A Combined Iteration Algorithm for the Implicit Cycles of Gold Price and the US Dollar Index

Haitao Zheng, Huiwen Wang, and Andi Zheng

***Abstract*—The relationship between gold and US dollar series, which are non-stationary, is commonly known to be negative from a periodic perspective. Is this really the case? This paper established a combined iteration algorithm using the theory of spectral analysis after subtracting the trend using penalized B-spline functions to obtain the implicit cycles in gold and US dollar series. This algorithm accurately separates the trend terms and periodic terms of the two series to produce more precise and complete periodic information. The results show that both series share three common implicit cycles: two long periods and one short one. Both long-period terms are negatively correlated, whereas the short-period terms are positively correlated.**

***Index Terms*—B-spline function, spectrum analysis, implicit cycles, gold price, US dollar index**

I. Introduction

GOLD is a financial asset that has always been a subject of interest and widely accepted as international currency. Currently, it is commonly used as an investing and hedging tool. Accurately measuring and forecasting future price trends are critical to mitigating risk and uncertainty when making hedging, future investment and evaluation decisions. Gold also attracts speculators seeking opportunities to engage in arbitrage. In quantitative finance, it is crucial to develop appropriate financial models based on the properties and mathematical descriptions of assets to determine their prices, develop investment portfolios, and manage risk.

Common models for predicting price series include the ARIMA and GARCH models. Neural network methodology has also been applied in this field. The asymmetric power GARCH model and likelihood ratio tests have been used to assess the significance of models [1]. Additionally, non-stationary and nonlinear economic time series of gold prices have been used to compare the GARCH model with a wavelet neural network model [2]. An auto-regression method was found to give the best overall performance in a comparison of 49 univariate forecasting methods [3]. The asymmetric GARCH method was used to improve the Bayesian option pricing model [4]. Other examples of studies examining financial time series using ARCH or GARCH models include [5], [6], [7] and [8]. Among studies applying neural network methods, most have used neural networks trained by features extracted from ARIMA analyses to model the behavior of time series, such as [9], [10], [11], [12], [13], [14] and [15].

Another widely discussed topic is the correlation between gold and other macroeconomic factors. Such research can offer investors the opportunity to obtain evidence of gold's hedging ability and protect their wealth in the event of negative market conditions. Several factors can influence the gold market, including the oil price, inflation rate, stock market, and exchange rate. Among these factors, one very important index is the US dollar. Studies relating gold price and the dollar have a long history, and of all the methods, regression models are the most commonly used in all financial market areas. Baker, Tassel and Roger(1985) [16] described a regression model relating the gold price and the US dollar exchange rate, the interest rate, the inflation rate, and commodity prices, and until now, most studies have used univariate or multivariate regression models to determine the correlation between the gold market and many other macroeconomic factors. For example, see [17] and Baur and McDermott(2010) [18] who used regression models to show that gold is both a safe haven and a hedge in the US. Related studies investigating whether gold is a hedge, a diversifier or a safe haven also include [19], who assessed the role of gold as a hedge against the US dollar by estimating gold's response elasticity to changes in the exchange rate. Additionally, a multivariate GARCH model of dynamic conditional correlations was used to demonstrate that gold has been a good hedge and a poor safe haven against the US dollar in the past and that, in recent years, gold has acted as an increasingly effective hedge against currency risk associated with the US dollar [20]. [1] also used the asymmetric power GARCH model to investigate macroeconomic influences on gold and found that the US dollar is the most significant. This model also revealed that the dollar's influence is negative, confirming the long-held notion of gold as an 'anti-dollar'. Statistical physics has also been applied to economic and financial series, such as gold and the US dollar. Examples include [21], who used a variability diagram technique to detect short-range correlations and decorrelations in financial data; the detrended fluctuation analysis performed by [22]; a multifractal detrended fluctuation analysis by [23] and [24]; a visibility graph network analysis reported by [25]; and a singular spectrum analysis conducted by [26].

Most of these analyses suggest that gold and the US dollar are very negatively correlated (see [20] and [27]). Because investors have traditionally used gold as a hedge against inflation, it seems reasonable to expect that the price of gold will tend to rise when the US dollar loses value, preserving gold's real value. However, this conclusion is not necessarily true over the long term. Figures 1 and 2 present monthly

Manuscript received October 26, 2015; revised March 04, 2016. This work was supported by the National Natural Science Foundation of China (Nos. 71420107025, 71371021, and 71333014), the National Key Technology Research and Development Program of China (No. 2012BAC20B08), the National High Technology Research and Development Program of China (No. SS2014AA012303) and the Generalized Virtual Economy Research Program of China (GX2014-1007(M)).

H. Zheng is with the School of Economics and Management, Beihang University, Beijing,100191, China.

H. Wang and A. Zheng are with the School of Economics and Management, Beihang University and the Beijing Key Laboratory of Emergency Support Simulation Technologies for City Operations, Beijing. H. Wang is the corresponding author.Email:wanghw@vip.sina.com

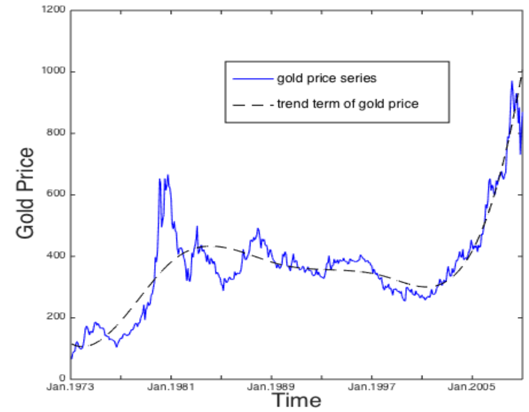

Fig. 1. Gold Price

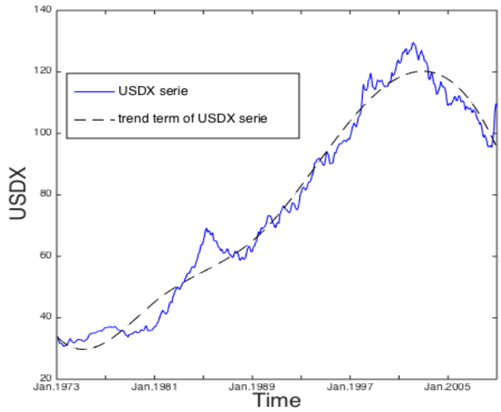

Fig. 2. US Dollar Price

gold price and US dollar index data between January 1973 and December 2008. These two figures also show the trend terms, which were obtained by order-4 B-spline fitting. From the graphical representation of the two series, we can observe that instead of being negatively related, these two trend terms both exhibit reasonably clear upward movement. Thus, these two series are non-stationary time series.

Based on this phenomenon and the fact that these two series are obviously non-stationary time series with both short and long periods, we came to suspect that the periodic components may better explain the negative correlation and that a spectrum analysis of the periodic terms could give us a clearer understanding because reconstructing the complete periodic terms, including the amplitude and phase, will help identify the extent to which gold and the dollar are negatively related. This suspicion led us to the key problem affecting the entire methodology, which is the accurate separation of trend terms and periodic terms, which can strongly affect the explanatory results. Thus, we conducted a series of studies resulting in some interesting findings. The contribution of the proposed method and this paper relates to the study of two non-stationary series–the gold price and the US dollar index–from a cyclical perspective to better explain their correlation. We believe these findings will also help investors make hedging, future investment and evaluation decisions.

The paper is structured as follows. Section 2 presents detailed information of the proposed methodology, Section 3 introduces the empirical studies and analyzes the results, and Section 5 concludes.

II. Methodology

Let Xt be a time series representing gold prices and the US dollar index. Because our methodology is based on the assumption that Xt is non-stationary, we assume it has the following form:

$$X_t = Z_t + \sum_{j=1}^{M} [\alpha_j \cos(\omega_j t) + \beta_j \sin(\omega_j t)] + \epsilon_t, \quad (1)$$

where Z_t denotes the trend term, ϵ_t denotes the random term, and the rest are periodic components.

As stated above, the major focus of this methodology is the accurate separation of the trend term Z_t and the periodic terms. To achieve this goal, we believe that the precise estimation of Z_t is key because it contains the most variance information, and thus, the estimation and extraction of Z_t will directly impact the analysis of periodic terms. This method should be able to suitably fit the movement of the gold price and USDX while containing the least periodic information possible. Our method mainly consists of fitting the trend components using non-parametric estimation and performing spectrum analysis of the periodic components. Individually, these two analysis methods are widely used and far from original. However, in this article, we combine them with an iteration algorithm, extracting one periodic term in each iteration to reduce its influence on the trend term during the fitting process. By using the iterative combination of order-4 B spline and spectrum analysis, we can accurately fit the trend movement while preserving complete periodic information.

Our proposed method consists of three major steps:
Step 1. Extraction of trend term

We use B-spline to estimate Z_t, and this method is non-parametric because the form of the model is typically unknown:

$$\hat{Z}_t = \sum_{j=-\nu}^{K} \delta_j N_{j,\nu+1}(t) = \mathbf{N}_{\nu+1}^T(t)\delta, \quad (2)$$

where $\delta = (\delta_{-\nu}, \cdots, \delta_K)^T$ is the parameter to be estimated and $\mathbf{N}_{\nu+1}(t) = \{N_{-\nu,\nu+1}(t), \cdots, N_{K,\nu+1}(t)\}$ is the $(\nu+1)$-order B-spline basis function, with knots of $t_0 = \tau_{-\nu} = \tau_{-\nu+1} = \cdots = \tau_0 < \cdots < \tau_K < \tau_{K+1} = \tau_{K+2} = \tau_{K+\nu+1} = t_T$, Therefore, we have the following:

$$\hat{\delta} = \arg\min \sum_{i=1}^{T} [X_i - \delta^T \mathbf{N}_{\nu+1}(t_i)]^2 + \lambda \int [(\delta^T \mathbf{N}_{\nu+1}(t))^{(2)}]^2 dt. \quad (3)$$

Here, λ is the penalty parameter, and $\int [(\delta^T \mathbf{N}_{\nu+1}(t))^{(2)}]^2 dt$ is the penalty term on the second derivative of the spline function, which can control the smoothness of the fitting and thus prevent the emergence of overfitting. The second derivative of the spline function can be described as follows:

$$(\delta^T \mathbf{N}_{\nu+1}(t))^{(2)} = \left(\sum_{j=-\nu}^{K} \delta_j N_{j,\nu+1}(t) \right)^{(2)} \quad (4)$$

$$= \sum_{j=-\nu+1}^{K} \delta^*_{j,i} N_{j,\nu+1-i}(t),$$

where $\delta^*_{j,i}$ is defined by

$$\delta^*_{j,i} = \frac{\delta^*_{j,i-1} - \delta^*_{j-1,i-1}}{\tau_{j+\nu+1-i} - \tau_j}(\nu + 1 - i), \quad i = 1, \cdots, K, \quad (5)$$

$$\delta^*_{j,0} = \delta_j.$$

Therefore, we have $\hat{Z}_t = \mathbf{N}^T_{\nu+1}(t)\hat{\delta}$.

Step 2. Spectral analysis of the residuals

If $(X_t - \hat{Z}_t)$ passes the stationary test, the residuals can be described as:

$$X_t - \hat{Z}_t = \sum_{j=1}^{M} [\alpha_j \cos(\omega_j t) + \beta_j \sin(\omega_j t)] + \epsilon_t. \quad (6)$$

Here, we apply the fast Fourier transformation to $(X_t - \hat{Z}_t)$ and obtain the spectrum of the residuals. Spectrum analysis has proven to be very useful in decomposing a time series to capture its periodic characteristics, and it has been widely applied in the natural sciences. However, a direct spectrum analysis of the original series is inappropriate because the trend term has the most covariance information, leaving the periodic components buried and difficult to estimate. Therefore, we choose to fit and extract the trend term before applying it. Subsequently, we extract and reconstruct one complete periodic term containing the most spectrum energy from the original series X_t, leaving $X_t - \alpha_1 \cos(\omega_1 t)$. Then, we denote it as the new X_t and repeat the steps beginning at step 1.

The reason that we combine these two methods with cyclical iteration is that each time steps 1 and 2 are repeated, we can exclude a major periodic term, thus reducing the periodic information contained in the trend term until the difference between the last two trend terms is minimized. Without the influence of periodic information, the trend term can be better fitted and, in turn, better estimate implicit cycles.

Step 3. Smoothing of the spectral density function

After extracting the trend term, we apply window spectrum analysis to the residuals, thereby reducing the frequency leakage phenomena and helping to obtain more accurate results without alternating the implicit cycles. The price series is discrete and so is its power spectral density. The spectrum frequency resolution depends on the sequence length, and limited frequency resolution inevitably leads to frequency leakage. Additionally, the leaked spectral components of different frequencies will superimpose on each other, resulting in a barrier effect. The leakage effect is strongly related to the side-lobes of the window function. If the heights of both side-lobes tend to zero, leaving the energy concentrated on the main-lobe, then we can obtain the real spectrum. In this step, we choose the Hamming window function to smooth the spectral density function, reduce energy leakage, and obtain more accurate periodic terms.

III. Data

We choose the monthly gold price data from the World Gold Council from January 1973 to December 2008, which are labeled as dollars per ounce, and monthly US dollar index data from the official website of the Federal Reserve over the same period. The US dollar index (USDX) reflects the changes of the US dollar in the international foreign exchange market. It is a weighted average of the foreign exchange value of the US dollar against the currencies of a large group of major US trading partners, and it is an important indicator of the US dollar's comprehensive strength. Therefore, in this paper, we study the gold price and USDX series to elucidate the correlation between gold and the US dollar according to 432 observations each, as graphically represented in Figures 1 and 2.

IV. Empirical Results

After decomposing the gold price and USDX series into trend terms and periodic terms based on our proposed methodology, we calculate the correlation coefficients of the two original series, two trend series and two periodic series: 0.3128, 0.4010 and -0.5719, respectively. These values roughly fit our hypothesis that the negative correlation between gold and USDX in fact derives from the periodic components. After characterizing the negative correlation, we further compare the complete implicit cycles of the two series, including their period length, amplitude and phase, to describe the extent to which they are negatively related.

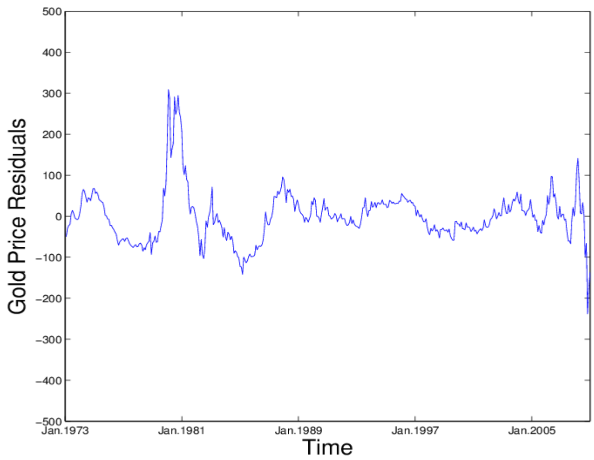

Fig. 3. Gold Price Residuals

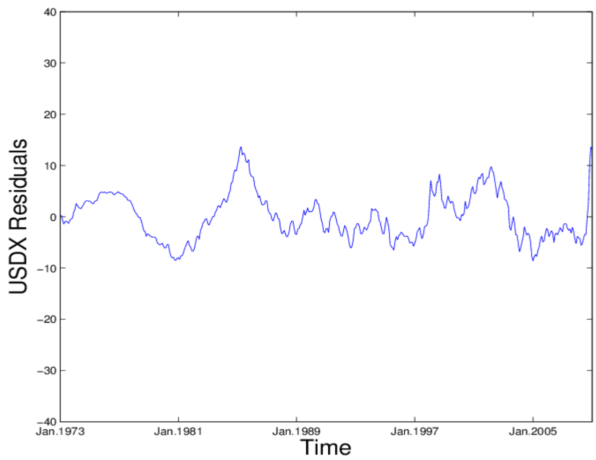

Fig. 4. USDX Residuals

As shown in Figures 1 and 2, the trend terms of both series are obtained by order-4 B-spline fitting. After extracting

TABLE I
COMPARISON OF THE GOLD PRICE AND USDX IMPLICIT CYCLES

	Gold Price			USDX		
	amplitude	period	phase	amplitude	period	phase
cycle 1	24.4329	108	1.1935	3.1105	108	-1.7015
cycle 2	24.5023	72	-2.3078	0.732	72	0.6331
cycle 3	13.972	43.2	-0.7292	0.365	43.2	-0.5216

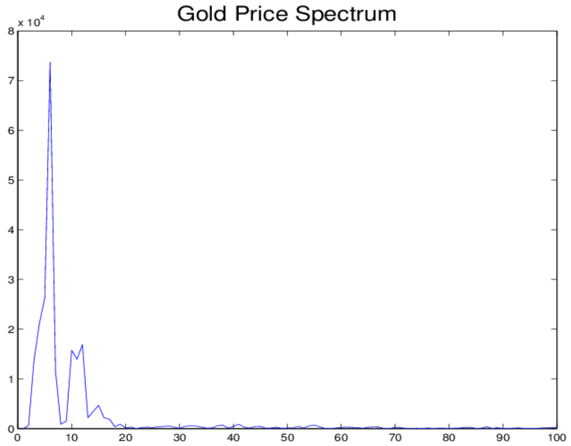

Fig. 5. Gold Price Spectrum

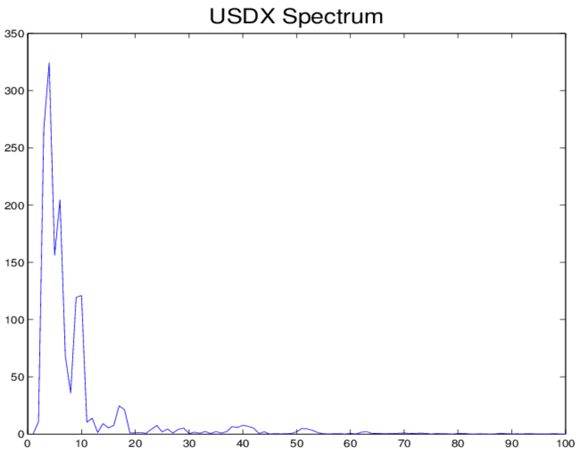

Fig. 6. USDX Spectrum

the final trend term from the original data, the residuals should be stationary time series, as shown in Figures 3 and 4. Next, we apply the Hamming window function to the residuals, calculate the spectrum density function using fast Fourier transformation, and observe the two series' periodic components. Both series show significant periodic characteristics, as seen in Figures 5 and 6 (we compress the X axis from 0-216 to 0-100 to present the data more clearly). We extract the three major implicit cycles containing the most spectrum energy and recreate both the amplitude and the phase of each periodic component. The results are shown in Table 1.

Table 1 shows that the gold price and US dollar index have surprisingly similar implicit cycles: both series can be decomposed into three major periodic terms, with two long cycles of 108 and 72 months and a relatively short cycle of 43.2 months. Furthermore, the phase difference of both long periods is approximately ± 2.90, which is very close to $\pm\pi$, indicating a negative correlation. In contrast, the phase difference of the short period is approximately ± 0.2, which is very close to 0, indicating a positive correlation. We also noted that compared to cycle 3, which is 43.2 months, the amplitudes of cycles 1 and 2 are relatively large, indicating that the first two cycles of gold and USDX, which are also negatively related, contain the most spectrum energy. This finding explains the overall negative correlation between gold and the US dollar.

Although gold no longer has any role in the monetary system, it still acts as an exchange rate hedge and serves as a protection against currency fluctuations. Additionally, it is always preferred by investors in response to global depression or severe market shocks over a period to minimize losses. Deep research into the correlation between gold and other macroeconomic factors can enable investors to diversify their portfolios and reduce the risk of heavy losses. Furthermore, the commonly accepted negative correlation between gold and the US dollar has led investors with dollar holdings to use gold as a hedge against exchange rate risks. We believe that our results more thoroughly explain the complicated long-term correlation between gold and the US dollar from a periodic perspective and serve as an indicator regarding whether to switch to or from gold as currencies weaken or strengthen. That is, although an increase in the price of gold has traditionally tended to be associated with a decrease in the value of the US dollar, this negative correlation does not hold for all implicit cycles shared by gold and the US dollar considering the positively related periodic term of 43.2 months. Furthermore, reconstructing the complete periodic terms can produce quantitative evidence, such as amplitude and phase. In general, this conclusion not only provides statistical confirmation of the hedging properties of gold but also reveals explanatory regular patterns of such hedging.

V. CONCLUSION

In this paper, we investigate the nature of the relationship between gold price and the US dollar from a periodic perspective using non-parametric estimation and spectrum analysis and investigate gold's role as an investment hedge against the dollar. We decompose the two time series into trend terms and periodic terms, use B-spline to estimate and extract the trend terms, and reconstruct the complete periodic terms after applying spectrum analysis. The gold price and US dollar index series share three major periodic terms containing the most spectrum energy: two long periods of 108 and 72 months and a short one of 43.2 months. These results indicate that the commonly acknowledged negative correlation between these two series is actually attributable to their periodic components because the two long periods are negatively related. Additionally, we reveal the existence of third relatively short period terms in both series, which are positively related and explain the extent to which gold and the US dollar are negatively correlated. This observation describes the nature of these two assets' relationship, and we hope that it will cast some new light on how to make hedging, future investment and evaluation decisions.

ACKNOWLEDGMENT

The authors would like to thank the anonymous referees for their helpful comments and valuable suggestions, which substantially improved the paper's content and composition.

REFERENCES

[1] E. Tully and B. M. Lucey, "A power garch examination of the gold market," *Research in International Business and Finance*, vol. 21, no. 2, pp. 316–325, 2007.

[2] M. Lineesh, K. Minu, and C. J. John, "Analysis of nonstationary nonlinear economic time series of gold price: A com-par-a-tive study," in *International— Mathematical Forum*, vol. 5, no. 34, 2010, pp. 1673–1683.

[3] J. H. Stock and M. W. Watson, "A comparison of linear and nonlinear univariate models for forecasting macroeconomic time series," National Bureau of Economic Research, Tech. Rep., 1998.

[4] L. Bauwens and M. Lubrano, "Bayesian option pricing using asymmetric garch models," *Journal of Empirical Finance*, vol. 9, no. 3, pp. 321–342, 2002.

[5] M.-H. Chiang and H.-Y. Huang, "Stock market momentum, business conditions, and garch option pricing models," *Journal of Empirical Finance*, vol. 18, no. 3, pp. 488–505, 2011.

[6] R. D. Brooks, R. W. Faff, M. D. McKenzie, and H. Mitchell, "A multi-country study of power arch models and national stock market returns," *Journal of International Money and Finance*, vol. 19, no. 3, pp. 377–397, 2000.

[7] R. Engle, "Garch 101: The use of arch/garch models in applied econometrics," *The Journal of Economic Perspectives*, vol. 15, no. 4, pp. 157–168, 2001.

[8] M. McKenzie and H. Mitchell, "Generalized asymmetric power arch modelling of exchange rate volatility," *Applied Financial Economics*, vol. 12, no. 8, pp. 555–564, 2002.

[9] Y. Yoon and G. Swales, "Predicting stock price performance: A neural network approach," in *System Sciences, 1991. Proceedings of the Twenty-Fourth Annual Hawaii International Conference on*, vol. 4. IEEE, 1991, pp. 156–162.

[10] G. Grudnitski and L. Osburn, "Forecasting s&p and gold futures prices: an application of neural networks," *Journal of Futures Markets*, vol. 13, no. 6, pp. 631–643, 1993.

[11] J.-H. Wang and J.-Y. Leu, "Stock market trend prediction using arima-based neural networks," in *Neural Networks, 1996., IEEE International Conference on*, vol. 4. IEEE, 1996, pp. 2160–2165.

[12] J. E. Kutsurelis, "Forecasting financial markets using neural networks: An analysis of methods and accuracy," DTIC Document, Tech. Rep., 1998.

[13] A. Parisi, F. Parisi, and D. Díaz, "Forecasting gold price changes: Rolling and recursive neural network models," *Journal of Multinational financial management*, vol. 18, no. 5, pp. 477–487, 2008.

[14] Z.-F. Guo and L. Cao, "An asymmetric smooth transition garch model," *IAENG International Journal of Applied Mathematics*, vol. 41, no. 4, pp. 349–351, 2011.

[15] S. P. Sidorov, A. Revutskiy, A. Faizliev, E. Korobov, and V. Balash, "Stock volatil-ity mod-elling with aug-mented garch model with jumps," *IAENG International Journal of Applied Mathematics*, vol. 44, no. 4, pp. 212–220, 2014.

[16] S. A. Baker, V. Tassel, and R. C, "Forecasting the price of gold: a fun-da-men-tal-ist approach," *Atlantic Economic Journal*, vol. 13, no. 4, pp. 43–51, 1985.

[17] Z. Ismail, A. Yahya, and A. Shabri, "Forecasting gold prices using multiple linear regression method," *American Journal of Applied Sciences*, vol. 6, no. 8, p. 1509, 2009.

[18] D. G. Baur and T. K. McDermott, "Is gold a safe haven? international evidence," *Journal of Banking & Finance*, vol. 34, no. 8, pp. 1886–1898, 2010.

[19] F. Capie, T. C. Mills, and G. Wood, "Gold as a hedge against the dollar," *Journal of International Financial Markets, Institutions and Money*, vol. 15, no. 4, pp. 343–352, 2005.

[20] M. Joy, "Gold and the us dollar: Hedge or haven?" *Finance Research Letters*, vol. 8, no. 3, pp. 120–131, 2011.

[21] M. Ivanova, K.and Ausloos, "Low-order variability diagrams for short-range correlation evidence in financial data: BGL-USD exchange rate, Dow jones industrial average, gold ounce price," *Physica A: Statistical Mechanics and its Applications*, vol. 265, no. 1, pp. 279–291, 1999.

[22] T. C. Mills, "Statistical analysis of daily gold price data," *Physica A: Statistical Mechanics and its Applications*, vol. 338, no. 3, pp. 559–566, 2004.

[23] Y. Wang, Y. Wei, and C. Wu, "Analysis of the efficiency and multifractality of gold markets based on multifractal detrended fluctuation analysis," *Physica A: Statistical Mechanics and its Applications*, vol. 390, no. 5, pp. 817–827, 2011.

[24] Y. Wang, C. Wu, and Z. Pan, "Multifractal detrending moving average analysis on the us dollar exchange rates," *Physica A: Statistical Mechanics and its Applications*, vol. 390, no. 20, pp. 3512–3523, 2011.

[25] Y. Long, "Visibility graph network analysis of gold price time series," *Physica A: Statistical Mechanics and its Applications*, vol. 392, no. 16, pp. 3374–3384, 2013.

[26] D. D. Thomakos, T. Wang, and L. T. Wille, "Modeling daily realized futures volatility with singular spectrum analysis," *Physica A: Statistical Mechanics and its Applications*, vol. 312, no. 3, pp. 505–519, 2002.

[27] S. C. Bae, T. H. Kwon, and M. Li, "Foreign exchange rate exposure and risk premium in international investments: Evidence from american depositary receipts," *Journal of Multinational Financial Management*, vol. 18, no. 2, pp. 165–179, 2008.