated in Fig. 5.

Example 4.6. Consider the following Nicholson-type system with time-varying delays

$$\begin{cases} \dot{x}_1(t) = -a_1x_1(t) + b_1x_2(t) \\ \qquad\quad + c_1(t)x_1(t-\tau)e^{-x_1(t-\tau)}, \\ \dot{x}_2(t) = -a_2x_2(t) + b_2x_1(t) \\ \qquad\quad + c_2(t)x_2(t-\tau)e^{-x_2(t-\tau)}, \end{cases} \tag{27}$$

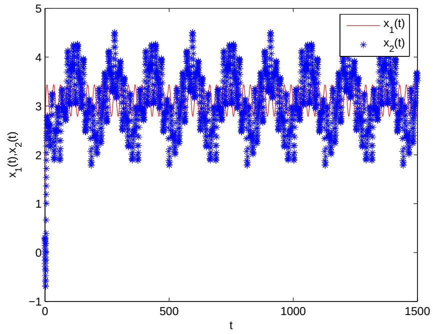

Fig. 5. Transient response of state variables $x_1(t)$ and $x_2(t)$.

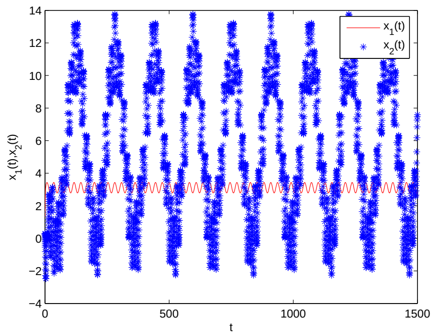

Fig. 6. Transient response of state variables $x_1(t)$ and $x_2(t)$.

where $a_1 = 8, a_2 = 3, b_1 = -4, b_2 = -6, c_1(t) = e^5(0.76 + 0.76\cos t), c_2(t) = e^4(0.76+0.67\sin t), \tau = 0.02$. It is easy to check that all the conditions (H1) and (H2) are satisfied. Thus system (27) has exactly one positive solution which is globally exponentially stable. The results are illustrated in Fig. 6.

Example 4.7. Consider the following Nicholson-type system with time-varying delays

$$\begin{cases} \dot{x}_1(t) = -a_1x_1(t) + b_1x_2(t) \\ \qquad\quad + c_1(t)x_1(t-\tau)e^{-x_1(t-\tau)}, \\ \dot{x}_2(t) = -a_2x_2(t) + b_2x_1(t) \\ \qquad\quad + c_2(t)x_2(t-\tau)e^{-x_2(t-\tau)}, \end{cases} \tag{28}$$

where $a_1 = 4, a_2 = 3, b_1 = -2.6, b_2 = -4, c_1(t) = e^2(0.2+0.2\cos t), c_2(t) = e^4(0.5 + 0.1\sin t), \tau = 0.29$. It is easy to check that all the conditions (H1) and (H2) are satisfied. Thus system (28) has exactly one positive solution which is globally exponentially stable. The results are illustrated in Fig. 7.

Example 4.8. Consider the following Nicholson-type system with time-varying delays

$$\begin{cases} \dot{x}_1(t) = -a_1x_1(t) + b_1x_2(t) \\ \qquad\quad + c_1(t)x_1(t-\tau)e^{-x_1(t-\tau)}, \\ \dot{x}_2(t) = -a_2x_2(t) + b_2x_1(t) \\ \qquad\quad + c_2(t)x_2(t-\tau)e^{-x_2(t-\tau)}, \end{cases} \tag{29}$$

where $a_1 = 5.002, a_2 = 4.902, b_1 = -2.305, b_2 = -3.002, c_1(t) = e^2(0.6012 + 0.6012\cos t), c_2(t) = e^2(0.6012+0.3\sin t), \tau = 0.3$. It is easy to check that all the conditions (H1) and (H2) are satisfied. Thus system (29) has

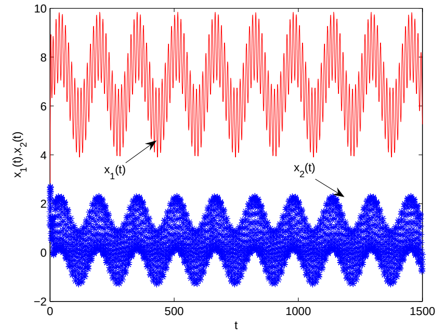

Fig. 7. Transient response of state variables $x_1(t)$ and $x_2(t)$.

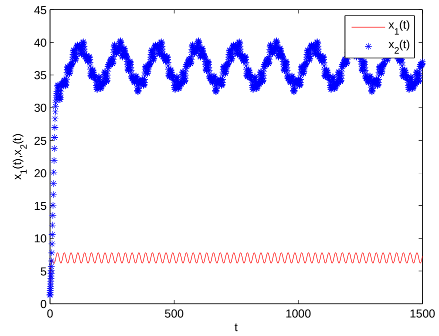

Fig. 8. Transient response of state variables $x_1(t)$ and $x_2(t)$.

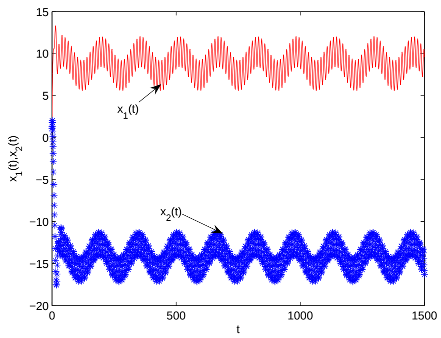

Fig. 9. Transient response of state variables $x_1(t)$ and $x_2(t)$.

exactly one positive solution which is globally exponentially stable. The results are illustrated in Fig. 8.

Example 4.9. Consider the following Nicholson-type system with time-varying delays

$$\begin{cases} \dot{x}_1(t) = -a_1x_1(t) + b_1x_2(t) \\ \qquad\qquad + c_1(t)x_1(t-\tau)e^{-x_1(t-\tau)}, \\ \dot{x}_2(t) = -a_2x_2(t) + b_2x_1(t) \\ \qquad\qquad + c_2(t)x_2(t-\tau)e^{-x_2(t-\tau)}, \end{cases} \tag{30}$$

where $a_1 = 5.7, a_2 = 3.9, b_1 = -2.9, b_2 = -2.98, c_1(t) = e^4(0.2 + 0.2\sin t), c_2(t) = e^2(0.75 + 0.35\sin t), \tau = 0.21$. It is easy to check that all the conditions (H1) and (H2) are satisfied. Thus system (30) has exactly one positive solution which is globally exponentially stable. The results are illustrated in Fig. 9.

V. Conclusions

In this paper, we investigated a class of Nicholson-type system with delays. Applying the fundamental solution matrix, inequality techniques, Lyapunov function and constructing fundamental function sequences, some sufficient conditions which ensure the boundedness and exponential stability of positive solutions of Nicholson-type delay system are established. The obtained conditions are easily checked in practice by simple algebraic methods. Our results are new and supplement some previously known ones. Recently, Nicholson-type delay system with stochastic perturbation have also paid more attention by many scholars. However, there are rare results on the stability of solutions of stochastic Nicholson-type delay system, which might be our future research topic.

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