Exponential Stability Analysis for Neutral Stochastic Systems with Distributed Delays

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Abstract—**The paper mainly studies the exponential stability analysis problem for a class of grey neutral stochastic systems with distributed delays. As we know, till now, the stability problem of grey neutral stochastic systems has not been intensively studied except some papers, which motives our research. In this paper, by using an appropriately constructed Lyapunov-Krasovskii functional and some stochastic analysis approaches, especially, by utilizing decomposition technique of continuous matrix-covered sets, sufficient stability criteria are proposed which ensure the mean-square exponential stability and almost surely exponentially robustly stable for the systems. Moreover, an example is provided to illustrate the effectiveness and correctness of the obtained results.**

Index Terms—**Grey Neutral Stochastic Systems, Distributed Delays, Lyapunov-Krasovskii Functional, Decomposition Technique, Exponential Stability**

I. Introduction

Stochastic systems have come to play an important role in many branches of science or engineering applications, and time delays are frequently encountered in many real-word control systems, which is the main causes of instability, oscillation, and poor performance of the systems. Therefore, during the past decades, stochastic systems with time delays have been extensively investigated, many important results have been reported in the literature, see [1-8], and the references therein. On the other hand, in practice, many dynamical systems can be effectively established or described by neutral functional differential equations, such as the distributed networks, population ecology, chemical reactors, water pipes and so on [9]. When the number of summands in a system equation is increased and the differences between neighboring argument values are decreased, another type of time-delays, namely, distributed delays will appear, which can be found in the modeling of feeding systems and combustion chambers in a liquid monopropellant rocket motor with pressure feeding [10,11]. Hence, the study of neutral stochastic systems with distributed delays has come to become a subject of intensive research activity in recent years, much effort has been devoted to the study of this kind of systems [9-20]. For example, in [12], authors investigate the problem of stability analysis of neutral type neural networks with both discrete and unbounded distributed delays. In terms of linear matrix inequalities, delay-dependent conditions are obtained, which guarantee the networks to have a unique equilibrium point.

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It is worth noting that, when the mathematical model of stochastic systems is built, the parameters are difficult to obtain. In addition, as pointed out in [21], if the parameters of the systems are evaluated by grey numbers, the systems should be established indeterminately and become grey systems. Therefore, the study of grey systems has received much attention from many scholars, some significant and innovative results have appeared in the literature [21-24]. For instance, in [22], two easily verified delay-dependent criteria of mean-square exponential robust stability were obtained. However, to the best of our knowledge, till now, the problem of exponential stability for grey neutral stochastic systems with distributed delays has not been full investigated, which is still open and remains challenging. this situation motives our present study.

In this paper, we investigate the exponential stability problem for a class of grey neutral stochastic systems with distributed delays. First, a new type of Lyapunov-Krasovskii functional is constructed. Then, by using the decomposition technique of the continuous matrix-covered sets of grey matrix (see [21-24]), we study the systems model directly with some well-known differential formulas, and sufficient criteria are obtained, which ensure the systems in the mean square exponential stability and almost surely exponentially robustly stable. Finally, an example is given to demonstrate the applicability of the proposed stability criteria.

Notations: The notations are quite standard. Throughout this paper, R^n and $R^{n\times n}$ denote the n-dimensional Euclidean space and the set of all n ×n real matrices. The superscript "T" represents matrix transposition, and the notation $X \ge Y$ (respectively $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively positive definite). The symbol $\|\cdot\|$ denotes the Euclidean norm for vector or the spectral norm of matrices. Moreover, Let $(\Omega, F, \{F_t\}_{t\ge 0}, P)$ be a complete probability space with a filtration $\{F_t\}_{t\ge 0}$ satisfying the usual conditions. Let $\tau > 0$ and $C([-\tau,0]; R^n)$ denote the family of all continuous R^n -valued functions φ on $[-\tau,0]$. Let $L^2_{F_0}([-\tau,0]; R^n)$ be the family of all F_0 -measurable bounded $C([-\tau,0]; R^n)$-valued random Variables $\xi = \{\xi(\theta) : -\tau \le \theta \le 0\}$.

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II. Preliminaries and Problem formulation

Consider a class of grey neutral stochastic systems with distributed delays:

$$\begin{cases} d[x(t)-G(\otimes)x(t-\tau)] \\ =[A(\otimes)x(t)+B(\otimes)x(t-\tau)+C(\otimes)\int_{t-\tau}^{t}x(s)ds]dt \\ +[D(\otimes)x(t)+E(\otimes)x(t-\tau)+F(\otimes)\int_{t-\tau}^{t}x(s)ds]dW(t),\ t\ge 0 \\ x_0=\xi,\ \xi\in L^2_{F_0}([-\tau,0];R^n), -\tau\le t\le 0 \end{cases} \tag{2.1}$$

Where $A(\otimes)$, $B(\otimes)$, $C(\otimes)$, $D(\otimes)$, $E(\otimes)$, $F(\otimes)$, $G(\otimes)$ are grey n ×n -matrices, and let

$A(\otimes)=(\otimes_{ij}^{a})$, $B(\otimes)=(\otimes_{ij}^{b})$, $C(\otimes)=(\otimes_{ij}^{c})$,

$D(\otimes)=(\otimes_{ij}^{d})$, $E(\otimes)=(\otimes_{ij}^{e})$, $F(\otimes)=(\otimes_{ij}^{f})$,

$G(\otimes)=(\otimes_{ij}^{g})$.

Here, $\otimes_{ij}^{a}$, $\otimes_{ij}^{b}$, $\otimes_{ij}^{c}$, $\otimes_{ij}^{d}$, $\otimes_{ij}^{e}$, $\otimes_{ij}^{f}$ and $\otimes_{ij}^{g}$ are said to be grey elements of

$A(\otimes)$, $B(\otimes)$, $C(\otimes)$, $D(\otimes)$, $E(\otimes)$, $F(\otimes)$ and $G(\otimes)$.

Now, we define

$$[L_a,U_a]=\{A(\hat{\otimes})=(a_{ij}):\underline{a_{ij}}\le a_{ij}\le\overline{a_{ij}}, i,j=1,2,..n\}$$

$$[L_b,U_b]=\{B(\hat{\otimes})=(b_{ij}):\underline{b_{ij}}\le b_{ij}\le\overline{b_{ij}}, i,j=1,2,..n\}$$

$$[L_c,U_c]=\{C(\hat{\otimes})=(c_{ij}):\underline{c_{ij}}\le c_{ij}\le\overline{c_{ij}}, i,j=1,2,..n\}$$

$$[L_d,U_d]=\{D(\hat{\otimes})=(d_{ij}):\underline{d_{ij}}\le d_{ij}\le\overline{d_{ij}}, i,j=1,2,..n\}$$

$$[L_e,U_e]=\{E(\hat{\otimes})=(e_{ij}):\underline{e_{ij}}\le e_{ij}\le\overline{e_{ij}}, i,j=1,2,..n\}$$

$$[L_f,U_f]=\{F(\hat{\otimes})=(f_{ij}):\underline{f_{ij}}\le f_{ij}\le\overline{f_{ij}}, i,j=1,2,..n\}$$

$$[L_g,U_g]=\{G(\hat{\otimes})=(g_{ij}):\underline{g_{ij}}\le g_{ij}\le\overline{g_{ij}}, i,j=1,2,..n\}$$

Which are said to be the continuous matrix-covered sets of $A(\otimes)$, $B(\otimes)$, $C(\otimes)$, $D(\otimes)$, $E(\otimes)$, $F(\otimes)$, and $G(\otimes)$.

Here,

$A(\hat{\otimes})$, $B(\hat{\otimes})$, $C(\hat{\otimes})$, $D(\hat{\otimes})$, $E(\hat{\otimes})$, $F(\hat{\otimes})$ and $G(\hat{\otimes})$

are whitened (deterministic) matrices of

$A(\otimes)$, $B(\otimes)$, $C(\otimes)$, $D(\otimes)$, $E(\otimes)$, $F(\otimes)$ and $G(\otimes)$.

Moreover,

$[\underline{a_{ij}},\overline{a_{ij}}]$, $[\underline{b_{ij}},\overline{b_{ij}}]$, $[\underline{c_{ij}},\overline{c_{ij}}]$, $[\underline{d_{ij}},\overline{d_{ij}}]$, $[\underline{e_{ij}},\overline{e_{ij}}]$, $[\underline{f_{ij}},\overline{f_{ij}}]$ and $[\underline{g_{ij}},\overline{g_{ij}}]$

are said to be the number-covered sets of

$\otimes_{ij}^{a}$, $\otimes_{ij}^{b}$, $\otimes_{ij}^{c}$, $\otimes_{ij}^{d}$, $\otimes_{ij}^{e}$, $\otimes_{ij}^{f}$ and $\otimes_{ij}^{g}$.

Definition 2.1. System(2.1)is said to be exponentially stable in mean square, if for all $\xi\in L^2_{F_0}([-\tau,0];R^n)$ and whitened matrices

$A(\hat{\otimes})\in[L_a,U_a]$, $B(\hat{\otimes})\in[L_b,U_b]$, $C(\hat{\otimes})\in[L_c,U_c]$,

$D(\hat{\otimes})\in[L_d,U_d]$, $E(\hat{\otimes})\in[L_e,U_e]$, $F(\hat{\otimes})\in[L_f,U_f]$,

$G(\hat{\otimes})\in[L_g,U_g]$,

there exist scalars $r>0$ and $C>0$, such that

$$E|x(t;\xi)|^2\le Ce^{-rt}\sup_{-\tau\le\theta\le 0}E|\xi(\theta)|^2, t\ge 0.$$

Definition 2.2. System(2.1)is almost surely exponentially robustly stable, if for all $\xi\in L^2_{F_0}([-\tau,0];R^n)$ and whitened matrices

$A(\hat{\otimes})\in[L_a,U_a]$, $B(\hat{\otimes})\in[L_b,U_b]$, $C(\hat{\otimes})\in[L_c,U_c]$,

$D(\hat{\otimes})\in[L_d,U_d]$, $E(\hat{\otimes})\in[L_e,U_e]$, $F(\hat{\otimes})\in[L_f,U_f]$,

$G(\hat{\otimes})\in[L_g,U_g]$,

the following inequality holds:

$$\lim_{t\to\infty}\sup\frac{1}{t}\ln|x(t;\xi)|\le-\frac{\hat{r}}{2},\quad a.s.$$

First, let us introduce the following lemmas, in particular, lemma 2.1, which will be important for the proof of our main results.

Lemma 2.1. [21] If $A(\otimes)=(\otimes_{ij}^{a})_{m\times n}$ is a grey m×n -matrix, $[\underline{a_{ij}},\overline{a_{ij}}]$ is a number-covered sets of grey element $\otimes_{ij}^{a}$, then for whitened matrix $A(\hat{\otimes})\in[L_a,U_a]$, it follows that

i) $A(\hat{\otimes})=L_a+\Delta A$

ii) $0\le\Delta A\le U_a-L_a$

iii) $\|A(\hat{\otimes})\|\le\|L_a\|+\|U_a-L_a\|$

Where $L_a=(\underline{a_{ij}})_{m\times n}$, $U_a=(\overline{a_{ij}})_{m\times n}$, $\Delta A=(\delta_{ij}\hat{r}_{ij})_{m\times n}$, $\delta_{ij}=\overline{a_{ij}}-\underline{a_{ij}}\ge 0$, $\hat{r}_{ij}$ is a whitened number of γ_{ij}, and γ_{ij} is said to be a unit grey number.

Lemma 2.2. [25] Let $x,\ y\in R^n$, $P\in R^{n\times n}$ is a symmetric positive definite matrix, $M,N\in R^{n\times n}$, constants $\varepsilon>0$, then one has the following inequality:

$$2x^TM^TPNy\le\varepsilon x^TM^TPMx+\varepsilon^{-1}y^TN^TPNy$$

Lemma 2.3. [26] (Schur complement). Given constant matrices $S=\begin{bmatrix}S_{11} & S_{12}\\ S_{12}^T & S_{22}\end{bmatrix}$, where $S_{11}=S_{11}^T$, $S_{22}=S_{22}^T$, the following conditions are equivalent:

i) $S<0$

ii) $S_{22}<0$, $S_{11}-S_{12}S_{22}^{-1}S_{12}^T<0$

III. Main Results and Proofs

In this section, we will discuss the stability problem of systems (2.1). In the following theorem, two sufficient criteria will be given, which guarantees the mean-square exponential stability and almost surely exponentially stable for the systems.

Theorem 3.1. Let $k=\|L_g\|+\|U_g-L_g\|<1$, system (2.1) is exponentially stable in mean square, if there exist symmetric matrices $P>0, Q>0, R>0$, and constants $\varepsilon_1>0, \varepsilon_2>0, \varepsilon_3>0$, such that

$$\begin{pmatrix} \Delta_1 & \Delta_2 & \Delta_3 & M \\ \Delta_2^T & \Delta_4 & \Delta_5 & 0 \\ \Delta_3^T & \Delta_5^T & \Delta_6 & 0 \\ M^T & 0 & 0 & -J \end{pmatrix}<0$$

Where

$$\Delta_1 = PL_a + L_a^T P + Q + \tau^2 R + k_1 I_n$$

$$\Delta_2 = PL_b - L_a^T PL_g + L_d^T PL_e$$

$$\Delta_3 = PL_c + L_d^T PL_f$$

$$\Delta_4 = -Q - L_g^T PL_b - L_b^T PL_g + k_2 I_n$$

$$\Delta_5 = L_e^T PL_f - L_g^T PL_c$$

$$\Delta_6 = -R + k_3 I_n$$

$$M = (P\ P\ P)$$

$$J = diag(\varepsilon_1 I_n \quad \varepsilon_2 I_n \quad \varepsilon_3 I_n)$$

with

$$k_1 = \mu_1 + \mu_2 + \mu_5 + \varepsilon_1 \|U_a - L_a\|^2$$

$$k_2 = \mu_1 + \mu_3 + \mu_4 + \mu_6 + \varepsilon_2 \|U_b - L_b\|^2$$

$$k_3 = \mu_2 + \mu_3 + \mu_7 + \varepsilon_3 \|U_c - L_c\|^2$$

$$\begin{aligned}\mu_1 = \lambda_{\max}(P)[&\|U_g - L_g\|\,\|L_a\| + \|U_a - L_a\|\,\|L_g\| \\ &+ \|U_g - L_g\|\,\|U_a - L_a\|] \\ &+ \lambda_{\max}(P)[\|U_d - L_d\|\,\|L_e\| + \|U_e - L_e\|\,\|L_d\| \\ &+ \|U_d - L_d\|\,\|U_e - L_e\|]\end{aligned}$$

$$\begin{aligned}\mu_2 = \lambda_{\max}(P)[&\|U_d - L_d\|\,\|L_f\| + \|U_f - L_f\|\,\|L_d\| \\ &+ \|U_d - L_d\|\,\|U_f - L_f\|]\end{aligned}$$

$$\begin{aligned}\mu_3 = \lambda_{\max}(P)[&\|U_g - L_g\|\,\|L_c\| + \|U_c - L_c\|\,\|L_g\| \\ &+ \|U_g - L_g\|\,\|U_c - L_c\|] \\ &+ \lambda_{\max}(P)[\|U_e - L_e\|\,\|L_f\| + \|U_f - L_f\|\,\|L_e\| \\ &+ \|U_e - L_e\|\,\|U_f - L_f\|]\end{aligned}$$

$$\begin{aligned}\mu_4 = \lambda_{\max}(P)[&\|U_g - L_g\|\,\|L_b\| + \|U_b - L_b\|\,\|L_g\| \\ &+ \|U_g - L_g\|\,\|U_b - L_b\|]\end{aligned}$$

$$\mu_5 = \lambda_{\max}(P)(\|L_d\| + \|U_d - L_d\|)^2$$

$$\mu_6 = \lambda_{\max}(P)(\|L_e\| + \|U_e - L_e\|)^2$$

$$\mu_7 = \lambda_{\max}(P)(\|L_f\| + \|U_f - L_f\|)^2$$

Then, for all $\xi \in L^2_{F_0}([-\tau,0];R^n)$,

$$\begin{aligned}&E|x(t;\xi)|^2 \\ &\le \frac{\lambda_{\max}(P)(2+2k^2)+\tau\lambda_{\max}(Q)+2\tau^3\lambda_{\max}(R)}{(1-k)(1-ke^{r\tau})} \\ &\sup_{-\tau\le\theta\le 0} E|\xi(\theta)|^2 e^{-rt}\end{aligned}$$

Here, r satisfies the following inequalities:

$$\begin{cases} r>0,\ ke^{r\tau}<1 \\ \lambda_{\max}(P)(r+rk^2)+\lambda_{\max}(\Psi)\le 0 \\ \lambda_{\max}(P)(r+rk^2)+(1+\tau)\lambda_{\max}(\Psi) \\ \qquad + r\tau(\lambda_{\max}(Q)+2\tau^2\lambda_{\max}(R))\le 0 \end{cases}$$

with

$$\psi := \begin{pmatrix} \Delta_1 + \sum_{i=1}^{3}\varepsilon_i^{-1}P^2 & \Delta_2 & \Delta_3 \\ \Delta_2^T & \Delta_4 & \Delta_5 \\ \Delta_3^T & \Delta_5^T & \Delta_6 \end{pmatrix}$$

Proof By applying Lemma 2.3, it follows that

$$\begin{pmatrix} \Delta_1 & \Delta_2 & \Delta_3 & M \\ \Delta_2^T & \Delta_4 & \Delta_5 & 0 \\ \Delta_3^T & \Delta_5^T & \Delta_6 & 0 \\ M^T & 0 & 0 & -J \end{pmatrix}<0$$

is equivalent to

$$\psi := \begin{pmatrix} \Delta_1 + \sum_{i=1}^{3}\varepsilon_i^{-1}P^2 & \Delta_2 & \Delta_3 \\ \Delta_2^T & \Delta_4 & \Delta_5 \\ \Delta_3^T & \Delta_5^T & \Delta_6 \end{pmatrix}<0$$

Fix $\xi \in L^2_{F_0}([-\tau,0];R^n)$ and whitened matrices $A(\hat{\otimes})\in[L_a,U_a], B(\hat{\otimes})\in[L_b,U_b], C(\hat{\otimes})\in[L_c,U_c]$, $D(\hat{\otimes})\in[L_d,U_d], E(\hat{\otimes})\in[L_e,U_e], F(\hat{\otimes})\in[L_f,U_f]$, $G(\hat{\otimes})\in[L_g,U_g]$ arbitrarily, and write $x(t,\xi)=x(t)$.

Now, we use the similar methods in[23] to proof the Theorem. First, we choose a Lyapunov-Krasovskii functional candidate for system (2.1) as follows:

$$\begin{aligned}&V(x(t),t) \\ &= V_1(x(t),t)+V_2(x(t),t)+V_3(x(t),t)+V_4(x(t),t)\end{aligned}$$

Where

$$\begin{aligned}&V_1(x(t),t) \\ &= [x(t)-G(\hat{\otimes})x(t-\tau)]^T P[x(t)-G(\hat{\otimes})x(t-\tau)]\end{aligned}$$

$$V_2(x(t),t) = \int_{t-\tau}^{t} x^T(\alpha)Qx(\alpha)d\alpha$$

$$V_3(x(t),t)=\int_{t-\tau}^{t}[\int_{\beta}^{t}x^T(\alpha)d\alpha]R[\int_{\beta}^{t}x(\alpha)d\alpha]d\beta$$

$$V_4(x(t),t)=\int_0^{\tau}[\int_{t-\beta}^{t}(\alpha-t+\beta)x^T(\alpha)Rx(\alpha)d\alpha d\beta$$

By Itô's formula and the definition of weak infinitesimal generator, we can obtain

$$\begin{aligned}&dV(x(t),t)\\&=LV(x(t),t)dt\\&+2[x(t)-G(\hat{\otimes})x(t-\tau)]^T P[D(\hat{\otimes})x(t)\\&+E(\hat{\otimes})x(t-\tau)+F(\hat{\otimes})\int_{t-\tau}^{t}x(s)ds]dW(t)\end{aligned}\tag{3.1}$$

with

$$\begin{aligned}&LV(x(t),t)\\&=2[x(t)-G(\otimes)x(t-\tau)]^T P[A(\otimes)x(t)\\&+B(\otimes)x(t-\tau)+C(\otimes)\int_{t-\tau}^{t}x(s)ds]\\&+[D(\hat{\otimes})x(t)+E(\hat{\otimes})x(t-\tau)+F(\hat{\otimes})\int_{t-\tau}^{t}x(s)ds]^T P\\&[D(\hat{\otimes})x(t)+E(\hat{\otimes})x(t-\tau)+F(\hat{\otimes})\int_{t-\tau}^{t}x(s)ds]\\&+\dot{V}_2(x(t),t)+\dot{V}_3(x(t),t)+\dot{V}_4(x(t),t)\end{aligned}\tag{3.2}$$

Here,

$$\begin{aligned}&\dot{V}_2(x(t),t)\\&=x^T(t)Qx(t)-x^T(t-\tau)Qx(t-\tau)\end{aligned}$$

$$\begin{aligned}&\dot{V}_3(x(t),t)\\&=\int_{t-\tau}^{t}x^T(t)R[\int_{\beta}^{t}x(\alpha)d\alpha]d\beta\\&+\int_{t-\tau}^{t}[\int_{\beta}^{t}x^T(\alpha)d\alpha]Rx(t)d\beta\\&-[\int_{t-\tau}^{t}x^T(\alpha)d\alpha]R[\int_{t-\tau}^{t}x(\alpha)d\alpha]\\&\le\int_{t-\tau}^{t}(\alpha-t+\tau)x^T(t)Rx(\alpha)d\alpha\\&+\int_{t-\tau}^{t}(\alpha-t+\tau)x^T(\alpha)Rx(t)d\alpha\\&-[\int_{t-\tau}^{t}x(\alpha)d\alpha]^T R[\int_{t-\tau}^{t}x(\alpha)d\alpha]\\&\le\int_{t-\tau}^{t}(\alpha-t+\tau)x^T(t)Rx(t)d\alpha\\&+\int_{t-\tau}^{t}(\alpha-t+\tau)x^T(\alpha)Rx(\alpha)d\alpha\\&-[\int_{t-\tau}^{t}x(\alpha)d\alpha]^T R[\int_{t-\tau}^{t}x(\alpha)d\alpha]\end{aligned}$$

and

$$\begin{aligned}&\dot{V}_4(x(t),t)\\&=\int_0^{\tau}\beta x^T(t)Rx(t)d\beta-\int_0^{\tau}[\int_{t-\beta}^{t}x^T(\alpha)Rx(\alpha)d\alpha d\beta\\&=\frac{\tau^2}{2}x^T(t)Rx(t)-\int_{t-\tau}^{t}(\alpha-t+\tau)x^T(\alpha)Rx(\alpha)d\alpha\end{aligned}$$

For convenience, let $z(t)=\int_{t-\tau}^{t}x(\alpha)d\alpha$, then

$$\dot{V}_3(x(t),t)+\dot{V}_4(x(t),t)\le\tau^2x^T(t)Rx(t)-z^T(t)Rz(t)$$

and (3.2) formula can be rewritten as

$$\begin{aligned}&LV(x(t),t)\\&\le 2[x(t)-G(\hat{\otimes})x(t-\tau)]^T P[A(\hat{\otimes})x(t)\\&+B(\hat{\otimes})x(t-\tau)+C(\hat{\otimes})z(t)]\\&+[D(\hat{\otimes})x(t)+E(\hat{\otimes})x(t-\tau)+F(\hat{\otimes})z(t)]^T P\\&[D(\hat{\otimes})x(t)+E(\hat{\otimes})x(t-\tau)+F(\hat{\otimes})z(t)]\\&+x^T(t)(Q+\tau^2R)x(t)-x^T(t-\tau)Qx(t-\tau)\\&-z^T(t)Rz(t)\\&=x^T(t)(Q+\tau^2R)x(t)-x^T(t-\tau)Qx(t-\tau)\\&-z^T(t)Rz(t)\\&+2x^T(t)PA(\hat{\otimes})x(t)+2x^T(t)PB(\hat{\otimes})x(t-\tau)\\&+2x^T(t)PC(\hat{\otimes})z(t)\\&-2x^T(t-\tau)G^T(\hat{\otimes})PA(\hat{\otimes})x(t)\\&-2x^T(t-\tau)G^T(\hat{\otimes})PB(\hat{\otimes})x(t-\tau)\\&-2x^T(t-\tau)G^T(\hat{\otimes})PC(\hat{\otimes})z(t)\\&+x^T(t)D^T(\hat{\otimes})PD(\hat{\otimes})x(t)\\&+x^T(t)D^T(\hat{\otimes})PE(\hat{\otimes})x(t-\tau)\\&+x^T(t)D^T(\hat{\otimes})PF(\hat{\otimes})z(t)\\&+x^T(t-\tau)E^T(\hat{\otimes})PD(\hat{\otimes})x(t)\\&+x^T(t-\tau)E^T(\hat{\otimes})PE(\hat{\otimes})x(t-\tau)\\&+x^T(t-\tau)E^T(\hat{\otimes})PF(\hat{\otimes})z(t)\\&+z^T(t)F^T(\hat{\otimes})PD(\hat{\otimes})x(t)\\&+z^T(t)F^T(\hat{\otimes})PE(\hat{\otimes})x(t-\tau)\\&+z^T(t)F^T(\hat{\otimes})PF(\hat{\otimes})z(t)\end{aligned}\tag{3.3}$$

By using Lemma 2.1 and Lemma 2.2, we can get

$$\begin{aligned}&2x^T(t)PA(\hat{\otimes})x(t)\\&=x^T(t)(PL_a+L_a^TP)x(t)+2x^T(t)P\Delta Ax(t)\\&\le x^T(t)PL_ax(t)+x^T(t)L_a^TPx(t)\\&+\varepsilon_1^{-1}x^T(t)P^2x(t)+\varepsilon_1\left\|U_a-L_a\right\|^2x^T(t)x(t)\end{aligned}\tag{3.4}$$

Similar to (3.4), we have

$$\begin{aligned}&2x^T(t)PB(\hat{\otimes})x(t-\tau)\\&\le x^T(t)PL_bx(t-\tau)+x^T(t-\tau)L_b^TPx(t)\\&+\varepsilon_2^{-1}x^T(t)P^2x(t)\\&+\varepsilon_2\left\|U_b-L_b\right\|^2x^T(t-\tau)x(t-\tau)\end{aligned}\tag{3.5}$$

and

$$\begin{aligned}&2x^T(t)PC(\hat{\otimes})z(t)\\ &\le x^T(t)PL_c z(t)+z^T(t)L_c^T Px(t)\\ &+\varepsilon_3^{-1}x^T(t)P^2x(t)+\varepsilon_3\|U_c-L_c\|^2 z^T(t)z(t)\end{aligned} \tag{3.6}$$

Moreover, by Lemma 2.1 and Lemma 2.2, we have

$$\begin{aligned}&-2x^T(t-\tau)G^T(\hat{\otimes})PA(\hat{\otimes})x(t)\\ &=-x^T(t-\tau)L_g^T PL_a x(t)\\ &-x^T(t)L_a^T PL_g x(t-\tau)\\ &-2x^T(t-\tau)\Delta G^T PL_a x(t)\\ &-2x^T(t-\tau)L_g^T P\Delta A x(t)\\ &-2x^T(t-\tau)\Delta G^T P\Delta A x(t)\end{aligned} \tag{3.7}$$

Where

$$\begin{aligned}&-2x^T(t-\tau)\Delta G^T PL_a x(t)\\ &\le 2\|\Delta G^T\|\|P\|\|L_a\|\|x^T(t-\tau)\|x(t)\|\\ &\le \lambda_{\max}(P)\|U_g-L_g\|\|L_a\|[x^T(t)x(t)+x^T(t-\tau)x(t-\tau)]\end{aligned} \tag{3.8}$$

$$\begin{aligned}&-2x^T(t-\tau)L_g^T P\Delta A x(t)\\ &\le \lambda_{\max}(P)\|U_a-L_a\|\|L_g\|[x^T(t)x(t)\\ &+x^T(t-\tau)x(t-\tau)]\end{aligned} \tag{3.9}$$

and

$$\begin{aligned}&-2x^T(t-\tau)\Delta G^T P\Delta A x(t)\\ &\le \lambda_{\max}(P)\|U_g-L_g\|\|U_a-L_a\|[x^T(t)x(t)\\ &+x^T(t-\tau)x(t-\tau)]\end{aligned} \tag{3.10}$$

Combining with (3.7)-(3.10) and computing them, we see

$$\begin{aligned}&-2x^T(t-\tau)G^T(\hat{\otimes})PA(\hat{\otimes})x(t)\\ &\le -x^T(t-\tau)L_g^T PL_a x(t)\\ &-x^T(t)L_a^T PL_g x(t-\tau)\\ &+\lambda_{\max}(P)[\|U_g-L_g\|\|L_a\|\\ &+\|U_a-L_a\|\|L_g\|\\ &+\|U_g-L_g\|\|U_a-L_a\|]\cdot\\ &[x^T(t)x(t)+x^T(t-\tau)x(t-\tau)]\end{aligned} \tag{3.11}$$

Using the similar method as in (3.11), the following inequalities hold:

$$\begin{aligned}&-2x^T(t-\tau)G^T(\hat{\otimes})PB(\hat{\otimes})x(t-\tau)\\ &\le -x^T(t-\tau)L_g^T PL_b x(t-\tau)\\ &-x^T(t-\tau)L_b^T PL_g x(t-\tau)\\ &+\lambda_{\max}(P)[\|U_g-L_g\|\|L_b\|+\|U_b-L_b\|\|L_g\|\\ &+\|U_g-L_g\|\|U_b-L_b\|]x^T(t-\tau)x(t-\tau)\end{aligned} \tag{3.12}$$

$$\begin{aligned}&-2x^T(t-\tau)G^T(\hat{\otimes})PC(\hat{\otimes})z(t)\\ &\le -x^T(t-\tau)L_g^T PL_c z(t)\\ &-z^T(t)L_c^T PL_g x(t-\tau)\\ &+\lambda_{\max}(P)[\|U_g-L_g\|\|L_c\|\\ &+\|U_c-L_c\|\|L_g\|+\|U_g-L_g\|\|U_c-L_c\|]\cdot\\ &[z^T(t)z(t)+x^T(t-\tau)x(t-\tau)]\end{aligned} \tag{3.13}$$

$$\begin{aligned}&x^T(t)D^T(\hat{\otimes})PE(\hat{\otimes})x(t-\tau)\\ &\le x^T(t)L_d^T\frac{1}{2}PL_e x(t-\tau)\\ &+x^T(t-\tau)L_e^T\frac{1}{2}PL_d x(t)\\ &+\frac{1}{2}\lambda_{\max}(P)[\|U_d-L_d\|\|L_e\|\\ &+\|U_e-L_e\|\|L_d\|+\|U_d-L_d\|\|U_e-L_e\|]\cdot\\ &[x^T(t)x(t)+x^T(t-\tau)x(t-\tau)]\end{aligned} \tag{3.14}$$

$$\begin{aligned}&x^T(t)D^T(\hat{\otimes})PF(\hat{\otimes})z(t)\\ &\le x^T(t)L_d^T\frac{1}{2}PL_f z(t)+z^T(t)L_f^T\frac{1}{2}PL_d x(t)\\ &+\frac{1}{2}\lambda_{\max}(P)[\|U_d-L_d\|\|L_f\|\\ &+\|U_f-L_f\|\|L_d\|+\|U_d-L_d\|\|U_f-L_f\|]\cdot\\ &[x^T(t)x(t)+z^T(t)z(t)]\end{aligned} \tag{3.15}$$

$$\begin{aligned}&x^T(t-\tau)E^T(\hat{\otimes})PD(\hat{\otimes})x(t)\\ &\le x^T(t-\tau)L_e^T\frac{1}{2}PL_d x(t)+x^T(t)L_d^T\frac{1}{2}PL_e x(t-\tau)\\ &+\frac{1}{2}\lambda_{\max}(P)[\|U_e-L_e\|\|L_d\|\\ &+\|U_d-L_d\|\|L_e\|+\|U_e-L_e\|\|U_d-L_d\|]\cdot\\ &[x^T(t)x(t)+x^T(t-\tau)x(t-\tau)]\end{aligned} \tag{3.16}$$

$$\begin{aligned}&x^T(t-\tau)E^T(\hat{\otimes})PF(\hat{\otimes})z(t)\\ &\le x^T(t-\tau)L_e^T\frac{1}{2}PL_fz(t)\\ &\quad+z^T(t)L_f^T\frac{1}{2}PL_ex(t-\tau)\\ &\quad+\frac{1}{2}\lambda_{\max}(P)[\|U_e-L_e\|\,\|L_f\|\\ &\quad+\|U_f-L_f\|\,\|L_e\|+\|U_e-L_e\|\,\|U_f-L_f\|]\cdot\\ &\quad[z^T(t)z(t)+x^T(t-\tau)x(t-\tau)]\end{aligned}\tag{3.17}$$

$$\begin{aligned}&z^T(t)F^T(\hat{\otimes})PD(\hat{\otimes})x(t)\\ &\le z^T(t)L_f^T\frac{1}{2}PL_dx(t)\\ &\quad+x^T(t)L_d^T\frac{1}{2}PL_fz(t)\\ &\quad+\frac{1}{2}\lambda_{\max}(P)[\|U_f-L_f\|\,\|L_d\|\\ &\quad+\|U_d-L_d\|\,\|L_f\|+\|U_f-L_f\|\,\|U_d-L_d\|]\cdot\\ &\quad[z^T(t)z(t)+x^T(t)x(t)]\end{aligned}\tag{3.18}$$

$$\begin{aligned}&z^T(t)F^T(\hat{\otimes})PE(\hat{\otimes})x(t-\tau)\\ &\le z^T(t)L_f^T\frac{1}{2}PL_ex(t-\tau)\\ &+x^T(t-\tau)L_e^T\frac{1}{2}PL_fz(t)\\ &+\frac{1}{2}\lambda_{\max}(P)[\|U_f-L_f\|\,\|L_e\|\\ &+\|U_e-L_e\|\,\|L_f\|+\|U_f-L_f\|\,\|U_e-L_e\|]\cdot\\ &[z^T(t)z(t)+x^T(t-\tau)x(t-\tau)]\end{aligned}\tag{3.19}$$

Furthermore,

$$\begin{aligned}&x^T(t)D^T(\hat{\otimes})PD(\hat{\otimes})x(t)\\ &\le\lambda_{\max}(P)(\|L_d\|+\|U_d-L_d\|)^2x^T(t)x(t)\end{aligned}\tag{3.20}$$

$$\begin{aligned}&x^T(t-\tau)E^T(\hat{\otimes})PE(\hat{\otimes})x(t-\tau)\\ &\le\lambda_{\max}(P)(\|L_e\|+\|U_e-L_e\|)^2x^T(t-\tau)x(t-\tau)\end{aligned}\tag{3.21}$$

$$\begin{aligned}&z^T(t)F^T(\hat{\otimes})PF(\hat{\otimes})z(t)\\ &\le\lambda_{\max}(P)(\|L_f\|+\|U_f-L_f\|)^2z^T(t)z(t)\end{aligned}\tag{3.22}$$

Then, substituting of (3.4) - (3.22) into (3.3), and noting the definition of Ψ , it is clear that

$$\begin{aligned}&LV(x(t),t)\\ &\le(x^T(t),x^T(t-\tau),z^T(t))\Psi\begin{pmatrix}x(t)\\ x(t-\tau)\\ z(t)\end{pmatrix}\\ &\le\lambda_{\max}(\Psi)(|x(t)|^2+|x(t-\tau)|^2+|z(t)|^2)\end{aligned}\tag{3.23}$$

It follows from (3.1) and (3.23) that, the following inequality holds:

$$\begin{aligned}&dV(x(t),t)\\ &\le\lambda_{\max}(\Psi)(|x(t)|^2+|x(t-\tau)|^2+|z(t)|^2)dt\\ &\quad+2[x(t)-G(\hat{\otimes})x(t-\tau)]^TP[D(\hat{\otimes})x(t)\\ &\quad+E(\hat{\otimes})x(t-\tau)+F(\hat{\otimes})\int_{t-\tau}^{t}x(s)ds]dW(t)\end{aligned}\tag{3.24}$$

On the other hand, from the definitions of $V_1(x(t),t)$, $V_2(x(t),t)$, $V_3(x(t),t)$, $V_4(x(t),t)$, we can obtain

$$\begin{aligned}&V_1(x(t),t)\\ &\le\lambda_{\max}(P)(1+\|G(\hat{\otimes})\|^2)(|x(t)|^2+|x(t-\tau)|^2)\end{aligned}\tag{3.25}$$

$$V_2(x(t),t)\le\lambda_{\max}(Q)\int_{t-\tau}^{t}|x(\alpha)|^2d\alpha\tag{3.26}$$

$$\begin{aligned}&V_3(x(t),t)\\ &\le\int_{t-\tau}^{t}(t-\beta)\int_{\beta}^{t}x^T(\alpha)Rx(\alpha)d\alpha d\beta\\ &\le\tau\lambda_{\max}(R)\int_{t-\tau}^{t}\int_{\beta}^{t}|x(\alpha)|^2d\alpha d\beta\\ &\le\tau^2\lambda_{\max}(R)\int_{t-\tau}^{t}|x(\alpha)|^2d\alpha\end{aligned}\tag{3.27}$$

$$V_4(x(t),t)\le\tau^2\lambda_{\max}(R)\int_{t-\tau}^{t}|x(\alpha)|^2d\alpha\tag{3.28}$$

By (3.25) - (3.28) and the definition of $V(x(t),t)$, we have

$$\begin{aligned}&V(x(t),t)\\ &\le\lambda_{\max}(P)(1+\|G(\hat{\otimes})\|^2)(|x(t)|^2+|x(t-\tau)|^2)\\ &\quad+[\lambda_{\max}(Q)+2\tau^2\lambda_{\max}(R)]\int_{t-\tau}^{t}|x(\alpha)|^2d\alpha\\ &\le\lambda_{\max}(P)(1+k^2)(|x(t)|^2+|x(t-\tau)|^2)\\ &\quad+[\lambda_{\max}(Q)+2\tau^2\lambda_{\max}(R)]\int_{-\tau}^{0}|x(t+\theta)|^2d\theta\end{aligned}\tag{3.29}$$

and

$$\begin{aligned}&V(x(0),0)\\ &\le[\lambda_{\max}(P)(2+2k^2)+\tau\lambda_{\max}(Q)\\ &\quad+2\tau^3\lambda_{\max}(R)]\sup_{-\tau\le\theta\le0}|\xi(\theta)|^2\end{aligned}\tag{3.30}$$

Using the integration-by-parts formula and (3.24), (3.29), we can get

$$
\begin{aligned}
& d\left[e^{rt} V(x(t),t)\right] \\
&= e^{rt}\left[r V(x(t),t)dt + dV(x(t),t)\right] \\
&\le re^{rt}\left[2\lambda_{\max}(P)(1+k^2)\left(|x(t)|^2 + |x(t-\tau)|^2\right)\right]dt \\
&+ re^{rt}\left[(\lambda_{\max}(Q) + 2\tau^2\lambda_{\max}(R))\int_{-\tau}^{0}|x(t+\theta)|^2 d\theta\right]dt \\
&+ e^{rt}\left[\lambda_{\max}(\Psi)\left(|x(t)|^2 + |x(t-\tau)|^2 + |z(t)|^2\right)\right]dt \\
&+ 2e^{rt}[x(t) - G(\otimes)x(t-\tau)]^T P[D(\hat{\otimes})x(t) + E(\hat{\otimes})x(t-\tau) \\
&+ F(\hat{\otimes})\int_{t-\tau}^{t} x(s)ds]dW(t)
\end{aligned} \tag{3.31}
$$

Integrating both sides of (3.31) from 0 to t> 0 and then taking the mathematical expectation, and considering (3.30), we have

$$
\begin{aligned}
& E\left[e^{rt} V(x(t),t)\right] \\
&\le [\lambda_{\max}(P)(2+2k^2) + \tau\lambda_{\max}(Q) \\
&+ 2\tau^3\lambda_{\max}(R)] \sup_{-\tau\le\theta\le 0} E|\xi(\theta)|^2 \\
&+ \lambda_{\max}(P)(r+rk^2)\int_0^t e^{rs}(E|x(s)|^2 + E|x(s-\tau)|^2)ds \\
&+ r(\lambda_{\max}(Q) + \\
& 2\tau^2\lambda_{\max}(R)) E\int_0^t e^{rs}\int_{-\tau}^{0}|x(s+\theta)|^2 d\theta ds \\
&+ \lambda_{\max}(\Psi)\int_0^t e^{rs}(E|x(s)|^2 + E|x(s-\tau)|^2 + E|z(s)|^2)ds
\end{aligned} \tag{3.32}
$$

In addition, the following inequalities hold:

$$
\begin{aligned}
& \int_0^t e^{rs} E|x(s-\tau)|^2 ds \\
&\le e^{r\tau}\int_0^t e^{ru} E|x(s)|^2 ds + e^{r\tau}\int_{-\tau}^{0} e^{r\theta} E|\xi(\theta)|^2 d\theta
\end{aligned} \tag{3.33}
$$

$$
\begin{aligned}
& E\int_0^t e^{rs}\int_{-\tau}^{0}|x(s+\theta)|^2 d\theta ds \\
&\le \tau e^{r\tau}(\int_0^t e^{rs} E|x(s)|^2 ds + \int_{-\tau}^{0} E|\xi(\theta)|^2 d\theta)
\end{aligned} \tag{3.34}
$$

and

$$
\begin{aligned}
& \int_0^t e^{rs} E|z(s)|^2 ds \\
&\le \tau e^{r\tau}\int_0^t e^{rs} E|x(s)|^2 ds + \tau^2 e^{r\tau} \sup_{-\tau\le\theta\le 0} E|\xi(\theta)|^2
\end{aligned} \tag{3.35}
$$

Then, substituting of (3.33)-(3.35) into (3.32) yield, and noting that

$$\lambda_{\max}(P)(r+rk^2) + \lambda_{\max}(\Psi) \le 0$$

and

$$
\begin{aligned}
& \lambda_{\max}(P)(r+rk^2) + (1+\tau)\lambda_{\max}(\Psi) \\
&\qquad + r\tau(\lambda_{\max}(Q) + 2\tau^2\lambda_{\max}(R)) \le 0
\end{aligned}
$$

Hence, we see

$$
\begin{aligned}
& E\left[e^{rt} V(x(t),t)\right] \\
&\le [\lambda_{\max}(P)(2+2k^2) + \tau\lambda_{\max}(Q) \\
&\quad + 2\tau^3\lambda_{\max}(R)] \sup_{-\tau\le\theta\le 0} E|\xi(\theta)|^2 \\
&\quad + [\lambda_{\max}(P)(r+rk^2) + (1+\tau)\lambda_{\max}(\Psi) \\
&\quad + r\tau(\lambda_{\max}(Q) + 2\tau^2\lambda_{\max}(R))]\tau e^{r\tau} \sup_{-\tau\le\theta\le 0} E|\xi(\theta)| \\
&\quad + \{\lambda_{\max}(P)(r+rk^2) + \lambda_{\max}(\Psi) \\
&\quad + [\lambda_{\max}(P)(r+rk^2) + (1+\tau)\lambda_{\max}(\Psi) \\
&\quad + r\tau(\lambda_{\max}(Q) + 2\tau^2\lambda_{\max}(R))]e^{r\tau}\}\int_0^t e^{rs} E|x(s)|^2 ds \\
&\le [\lambda_{\max}(P)(2+2k^2) + \tau\lambda_{\max}(Q) \\
&\quad + 2\tau^3\lambda_{\max}(R)] \sup_{-\tau\le\theta\le 0} E|\xi(\theta)|^2
\end{aligned} \tag{3.36}
$$

Furthermore, noting that the following inequality holds:

$$
\begin{aligned}
& E[e^{rt} V(x(t),t)] \\
&\ge e^{rt}[(1-k)E|x(t)|^2 - (\frac{1}{k}-1)k^2 E|x(t-\tau)|^2]
\end{aligned} \tag{3.37}
$$

Since $\left|G(\hat{\otimes})\right| \le k = \|L_g\| + \|U_g - L_g\| < 1$, from (3.37), it follows that

$$
\begin{aligned}
& e^{rt} E|(x(t),t)|^2 \\
&\le \frac{1}{1-k} E[e^{rt} V(x(t),t)] + ke^{rt} \sup_{-\tau\le\theta\le 0} E|x(t+\theta)|^2
\end{aligned} \tag{3.38}
$$

Then, for $0 \le t \le T$, from (3.36) and (3.38), we also have

$$
\begin{aligned}
& e^{rt} E|(x(t),t)|^2 \\
&\le \frac{1}{1-k} \sup_{0\le t\le T} E[e^{rt} V(x(t),t)] \\
&\quad + k \sup_{0\le t\le T}(e^{rt} \sup_{-\tau\le\theta\le 0} E|x(t+\theta)|^2) \\
&\le \frac{\lambda_{\max}(P)(2+2k^2) + \tau\lambda_{\max}(Q) + 2\tau^3\lambda_{\max}(R)}{1-k} \\
&\quad \sup_{-\tau\le\theta\le 0} E|\xi(\theta)|^2 + ke^{r\tau} \sup_{-\tau\le t\le T}(e^{rt} E|x(t)|^2)
\end{aligned} \tag{3.39}
$$

Noting that, for $0 \le t \le T$, (3.39) also holds. Therefore, we have

$$
\begin{aligned}
& \sup_{-\tau\le t\le T} e^{rt} E|(x(t),t)|^2 \\
&\le \frac{\lambda_{\max}(P)(2+2k^2) + \tau\lambda_{\max}(Q) + 2\tau^3\lambda_{\max}(R)}{1-k} \\
& \sup_{-\tau\le\theta\le 0} E|\xi(\theta)|^2 + ke^{r\tau} \sup_{-\tau\le t\le T}(e^{rt} E|x(t)|^2)
\end{aligned} \tag{3.40}
$$

Since $ke^{r\tau} < 1$, and (3.40) can be rewritten as

$$\sup_{-\tau\le t\le T} e^{rt}E\left|(x(t),t)\right|^2 \le \frac{\lambda_{\max}(P)(2+2k^2)+\tau\lambda_{\max}(Q)+2\tau^3\lambda_{\max}(R)}{(1-k)(1-ke^{r\tau})} \sup_{-\tau\le\theta\le 0} E\left|\xi(\theta)\right|^2 \quad (3.41)$$

which indicates that the system(2.1) is exponentially stable in the mean square.

Remark 3.1. if $A(\otimes)\equiv A$, $B(\otimes)\equiv B$, $C(\otimes)\equiv C$, $D(\otimes)\equiv D$, $E(\otimes)\equiv E$, $F(\otimes)\equiv F$ and $G(\otimes)\equiv G$, system (2.1) becomes the deterministic stochastic neutral systems with distributed-delays (3.42).

$$\begin{cases} d[x(t)-Gx(t-\tau)] \\ =\left[Ax(t)+Bx(t-\tau)+C\int_{t-\tau}^{t}x(s)ds\right]dt \\ +\left[Dx(t)+Ex(t-\tau)+F\int_{t-\tau}^{t}x(s)ds\right]dW(t), t\ge 0 \\ x_0=\xi,\ \xi\in L_{F_0}^2([-\tau,0];R^n), -\tau\le t\le 0 \end{cases} \quad (3.42)$$

Let

$L_a=U_a\equiv A$, $L_b=U_b\equiv B$, $L_c=U_c\equiv C$,

$L_d=U_d\equiv D$, $L_e=U_e\equiv E$, $L_f=U_f\equiv F$,

$L_g=U_g\equiv G$.

Now, following the similar line of the proof of Theorem 3.1, we also obtain the following exponential stability criterion for the deterministic stochastic system (3.42).

Corollary 3.1. Let $m=\|G\|<1$, system (3.42) is exponentially robustly stable in mean square, if there exist symmetric matrices $P>0, Q>0, R>0$, and constants $\varepsilon_1>0, \varepsilon_2>0, \varepsilon_3>0$, such that

$$\begin{pmatrix} \Sigma_1 & \Sigma_2 & \Sigma_3 & M \\ \Sigma_2^T & \Sigma_4 & \Sigma_5 & 0 \\ \Sigma_3^T & \Sigma_5^T & \Sigma_6 & 0 \\ M^T & 0 & 0 & -J \end{pmatrix}<0$$

Where

$\Sigma_1=PA+A^TP+Q+\tau^2R+w_1I_n$

$\Sigma_2=PB-A^TPG+D^TPE$

$\Sigma_3=PC+D^TPF$

$\Sigma_4=-Q-G^TPB-B^TPG+w_2I_n$

$\Sigma_5=E^TPF-G^TPC$

$\Sigma_6=-R+w_3I_n$

$M=(P\ P\ P)$,

$J=diag(\varepsilon_1I_n\ \ \varepsilon_2I_n\ \ \varepsilon_3I_n)$,

with

$w_1=\lambda_{\max}(P)\|D\|^2$,

$w_2=\lambda_{\max}(P)\|E\|^2$,

$w_3=\lambda_{\max}(P)\|F\|^2$

Theorem 3.2. Under the conditions of Theorem 3.1, system (2.1) is said to be almost surely exponentially robustly stable. In other words, for all $\xi\in L_{F_0}^2([-\tau,0];R^n)$, then one has the following inequality:

$$\lim_{t\to\infty}\sup\frac{1}{t}\ln|x(t;\xi)|\le-\frac{\hat{r}}{2},\quad a.s.$$

where, $\hat{r}=\min\{r,\tau^{-1},\ln k^{-1}\}$.

Proof By Doob's martingale inequality, Cauchy inequality and Borel-Cantelli lemma, the result can be worked out easily along the same line as in the proof of Theorem 2 in [23,24], and thus is omitted.

IV. Examples

In this section, an example is provided to demonstrate the effectiveness of the obtained results.

Consider the following grey neutral stochastic systems with distributed delays

$$\begin{cases} d[x(t)-G(\otimes)x(t-\tau)] \\ =[A(\otimes)x(t)+B(\otimes)x(t-0.5)+C(\otimes)\int_{t-\tau}^{t}x(s)ds]dt \\ +[D(\otimes)x(t)+E(\otimes)x(t-0.5)+F(\otimes)\int_{t-\tau}^{t}x(s)ds]dW(t),\ t\ge 0 \\ x_0=\xi,\ \xi\in L_{F_0}^2([-0.5,0];R^n), -0.5\le t\le 0 \end{cases} \quad (4.1)$$

where

$$L_a=\begin{bmatrix}-3.35 & 0.22\\ 0.23 & -3.34\end{bmatrix};\quad U_a=\begin{bmatrix}-3.15 & 0.32\\ 0.31 & -3.45\end{bmatrix}$$

$$L_b=\begin{bmatrix}-1.15 & 0.20\\ 0.23 & -1.16\end{bmatrix};\quad U_b=\begin{bmatrix}-1.12 & 0.22\\ 0.31 & -1.09\end{bmatrix}$$

$$L_c=\begin{bmatrix}-0.35 & 0.02\\ 0.12 & -1.24\end{bmatrix};\quad U_c=\begin{bmatrix}-0.25 & 0.04\\ 0.16 & -1.15\end{bmatrix}$$

$$L_d=\begin{bmatrix}0.19 & 0.60\\ 0.29 & -2.56\end{bmatrix};\quad U_d=\begin{bmatrix}0.20 & 0.62\\ 0.31 & -1.99\end{bmatrix}$$

$$L_e=\begin{bmatrix}2.15 & 0.92\\ 1.29 & 1.34\end{bmatrix};\quad U_e=\begin{bmatrix}2.25 & 0.82\\ 1.31 & 1.45\end{bmatrix}$$

$$L_f=\begin{bmatrix}-0.15 & 1.20\\ 1.53 & -1.56\end{bmatrix};\quad U_f=\begin{bmatrix}-0.19 & 1.22\\ 1.39 & -1.69\end{bmatrix}$$

$$L_g=\begin{bmatrix}-2.35 & 0.23\\ 0.43 & -2.84\end{bmatrix};\quad U_g=\begin{bmatrix}-2.15 & 0.22\\ 0.41 & -2.75\end{bmatrix}$$

Here,

L_a, U_a ; L_b, U_b ; L_c, U_c ; L_d, U_d ; L_e, U_e ; L_f, U_f ; L_g, U_g

are the lower bound and upper bound matrices of $A(\otimes), B(\otimes), C(\otimes), D(\otimes), E(\otimes), F(\otimes)$ and $G(\otimes)$.

Using the programmed procedure (see [24]), it is easy to calculate and optimize $\varepsilon_1, \varepsilon_2, \varepsilon_2$, and we can obtain that r =0.8216. It follows from Theorem 3.1 that the system (4.1) is exponentially stable in mean square.

V. Conclusion

In this paper, exponential stability problem for a class of grey neutral stochastic systems with distributed delays has been studied. Based on the Lyapunov stability theory and some well-known differential formulas, in particular, using decomposition approach of the continuous matrix-covered sets, the stability criteria have been derived to guarantee the exponential stability in mean square and almost surely exponentially robustly stable for our considered systems. In addition, an example is given to illustrate the effectiveness of the obtained results.

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