A Combined Iteration Algorithm for the Implicit Cycles of Gold Price and the US Dollar Index

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Abstract—The relationship between gold and US dollar series, which are non-stationary, is commonly known to be negative from a periodic perspective. Is this really the case? This paper established a combined iteration algorithm using the theory of spectral analysis after subtracting the trend using penalized B-spline functions to obtain the implicit cycles in gold and US dollar series. This algorithm accurately separates the trend terms and periodic terms of the two series to produce more precise and complete periodic information. The results show that both series share three common implicit cycles: two long periods and one short one. Both long-period terms are negatively correlated, whereas the short-period terms are positively correlated.

Index Terms—B-spline function, spectrum analysis, implicit cycles, gold price, US dollar index

I. INTRODUCTION

Gold is a financial asset that has always been a subject of interest and widely accepted as international currency. Currently, it is commonly used as an investing and hedging tool. Accurately measuring and forecasting future price trends are critical to mitigating risk and uncertainty when making hedging, future investment and evaluation decisions. Gold also attracts speculators seeking opportunities to engage in arbitrage. In quantitative finance, it is crucial to develop appropriate financial models based on the properties and mathematical descriptions of assets to determine their prices, develop investment portfolios, and manage risk.

Common models for predicting price series include the ARIMA and GARCH models. Neural network methodology has also been applied in this field. The asymmetric power GARCH model and likelihood ratio tests have been used to assess the significance of models [1]. Additionally, non-stationary and nonlinear economic time series of gold prices have been used to compare the GARCH model with a wavelet neural network model [2]. An auto-regression method was found to give the best overall performance in a comparison of 49 univariate forecasting methods [3]. The asymmetric GARCH method was used to improve the Bayesian option pricing model [4]. Other examples of studies examining financial time series using ARCH or GARCH models include [5], [6], [7] and [8]. Among studies applying neural network methods, most have used neural networks trained by features extracted from ARIMA analyses to model the behavior of time series, such as [9], [10], [11], [12], [13], [14] and [15].

Another widely discussed topic is the correlation between gold and other macroeconomic factors. Such research can offer investors the opportunity to obtain evidence of gold’s hedging ability and protect their wealth in the event of negative market conditions. Several factors can influence the gold market, including the oil price, inflation rate, stock market, and exchange rate. Among these factors, one very important index is the US dollar. Studies relating gold price and the dollar have a long history, and of all the methods, regression models are the most commonly used in all financial market areas. Baker, Tassel and Roger(1985) [16] described a regression model relating the gold price and the US dollar exchange rate, the interest rate, the inflation rate, and commodity prices, and until now, most studies have used univariate or multivariate regression models to determine the correlation between the gold market and many other macroeconomic factors. For example, see [17] and Baur and McDermott(2010) [18] who used regression models to show that gold is both a safe haven and a hedge in the US. Related studies investigating whether gold is a hedge, a diversifier or a safe haven also include [19], who assessed the role of gold as a hedge against the US dollar by estimating gold’s response elasticity to changes in the exchange rate. Additionally, a multivariate GARCH model of dynamic conditional correlations was used to demonstrate that gold has been a good hedge and a poor safe haven against the US dollar in the past and that, in recent years, gold has acted as an increasingly effective hedge against currency risk associated with the US dollar [20]. [1] also used the asymmetric power GARCH model to investigate macroeconomic influences on gold and found that the US dollar is the most significant. This model also revealed that the dollar’s influence is negative, confirming the long-held notion of gold as an ‘anti-dollar’. Statistical physics has also been applied to economic and financial series, such as gold and the US dollar. Examples include [21], who used a variability diagram technique to detect short-range correlations and decorrelations in financial data; the detrended fluctuation analysis performed by [22]; a multifractal detrended fluctuation analysis by [23] and [24]; a visibility graph network analysis reported by [25]; and a singular spectrum analysis conducted by [26].

Most of these analyses suggest that gold and the US dollar are very negatively correlated (see [20] and [27]). Because investors have traditionally used gold as a hedge against inflation, it seems reasonable to expect that the price of gold will tend to rise when the US dollar loses value, preserving gold’s real value. However, this conclusion is not necessarily true over the long term. Figures 1 and 2 present monthly...
We believe these findings will also help investors make non-stationary series—the gold price and the US dollar index—proposed method and this paper relates to the study of two resulting in some interesting findings. The contribution of the explanatory results. Thus, we conducted a series of studies trend terms and periodic terms, which can strongly affect the entire methodology, which is the accurate separation of related. This suspicion led us to the key problem affecting identify the extent to which gold and the dollar are negatively periodic terms, including the amplitude and phase, will help a clearer understanding because reconstructing the complete components may better explain the negative correlation and short and long periods, we came to suspect that the periodic terms, which were obtained by order-4 B-spline fitting. From a spectrum analysis of the periodic terms could give us that instead of being negatively related, these two trend terms both exhibit reasonably clear upward movement. Thus, these two series are non-stationary time series.

Based on this phenomenon and the fact that these two series are obviously non-stationary time series with both short and long periods, we came to suspect that the periodic components may better explain the negative correlation and that a spectrum analysis of the periodic terms could give us a clearer understanding because reconstructing the complete periodic terms, including the amplitude and phase, will help identify the extent to which gold and the dollar are negatively related. This suspicion led us to the key problem affecting the entire methodology, which is the accurate separation of trend terms and periodic terms, which can strongly affect the explanatory results. Thus, we conducted a series of studies resulting in some interesting findings. The contribution of the proposed method and this paper relates to the study of two non-stationary series—the gold price and the US dollar index—from a cyclical perspective to better explain their correlation. We believe these findings will also help investors make hedging, future investment and evaluation decisions.

The paper is structured as follows. Section 2 presents detailed information of the proposed methodology, Section 3 introduces the empirical studies and analyzes the results, and Section 5 concludes.

II. METHODOLOGY

Let $X_t$ be a time series representing gold prices and the US dollar index. Because our methodology is based on the assumption that $X_t$ is non-stationary, we assume it has the following form:

$$
X_t = Z_t + \sum_{j=1}^{M} [\alpha_j \cos(\omega_j t) + \beta_j \sin(\omega_j t)] + \epsilon_t,
$$

where $Z_t$ denotes the trend term, $\epsilon_t$ denotes the random term, and the rest are periodic components.

As stated above, the major focus of this methodology is the accurate separation of the trend term $Z_t$ and the periodic terms. To achieve this goal, we believe that the precise estimation of $Z_t$ is key because it contains the most variance information, and thus, the estimation and extraction of $Z_t$ will directly impact the analysis of periodic terms. This method should be able to suitably fit the movement of the gold price and USDX while containing the least periodic information possible. Our method mainly consists of fitting the trend components using non-parametric estimation and performing spectrum analysis of the periodic components. Individually, these two analysis methods are widely used and far from original. However, in this article, we combine them with an iteration algorithm, extracting one periodic term in each iteration to reduce its influence on the trend term during the fitting process. By using the iterative combination of order-4 B spline and spectrum analysis, we can accurately fit the trend movement while preserving complete periodic information.

Our proposed method consists of three major steps:

**Step 1. Extraction of trend term**

We use B-spline to estimate $Z_t$, and this method is non-parametric because the form of the model is typically unknown:

$$
\hat{Z}_t = \sum_{j=-\nu}^{K} \delta_j N_{j,\nu+1}(t) = N_{\nu+1}^T(t)\delta,
$$

where $\delta = (\delta_{-\nu}, \ldots, \delta_K)^T$ is the parameter to be estimated and $N_{\nu+1}(t) = \{N_{-\nu,\nu+1}(t), \ldots, N_{K,\nu+1}(t)\}$ is the $(\nu+1)$-order B-spline basis function, with knots of $t_0 = \tau_{-\nu} = \tau_{-\nu+1} = \cdots = \tau_0 < \cdots < \tau_K < \tau_{K+1} = \tau_{K+2} = t_T$. Therefore, we have the following:

$$
\hat{\delta} = \arg \min \sum_{t=1}^{T} [X_t - \delta^T N_{\nu+1}(t_i)]^2 + \lambda \int [\delta^T N_{\nu+1}(t)]^{(2)}^2 dt.
$$

Here, $\lambda$ is the penalty parameter, and $\int [\delta^T N_{\nu+1}(t)]^{(2)}^2 dt$ is the penalty term on the second derivative of the spline function, which can control the smoothness of the fitting and thus prevent the emergence of overfitting. The second derivative of the spline function can be described as follows:

$$
(\delta^T N_{\nu+1}(t))^{(2)} = \left(\sum_{j=-\nu}^{K} \delta_j N_{j,\nu+1}(t)\right)^{(2)}
$$
Step 3. Smoothing of the spectral density function

Without the influence of periodic information, the trend term difference between the last two trend terms is minimized. We can exclude a major periodic term, thus reducing the cyclical iteration is that each time steps 1 and 2 are repeated, at step 1.

\[X_t = \hat{Z}_t\]

We denote it as the new index data from the official website of the Federal Reserve are labeled as dollars per ounce, and monthly US dollar index data from the World Gold Council from January 1973 to December 2008, which more accurate periodic terms. After extracting the trend term, we apply window spectrum analysis to the residuals, thereby reducing the frequency leakage. Additionally, the leaked spectral components of different frequencies will superimpose on each other, resulting in a barrier effect. The leakage effect is strongly related to the side-lobes of the window function. If the heights of both side-lobes tend to zero, leaving the energy concentrated on the main-lobe, then we can obtain the real spectrum. In this step, we choose the Hamming window function to smooth the spectral density function, reduce energy leakage, and obtain more accurate periodic terms.

Step 2. Spectral analysis of the residuals

If \((X_t - \hat{Z}_t)\) passes the stationary test, the residuals can be described as:

\[X_t - \hat{Z}_t = \sum_{j=1}^{M} \left[ \alpha_j \cos(\omega_j t) + \beta_j \sin(\omega_j t) \right] + \epsilon_t. \tag{6}\]

Here, we apply the fast Fourier transformation to \((X_t - \hat{Z}_t)\) and obtain the spectrum of the residuals. Spectrum analysis has proven to be very useful in decomposing a time series to capture its periodic characteristics, and it has been widely applied in the natural sciences. However, a direct spectrum analysis of the original series is inappropriate because the trend term has the most covariance information, leaving the periodic components buried and difficult to estimate. Therefore, we choose to fit and extract the trend term before applying it. Subsequently, we extract and reconstruct one complete periodic term containing the most spectrum energy from the original series \(X_t\), leaving \(X_t - \alpha_1 \cos(\omega_1 t)\). Then, we denote it as the new \(X_t\) and repeat the steps beginning at step 1.

The reason that we combine these two methods with cyclical iteration is that each time steps 1 and 2 are repeated, we can exclude a major periodic term, thus reducing the periodic information contained in the trend term until the difference between the last two trend terms is minimized. Without the influence of periodic information, the trend term can be better fitted and, in turn, better estimate implicit cycles.

Step 3. Smoothing of the spectral density function

After extracting the trend term, we apply window spectrum analysis to the residuals, thereby reducing the frequency leakage phenomena and helping to obtain more accurate results without alternating the implicit cycles. The price series is discrete and so is its power spectral density. The spectrum frequency resolution depends on the sequence length, and limited frequency resolution inevitably leads to frequency leakage. Additionally, the leaked spectral components of different frequencies will superimpose on each other, resulting in a barrier effect. The leakage effect is strongly related to the side-lobes of the window function. If the heights of both side-lobes tend to zero, leaving the energy concentrated on the main-lobe, then we can obtain the real spectrum. In this step, we choose the Hamming window function to smooth the spectral density function, reduce energy leakage, and obtain more accurate periodic terms.

III. DATA

We choose the monthly gold price data from the World Gold Council from January 1973 to December 2008, which are labeled as dollars per ounce, and monthly US dollar index data from the official website of the Federal Reserve over the same period. The US dollar index (USDX) reflects the changes of the US dollar in the international foreign exchange market. It is a weighted average of the foreign exchange value of the US dollar against the currencies of a large group of major US trading partners, and it is an important indicator of the US dollar’s comprehensive strength. Therefore, in this paper, we study the gold price and USDX series to elucidate the correlation between gold and the US dollar according to 432 observations each, as graphically represented in Figures 1 and 2.

IV. EMPIRICAL RESULTS

After decomposing the gold price and USDX series into trend terms and periodic terms based on our proposed methodology, we calculate the correlation coefficients of the two original series, two trend series and two periodic series: 0.3128, 0.4010 and -0.5719, respectively. These values roughly fit our hypothesis that the negative correlation between gold and USDX in fact derives from the periodic components. After characterizing the negative correlation, we further compare the complete implicit cycles of the two series, including their period length, amplitude and phase, to describe the extent to which they are negatively related.

As shown in Figures 1 and 2, the trend terms of both series are obtained by order-4 B-spline fitting. After extracting
TABLE I
COMPARISON OF THE GOLD PRICE AND USDX IMPLICIT CYCLES

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<th>Gold Price</th>
<th>USDX</th>
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<td></td>
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<td>0.6331</td>
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<td>13.972</td>
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<tr>
<td></td>
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</tr>
</tbody>
</table>

Fig. 5. Gold Price Spectrum

Fig. 6. USDX Spectrum

In this paper, we investigate the relationship between gold price and the US dollar from a periodic perspective using non-parametric estimation and spectrum analysis to investigate gold’s role as an investment hedge against the dollar. We decompose the two time series into trend terms and periodic terms, use B-spline to estimate and extract the trend terms, and reconstruct the complete periodic terms after applying spectrum analysis. The gold price and US dollar index series share three major periodic terms containing the most spectrum energy: two long periods of 108 and 72 months and a short one of 43.2 months. These results indicate that the commonly acknowledged negative correlation between these two series is actually attributable to their periodic components because the two long periods are negatively related. Additionally, we reveal the existence of long periods is approximately \( \pm 2.90 \), which is very close to \( \pm \pi \), indicating a negative correlation. In contrast, the phase difference of the short period is approximately \( \pm 0.2 \), which is very close to 0, indicating a positive correlation. We also noted that compared to cycle 1 and cycle 3, which is 43.2 months, the amplitudes of cycles 1 and 2 are relatively large, indicating that the first two cycles of gold and USDX, which are also negatively related, contain the most spectrum energy. This finding explains the overall negative correlation between gold and the US dollar.

Although gold no longer has any role in the monetary system, it still acts as an exchange rate hedge and serves as a protection against currency fluctuations. Additionally, it is always preferred by investors in response to global depression or severe market shocks over a period to minimize losses. Deep research into the correlation between gold and other macroeconomic factors can enable investors to diversify their portfolios and reduce the risk of heavy losses. Furthermore, the commonly accepted negative correlation between gold and the US dollar has led investors with dollar holdings to use gold as a hedge against exchange rate risks. We believe that our results more thoroughly explain the complicated long-term correlation between gold and the US dollar from a periodic perspective and serve as an indicator regarding whether to switch to or from gold as currencies weaken or strengthen. That is, although an increase in the price of gold has traditionally tended to be associated with a decrease in the value of the US dollar, this negative correlation does not hold for all implicit cycles shared by gold and the US dollar considering the positively related periodic term of 43.2 months. Furthermore, reconstructing the complete periodic terms can produce quantitative evidence, such as amplitude and phase. In general, this conclusion not only provides statistical confirmation of the hedging properties of gold but also reveals explanatory regular patterns of such hedging.

V. CONCLUSION

In this paper, we investigate the nature of the relationship between gold price and the US dollar from a periodic perspective using non-parametric estimation and spectrum analysis and investigate gold’s role as an investment hedge against the dollar. We decompose the two time series into trend terms and periodic terms, use B-spline to estimate and extract the trend terms, and reconstruct the complete periodic terms after applying spectrum analysis. The gold price and US dollar index series share three major periodic terms containing the most spectrum energy: two long periods of 108 and 72 months and a short one of 43.2 months. These results indicate that the commonly acknowledged negative correlation between these two series is actually attributable to their periodic components because the two long periods are negatively related. Additionally, we reveal the existence of
of third relatively short period terms in both series, which are positively related and explain the extent to which gold and the US dollar are negatively correlated. This observation describes the nature of these two assets’ relationship, and we hope that it will cast some new light on how to make hedging, future investment and evaluation decisions.

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REFERENCES


