Abstract—This paper is concerned with cellular neural networks with proportional delays. The proportional delay is a time-varying unbounded delay which is different from the constant delay, bounded time-varying delay and distributed delay. Using matrix measure and generalized Halanay inequality, some sufficient conditions are obtained to ensure the $\pi$th exponential stability of cellular neural networks with proportional delays. The results obtained are simple and easy to be verified. An example is given to illustrate the effectiveness of the obtained results. This paper ends with a brief conclusion.

Index Terms—Cellular neural networks, exponential stability, proportional delays, matrix measure.

I. INTRODUCTION

It is well known that considerable attention has been paid to cellular neural networks as well as various generalizations for their potential applications in many fields such as associative memories, pattern recognition, optimization and image processing and so on [1-13]. On the one hand, the existence and stability of the equilibrium point of cellular neural networks plays an important role in practical application. On the other hand, time delay is inevitable due to the finite switching speed of information processing and the inherent communication time of neurons, moreover, its existence may cause the instability of networks [14]. Thus many interesting stability results on cellular neural networks with delays have been available [14-19]. At present, the time delays considered for cellular neural networks can be classified as constant delays [4,16,21], time-varying delays [3,15,19,21], and distributed delays [22-24]. Here we would like to point out that the proportional delay which is a special delay type exists in many fields such as physics, biology systems, control theory and Web quality of service (QoS) routing decision. Since the presence of an amount of parallel pathways of a variety of axon sizes and lengths, a neural network usually has a spatial structure, it is reasonable to introduce the proportional delays into the neural networks. In an amount of parallel pathways, affected by different materials and topology, there may be some unbounded delays that is proportional to the time, thus we should choose suitable proportional delays factors in view of different cases and adopt proportional delays to characterize these unbounded delays [25]. Recently, there are only very few papers that focus on this aspect. For example, Zhou et al. [14] considered the asymptotic stability of cellular neural networks with multiple proportional delays, Zheng et al. [26] established the stability criteria for high-order networks with proportional delay. Zhou [27] addressed the delay-dependent exponential stability of cellular neural networks with multi-proportional delays, Zhou [28] discussed the delay-dependent exponential synchronization of recurrent neural works with multiple proportional delays, Zhou [29] analyzed the global asymptotic stability of cellular neural networks with proportional delays. In details, one can see [34,35]. We must point out that cellular neural networks with proportional delays have been widely applied in many fields such as light absorption in the star substance and nonlinear dynamic systems. Therefore, the study on the cellular neural networks with proportional delays has important theoretical and practical value.

In 2015, Zhou and Zhang [31] investigated the global exponential stability of the following cellular neural networks with multi-proportional delays

$$\begin{cases}
\dot{x}_i(t) = -d_i x_i(t) + \sum_{j=1}^{n} (a_{ij} f_j(x_j(t)) + b_{ij} g_j(x_j(t))), \\
x_i(s) = x_{i0}, s \in [q, 1],
\end{cases}$$

(1)

where $i = 1, 2, \cdots, n, t > 1$, $x_i(t)$ stands for the state variable of the $i$-th cell at time $t$, $d_i > 0$ is a constant, $a_{ij}$, $b_{ij}$ and $c_{ij}$ represent the connection weights between the $i$-th cell and the $j$-th cell at time $t$, $q_1 t, q_2 t$, respectively. $q_1$ and $q_2$ are proportional delay factors and satisfy $0 < q_1 < q_2 \leq 1.1, \rho = \min\{q_1, q_2\}, q_1 t = t - (1 - q_1) t, q_2 t = t - (1 - q_2) t$ in which $(1 - q_1) t$ and $(1 - q_2) t$ denote the transmission delays, $(1 - q_1) t \rightarrow +\infty, (1 - q_2) t \rightarrow +\infty$ as $t \rightarrow +\infty$. $x_{i0}$ is a constant which denotes the initial value of $x_i(t) \in [q, 1]$ and $x(0) = (x_{10}, x_{20}, \cdots, x_{n0})'$. $I_i(t)$ is the external input, $f_i(\cdot), g_i(\cdot)$ and $h_i(\cdot)$ are the nonlinear activation functions, and satisfy the following conditions:

$$\begin{cases}
f_i(u) - f_i(v), g_i(u) - g_i(v), h_i(u) - h_i(v) : \mathbb{R} \rightarrow \mathbb{R}, \\
f_i(u) - f_i(v) \leq L_i |u - v|, |f_i(u)| \leq q_i, \\
g_i(u) - g_i(v) \leq M_i |u - v|, |g_i(u)| \leq r_i, \\
h_i(u) - h_i(v) \leq N_i |u - v|, |h_i(u)| \leq s_i,
\end{cases}$$

(2)

where $i = 1, 2, \cdots, n, u, v \in \mathbb{R}$ and $L_i, M_i, N_i, q_i, r_i$ and $s_i$ are non-negative constants. By applying Brouwer fixed point theorem and constructing the delay differential inequality, Zhou and Zhang [31] obtained some delay-independent and delay-dependent sufficient conditions to ensure the existence, uniqueness and global exponential stability of equilibrium of
where the exponential stability of the following system is said to be 

\[
\dot{x}(t) = -d_i x_i(t) + \sum_{j=1}^{n} [a_{ij} f_j(x_j(t)) + b_{ij} g_j(x_j(\tilde{q}t))] + I_i,
\]

\[x_i(s) = x_{i0}, s \in [\tilde{q}t_0, t_0], \]

which is a revised version of system (1). Here for simplification, we let \(q = q_2 = \tilde{q}\) in system (1), \(t_0\) is a constant. If \(t_0 = 0, s \in [\tilde{q}t_0, t_0]\) is equal to \(s = t_0 = 0\). Different from the work of Zhou and Zhang [31], we will obtain some sufficient conditions to ensure the \(p\)th exponential stability of system (3) by applying matrix measure and generalized Halanay inequality. The results of this paper are completely new and complement those of the previous studies in [31]. The approach is new.

The organization of the rest of this paper is as follows. In Section 2, some preliminaries are presented. In Section 3, some sufficient conditions are derived for the exponential stability of (3) by matrix measure and generalized Halanay inequality. In Section 4, we present three examples to illustrate the feasibility and effectiveness of our main theoretical findings in previous sections. A brief conclusion is drawn in Section 5.

II. PRELIMINARY RESULTS

First we give some notations. Let \(x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T\), 
\[D = \text{diag}[d_1, d_2, \cdots, d_n],\]
\[A = (a_{ij})_{n \times n}, B = (b_{ij})_{n \times n}, C = (c_{ij})_{n \times n},\]
\[f(x(t)) = (f_1(x_1(t)), f_2(x_1(t)), \cdots, f_n(x_1(t)))^T,\]
\[g(x(t)) = (g_1(x_1(t)), g_2(x_1(t)), \cdots, g_n(x_1(t)))^T,\]
\[h(x(t)) = (h_1(x_1(t)), h_2(x_1(t)), \cdots, h_n(x_1(t)))^T,\]
\[I = (I_1, I_2, \cdots, I_n)^T.\]

Then system (3) can be rewritten as follows:
\[
\dot{x}(t) = -Dx(t) + [Af(x(t)) + Bg(x(\tilde{q}t)) + Ch(x(\tilde{q}t))] + I. \tag{4}
\]

Using the transformation
\[y_i(t) = x_i(e^t), i = 1, 2, \cdots, n, \tag{5}\]
and letting \(y(t) = (y_1(t), y_2(t), \cdots, y_n(t))^T,\) \(\tau = -\ln \tilde{q} > 0,\)
\[
y(t) = e^t \{ -Dy(t) + [Af(y(t)) + Bg(y(t - \tau)) + Ch(y(t - \tau))] + I}, \tag{6}
\]
with the initial condition
\[y_i(s) = \varphi_i(s), t_0 - \tau \leq s \leq t_0, i = 1, 2, \cdots, n, \tag{7}\]
where \(\varphi_i(s) \in C([t_0 - \tau, t_0], \mathbb{R})\) is a continuous function.

In addition, we need the following definitions and lemmas.

Definition 2.1 An equilibrium point \(x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T\) of the system (8) is said to be \(p\)th (\(p = 1, 2, \infty\)) globally exponentially stable, if there exist two positive constants \(M > 0\) and \(\lambda > 0\) such that \(||x(t, t_0, x_0) - x^*||_p \leq M||x_0 - x^*||_p e^{-\lambda t}\) holds, where \(x_0 = (x_{10}, x_{20}, \cdots, x_{n0})^T\) is the initial condition of the system (8), \(x(t, t_0, x_0)\) is the solution of system (8).

Lemma 2.1 ([32]) For any real matrix \(A = (a_{ij})_{n \times n},\) its matrix measure is defined as
\[\mu_p(A) = \lim_{\varepsilon \to 0^+} \frac{||E + \varepsilon A||_p - 1}{\varepsilon},\]
where \(||.||_p\) denotes the matrix norm in \(\mathbb{R}^{n \times n}, E\) is the identity matrix, \(p \in [1, 2, \infty].\)

Let the matrix norm be as follows:
\[||A||_1 = \max_j \left\{ \sum_{i=1}^{n} |a_{ij}| \right\}, \quad ||A||_2 = \sqrt{\lambda_{\max}(A^T A)},\]
\[||A||_\infty = \max_j \left\{ \sum_{i=1}^{n} |a_{ij}| \right\}.\]

Then we get
\[\mu_1(A) = \max_j \left\{ a_{jj} + \sum_{i=1, i \neq j}^{n} |a_{ij}| \right\}, \quad \mu_2(A) = \frac{1}{2} \lambda_{\max}(A^T + A),\]
\[\mu_\infty(A) = \max_i \left\{ a_{ii} + \sum_{j=1, j \neq i}^{n} |a_{ij}| \right\}.\]

Lemma 2.2 ([33], Generalized Halanay’s inequality) Suppose
\[
\dot{x}(t) \leq \gamma(t) - \alpha(t)x(t) + \beta(t) \sup_{t-\tau \leq \sigma \leq t} x(\sigma) \tag{8}
\]
holds for any \(t \geq t_0.\) Here \(\tau \geq 0, \) and \(\gamma(t), \alpha(t), \beta(t)\) are continuous functions such that \(0 \leq \gamma(t) \leq \gamma^*, \alpha(t) \geq \alpha_0, 0 \leq \beta(t) \leq \beta_0,\) for any \(t \geq t_0\) with constants \(\gamma^* > 0, \alpha_0 > 0, 0 \leq \beta_0 < 1.\) Then we have
\[x(t) \leq \frac{\gamma^*}{(1 - \tilde{q})\alpha_0} + Ge^{-\mu^*(t-t_0)}\]
holds for \(t \geq t_0.\) Here \(G = \sup_{t_0 - \tau \leq \sigma \leq t_0} x(\sigma)\) and \(\mu^* > 0\) is defined as
\[\mu^* = \inf_{t_0 \leq t} \left\{ \mu(t) : \mu(t) - \alpha(t) + \beta(t)e^{\mu(t)} = 0 \right\}.\]

In Lemma 2.2, Letting \(\gamma(t) = 0, \gamma^* \to 0,\) then we can obtain the following lemma.

Lemma 2.3 Suppose
\[
\dot{x}(t) \leq -\alpha(t)x(t) + \beta(t) \sup_{t-\tau \leq \sigma \leq t} x(\sigma) \tag{9}
\]
holds for any \(t \geq t_0.\) Here \(\tau \geq 0, \) and \(\alpha(t), \beta(t)\) are continuous functions such that \(\alpha(t) \geq \alpha_0, 0 \leq \beta(t) \leq \beta_0\)
for any $t \geq t_0$ with constants $\alpha_0 > 0, 0 \leq q < 1$. Then we have

$$x(t) \leq Ge^{-\mu^*(t-t_0)}$$

for $t \geq t_0$, where $G = \sup_{t_0 - \tau \leq t \leq t_0} x(t)$ and $\mu^* > 0$ is defined as

$$\mu^* = \inf_{t \geq t_0} \{ \mu(t) : \mu(t) - \alpha(t) + \beta(t)e^{\nu(t)} = 0 \}.$$

In order to obtain the main results, we make the following assumptions.

(H1) For $i = 1, 2, \cdots, n$, there exist positive constants $\alpha_i, \beta_i$ and $\gamma_i$ such that $|f_i(u) - f_i(v)| \leq \alpha_i|u - v|, g_i(u) - g_i(v) \leq \beta_i|u - v|, |h_i(u) - h_i(v)| \leq \gamma_i|u - v|$ for all $u, v \in \mathbb{R}$.

Denote

$$\alpha = \max_{1 \leq i \leq n} \{ \alpha_i \}, \beta = \max_{1 \leq i \leq n} \{ \beta_i \}, \gamma = \max_{1 \leq i \leq n} \{ \gamma_i \}.$$

III. MAIN RESULTS

In this section, we consider the global exponential stability of system (4) by applying the matrix norm and matrix measure.

**Theorem 3.1** Under the condition (H1), let $\Theta_1 = -\mu_p(-D)$ and $\Theta_2 = \alpha||A||_p + \beta||B||_p + \gamma||C||_p$. If $\Theta_1 > \Theta_2 > 0$, then the equilibrium point $y^*$ of system (4) is $p$th globally exponentially stable.

**Proof** Assume that $y^* = (y_1^*, y_2^*, \cdots, y_n^*)^T$ is the equilibrium point of system (6), then $y^*$ satisfies

$$-Dy^* + [Af(y^*) + Bg(y^*) + Ch(y^*)] = 0.$$

Let $u(t) = y(t) - y^*$, then

$$\dot{u}(t) = e^t(-Du(t) + Af(y^*) - f(y^*) + B[g(y(t) - \tau)] - g(y^*)) + C[h(y(t) - \tau)] - h(y^*)).$$

Consider the following nonnegative function

$$V(t) = ||u(t)||_p.$$

Calculating the derivative of $V(t)$ along the trajectories of (9) leads to

$$D^+V(t) = \lim_{\epsilon \to 0^+} \frac{||u(t + \epsilon)|| - ||u(t)||}{\epsilon} = \lim_{\epsilon \to 0^+} \frac{E + \epsilon e^t(-D) ||u(t)||_p - ||u(t)||_p}{\epsilon} = \epsilon ||e^tAf(y(t)) - f(y^*) + B[g(y(t) - \tau)] - g(y^*) + C[h(y(t) - \tau)] - h(y^*)||_p.$$

In view of (H1), we get

$$\begin{align*}
||f(y(t)) - f(y^*)|| &\leq \alpha||u(t)||_p, \\
||g(y(t) - \tau) - g(y^*)|| &\leq \beta||u(t - \tau)||_p, \\
||h(y(t) - \tau) - h(y^*)|| &\leq \gamma||u(t - \tau)||_p.
\end{align*}$$

It follows from (10) and (11) that

$$D^+V(t) \leq \mu_p(-e^tD)||u(t)||_p + e^t(\alpha||A||_p + \beta||B||_p + \gamma||C||_p)||u(t - \tau)||_p$$

$$\leq -\Theta_1 e^t||u(t)||_p + \Theta_2 e^t\sup_{t-\tau \leq s \leq t}||u(s)||_p.$$ (12)

By Lemma 2.3, we have

$$V(t) \leq \sup_{t_0 - \tau \leq s \leq t_0} V(s)e^{-\mu^*(t-t_0)},$$ (13)

where

$$\mu^* = \inf_{t \geq t_0} \{ \mu(t) : \mu(t) - \Theta_1 e^t + \Theta_2 e^t\tau \beta(t) = 0 \} > 0.$$

Then the zero solution of system (8) is $p$th globally exponentially stable, i.e., the equilibrium point $y^*$ of system (4) is $p$th globally exponentially stable. The proof of Theorem 3.1 is complete.

IV. EXAMPLES

In this section, we give three examples to illustrate our main results derived in previous sections. Consider the following three cellular neural networks with proportional delays

**Example 4.1** Consider the following cellular neural networks with proportional delays

$$\begin{align*}
\dot{x}_1(t) &= -d_1 x_1(t) + \sum_{j=1}^3 (a_{1j} f_j(x_j(t)) + b_{1j} g_j(x_j(q))) + c_{1j} h_j(x_j(q)) + I_1, \\
\dot{x}_2(t) &= -d_2 x_2(t) + \sum_{j=1}^3 (a_{2j} f_j(x_j(t)) + b_{2j} g_j(x_j(q))) + c_{2j} h_j(x_j(q)) + I_2, \\
\dot{x}_3(t) &= -d_3 x_3(t) + \sum_{j=1}^3 (a_{3j} f_j(x_j(t)) + b_{3j} g_j(x_j(q))) + c_{3j} h_j(x_j(q)) + I_3,
\end{align*}$$ (14)

where

$$\begin{align*}
&d_1 \begin{bmatrix} 0 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} = \begin{bmatrix} 0.3 & 0.1 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \\
&a_{11} \begin{bmatrix} a_{12} & a_{13} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} 0.4 & 0.4 & 0.4 \\ 0.1 & 0.8 & 0.2 \\ 0.2 & 0.5 & 0.3 \end{bmatrix}, \\
&b_{11} \begin{bmatrix} b_{12} & b_{13} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.6 & 0.1 \\ 0.8 & 0.3 & 0.5 \\ 0.5 & 0.4 & 0.2 \end{bmatrix}, \\
&c_{11} \begin{bmatrix} c_{12} & c_{13} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix} = \begin{bmatrix} 0.2 & 0.6 & 0.1 \\ 0.5 & 0.4 & 0.6 \\ 0.1 & 0.6 & 0.5 \end{bmatrix},
\end{align*}$$

$$\begin{align*}
&I_1 = \begin{bmatrix} 0.2 \\ 0.5 \\ 0.7 \end{bmatrix}, \\
&f_i(x) = \frac{1}{2}(|x_i + 1| - |x_i|), \\
&g_i(x) = \tanh\left(\frac{5}{7}x\right), h_i(x) = \tanh\left(\frac{2}{5}x\right)(i = 1, 2, 3).
\end{align*}$$

Set $\tau = 0.5$ Then $\alpha = \alpha_1 = 1, \beta = \beta_1 = \frac{2}{3}, \gamma = \gamma_1 = \frac{2}{7}$. It is easy to verify that $\Theta_1 = -\mu_p(-D) = 9.0703$ and...
$\Theta_2 = \alpha ||A||_p + \beta ||B||_p + \gamma ||C||_p = 6.3423$. It follows that $\Theta_1 > \Theta_2 > 0$. Then all the conditions (H1)-(H2) of Theorem 3.1 hold. Thus system (14) has a unique equilibrium (0.0921, 0.3532, 0.2036) which is globally exponentially stable. The results are illustrated in Fig. 1.

**Example 4.2** Consider the following cellular neural networks with proportional delays

$$
\begin{align*}
\dot{x}_1(t) &= -d_1 x_1(t) + \sum_{j=1}^{3} [a_{1j} f_j(x_j(t)) + b_{1j} g_j(x_j(\tilde{q}t))] + I_1, \\
\dot{x}_2(t) &= -d_2 x_2(t) + \sum_{j=1}^{3} [a_{2j} f_j(x_j(t)) + b_{2j} g_j(x_j(\tilde{q}t))] + I_2, \\
\dot{x}_3(t) &= -d_3 x_3(t) + \sum_{j=1}^{3} [a_{3j} f_j(x_j(t)) + b_{3j} g_j(x_j(\tilde{q}t))] + I_3,
\end{align*}
$$

(15)

where

$$
\begin{bmatrix}
d_1 & 0 & 0 \\
0 & d_2 & 0 \\
0 & 0 & d_3
\end{bmatrix} = 
\begin{bmatrix}
0.8 & 0 & 0 \\
0 & 0.4 & 0 \\
0 & 0 & 0.7
\end{bmatrix},
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix} = 
\begin{bmatrix}
0.2 & 0.5 & 0.7 \\
0.2 & 0.3 & 0.5 \\
0.7 & 0.4 & 0.8
\end{bmatrix},
\begin{bmatrix}
b_{11} & b_{12} & b_{13} \\
b_{21} & b_{22} & b_{23} \\
b_{31} & b_{32} & b_{33}
\end{bmatrix} = 
\begin{bmatrix}
0.9 & 0.3 & 0.7 \\
0.3 & 0.8 & 0.7 \\
0.8 & 0.1 & 0.5
\end{bmatrix},
\begin{bmatrix}
c_{11} & c_{12} & c_{13} \\
c_{21} & c_{22} & c_{23} \\
c_{31} & c_{32} & c_{33}
\end{bmatrix} = 
\begin{bmatrix}
0.1 & 0.9 & 0.4 \\
0.7 & 0.3 & 0.4 \\
0.8 & 0.9 & 0.9
\end{bmatrix},
\begin{bmatrix}
f_1(x) = \frac{1}{2}(|x| + |x_1|) \\
f_2(x) = \frac{1}{2}(|x| + |x_1|) \\
f_3(x) = \frac{1}{2}(|x| + |x_1|)
\end{bmatrix},
\begin{bmatrix}
g_1(x) = \tanh\left(\frac{7}{8}x\right) \\
g_2(x) = \tanh\left(\frac{5}{9}x\right) \\
g_3(x) = \tanh\left(\frac{3}{4}x\right)
\end{bmatrix}.
\end{equation}

Set $\tilde{q} = 0.3$ Then $\alpha = \alpha_1 = 1, \beta_1 = \beta_2 = \beta_3 = \frac{5}{7}, \gamma = \gamma_1 = \frac{2}{7}$. It is easy to verify that $\Theta_1 = -\mu_p(-D) = 14.5008$ and $\Theta_2 = \alpha ||A||_p + \beta ||B||_p + \gamma ||C||_p = 9.8402$. It follows that $\Theta_1 > \Theta_2 > 0$. Then all the conditions (H1)-(H2) of Theorem 3.1 hold. Thus system (14) has a unique equilibrium (0.5902, 0.7023, 1.0931) which is globally exponentially stable. The results are illustrated in Fig. 2.

**Example 4.3** Consider the following cellular neural networks with proportional delays

$$
\begin{align*}
\dot{x}_1(t) &= -d_1 x_1(t) + \sum_{j=1}^{3} [a_{1j} f_j(x_j(t)) + b_{1j} g_j(x_j(\tilde{q}t))] + c_{1j} h_j(x_j(\tilde{q}t)) + I_1, \\
\dot{x}_2(t) &= -d_2 x_2(t) + \sum_{j=1}^{3} [a_{2j} f_j(x_j(t)) + b_{2j} g_j(x_j(\tilde{q}t))] + c_{2j} h_j(x_j(\tilde{q}t)) + I_2, \\
\dot{x}_3(t) &= -d_3 x_3(t) + \sum_{j=1}^{3} [a_{3j} f_j(x_j(t)) + b_{3j} g_j(x_j(\tilde{q}t))] + c_{3j} h_j(x_j(\tilde{q}t)) + I_3,
\end{align*}
$$

(16)

where

$$
\begin{bmatrix}
d_1 & 0 & 0 \\
0 & d_2 & 0 \\
0 & 0 & d_3
\end{bmatrix} = 
\begin{bmatrix}
0.5 & 0 & 0 \\
0 & 0.3 & 0 \\
0 & 0 & 0.5
\end{bmatrix},
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix} = 
\begin{bmatrix}
0.5 & 0.9 & 0.3 \\
0.5 & 0.8 & 0.4 \\
0.7 & 0.4 & 0.7
\end{bmatrix},
\begin{bmatrix}
b_{11} & b_{12} & b_{13} \\
b_{21} & b_{22} & b_{23} \\
b_{31} & b_{32} & b_{33}
\end{bmatrix} = 
\begin{bmatrix}
0.2 & 0.5 & 0.5 \\
0.8 & 0.5 & 0.4 \\
0.8 & 0.7 & 0.6
\end{bmatrix},
\begin{bmatrix}
c_{11} & c_{12} & c_{13} \\
c_{21} & c_{22} & c_{23} \\
c_{31} & c_{32} & c_{33}
\end{bmatrix} = 
\begin{bmatrix}
0.1 & 0.7 & 0.7 \\
0.3 & 0.5 & 0.8 \\
0.7 & 0.5 & 0.6
\end{bmatrix},
\begin{bmatrix}
f_1(x) = \frac{1}{2}(|x| + |x_1|) \\
f_2(x) = \frac{1}{2}(|x| + |x_1|) \\
f_3(x) = \frac{1}{2}(|x| + |x_1|)
\end{bmatrix},
\begin{bmatrix}
g_1(x) = \tanh\left(\frac{5}{6}x\right) \\
g_2(x) = \tanh\left(\frac{3}{4}x\right) \\
g_3(x) = \tanh\left(\frac{3}{4}x\right)
\end{bmatrix}.
\end{equation}

Set $\tilde{q} = 0.2$ Then $\alpha = \alpha_1 = 1, \beta_1 = \beta_2 = \beta_3 = \frac{5}{7}, \gamma = \gamma_1 = \frac{2}{7}$. It is easy to verify that $\Theta_1 = -\mu_p(-D) = 11.2323$ and $\Theta_2 = \alpha ||A||_p + \beta ||B||_p + \gamma ||C||_p = 11.2323$.
Fig. 3. Transient response of state variables $x_1(t), x_2(t)$ and $x_3(t)$.

$$\Theta_2 = \alpha||A||_p + \beta||B||_p + \gamma||C||_p = 8.7865.$$ It follows that $\Theta_1 > \Theta_2 > 0$. Then all the conditions (H1)-(H2) of Theorem 3.1 hold. Thus system (16) has a unique equilibrium (0.9034, 0.3215, 1.7213) which is globally exponentially stable. The results are illustrated in Fig. 3.

V. CONCLUSIONS

In this paper, we have investigated the global exponential stability of cellular neural networks with proportional delays. Applying matrix measure and generalized Halanay inequality, a series of new sufficient conditions to guarantee the $p$th exponential stability of cellular neural networks with proportional delays are established. The obtained conditions are easily to check in practice. Finally, three examples are included to illustrate the feasibility and effectiveness. To the best of our knowledge, there are no results on the anti-periodic solution and synchronization for cellular neural networks with proportional delays, which might be our future research topic.

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