Pricing Model for Owner's Payment Bond in China

Hui Wang, Xiaomei Deng, and Weihua Yang

Abstract—**To reveal the pricing mechanism of an owner's payment bond and improve warrantor risk management, a pricing model of an owner's payment bond was developed in a risk neutral valuation framework. The pricing model combines the unique characteristics of an owner's payment bond using a binomial tree method. A numerical simulation using a real estate project as an example was created by compiling a program in the *C#* language based on the model. The findings of the simulation not only verified the reliability of the model but also described explicitly the relationship between bond prices and the main influencing factors.**

Index Terms—**Bermudan put option, binomial tree method, owner's payment bond, pricing model.**

I. INTRODUCTION

THE Chinese construction market is a buyer's market. If owners default on and fail to make on-time payments regarding the contractors' engineering costs according to the construction contract, the contractors' interests will be seriously violated, and the sound development of the construction industry will also be hampered [1], [2]. To avoid owners' payment defaults, the Chinese government has focused on the implementation of an owner's payment bond system since 2003. In the agreement of the owner's payment bond, the warrantor provides the contractor with a written bond that the project owner will pay on time as agreed in the construction contracts and will pay the contractor if the owner defaults. This creation in the history of construction contract bonds focuses on solving construction payment arrears [1], [3].

However, the premium of an owner's payment bond continues to decline after decades of development, which leads to a situation in which the income and risk of warrantors are unequal, and many warrantors refuse to underwrite owner's payment bonds. These phenomena restrict the development of the surety industry and pose an impediment to the establishment of a construction market's credit system. To standardize the bond premium in the surety industry, some provincial and municipal authorities have issued specific regulations. For example, the city of Xiamen requires that the premium be determined according to the length of the bond validity period [4]. Henan Province refers to the determination method of the loan interest rate and charges the bond premium based on different values of collaterals that warrantees provide to warrantors [5]. The vast differences in bond premium regulation in different regions have shown that the pricing methods of owner's payment bonds have not yet reached consensus; moreover, different regulations have led to confusion in bond pricing. Therefore, the performance of premium regulations is not satisfactory. To create a fair trading environment and use the owner's payment bond to reasonably share the construction risks, it is highly necessary to study the pricing mechanism of the owner's payment bond.

This study aimed to develop a pricing model for an owner's payment bond based on the binomial tree method to reveal the bond pricing mechanism in a risk neutral valuation framework. The developed model can help the different parties involved in the bond transactions to achieve a consensus regarding the owner's payment bond pricing method. After numerical simulations using an example and compiling a program in the C# language based on the model, this study used impact analysis to describe explicitly the relationship between the bond price and the main influencing factors.

The paper is organized as follows. After a brief introduction of the Chinese construction contract bond system in Section 2, the manuscript presents related studies in Section 3. Section 4 introduces the research method, and Section 5 develops a pricing model for the owner's payment bond based on the binomial tree method. Section 6 focuses on a numerical simulation with an application created by compiling a program in the C# language based on the model. Section 7 discusses the practical and theoretical implications of the findings. The final section summarizes the paper and offers suggestions for further research.

II. BACKGROUND INFORMATION

It has been hypothesized that the Chinese construction contract bond system is the mechanism through which the demand of the construction market credit can be met. Owners and contractors are the two basic subjects in the construction market. Therefore, the construction contract bond system should avoid default risks of owners and contractors and extend their credit.

On the one hand, to avoid default risks of contractors,

Manuscript received Mar. 31, 2015; revised Jun. 20, 2015. This work is supported by the National Natural Science Foundation of China (NNSFC) (7147314).

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they (contractors) can be required to provide bid bonds, performance bonds, payment bonds, advance payment bonds, maintenance bonds, etc. Under these contracts, contractors will perform as agreed in the plans, specifications, and construction contracts. The bid bonds, performance bonds, and payment bonds are mandatory bond products in real estate development projects beyond a certain scale in China. Of course, they can also be used in other construction projects.

On the other hand, owners' payment defaults became the primary origin of chaos in the Chinese construction market at the beginning of the 21st century. According to the information of the National Bureau of Statistics of the People's Republic of China, the cumulative total of construction payment arrears in the whole construction industry was 136 billion yuan in 1996 and increased to 222.1 billion yuan in 1999. At the end of 2001, the total exceeded 278 billion yuan. Furthermore, it should be noted that part of the arrears were due to by unscrupulous owners who delayed payment intentionally. Therefore, the Chinese government placed emphasis on the implementation of owners' payment bonds to avoid the problem arising from owners' payment defaults. The following describes the basic components of the owner's payment bond in China.

According to the regulations of the MOHURD (Ministry of Housing and Urban-Rural Development of the People's Republic of China, formerly known as the China Ministry of Construction), if the contract value of a real estate project exceeds 10 million yuan (approximately $1.63 million), real estate developers must submit an owner's payment bond to the contractor. Some regions have made other regulations based on this, e.g., the city of Xiamen requires an owner's payment bond for all construction projects with a construction area over 1000 square meters or total project costs above 1 million yuan (approximately $0.16 million). An owner's payment bond, which is an accessory contract based on a construction contract, can be written by a surety company or bank. The owners first submit an application to the warrantor, and then, by examining the credit status and paying a certain amount of premium, the warrantor provides a bond. The bonded amount shall not be lower than a certain proportion of the contract price, such as at least 10% in many cities of China, and no less than the maximum amount of the progress payments agreed upon in the contract. When a scheduled payment is completed, the bond will be automatically converted to the next stage, and this process continues until the owner makes all of the payments under the secured construction contract. The warrantor will implement payment obligation to ensure the payment on behalf of the principal (owner) if he/she defaults. If a warrantor paid, he/she has the right of recourse against the owner. If the entire bonded amount has been used up because of the owner's default, he/she shall resubmit another owner's payment bond with the same amount as the previous one within an allotted time. Otherwise, the contractor shall exercise the right to suspend work and claim damages. If the guarantors paid, they may recover from the owner. In essence, contractors are empowered with rights (not obligations) regarding the agreement of an owner's payment bond. If the contractor does not receive the construction payment on time, he/she can choose to exercise the owner's payment bond to protect the contractor's own rights and interests.

III. RELATED STUDIES

The pricing of bonds is different from the pricing of ordinary commodities. The pricing of ordinary commodities has been a research focus in the field of micro-economics. The main theories include the labor theory of value, the utility theory of value and the general equilibrium theory in neoclassical economics, etc. Although deriving from different aspects, these theories imply one common intrinsic character: they are based on historical costs. The concept of "future" can hardly be found in the pricing of ordinary commodities, which therefore avoids the impact of uncertainty regarding the future values of assets. However, bond pricing must face the future. The theory of bond pricing did not make much breakthrough until the creation of the credit rationing theory, the adverse selection of loan interest rates and the incentive effect theory, which provides a theoretical basis for bond pricing. Merton (1973) [6] first illustrated that the theory of warrant pricing is analogous to that of a put option. Hence, the theory of options has been widely used in the pricing of credit loan guarantees. Jones and Mason (1980) [7] used the method of Markov chains to develop numerical solutions of loan guarantees based on the option method. Lai (1992) [8] constructed a discrete-time model to value private loan guarantees according to the option-pricing theory. Dietsch and Petey (2002) [9] estimated the credit risk value of small commercial loans using a VaR (Value at Risk) model. Lu and Kuo (2005) [10] gauged the credit risk of guarantee issues using a market-based risk neutral approach. Chang et al. (2006) [11] constructed models to analyze the values and default probability of joint loan guarantees using an option approach.

In contrast to credit loan guarantee pricing, hardly any studies have investigated the issue of construction contract bond pricing. Because this conventional bond is regarded as a system to avoid contract default through the screening of construction risks [12], the most important link is the underwriting process. Therefore, many scholars have focused on underwriting technologies and methods to help underwriters assess the default risks of contractors [13], [14], [15], [16]. In reality, many warrantors would have to pay compensation for the defaults of contractors and even go bankrupt. Such phenomena are more common when economies go downwards [17]. Thus, the default risk value should be taken into account when calculating the bond premium. In spite of this problem, the literature on estimating the default risk value of construction contract bonds is sparse. Huang (2008) [18] referred to the credit default swap (CDS) model to determine the price of contractor performance bonds, and Cui (2008) [19] used the real option theory to evaluate the warranty terms of the No. 44 highway in New Mexico.

Based on the characteristics of the terms of exercise in the agreement of bonds, credit loan guarantees can be exercised only upon the maturity date of the guaranteed loan, similarly to the European put option. Contractors guaranteed by performance bonds or warranty bonds may default at any time during the entire guarantee period. From the view of warrantors, the performance or warranty bond can be exercised at any time, similarly to the American put option. However, the owner's payment bond can only be exercised at finite time points predetermined in the construction contract, which is similar to the Bermudan put option.

Although similar to the Bermudan put option, an owner's payment bond has three distinct features. First, a construction payment sequence exists in the owner's payment bond agreement, which indicates the existence of a strike price sequence. For the financial option contract, there is usually only one strike price, namely the predetermined future price of the underlying asset [20], [21]. However, for the construction project paid according to the construction schedule, the whole construction project is divided into finite intermediate construction products that contractors finish during each payment period. In addition, the values of each intermediate construction product, i.e., the construction payment required in each period, form the strike price sequence agreed to in the owner's payment bond. Each element of the strike price sequence is an additional factor that determines whether the owner's payment bond will be exercised.

Second, the risk value of the owner's payment bond originates mainly from the uncertainty of the value of the owner's assets, and whether the bond is exercised mainly depends on the comparison between the value of the owner's assets and the payment sequence. In a financial option contract, the option value (i.e., the fair price of an option) mainly derives from the uncertainty of the underlying asset's future value. The comparison between the asset's future price and the strike price determines whether the option will be exercised. The future price and strike price are based on the same underlying asset, such as stocks or foreign currencies. However, owner's payment bonds are different. The payment sequence (i.e., strike price sequence) is based on the value of the construction project proposed. However, the value of the owner's assets depends on all of the assets of the owner, including the construction project. Thus, an owner's payment bond involves two subject matters: one is the construction project, and the other is the owner's assets. Although the value of the construction project may change during the construction process due to changes in prices, policy, engineering change, etc., this will also change the value of the owner's assets. For example, if the value of the construction project rises, so will the value of the owner's assets, and the owner can refinance based on the project's incremental value to make up the additional amount of the current payment. Therefore, to simplify the research question, the value of the construction project proposed can be viewed as constant, and the risk value of the owner's payment bond mainly derives from the uncertainty of the value of the owner's assets.

Third, the change in the value of the owner's assets is a specific jump-down process due to payment in addition to the diffusion process. In addition to general fluctuations (often referred to as the diffusion process), the future value change of the underlying asset will also constitute a jump process due to policy and market changes. Furthermore, this jump-diffusion process is random. However, the changes in owner asset value include a specific jump-down process due to payment for the current projects, in addition to the general random diffusion process. The time and size of the "value jumps" can be predetermined according to the proposed construction organization design, construction contract, and other documents, and provides convenient pricing.

To summarize, based on the characteristics of the terms of exercise in the bond agreement, the pricing method for existing credit loan guarantees and contractor's performance bonds cannot be applied directly to the pricing of owner's payment bonds. Moreover, the owner's payment bond has its own features that distinguish it from the Bermudan put option; thus, the pricing method of the Bermudan put option also cannot be applied directly to the pricing of owner's payment bonds. Therefore, this study constructs a pricing model for owner's payment bonds using a binomial tree method with reference to the pricing theory of the Bermudan put option, verifies the reliability of the model, and describes explicitly the relationship between the bond price and the main influencing factors through a numerical simulation application.

IV. RESEARCH METHOD

The Bermudan option is a type of exotic option [22]. Lattice and finite difference methods, such as the binomial tree method [23], a lattice method based on an approximation to the transition density function of the Lévy process [24], and low-discrepancy mesh methods [25], are usually used to price Bermudan options [26]. The binomial tree method can clearly deduce the possible future values of underlying assets [27]. It is a simple and direct method to describe the diffusion and jump-down processes of the future value of owner's assets, and it is easy for all parties of the bond transaction to accept. Therefore, this paper uses the binomial tree method to price the owner's payment bond.

The binomial tree method is a common approach for pricing various complicated options and other financial derivatives. It is assumed that the value changes of an underlying asset can be described by many one-period and two-state models during the entire contract period. Backward deduction can then be applied to calculate the value of the financial derivative at the initial moment [28]. This constitutes the basic idea of the binomial tree method.

The one-period and two-state models assume that the market is completely non-arbitrage. Underlying assets are traded at the initial time t=0 and at the terminal time t=T only during the valid period [0, T] of the option contract. In addition, the asset has two possible states at the terminal time t=T: $S_T^u = uS_0$, $S_T^d = dS_0$, where, S_T^u and S_T^d represent the up-move and down-move of the asset's value, respectively, u is the *up factor*, and d is the *down factor*. The *up* and *down* factors can be calculated using the underlying asset's historical volatility σ and the time length Δt : $u = \exp(\sigma\sqrt{\Delta t})$, $d = \exp(-\sigma\sqrt{\Delta t})$. The Δ -Hedging technique is then applied to define the risk-neutral probability measure Q, i.e., in the future at time

T.

The increasing probability of S_0 is shown in Eq. (1):

$$q = \Pr ob_Q\{S_T = S_T^u\} = \frac{\rho - d}{u - d} \quad (1)$$

The decreasing probability of S_0 is shown in Eq. (2):

$$1 - q = \Pr ob_Q\{S_T = S_T^d\} = \frac{u - \rho}{u - d} \quad (2)$$

where the *interest rate factor* is $\rho = \exp(r\Delta t)$, and r is the *risk free interest rate*, which is usually determined by referring to the current Treasury rate.

The asset's values S_T^u and S_T^d are compared with the striking price to assess the bond values V_T^u and V_T^d at the terminal time, and the backward deduction and discounted methods are then used to achieve the bond fair price V_0 at the initial moment, as shown in Eq. (3):

$$V_0 = \frac{1}{\rho} E^Q(V_T) = \frac{1}{\rho}[qV_T^u + (1-q)V_T^d] \quad (3)$$

If the lifetime of the bond contract is divided into finite continuous subintervals, all of the possible development paths of the owner's asset value can be simulated by assuming that the values of the owner's assets are in line with the one-period and two-state models in each subinterval and considering the jump-down process at every construction payment time point. The possible bond values at the terminal moment (i.e., possible compensation) can then be calculated. Based on these possible bond values, we can obtain the initial value of the payment bond (i.e., the fair price of the bond) using backward deduction and the discounted method. For an owner's payment bond, we should compare the warrantor's possible compensations with the expected values calculated through the discounted method at each construction payment point. The bond value should be the greater value at this time point.

V. PRICING MODEL FOR OWNER'S PAYMENT BOND

A. Basic Assumptions

Assumption I: The capital market is a complete, frictionless, and arbitrage-free market, which means when the owner is using the current assets to finance, the transaction costs do not need to be taken into consideration, and the owner can readily obtain the equivalent amount of capital with the assets.

Assumption II: Once the owner's payment bond has been executed, the agreement between the warrantor and warrantee is terminated, even if the amount paid is lower than the amount of the bond.

Assumption III: Value changes in the owner's assets are subject to the binomial process. The rising or falling process of its value can be described through a large number of one-period and two-state models.

In an agreement of an owner's payment bond, the valid date is denoted as [0, T]. According to the construction contract, the payment is to be completed by time m. The time sequence is recorded as $T_M=\{T_1, T_2, \ldots, T_i, \ldots, T_m\}$, where $0<T_1<\ldots<T_i<\ldots<T_m=T$. The corresponding payment sequence is denoted as $C=\{C_1, C_2, \ldots, C_i, \ldots, C_m\}$, where the maximum value in the sequence is recorded as C_M. At the time point T_m, the value of the owner's assets is recorded as S_{T_m}. The contract amount of the guaranteed project is P, and the required minimum security percentage is l. Therefore, the bonded amount of the owner payment bond can be described as $G = \max(C_M, lP)$.

B. Binomial Evolution of Value of Owner's Assets

The valid date [0, T] of the owner's payment bond is first divided into N subintervals:

$0 = t_0<t_1<t_2< \ldots<t_n=T_m=T$, where $N \geq m$.

The number of segments Δt in each subinterval is denoted as $N_M= \{N_1, N_2, \ldots, N_i, \ldots, N_m\}$. The initial value of the owner's assets is S_0. In the first payment period [0, T_1], the value of the assets would evolve into a binomial model, as shown in Figure 1.

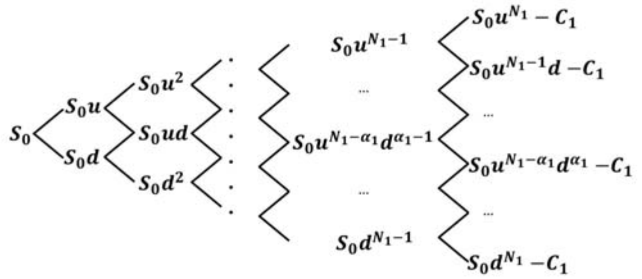

Fig. 1. Binomial tree model for value of owner's assets in the first payment period

At time T_1, these possible values of the owner's assets will jump down immediately after the owner pays an amount of C_1, as shown in Eq. (4):

$$S_{T_1}^{(\alpha_1)} = S_0u^{N_1-\alpha_1}d^{\alpha_1} - C_1, 0 \leq \alpha_1 \leq N_1 \quad (4)$$

If $S_0u^{N_1-\alpha_1}d^{\alpha_1} - C_1 \geq 0$, the value of the owner's assets is greater than the current project payment. The owners will choose to pay, and the assets' value will continue to evolve to the next stage. If $S_0u^{N_1-\alpha_1}d^{\alpha_1} - C_1 < 0$, the value of the owner's assets is not able to cover the current project payment, and the owner will fail to pay. If the contractor chooses to exercise the bond, the evolving process of the value of the owner's assets ceases. To calculate the value of the bond easily using backward induction, we can assume that the evolution of the value of the owner's assets continues, but the value is negative.

In the second payment period (T_1, T_2], the possible values $S_{T_1}^{(\alpha_1)}$ at time T_1 become the initial value. The values of the owner's assets evolve into a group of new binomial branches. At time T_2, the possible values of the owner's assets are described as in Eq. (5) after the payment of C_2.

$$S_{T_2}^{(\alpha_1,\alpha_2)} = S_{T_1}^{(\alpha_1)}u^{N_2-\alpha_2}d^{\alpha_2} - C_2 \quad (5)$$

where

$$0 \leq \alpha_1 \leq N_1, \ 0 \leq \alpha_2 \leq N_2.$$

Similar evolutions continue to the last payment period (T_{m-1}, T_m]. At time T_m, the possible values of the owner's assets are described in Eq. (6) after the payment of C_m.

$$S_{T_m}^{(\alpha_1,\alpha_2,\cdots,\alpha_m)} = S_{T_{m-1}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1})}u^{N_m-\alpha_m}d^{\alpha_m} - C_m \quad (6)$$

where

$$0 \le \alpha_1 \le N_1,\ 0 \le \alpha_2 \le N_2, \ldots,$$

$$0 \le \alpha_{m-1} \le N_{m-1}, 0 \le \alpha_m \le N_m .$$

C. Backward Induction of The Owner's Payment Bond

According to the whole binomial tree model for the value of the owner's assets, the fair price of the owner's payment bond can be calculated using backward induction.

The value of the owner's payment bond at the final time T_m

At time T_m, the contractor has the right to exercise the owner's bond payment option. Whether the right is implemented depends on the comparison between the assets' value and the current payment.

If $S_{T_{m-1}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1})} u^{N_m-\alpha_m} d^{\alpha_m} - C_m \ge 0$, indicating that the value of the owner's assets is greater than the current project payment, the owner will choose to pay; otherwise, the contractor could exercise the owner's payment bond, and the warrantor would pay on behalf of the principal. The warrantor is entitled to recourse from the owner. In the end, the owner would not only need to recover the losses of the warrantor but also lose the owner's own credit. At this time, the value of the owner's payment bond is $V_{T_m}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1},\alpha_m)} = 0$.

If $S_{T_{m-1}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1})} u^{N_m-\alpha_m} d^{\alpha_m} - C_m < 0$, indicating that the owner is not able to pay the current project costs, the contractor chooses to exercise the bond because of the owner's payment default. The value of the bond is the amount actually paid at that moment. If $C_m - S_{T_{m-1}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1})} u^{N_m-\alpha_m} d^{\alpha_m} \le G$, the paid amount is $C_m - S_{T_{m-1}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1})} u^{N_m-\alpha_m} d^{\alpha_m}$; otherwise, the paid amount is G.

Thus, at the final payment time T_m, the value of the owner's payment bond is described in Eq. (7):

$$V_{T_m}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1},\alpha_m)} = \begin{cases} 0 & , \ C_m \le S_{T_{m-1}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1})} u^{N_m-\alpha_m} d^{\alpha_m} \\ \min(C_m - S_{T_{m-1}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1})} u^{N_m-\alpha_m} d^{\alpha_m}, G), & \ C_m > S_{T_{m-1}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1})} u^{N_m-\alpha_m} d^{\alpha_m} \end{cases} \tag{7}$$

where

$$0 \le \alpha_1 \le N_1,\ 0 \le \alpha_2 \le N_2, \ldots,$$

$$0 \le \alpha_{m-1} \le N_{m-1}, 0 \le \alpha_m \le N_m .$$

The value of the owner's payment bond at the penultimate time T_{m-1}

Based on the bond value at T_m, we can obtain the bond's possible values at time $t_{N_m-1} = T_m - \Delta t$, i.e., closely preceding $T_m = t_{N_m}$, as shown in Eq. (8):

$$V_{t_{N_m-1}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1},\alpha)} = \frac{1}{\rho}[q V_{T_m}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1},\alpha)} + (1-q) V_{T_m}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1},\alpha+1)}] \tag{8}$$

Based on the bond value of $V_{t_{N_m-1}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1},\alpha)}$ at time t_{N_m-1}, the bond value at time T_{m-1} can be obtained through backward induction. However, at time T_{m-1}, the bond value should be compared with the corresponding value of the owner's assets $S_{T_{m-1}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1})}$. If $S_{T_{m-1}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1})} \le 0$, indicating that the current value of the owner's assets is insufficient to pay C_{m-1}, the owner's payment bond will be exercised, and the bond value would be $-S_{T_{m-1}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1})}$ at this moment; otherwise, the bond value is the amount calculated by backward induction, as shown in Eq. (9):

$$V_{T_{m-1}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-2},\alpha_{m-1})} = \begin{cases} \frac{1}{\rho^{N_m}} \sum_{l=0}^{N_m} \binom{N_m}{l} q^{N_m-l}(1-q)^l V_{T_m}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1},l)}, & S_{T_{m-1}}^{(\alpha_{m-1})} > 0 \\ -S_{T_{m-1}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-1})} & , \ S_{T_{m-1}}^{(\alpha_{m-1})} \le 0 \end{cases} \tag{9}$$

where $\binom{N_m}{l} = \frac{N_m!}{l!(N_m-l)!}$ represents the number of combinations; $0 \le \alpha_1 \le N_1$, $0 \le \alpha_2 \le N_2$, …, $0 \le \alpha_{m-1} \le N_{m-1}$, $0 \le \alpha_m \le N_m$.

The bond values of $V_{T_{m-2}}^{(\alpha_1,\alpha_2,\cdots,\alpha_{m-2})}$, …, $V_{T_i}^{(\alpha_1,\alpha_2,\cdots,\alpha_i)}$, …, $V_{T_2}^{(\alpha_1,\alpha_2)}$, $V_{T_1}^{(\alpha_1)}$ can be obtained through similar backward induction. Finally, we can calculate the initial value of the owner's payment bond (i.e., the fair price) on the basis of $V_{T_1}^{(\alpha_1)}$, as shown in Eq. (10):

$$V_0 = \frac{1}{\rho^{N_1}} \sum_{l=\alpha_1=0}^{N_1} \binom{N_1}{l} q^{N_1-l}(1-q)^l V_{T_1}^{(\alpha_1)} \tag{10}$$

VI. SIMULATION APPLICATION OF THE PRICING MODEL

A. Project Profile

A is a real estate development company that is planning to build a residential building. Through public bidding, construction company Z is selected as the successful tender to sign the contract. The contract amount P is 12 million yuan. The time limit of the project is 1 year, and a progress payment will be made each quarter. Company Z has calculated the payment sequence according to the proposed construction organization design: C= {C_1, C_2, C_3, C_4} = {3 million yuan, 3 million yuan, 3 million yuan, 3 million yuan}. Before signing the construction contract, company Z provides a contractor's performance bond to company A, and company A is required to offer an owner's payment bond in return. According to the requirements of local construction authorities, the owner's payment bond shall not be less than 10% of the contract amount, and shall not be less than the maximum amount of the progress payments. Hence, the bond amount of this owner's payment bond can be described as

$$G = \max(C_M, 10\%P) = \max(3,1.2) = 3.00 \quad \text{(million yuan)}$$

We assumed that company *A* filed an application to the surety company *HM* to issue the owner's payment bond and that, through an underwriting investigation, *HM* then determines that company *A*'s current asset value is 12 million yuan. The monthly standard deviation of changes in assets (i.e., volatility of owner's assets) is 0.05, and the current one-year treasury rate *r* is 4.01%. If company *HM* agreed to underwrite an owner's payment bond with a 1-year period of validity for company *A*, we then aim to determine the fair price of the owner's payment bond.

Through numerical simulation by compiling a program in the *C#* language based on the previous pricing model, the fair price of the owner's payment bond is determined to be 54,200 yuan and the premium rate is 1.81%, which is in line with the experience of previous charges.

B. Impact Analysis of Major Parameter Changes

According to the pricing model for the owner's payment bond, the fair price of the owner's payment bond is determined by the initial value of the owner's assets S_0, the owner's asset volatility σ, the risk-free interest rate r, the bonded amount G, and the period of validity. An impact analysis was conducted to evaluate the effects of these parameter changes on the fair price of the bond based on the previous example.

Impact of Change in The Initial Value of The Owner's Assets on Pricing

The relation curve of the initial value of the owner's assets S_0 and the fair price of the owner's payment bond is shown in Figure 2. When S_0 is 11 million yuan, the price of the payment bond is 705.9 thousand yuan and the premium rate is 23.53%, indicating that the owner's default risks increase sharply with the decline of S_0 when the initial value of the owner's assets is lower than the contract price, and the price of the bond increases rapidly in parallel. In contrast, when S_0 is 13 million yuan, the price of the bond is 67 yuan, and the premium rate is 0.002%, indicating that the owner's default risks are reduced rapidly with the increase in S_0 when the initial value of the owner's assets is higher than the contract price and the price declines during this period. Thus, the initial value of the owner's assets has a significant influence on the pricing of the owner's payment bond. When S_0 moves toward the direction that is lower than the contract price, the owner's default risks increase rapidly, in parallel to the price of the bond. When S_0 moves toward the direction that is higher than the contract price, the owner's default risks decline rapidly, and the price of the bond will go down.

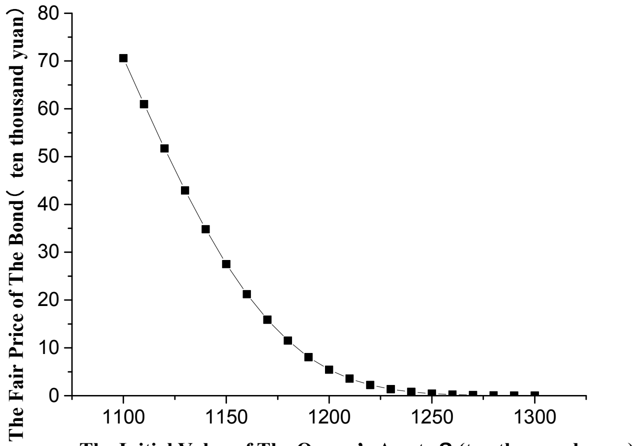

Fig. 2. Impact of the initial value of the owner's assets S_0 on the pricing of the owner's payment bond

Impact of change in the owner's asset volatility on pricing

The relation curve between the owner's asset volatility σ and the pricing of the payment bond is shown in Figure 3. When σ is 0.02, the price is 1.30 thousand yuan; when σ is 0.2, the price increases to 495.50 thousand yuan. This finding indicates that when the owner's future asset volatility σ declines, the price of the bond will decrease as well, whereas when the owner's future asset volatility σ increases, the probability of default increases, and the bond price will increase.

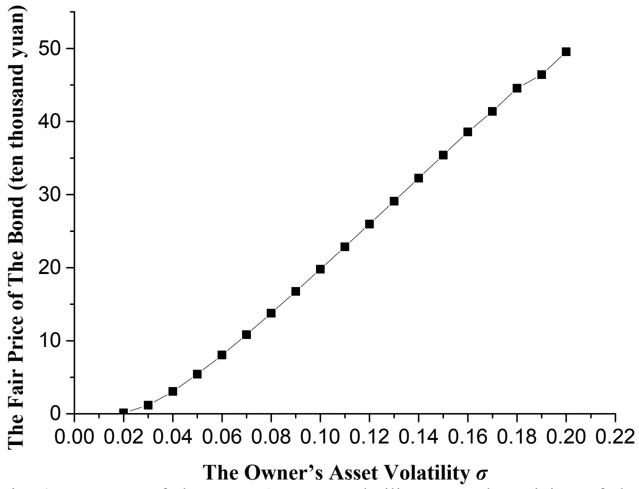

Fig. 3. Impact of the owner's asset volatility σ on the pricing of the owner's payment bond

Impact of change in risk-free interest rate on pricing

The relation curve between the risk-free rate r and the pricing of bonds is shown in Figure 4. When r is 15%, the price is 10 yuan, and when r is 1%, the price is 129.8 thousand yuan, indicating that the price of the owner's payment bond declines as the risk-free interest rate rises.

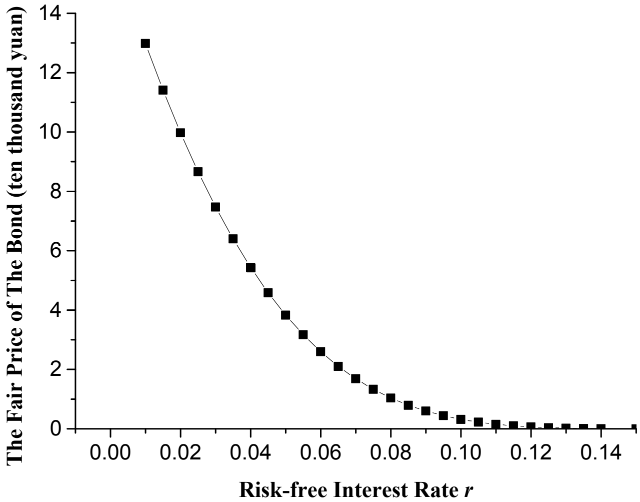

Fig. 4. Impact of risk-free interest rate r on the pricing of the owner's payment bond

Impact of change in bonded amount on pricing

The relation curve between the bonded amount G and the pricing of bonds is shown in Figure 5. When G is 200 thousand yuan, the price is 32.60 thousand yuan, and as the bonded amount increases, the bond price goes up, whereas when G increases to approximately 1.1 million yuan, the bond price stabilizes at 54.20 thousand yuan despite any further increase in G. By examining the evolution of the value of the owner's assets, it can be determined that even if the value of the owner's assets continued to decrease, the maximum amount of compensation would be 1.17 million yuan after the final payment. Therefore, the expected loss of the bond does not change even if the bonded amount exceeds 1.17 million yuan, which means that the bond price will increase as the bonded amount increases; however, the price will remain unchanged when the bonded amount reaches a certain level.

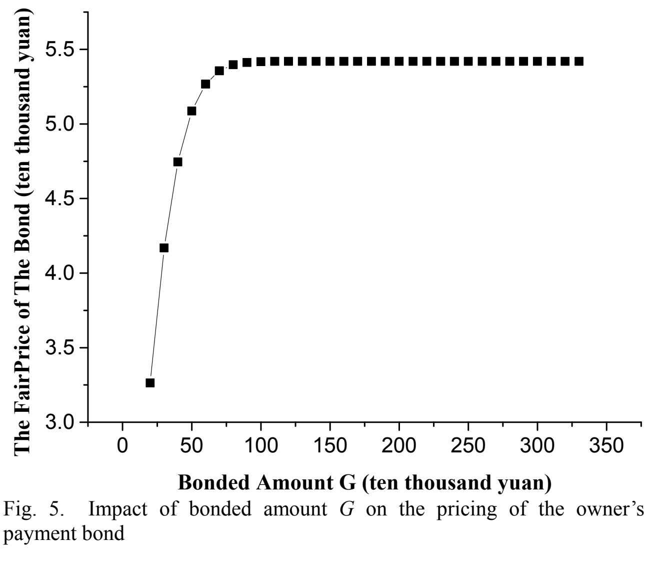

Fig. 5. Impact of bonded amount G on the pricing of the owner's payment bond

Impact of change in the bond valid period T on pricing

To compare the effects of the valid period T on the owner's payment bond as the other model variables remain unchanged, the bond prices were calculated when the valid periods were 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, and 48 months. The corresponding bond prices are shown in Figure 6. The lowest price, approximately 53.40 thousand yuan, is obtained when T is 4 months, whereas highest price, i.e., reaching 54.6 thousand yuan, is found when T is 48 months. The bond price increases with an increase in the period of validity, but the range of this change is small.

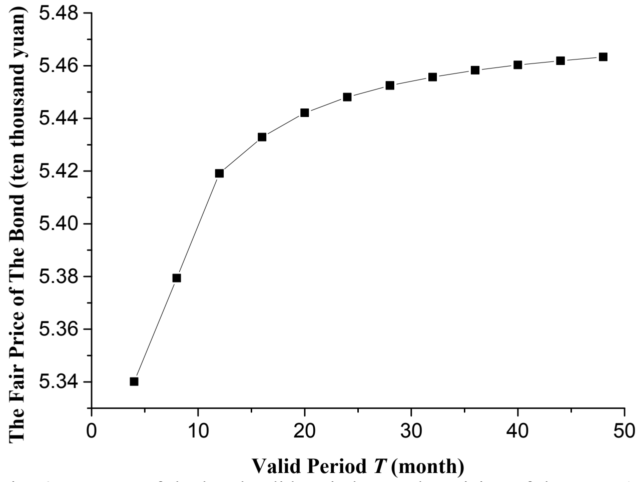

Fig. 6. Impact of the bond valid period T on the pricing of the owner's payment bond

VII. DISCUSSION

Based on the results of the simulation, the relationship between bond price and the variables in the pricing model, such as the initial value of the owner's assets, the volatility of the owner's assets, the bonded amount, and the valid period, are in line with people's common understanding. This finding illustrates that the pricing model for owner's payment bonds agrees with reality. In China's construction contract bond practice, it is generally believed that high-risk-free rates come along with high opportunity costs, which should bring more profit to the warrantor. However, the relationship between risk-free rates and bond price described by the model appears different from the usual understanding of this relationship. In fact, it should be noted that the construction contract bond is different from common investments. When risk-free rates increase, on the one hand, the owner's ROE (return on asset) increases and thus reduces the risk of payment default; on the other hand, the present value of future engineering payment declines, and the corresponding present value of the bonded amount also decreases. Thus, the price of the owner's payment bond decreases. The influence of different variables revealed from the pricing model for owner's payment bonds can offer reference for the risk management of warrantors.

Meanwhile, there are multiple factors that affect the pricing of owner's payment bonds. If the bond price is regulated only by a certain factor, such as the length of the construction period or the amount of counter guarantee, it is unreasonable and would even lead to supervisory disarray. Thus, it is recommended to start with the promotion of the owner's payment bond pricing method to achieve a consensus among the different parties involved. The fair price of the owner's payment bond, as determined by the pricing mechanism, could provide a theoretical basis for fair trade and a measurement of the fair value of warrantor incomes.

In addition, when the bonded amount exceeds the largest possible compensation of the owner, the risk value of the owner's payment bond does not increase. According to the regulations on owner's payment bonds, i.e., the bonded amount shall not be lower than a certain proportion of the contract price and no less than the maximum amount of installments agreed upon in the construction contract, it appears that the regulations lead to a guaranteed surplus for an honest owner. However, there are many dishonest (or bounded rationality) owners, as mentioned in Section 2. This surplus can contribute to the prevention of malicious arrears.

VIII. CONCLUSIONS AND FURTHER RESEARCH

This study revealed the pricing mechanism of owner's payment bonds through a risk-neutral pricing model based on the binomial tree method. The simulation of an application case not only verified the reliability of the model but also described explicitly the relationship between the bond price and the main influencing factors. These findings not only establish a theoretical basis for the fair trade and regulation of owner's payment bonds but also offer reference for the risk management of warrantors. To simplify the model, a series of assumptions were made: the market is arbitrage-free, the volatility of the owner's assets is known, and there are no transaction costs, etc. In reality, the actual transaction price of an owner's payment bond would be affected by many other factors, such as supply-demand relationships, transaction costs, and risk preferences. In particular, the prediction of the volatility of an owner's future assets is a complicated process because asset volatility is associated with the characteristics of the industry in which an owner operates, the historical information of the owner's assets, and the historical prices of the transaction of the bond. Therefore, further research will focus on relaxing the assumptions in the owner's payment bond pricing model and including more reality factors based on data accumulation to gradually improve the pricing theory system of owner's payment bonds.

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