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Dynamical Analysis of Fuzzy Cellular Neural Networks with Periodic Coefficients and Time-varying Delays

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Abstract—**By employing continuation theorem of the coincidence degree, and inequality technique, some sufficient conditions are established for the existence of periodic solution and global exponential stability of fuzzy cellular neural networks with time-varying delays. These results have important leading significance in the design and applications of globally stable neural networks. Moreover an example is given to illustrate the effectiveness and feasible of results obtained.**

Index Terms—**fuzzy cellular neural networks, periodic solution, global exponential stability, time-varying delays.**

I. Introduction

CELLULAR neural network is formed by many units named cells, and that cell contains linear and nonlinear circuit elements, which typically are linear capacitor, linear resistor, linear and nonlinear controlled source, and independent sources. Nowadays, cellular neural networks(CNNs) are widely used in signal and image processing, associative memories, pattern classification ([1], [2], [3], [4], [5], [6]). In the last decade, dynamic behaviors of CNNs have been intensively studied because of the successful hardware implementation and their widely application (see, for example, [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25]).

In recent years, many researchers pay much attention to dynamical behavior and application of fuzzy neural network (see [27], [28]). In this paper, we would like to integrate fuzzy operations into cellular neural networks. Speaking of fuzzy operations, Yang and Yang [29] first introduced fuzzy cellular neural networks (FCNNs) combining those operations with cellular neural networks. So far researchers have founded that FCNNs are useful in image processing, and some results have been reported on stability and periodicity of FCNNs ([29], [30], [31], [32], [33], [34], [35], [36], [37], [38], [39]). However, to the best of our knowledge, few author investigated the stability of fuzzy cellular neural networks with time-varying delays.

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This paper is concerned with the existence, and the global exponential stability of periodic solution for the following fuzzy cellular neural networks

$$\begin{aligned} x_i'(t) &= -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) \\ &\quad + \bigwedge_{j=1}^{n} \alpha_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) \\ &\quad + \bigvee_{j=1}^{n} \beta_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) + I_i(t) \qquad (1) \\ &\quad + \bigwedge_{j=1}^{m} T_{ij}(t)u_j(t) + \bigvee_{j=1}^{n} H_{ij}(t)u_j(t), \end{aligned}$$

$i = 1, 2, \cdots, n$, where n corresponds to the number of neurons in neural networks. $x_i(t)$ is the activations of the ith neuron at time t. $c_i(t)$ denotes the rate with which the ith neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operations. $a_{ij}(t)$ denotes the strengths of connectivity between cell i and cell j at time t. $\alpha_{ij}(t), \beta_{ij}(t), T_{ij}(t)$ and $H_{ij}(t)$ are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, fuzzy feed-forward MIN template and fuzzy feed-forward MAX template between cell i and j at time t. $\tau_{ij}(t)$ corresponds to the time delay required in processing and transmitting a signal from the jth cell to the ith cell at time t. $u_j(t)$ and $I_i(t)$ denote the external input, bias of the ith neurons at time t, respectively. $f_j(\cdot)$ is signal transmission functions.

Throughout the paper, we give the following assumptions
(A1) $|f_j(x)| \le p_j|x| + q_j$ for all $x \in R, j = 1, 2, \cdots, n$, where p_j, q_j are nonnegative constants.
(A2) The signal transmission functions $f_j(\cdot)(j = 1, 2, \cdots, n)$ are Lipschtiz continuous on R with Lipschtiz constants p_j, namely, for any $x, y \in R$

$$|f_j(x) - f_j(y)| \le p_j|x - y|, \quad f_j(0) = g_i(0) = 0.$$

Definition 1.1 *If $f(t) : R \to R$ is a continuous function, then the upper right derivative of $f(t)$ is defined as*

$$D^+f(t) = \lim_{h\to 0^+} \sup \frac{1}{h}(f(t+h) - f(t)).$$

Let $\tau = \max_{1\le i,j\le n} \sup_{t\ge 0}\{\tau_{ij}(t)\}$. For continuous functions $\varphi_i(i = 1, 2, \cdots, n)$ defined on $[-\tau, 0]$, we set $\Psi = (\varphi_1, \varphi_2, \cdots, \varphi_n)^T$. If $\overline{x}(t) = (\overline{x}_1(t), \overline{x}_2(t), \cdots, \overline{x}_n(t))^T$ is an $\omega-$periodic solution of system (1), then we denote

$$\|\Psi-\overline{x}\|=\sum_{i=1}^{n}(\sup_{-\tau\le t\le 0}|\varphi_i(t)-\overline{x}_i(t)|).$$

Assume that system (1) is supplemented with initial value of type

$$x_i(t)=\varphi_i(t),\quad -\tau\le t\le 0.$$

Definition 1.2 *The periodic solution* $\overline{x}(t)=(\overline{x}_1(t),\overline{x}_2(t),\cdots,\overline{x}_n(t))^T$ *is said to be globally exponentially stable. If there exist constants* $\lambda>0$ *and* $M\ge 1$ *such that for any solution* $x(t)=(x_1(t),x_2(t),\cdots,x_n(t))^T$ *of system (1)*

$$|x_i(t)-\overline{x}_i(t)|\le M\|\Psi-\overline{x}\|e^{-\lambda t},\quad t\ge 0. \tag{2}$$

Lemma 1.1(see [26]) *If* $\rho(K)<1$ *for matrix* $K=(k_{ij})_{n\times n}\ge 0$*, then* $(E-K)^{-1}\ge 0$*, where* E *denotes the identity matrix of size* n.

Lemma 1.2(see [29]) *Suppose* x *and* y *are two states of system (1), then we have*

$$\begin{aligned}\left|\bigwedge_{j=1}^{n}\alpha_{ij}(t)f_j(x)\right.&-\left.\bigwedge_{j=1}^{n}\alpha_{ij}(t)f_j(y)\right|\\ &\le\sum_{j=1}^{n}|\alpha_{ij}(t)||f_j(x)-f_j(y)|,\end{aligned}$$

and

$$\begin{aligned}\left|\bigvee_{j=1}^{n}\beta_{ij}(t)f_j(x)\right.&-\left.\bigvee_{j=1}^{n}\beta_{ij}(t)f_j(y)\right|\\ &\le\sum_{j=1}^{n}|\beta_{ij}(t)||f_j(x)-f_j(y)|.\end{aligned}$$

The remainder of this paper is organized as follows. In Section 2, we will give the sufficient conditions to ensure the existence of periodic oscillatory solution for fuzzy cellular neural networks with time-varying delays, and show that all other solutions converge exponentially to it as $n\to\infty$. In Section 3, an example will be given to illustrate effectiveness of results obtained. We will give a general conclusion in Section 4.

II. Periodic oscillatory solutions

In this section, we will consider the existence and global stability of periodic oscillatory solutions of system (1) with $\tau_{ij}(t),c_i(t),a_{ij}(t),\ \alpha_{ij}(t),\beta_{ij}(t),T_{ij}(t),H_{ij}(t),u_j(t)$ and $I_i(t)$ satisfying the following assumptions:

(A3) $\tau_{ij}(\cdot)\in C(R,[0,\infty))$ are periodic with common period ω for $i,j=1,2,\cdots,n$.
(A4) $c_i(\cdot)\in C(R,(0,\infty)),a_{ij}(\cdot),\alpha_{ij}(\cdot),\beta_{ij}(\cdot),T_{ij}(\cdot),H_{ij}(\cdot),u_j(\cdot),I_i(\cdot)\in C(R,R)$ are periodic with common periodic ω, and $f_i(\cdot)\in C(R,R),i,j=1,2,\cdots,n$.

We will use the coincidence degree theory to obtain the existence of an ω-periodic solution to system (1). For the sake of convenience, we briefly summarize the theory as below.

Let X and Z be normed spaces, $L:\mathrm{Dom}L\subset X\mapsto Z$ be a linear mapping and $N:X\mapsto Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim\mathrm{Ker}L=\mathrm{codim}\ \mathrm{Im}L<\infty$ and $\mathrm{Im}L$ is closed in Z. If L is a Fredholm mapping of index zero, then there exist continuous projectors $P:X\mapsto X$ and $Q:Z\mapsto Z$ such that $\mathrm{Im}P=\mathrm{Ker}L$ and $\mathrm{Im}L=\mathrm{Ker}Q=\mathrm{Im}(I-Q)$. It follows that $L|\mathrm{Dom}L\cap\mathrm{Ker}P:(I-P)X\mapsto\mathrm{Im}L$ is invertible. We denote the inverse of this map by K_p. If Ω is a bounded open subset of X, the mapping N is called L-compact on $\overline{\Omega}$. if $QN(\overline{\Omega})$ is bounded and $\mathrm{K}_p(I-Q)N:\overline{\Omega}\mapsto X$ is compact. Because $\mathrm{Im}Q$ is isomorphic to $\mathrm{Ker}L$, there exists an isomorphism $J:\mathrm{Im}Q\mapsto\mathrm{Ker}L$.

Let $\Omega\subset R^n$ be open and bounded, $f\in C^1(\Omega,R^n)\bigcap C(\overline{\Omega},R^n)$ and $y\in R^n\backslash f(\partial\Omega\bigcup S_f)$, i.e., y is a regular value of f. Here, $S_f=\{x\in\Omega:J_f(x)=0\}$, the critical set of f , and J_f is the Jacobian of f at x. Then the degree ($\deg\{f,\Omega,y\}$) is defined by

$$\deg\{f,\Omega,y\}=\sum_{x\in f^{-1}(y)}\mathrm{sgn}J_f(x)$$

with the agreement that the above sum is zero if $f^{-1}(y)=\Phi$.

Lemma 2.1 *Let* L *be a Fredholm mapping of index zero and let* N *be* L*-compact on* $\overline{\Omega}$*. Suppose that*
(a) for each $\lambda\in(0,1)$*, every solution* x *of* $Lx=\lambda Nx$ *is such that* $x\notin\partial\Omega$*.*
(b) $QNx\neq 0$ *for each* $x\in\partial\Omega\bigcap KerL$ *and*

$$deg\{JQN,\Omega\bigcap KerL,0\}\neq 0.$$

Then the equation $Lx=Nx$ *has at least one solution lying in* $DomL\bigcap\overline{\Omega}$*.*

To be convenience, in the rest of paper, for a continuous function $g:[0,\omega]\mapsto R$, we denote

$$g^+=\max_{t\in[0,\omega]}g(t),\ g_-=\min_{t\in[0,\omega]}g(t),\ \ \bar g=\frac{1}{\omega}\int_0^\omega g(t)dt.$$

Theorem 2.1 *Under assumptions* $(A1),(A3)$ *and* $(A4)$*,* $k_{ij}=\left(\frac{1}{\bar c}+\omega\right)(|\overline{a_{ij}}|+|\overline{\alpha_{ij}}|+|\overline{\beta_{ij}}|)p_j,\ K=(k_{ij})_{n\times n}$*. Suppose that* $\rho(K)<1$*, then system (1) has at least an* ω*-periodic solution.*

Proof. Take $X=Z=\{x(t)=(x_1(t),x_2(t),\cdots,x_n(t))^T\in C(R,R^n):x(t+\omega)=x(t),t\in R\}$ and denote $\|x\|=\max_{1\le i\le n}\max_{t\in[0,\omega]}|x_i(t)|$. Equipped with the norm $\|\cdot\|$, both X and Z are Banach space. For any $x(t)\in X$, it is easy to check that

$$\begin{aligned}\Theta(x_i,t)\ \ :=\ \ &-c_i(t)x_i(t)+\sum_{j=1}^{n}a_{ij}(t)f_j(x_j(t))\\ &+\bigwedge_{j=1}^{n}\alpha_{ij}(t)f_j(x_j(t-\tau_{ij}(t)))\\ &+\bigvee_{j=1}^{n}\beta_{ij}(t)f_j(x_j(t-\tau_{ij}(t)))+I_i(t)\\ &+\bigwedge_{j=1}^{m}T_{ij}(t)u_j(t)+\bigvee_{j=1}^{n}H_{ij}(t)u_j(t)\in Z.\end{aligned}$$

Let

$$L:\mathrm{Dom\ L}=\{x\in X:x\in C(R,R^n)\}\ni x\mapsto\dot x(\cdot)\in Z.$$

$$P : X \ni x \mapsto \frac{1}{\omega}\int_0^{\omega} x(t)dt \in X,$$

$$Q : Z \ni z \mapsto \frac{1}{\omega}\int_0^{\omega} z(t)dt \in Z,$$

$$N : X \ni x \mapsto \Theta(x, \cdot) \in Z.$$

For any $V = (v_1, v_2, \cdots, v_n) \in R^n$, we identify it as the constant function in X or Z with the value vector $V = (v_1, v_2, \cdots, v_n)$. Then system (1) can be reduced to operator equation $Lx = Nx$. It is easy to see that $\mathrm{Ker}L = R^n$, $\mathrm{Im}L = \{z \in Z : \frac{1}{\omega}\int_0^{\omega} z(t)dt = 0\}$. Which is closed in Z, $\dim\mathrm{Ker}L = \mathrm{codim}\mathrm{Im}L = n < \infty$, and P, Q are continuous projectors such that $\mathrm{Im}P = \mathrm{Ker}L, \mathrm{Ker}Q = \mathrm{Im}L = \mathrm{Im}(I - Q)$. It follows that L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) $K_p : \mathrm{Im}L \mapsto \mathrm{Ker}P \cap \mathrm{Dom}L$ given by

$$(K_p(z))_i(t) = \int_0^t z_i(s)ds - \frac{1}{\omega}\int_0^{\omega}\int_0^s z_i(v)dvds.$$

Therefore, applying the Arzela-Ascoli theorem, one can easily show that N is L-compact on $\bar{\Omega}$ with any bounded open subset $\Omega \subset X$. Since $\mathrm{Im}Q = \mathrm{Ker}L$, we take the isomorphism J of $\mathrm{Im}Q$ onto $\mathrm{Ker}L$ to be the identity mapping.

Now we need only to show that, for an appropriate open bounded subset Ω, application of the continuation theorem corresponding to the operator equation $Lx = \lambda Nx, \lambda \in (0, 1)$, Let

$$\dot{x}_i(t) = \lambda\Theta(x_i, t) \quad i = 1, 2, \cdots, n. \tag{3}$$

Assume that $x = x(t) \in X$ is a solution of system (3) for some $\lambda \in (0, 1)$. Integrating (3) over the interval $[0, \omega]$, we obtain that

$$0 = \int_0^{\omega} \dot{x}_i(t)dt = \lambda\int_0^{\omega} \Theta(x_i, t)dt. \tag{4}$$

Hence

$$\begin{aligned} \int_0^{\omega} & c_i(t)x_i(t)dt \\ = & \int_0^{\omega}\Bigg\{\sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) \\ & + \bigwedge_{j=1}^{n} \alpha_{ij}(t)f_j(x_j(t - \tau_{ij}(t))) \\ & + \bigwedge_{j=1}^{m} T_{ij}(t)u_j(t) + \bigvee_{j=1}^{n} \beta_{ij}(t)f_j(x_j(t - \tau_{ij}(t))) \\ & + \bigvee_{j=1}^{n} H_{ij}(t)u_j(t) + I_i(t)\Bigg\} dt. \end{aligned} \tag{5}$$

Noting assumption $(A1)$, we get

$$\begin{aligned} |x_i|_{-}\overline{c_i} \leq & \sum_{j=1}^{n}(|\overline{a_{ij}}| + |\overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)p_j|x_j|^{+} \\ & + \sum_{j=1}^{n}(|\overline{a_{ij}}| + \overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)q_j + |\overline{I_i}| \\ & + \bigwedge_{j=1}^{n} |\overline{T_{ij}}||u_j|^{+} + \bigvee_{j=1}^{n} |\overline{H_{ij}}||u_j|^{+}. \end{aligned} \tag{6}$$

It follows that

$$\begin{aligned} |x_i|_{-} \leq & \frac{1}{\overline{c_i}}\sum_{j=1}^{n}(|\overline{a_{ij}}| + |\overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)p_j|x_j|^{+} \\ & + \frac{1}{\overline{c_i}}\left\{\sum_{j=1}^{n}(|\overline{a_{ij}}| + \overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)q_j\right\} \\ & + \frac{1}{\overline{c_i}}\left\{\bigwedge_{j=1}^{n} |\overline{T_{ij}}||u_j|^{+} + \bigvee_{j=1}^{n} |\overline{H_{ij}}||u_j|^{+} + |\overline{I_i}|\right\}. \end{aligned} \tag{7}$$

Note that each $x_i(t)$ is continuously differentiable for $i = 1, 2, \cdots, n$, it is certain that there exists $t_i \in [0, \omega]$ such that $|x_i(t_i)| = |x_i(t)|_{-}$. Set $F = (F_1, F_2, \cdots, F_n)^T$, where

$$\begin{aligned} F_i = & (\frac{1}{\overline{c_i}} + \omega)\Bigg\{\sum_{j=1}^{n}(|\overline{a_{ij}}| + \overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)q_j \\ & + \bigwedge_{j=1}^{n} |\overline{T_{ij}}||u_j|^{+} + \bigvee_{j=1}^{n} |\overline{H_{ij}}||u_j|^{+} + |\overline{I_i}|\Bigg\}. \end{aligned} \tag{8}$$

In view of $\rho(K) < 1$ and Lemma 1.1, we have $(E - K)^{-1}F = h = (h_1, h_2, \cdots, h_n)^T \geq 0$, where h_i is given by

$$h_i = \sum_{j=1}^{n} k_{ij}h_j + F_i, \ i = 1, 2, \cdots, n. \tag{9}$$

Set

$$\Omega = \{(x_1, x_2, \cdots, x_n)^T \in R^n : |x_i| < h_i, i = 1, 2, \cdots, n\}. \tag{10}$$

Then, for $t \in [t_i, t_i + \omega]$, we have

$$\begin{aligned} |x_i(t)| \leq & |x_i(t_i)| + \int_{t_i}^{t} D^{+}|x_i(t)|dt \\ \leq & |x_i(t)|_{-} + \int_{t_i}^{t+\omega} D^{+}|x_i(t)|dt \\ \leq & \frac{1}{\overline{c_i}}\sum_{j=1}^{n}(|\overline{a_{ij}}| + |\overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)p_j|x_j|^{+} \\ & + \frac{1}{\overline{c_i}}\Bigg\{\sum_{j=1}^{n}(|\overline{a_{ij}}| + \overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)q_j \\ & + \bigwedge_{j=1}^{n} |\overline{T_{ij}}||u_j|^{+} + \bigvee_{j=1}^{n} |\overline{H_{ij}}||u_j|^{+} + |\overline{I_i}|\Bigg\} \\ & + \int_{t_i}^{t+\omega} D^{+}|x_i(t)|dt \\ \leq & (\frac{1}{\overline{c_i}} + \omega)\left\{\sum_{j=1}^{n}(|\overline{a_{ij}}| + |\overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)p_j|x_j|^{+}\right\} \\ & + (\frac{1}{\overline{c_i}} + \omega)\Bigg\{\sum_{j=1}^{n}(|\overline{a_{ij}}| + \overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)q_j \\ & + \bigwedge_{j=1}^{n} |\overline{T_{ij}}||u_j|^{+} + \bigvee_{j=1}^{n} |\overline{H_{ij}}||u_j|^{+} + |\overline{I_i}|\Bigg\} \\ \leq & \sum_{j=1}^{n} k_{ij}h_j + F_i = h_i. \end{aligned} \tag{11}$$

Clearly, $h_i, i = 1, 2, \cdots, n$, are independent of λ. Then for $\forall\lambda \in (0,1), x \in \partial\Omega$ such that $Lx \neq \lambda Nx$. When $u = (x_1, x_2, \cdots, x_n)^T \in \partial\Omega \cap \mathrm{Ker}L = \partial\Omega \cap R^n$, u is a constant vector with $|x_i| = h_i, i = 1, 2, \cdots, n$. Note that $QNu = JQNu$, when $u \in \mathrm{Ker}L$, it must be

$$\begin{aligned}(QN)u_i &= -\overline{c_i}x_i + \sum_{j=1}^{n}(\overline{a_{ij}} + \overline{\alpha_{ij}} + \overline{\beta_{ij}})f(x_j) \\ &\quad + \bigwedge_{j=1}^{n} \overline{T_{ij}}\overline{u_j} + \bigvee_{j=1}^{n} \overline{H_{ij}}\overline{u_j} + \overline{I_i}. \qquad (12)\end{aligned}$$

We claim that

$$\|(QNu)_i\| > 0, \ i = 1, 2, \cdots, n. \qquad (13)$$

On the contrary, suppose that there are some i such that$\|(QNu)_i\| = 0$, namely

$$\overline{c_i}x_i = \sum_{j=1}^{n}(\overline{a_{ij}}+\overline{\alpha_{ij}}+\overline{\beta_{ij}})f(x_j)+\bigwedge_{j=1}^{n} \overline{T_{ij}}\overline{u_j}+\bigvee_{j=1}^{n} \overline{H_{ij}}\overline{u_j}+\overline{I_i}. \qquad (14)$$

Then we have

$$\begin{aligned}h_i &= |x_i| = \frac{1}{c_i}\Bigg|\sum_{j=1}^{n}(\overline{a_{ij}} + \overline{\alpha_{ij}} + \overline{\beta_{ij}})f(x_j) \\ &\quad + \bigwedge_{j=1}^{n} \overline{T_{ij}}\overline{u_j} + \bigvee_{j=1}^{n} \overline{H_{ij}}\overline{u_j} + \overline{I_i}\Bigg| \\ &\le \frac{1}{c_i}\Bigg\{\sum_{j=1}^{n}(|\overline{a_{ij}}| + |\overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)p_j h_j \\ &\quad + \sum_{j=1}^{n}(|\overline{a_{ij}}| + |\overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)q_j \\ &\quad + \bigwedge_{j=1}^{n} |\overline{T_{ij}}||u_j|^+ + \bigvee_{j=1}^{n} |\overline{H_{ij}}||u_j|^+ + |\overline{I_i}|\Bigg\} \\ &< (\frac{1}{\overline{c_i}} + \omega)\Bigg\{\sum_{j=1}^{n}(|\overline{a_{ij}}| + |\overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)p_j|h_j|^+\Bigg\} \\ &\quad +(\frac{1}{\overline{c_i}} + \omega)\Bigg\{\sum_{j=1}^{n}(|\overline{a_{ij}}| + \overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)q_j \\ &\quad + \bigwedge_{j=1}^{n} |\overline{T_{ij}}||u_j|^+ + \bigvee_{j=1}^{n} |\overline{H_{ij}}||u_j|^+ + |\overline{I_i}|\Bigg\} \\ &= \sum_{j=1}^{n} k_{ij}h_j + F_j. \qquad (15)\end{aligned}$$

Which is a contradiction. Therefore (13) holds and

$$QNu \neq 0, \quad \text{for } u \in \partial\Omega \cap \ker L = \partial\Omega \cap R^n. \qquad (16)$$

Consider the homotopy $\Phi : (\Omega \cap \mathrm{Ker}L) \times [0,1] \mapsto \Omega \cap \mathrm{Ker}L$ defined by

$$\Phi(u,\mu) = \mu\mathrm{diag}(-\overline{c_1}, -\overline{c_2}, \cdots, -\overline{c_n})u+(1-\mu)QNu. \qquad (17)$$

Note that $\Phi(\cdot,0) = JQN$, if $\Phi(u,\mu) = 0$, then we have

$$\begin{aligned}|x_i| &= \frac{1-\mu}{c_i}\Bigg|\sum_{j=1}^{n}(\overline{a_{ij}} + \overline{\alpha_{ij}} + \overline{\beta_{ij}})f(x_j) \\ &\quad + \bigwedge_{j=1}^{n} \overline{T_{ij}}\overline{u_j} + \bigvee_{j=1}^{n} \overline{H_{ij}}\overline{u_j} + \overline{I_i}\Bigg| \\ &\le \frac{1}{c_i}\Bigg\{\sum_{j=1}^{n}(|\overline{a_{ij}}| + |\overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)p_j h_j \\ &\quad + \sum_{j=1}^{n}(|\overline{a_{ij}}| + |\overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)q_j \\ &\quad + \bigwedge_{j=1}^{n} |\overline{T_{ij}}||u_j|^+ + \bigvee_{j=1}^{n} |\overline{H_{ij}}||u_j|^+ + |\overline{I_i}|\Bigg\} \\ &< (\frac{1}{\overline{c_i}} + \omega)\Bigg\{\sum_{j=1}^{n}(|\overline{a_{ij}}| + |\overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)p_j|h_j|^+\Bigg\} \\ &\quad +(\frac{1}{\overline{c_i}} + \omega)\Bigg\{\sum_{j=1}^{n}(|\overline{a_{ij}}| + \overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)q_j \\ &\quad + \bigwedge_{j=1}^{n} |\overline{T_{ij}}||u_j|^+ + \bigvee_{j=1}^{n} |\overline{H_{ij}}||u_j|^+ + |\overline{I_i}|\Bigg\} \\ &= \sum_{j=1}^{n} k_{ij}h_j + F_j. \qquad (18)\end{aligned}$$

Therefore $\Phi(u,\mu) \neq 0$ for any $(u,\mu) \in (\Omega \cap \mathrm{Ker}L)$. It follows from the property of invariance under homotopy that

$$\begin{aligned}&\deg \quad \{JQN, \Omega \cap \mathrm{Ker}L, 0\} \\ &= \quad \deg\{\Phi(\cdot,0), \Omega \cap \mathrm{Ker}L, 0\} \\ &= \quad \deg\{\Phi(\cdot,1), \Omega \cap \mathrm{Ker}L, 0\} \\ &= \quad \deg\{\mathrm{diag}(-\overline{c_1}, -\overline{c_2}, \cdots, -\overline{c_n})\} \\ &\neq \quad 0.\end{aligned}$$

Thus, we have shown that Ω satisfies all the assumptions of Lemma 2.1. Hence, $Lu = Nu$ has at least one ω-periodic solution on $\mathrm{Dom}L \cap \overline{\Omega}$. This completes the proof.

Theorem 2.2 *Let* $\tau = \max_{1\le i,j\le n, t\in[0,\omega]}\{\tau_{ij}(t)\}$. *Suppose that* $(A2), (A3)$ *and* $(A4)$ *hold,* $\rho(K) < 1$, *and that*

$$\overline{c_i} - \sum_{j=1}^{n}(|\overline{a_{ij}}| + |\overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|)p_j e^{c^+\tau} > 0, \qquad (19)$$

then system (1) has exactly one $\omega-$*periodic solution* $\tilde{x}(t)$. *Moreover it is globally exponentially stable.*

Proof. Let $C = C([-\tau,0], R^n)$ with the norm $\|\varphi\| = \sup_{s\in[-\tau,0],1\le i\le n}|\varphi_i(s)|$. From $(A2)$, we can get $|f_j(u)| \le p_j|u| + |f_j(0)| = p_j|u|, j = 1, 2, \cdots, n$. Hence al-l hypotheses in Theorem 2.1 with $q_j = f_j(0) = 0$ hold. Thus system (1) has at least one ω-periodic solution, say $\overline{x}(t) = (\overline{x}_1(t), \overline{x}_2(t), \cdots, \overline{x}_n(t))^T$. Let $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$ be an arbitrary solution of system (1). Calculating the right derivative $D^+|x_i(t) - \overline{x}_i(t)|$ of

$|x_i(t) - \overline{x}_i(t)|$ along the solutions of system (1)

$$
\begin{aligned}
D^+ \quad & |x_i(t) - \overline{x}_i(t)| \\
\leq \quad & -c_i(t)|x_i(t) - \overline{x}_i(t)| \\
& + \sum_{j=1}^{n} |a_{ij}(t)[f_j(x_j(t)) - f_j(\overline{x}_j(t))]| \\
& + \left| \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j(x_j(t - \tau_{ij}(t))) \right. \\
& \left. - \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j(\overline{x}_j(t - \tau_{ij}(t))) \right| \\
& + \left| \bigvee_{j=1}^{n} \beta_{ij}(t) f_j(x_j(t - \tau_{ij}(t))) \right. \\
& \left. - \bigvee_{j=1}^{n} \beta_{ij}(t) f_j(\overline{x}_j(t - \tau_{ij}(t))) \right| \\
\leq \quad & -c_i(t)|x_i(t) - \tilde{x}_i(t)| + \sum_{j=1}^{n} |a_{ij}(t)| p_j |x_j(t) - \overline{x}_j(t)| \\
& + \sum_{j=1}^{n} |\alpha_{ij}(t)| p_j |x_j(t - \tau_{ij}(t)) - \overline{x}_j(t - \tau_{ij}(t))| \\
& + \sum_{j=1}^{n} |\beta_{ij}(t)| p_j |x_j(t - \tau_{ij}(t)) - \overline{x}_j(t - \tau_{ij}(t))| \\
\leq \quad & -c_i(t)|x_i(t) - \overline{x}_i(t)| + \sum_{j=1}^{n} (|a_{ij}(t)| + |\alpha_{ij}(t)| \\
& + |\beta_{ij}(t)|) p_j \sup_{t-\tau \leq s \leq t} |x_j(s) - \overline{x}_j(s)|. \qquad (20)
\end{aligned}
$$

Let $z_i(t) = |x_i(t) - \overline{x}_i(t)|$. Then (20) can be transformed into

$$
\begin{aligned}
D^+ z_i(t) \quad \leq \quad & -c_i(t) z_i(t) + \sum_{j=1}^{n} (|a_{ij}(t)| + |\alpha_{ij}(t)| \\
& + |\beta_{ij}(t)|) p_j \sup_{t-\tau \leq s \leq t} z_j(s). \qquad (21)
\end{aligned}
$$

Thus, for $t > t_0$, we have

$$
\begin{aligned}
D^+ \quad & (z_i(t) e^{\int_{t_0}^{t} c_i(s) ds}) \leq \sum_{j=1}^{n} (|a_{ij}(t)| + |\alpha_{ij}(t)| \\
& + |\beta_{ij}(t)|) p_j \|z_t\| e^{\int_{t_0}^{t} c_i(s) ds}. \qquad (22)
\end{aligned}
$$

It follows that

$$
\begin{aligned}
z_i(t) \quad & e^{\int_{t_0}^{t} c_i(s) ds} \leq |z_i(t_0)| \\
& + \int_{t_0}^{t} \left\{ \sum_{j=1}^{n} (|a_{ij}(u)| + |\alpha_{ij}(u)| \right. \\
& \left. + |\beta_{ij}(u)|) p_j \|z_u\| e^{\int_{t_0}^{u} c_i(s) ds} \right\} du. \qquad (23)
\end{aligned}
$$

Thus, for any $\theta \in [-\tau, 0]$, we have

$$
e^{\int_{t_0}^{t+\theta} c_i(s) ds} = e^{(\int_{t_0}^{t} + \int_{t}^{t+\theta}) c_i(s) ds} \geq e^{\int_{t_0}^{t} c_i(s) ds - c_i^+ \tau}. \qquad (24)
$$

Therefore

$$
\begin{aligned}
& e^{\int_{t_0}^{t} c_i(s) ds - c_i^+ \tau} z_i(t+\theta) \leq e^{\int_{t_0}^{t+\theta} c_i(s) ds} z_i(t+\theta) \\
\leq \quad & \|z_{t_0}\| + \int_{t_0}^{t} \left\{ \sum_{j=1}^{n} (|a_{ij}(u)| + |\alpha_{ij}(u)| + |\beta_{ij}(u)|) \right. \\
& \left. \times p_j \|z_u\| e^{\int_{t_0}^{u} c_i(s) ds} \right\} du.
\end{aligned}
$$

It follows that

$$
\begin{aligned}
& e^{\int_{t_0}^{t} c_i(s) ds} \|z_t\| \leq e^{c_i^+ \tau} \|z_{t_0}\| + \int_{t_0}^{t} e^{c_i^+ \tau} \left\{ \sum_{j=1}^{n} (|a_{ij}(u)| \right. \\
& \left. + |\alpha_{ij}(u)| + |\beta_{ij}(u)|) p_j \|z_u\| e^{\int_{t_0}^{u} c_i(s) ds} \right\} du. \qquad (25)
\end{aligned}
$$

By Gronwall's inequality, we obtain

$$
\begin{aligned}
\|z_t\| \quad \leq \quad & e^{c^+ \tau} \|z_{t_0}\| \\
\times \quad & e^{\int_{t_0}^{t} e^{c^+ \tau} \sum_{j=1}^{n} (|a_{ij}(u)| + |\alpha_{ij}(u)| + |\beta_{ij}(u)|) p_j du} \\
\times \quad & e^{\int_{t_0}^{t} -c_i(s) ds}, \quad t \geq t_0. \qquad (26)
\end{aligned}
$$

Let $t_0 = 0$, for $t \geq 0$, $[s]$ denotes the largest integer less than or equal to s. Noting that $[\frac{t}{\omega}] \geq \frac{t}{\omega} - 1$ and (19), we get

$$
\begin{aligned}
\|z_t\| \quad & \leq e^{c^+ \tau} \|z_0\| \\
\times \quad & e^{\int_{0}^{t} e^{c^+ \tau} \sum_{j=1}^{n} (|a_{ij}(u)| + |\alpha_{ij}(u)| + |\beta_{ij}(u)|) p_j du} \\
\times \quad & e^{\int_{0}^{t} -c_i(s) ds} \\
= \quad & e^{c^+ \tau} \|z_0\| e^{H(t)} \\
\leq \quad & e^{c^+ \tau + (-\overline{c_i} + \sum_{j=1}^{n} (|\overline{a_{ij}}| + |\overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|) p_j e^{c^+ \tau}) \omega [\frac{t}{\omega}]} \|z_0\| \\
\times \quad & \exp \left\{ \int_{\omega[\frac{t}{\omega}]}^{t} \left(-c_i(s) + \sum_{j=1}^{n} (|a_{ij}(s)| + |\alpha_{ij}(s)| \right. \right. \\
& \left. \left. + |\beta_{ij}(s)|) \, p_j e^{c_i^+ \tau} \right) ds \right\} \\
\leq \quad & e^{c^+ \tau} \|z_0\| e^{-(\overline{c_i} - \sum_{j=1}^{n} (|\overline{a_{ij}}| + |\overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|) p_j e^{c^+ \tau}) t} \\
= \quad & M \|z_0\| e^{-\lambda t}, \qquad (27)
\end{aligned}
$$

where $H(t) = \left(\int_0^{\omega[\frac{t}{\omega}]} + \int_{\omega[\frac{t}{\omega}]}^{t} \right) e^{c^+ \tau} \sum_{j=1}^{n} ((|a_{ij}(s)| + |\alpha_{ij}(s)| + |\beta_{ij}(s)|) p_j - c_i(s)) ds$, $M = \max_{1 \leq i \leq n} \{ e^{c_i^+ \tau} \}$, $\lambda = \min_{1 \leq i \leq n} \{ \overline{c_i} - \sum_{j=1}^{n} (|\overline{a_{ij}}| + |\overline{\alpha_{ij}}| + |\overline{\beta_{ij}}|) p_j e^{c^+ \tau} \}$. From (27), it is clear that periodic solution $\overline{x}_i(t)$ is global exponentially stable. This completes the proof of Theorem 2.2.

III. An illustrative example

Example 3.1 Consider the following fuzzy cellular neural networks with delays

$$\begin{cases} x_1'(t) &= -c_1(t)x_1(t) + \sum_{j=1}^{2} a_{1j}(t)f_j(x_j(t)) \\ & + \bigwedge_{j=1}^{2} \alpha_{1j}(t)f_j(x_j(t-\tau_{1j}(t))) + I_1(t) \\ & + \bigvee_{j=1}^{2} \beta_{1j}(t)f_j(x_j(t-\tau_{1j}(t))) \\ & + \bigwedge_{j=1}^{2} T_{1j}(t)u_j(t) + \bigvee_{j=1}^{2} H_{1j}(t)u_j(t) \\ x_2'(t) &= -c_2(t)x_2(t) + \sum_{j=1}^{2} a_{2j}(t)f_j(x_j(t)) \\ & + \bigwedge_{j=1}^{2} \alpha_{2j}(t)f_j(x_j(t-\tau_{2j}(t))) + I_2(t) \\ & + \bigvee_{j=1}^{2} \beta_{2j}(t)f_j(x_j(t-\tau_{2j}(t))) \\ & + \bigwedge_{j=1}^{2} T_{2j}(t)u_j(t) + \bigvee_{j=1}^{2} H_{2j}(t)u_j(t), \end{cases} \tag{28}$$

where

$$c_1(t) = 4 + \sin t, c_2(t) = 3 + \cos t,$$
$$f_1(x) = \sin(\frac{1}{3}x) + \frac{1}{4}, f_2(x) = \cos(\frac{1}{3}x) + \frac{1}{3},$$
$$a_{11}(t) = \alpha_{11}(t) = \beta_{11}(t) = \frac{1}{4} + \sin t,$$
$$a_{12}(t) = \alpha_{12}(t) = \beta_{12}(t) = \frac{1}{5} + \cos t,$$
$$a_{21}(t) = \alpha_{21}(t) = \beta_{21}(t) = \frac{1}{6} + \sin t,$$
$$a_{22}(t) = \alpha_{22}(t) = \beta_{22}(t) = \frac{1}{7} + \cos t,$$
$$T_{ij}(t) = H_{ij}(t) = \sin t, (i, j = 1, 2),$$
$$\tau_{11}(t) = \tau_{12}(t) = \frac{1}{3}\sin t, \tau_{21}(t) = \tau_{22}(t) = \frac{1}{3}\cos t,$$
$$I_1(t) = 2\sin t, I_2(t) = 3\cos t, u_1(t) = u_2(t) = 4\sin t.$$

By simple computation, we have

$$\overline{c_1} = 4, \overline{c_2} = 3, \overline{a_{11}} = \overline{\alpha_{11}} = \overline{\beta_{11}} = \frac{1}{4},$$
$$\overline{a_{12}} = \overline{\alpha_{12}} = \overline{\beta_{12}} = \frac{1}{5}, c^+ = 5,$$
$$\overline{a_{21}} = \overline{\alpha_{21}} = \overline{\beta_{21}} = \frac{1}{6}, \overline{a_{22}} = \overline{\alpha_{22}} = \overline{\beta_{22}} = \frac{1}{7},$$
$$c^+ = 5, p_1 = p_2 = \frac{1}{3}, \tau = \frac{1}{3}.$$

It can be easily check that all the conditions of Theorem 2.1 and Theorem 2.2 are satisfied. Thus system (28) has at least $2\pi-$periodic solution which is global exponentially stable.

IV. Conclusion

In this paper, we have studied the existence, and exponential stability of the periodic solution for fuzzy cellular neural networks with time-varying delays. Some sufficient conditions set up here are easily verified and these conditions are correlated with parameters and time delays of the system (1). The obtained criteria can be applied to design globally exponentially periodic oscillatory fuzzy cellular neural networks.

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