

# Active Fault-tolerant Control of a CSTR System Based on PWA Model

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**Abstract**—In this paper, we present an active fault-tolerant control method for a continuous stirred tank reactor (CSTR) system. In the proposed method, a piecewise affine (PWA) form of the system is modeled in both the normal and fault situations, and then an active fault-tolerant controller is designed using an explicit model predictive control algorithm. By this way, the control objective can be achieved simply by changing the controller parameters without re-computing the controller online when the system faults occur. So the method greatly reduces the computational burden and has a better real-time performance. Finally, simulation experiments for the system exposed to multiple sensor or actuator faults are carried out and show the effectiveness of the method.

**Index Terms**—piecewise affine model, active fault-tolerant control, explicit model predictive control, continuous stirred tank reactor system.

## I. INTRODUCTION

THE continuous stirred tank reactor (CSTR) system plays a vital role in the polymerization reaction and therefore is widely used in the chemical industry. However, once faults of the system occur, such as actuator or sensor faults, it will lead to poor quality and low yield of the products. So finding an appropriate fault-tolerance control technique of the system is very necessary and important.

The fault-tolerant control technique is capable of achieving the system acceptable performance and stability properties in both the normal and fault situations and can be classified into two types: passive and active. In the passive fault-tolerant control approach, it takes into account of all the expected component faults during the design of a controller, so that the system can maintain its expected performance when these faults occur. It doesn't change but uses the same robust controller during the whole operation period which will sacrifice parts of system performance and have great conservativeness. This method can be found in [1-4]. Contrary to the passive approach, the active approach uses a detection and diagnosis module (FDD) to get the real-time information of system faults and changes the control strategies with types of the faults, such as reconfiguration of the current controller, re-scheduling of

the control law and so on. So it is able to achieve the control goal even in the situation of unexpected system failure. This method is received more attention and can be found in [5-9].

For the CSTR system, several fault-tolerant control methods have already been proposed. In [10], a fault-tolerant controller is designed for a CSTR system subject to constraints and sensor data losses faults via a reconfiguration-based approach which can always preserve closed-loop stability. In [11], a CSTR system is modeled in an adaptive neural network form. When faults occur, it compensates the fault effects by employing an auto-tuning PID controller based on the established model. In [12], it proposes a method to design a controller on the basis of an adaptive learning and a switching function mechanism and then applies this method to a CSTR system with actuator faults successfully. In [13], it provides a new fault-tolerant method to control a CSTR system with multiple control failures relying on the coordination of a multi-loop proportional controller and a decentralized unconditionally stabilizing controller. However, these research results are mainly concentrated in passive fault-tolerant control of the CSTR system. For active methods, to the knowledge of the authors it is still lacking in studies. In this paper, an active fault-tolerant control method is proposed for a CSTR system. As some literatures, such as [14-18], has already presented fault detection and diagnosis strategies and their available for the CSTR system, we mainly focus our attention on active fault-tolerant controller design.

The idea of the proposed method combines active fault-tolerant control strategies with explicit model predictive control algorithms based on piecewise affine (PWA) model. In the method, a CSTR system is modeled in a PWA form which not only describes the system characteristics very well, but is convenient to be used to design the controller as well. Then an active fault-tolerant controller is designed using an explicit model predictive control approach and it can remain stable and feasible by properly choosing the design parameters. The method enables the system to make corresponding response to faults, mainly considering actuator or sensor faults here, quickly. The rest of this paper is organized as follows. In Section II, a brief description of PWA model and a PWA form of the CSTR system are presented. In Section III, an active fault-tolerant control algorithm is researched in detail. In Section IV, the proposed method is applied to the CSTR system subjected to actuator and sensor faults. Finally, conclusions are drawn in Section V.

Manuscript received January 16, 2016; revised April 11, 2016. This work was supported by the Shandong Provincial Natural Science Foundation of China (No. 2013ZRE28089).

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II. CSTR SYSTEM BASED ON PWA MODEL

A. PWA Model

PWA model is a typical model which contains a finite number of continuous dynamic submodels and can be switched among the submodels according to a specific switching law. It can describe a large number of physical systems very well, especially for nonlinear systems. In the model, the extended state+input space is partitioned into several polyhedral regions and each region is associated with a different linear state-update equation. It can be expressed by the following form:

$$\begin{aligned} x(k+1) &= f_{PWA}(x(k), u(k)) = A_i x(k) + B_i u(k) + f_i \\ y(k) &= C_i x(k) + D_i u(k) + g_i \\ \text{if } \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} &\in \Omega_i \quad i = 1, \dots, s \end{aligned} \tag{1}$$

where  $k \geq 0$ ,  $x \in \mathbb{R}^n$  is the state,  $u \in \mathbb{R}^m$  and  $y \in \mathbb{R}^p$  is the input and the output respectively.  $\{\Omega_i\}_{i=1}^s \triangleq \{(x, u) | H_{ix}x + H_{iu}u \leq K_i\}$ ,  $i = 1, \dots, s$  is the polyhedral partition of the sets which are in the extended space  $(x, u) \in \mathbb{R}^{n+m}$ . It should be noted that linear state and input constraints in the form of  $Kx(k) + Lu(k) \leq M$  can be easily incorporated in the description of  $\Omega_i$  [25].

B. CSTR System in PWA Form

In this paper, a schematic of a standard two-state CSTR system is shown in Figure 1.

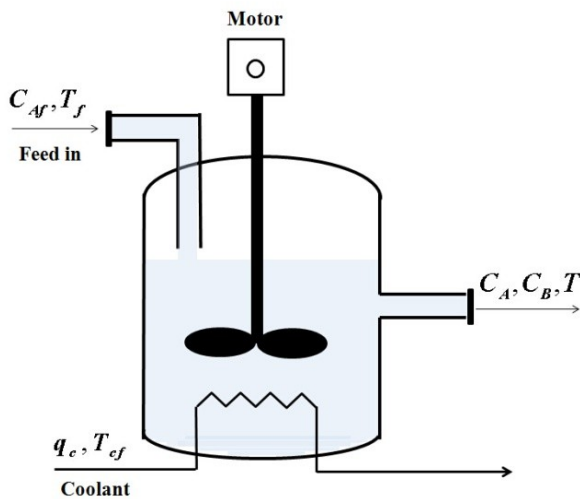


Fig. 1. Continuous stirred tank reactor system

It is assumed that a single irreversible, exothermic reaction  $A \rightarrow B$  occurs in this reactor. With concentration of  $A$  ( $C_A$ ) and the reactor temperature ( $T$ ) as states  $x = [x_1, x_2]^T$ , the coolant temperature ( $T_{cf}$ ) as input  $u$ ,  $C_A$  as output  $y$ , a set of nonlinear equations can be obtained to describe the system as follows according to [20,22]:

$$\begin{aligned} \frac{dx_1}{dt} &= -\theta x_1 \kappa(x_2) + q(x_{1f} - x_1) \\ \frac{dx_2}{dt} &= \beta \theta x_1 \kappa(x_2) - (q + \delta)x_2 + \delta u + q x_{2f} \\ y &= x_1 \end{aligned}$$

Where  $\kappa(x_2) = \exp(\frac{x_2}{1+x_2/\lambda})$  and the ranges of the variables are  $x \in [0,1] \times [0,6]$ ,  $u \in [-2,2]$ . Values of parameters in the nonlinear equations are shown in Table 1.

TABLE I  
VALUES OF PARAMETERS

$\lambda$	$\theta$	$q$	$\beta$	$\delta$	$x_{1f}$	$x_{2f}$
20.0	0.072	1.0	8.0	0.3	1.0	0

Apparently, the system is highly nonlinear and multi-operating points. If it is modeled in PWA form, these characteristics can be well captured.

For the nominal parameters, it has three steady states (steady operating points):  $x_{s_1} = (0.856, 0.886)$ ,  $x_{s_2} = (0.5528, 2.7517)$ ,  $x_{s_3} = (0.2353, 4.7050)$ . By linearizing the system around each steady state point and then discretizing it with a sampling time of 0.1 sec, we can get the PWA model of the CSTR system as follows:

$$\begin{aligned} x(k+1) &= \begin{cases} \begin{bmatrix} 0.8889 & -0.0123 \\ 0.1254 & 0.9751 \end{bmatrix} x(k) + \begin{bmatrix} -0.0002 \\ 0.0296 \end{bmatrix} u(k) + \begin{bmatrix} 0.1060 \\ -0.0852 \end{bmatrix}, x \in \Omega_1 \\ \begin{bmatrix} 0.8241 & -0.0340 \\ 0.6365 & 1.1460 \end{bmatrix} x(k) + \begin{bmatrix} -0.0005 \\ 0.0322 \end{bmatrix} u(k) + \begin{bmatrix} 0.1907 \\ -0.7537 \end{bmatrix}, x \in \Omega_2 \\ \begin{bmatrix} 0.6002 & -0.0463 \\ 2.4016 & 1.2430 \end{bmatrix} x(k) + \begin{bmatrix} -0.0007 \\ 0.0338 \end{bmatrix} u(k) + \begin{bmatrix} 0.3119 \\ -1.7083 \end{bmatrix}, x \in \Omega_3 \end{cases} \\ y(k) &= [1 \quad 0] x(k) \end{aligned}$$

where  $\{\Omega_i\}_{i=1}^3 \triangleq \{(x, u) | H_{ix}x + H_{iu}u \leq K_i\}$ ,  $i = 1, 2, 3$  is the polyhedral partition of the sets of the state+input space.  $H_{ix}, H_{iu}, K_i, i = 1, 2, 3$  are, respectively,  $6 \times 2$ ,  $6 \times 1$ ,  $6 \times 1$  corresponding matrices.

III. ACTIVE FAULT-TOLERANT CONTROL ALGORITHM

A. Active Fault-tolerant Control Based on PWA Model

Active fault-tolerant control of a system with PWA form achieves control objectives by way of changing control strategies when system faults, mainly considering actuator or sensor faults here, are detected. In detail, it needs to use a FDD module to real-time monitor the system and get the information of the faults in time once it occurs. Then the information is passed to a supervision module and corresponding control actions are subsequently taken to accommodate and recover the faulty system according to the fault messages. The general structure of this active fault-tolerant control system can be depicted in Figure 2.

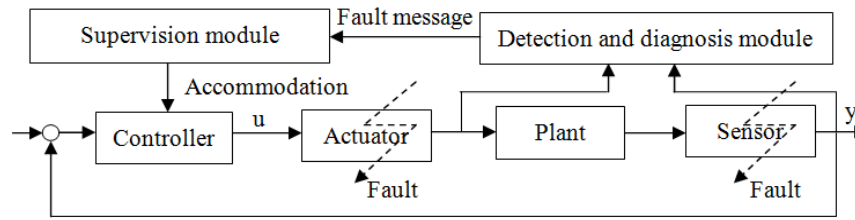


Fig. 2. Structure of the active fault-tolerant control system

One way of achieving the above fault-tolerant control process is to employ the controller reconfiguration technique. The basic idea of this technology is to use the FDD module to gain the fault information and generate a discrete event signal passing to the supervision module once the system failure occurs. Then the supervision module reconstructs the controller online to obtain an appropriate control law for the system so that the system will not be greatly influenced by the faults and can still achieve the control goal. If we assume the FDD module works normally, it is not difficult to see that a kind of control algorithms selected to reconfigure the controller is the key to guarantee the successful application of the proposed technique. As the characteristics of the system and the system's internal model and constraints may be changed with the types of faults, choosing the model predictive control algorithm seems to be quite suitable because of its outstanding ability to deal with these kinds of situations. However, it needs to receding-horizon optimization which solves the optimization problem online repeatedly. In the case of complex, frequent or multiple faulty systems, it may fail. To some extent, an explicit model predictive control algorithm will be introduced in the following section which can solve these problems perfectly.

*B. Active Fault-tolerant Control Using an Explicit Model Predictive Control Approach*

Explicit model predictive control inherits almost all characteristics of model predictive control. Besides these, it solves the optimization problem offline by using multi-parametric program which greatly reduces the on-line computational burden and leads to be excellent in real-time performance. Because of these advantages, this algorithm can be used to active fault-tolerant control well even in some extreme situations.

Faults of a plant are treated as additional states and added to the plant model. This relative uniform structure of model is called PWA fault model which is similar to (1) with the following form:

$$\begin{aligned} \tilde{x}(k+1) &= f_{PWA}(\tilde{x}(k), u(k)) = \begin{bmatrix} A_i \\ A_f \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} B_i \\ B_f \end{bmatrix} u(k) + \begin{bmatrix} f_i \\ f_f \end{bmatrix} \\ \tilde{y}(k) &= \begin{bmatrix} C_i \\ C_f \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} D_i \\ D_f \end{bmatrix} u(k) + \begin{bmatrix} g_i \\ g_f \end{bmatrix} \\ \text{if } \begin{bmatrix} \tilde{x}(k) \\ u(k) \end{bmatrix} &\in \Omega_i \quad i = 1, \dots, s \end{aligned}$$

Where  $\tilde{x}(k) = [x(k)^T, f_1(k), \dots, f_j(k)]^T \in \mathbb{R}^{n+l}$ ,  $f_j(k) (j=1, \dots, l)$  represents the  $j$ -th possible fault of the system during its run time.  $A_f, B_f, f_f, C_f, D_f$  and  $g_f$  are the corresponding matrices based on the fault types.

If the control objective is to track a constant reference state of the system, the following optimal control problem should be considered:

$$\begin{aligned} J_N^*(\tilde{x}(0)) &= \min_{U_0^{N-1}} J(U_0^{N-1}, \tilde{x}(0)) = \min_{U_0^{N-1}} \left\{ \left\| \tilde{x}(N) - \tilde{x}_r \right\|_p^{Q_N} + \right. \\ &\left. \sum_{k=0}^{N-1} \left[ \left\| \tilde{x}(k) - \tilde{x}_r \right\|_p^Q + \left\| u(k) - u_r \right\|_p^R \right] \right\} \\ \text{subj. to } &\begin{cases} \tilde{x}(k+1) = f_{PWA}(\tilde{x}(k), u(k)) \\ \tilde{x}(N) \in T_{set} \end{cases} \end{aligned} \tag{2}$$

Where  $N$  is the prediction horizon, the input sequence  $U_0^{N-1} \triangleq [u(0)^T, \dots, u(N-1)^T]^T \in \mathbb{R}^{mN}$  is the optimizer variable,  $T_{set} \in \mathbb{R}^{n+l}$  is a compact terminal set,  $p \in \{1, 2, \infty\}$  defines the norm type of the objective function and  $Q_N, Q, R$  are weight matrices with full rank.

The above optimization problem can be solved by transforming it into a multi-parametric program. If  $p=1$  or  $p=\infty$ , it can be converted into multi-parametric linear program. If  $p=2$ , it can be converted into multi-parametric quadratic program. As linear program needs less amount of calculations and a higher speed of computation compared with quadratic program,  $\infty$ -norm is chosen as the norm of the objective function in this paper.

According to Bellman's optimality principle, the optimal control problem (2) can be transformed into an equivalent dynamic program (DP) as follows [23,25]:

$$\begin{aligned} J_j^*(\tilde{x}(j)) &= \min_{u(j)} \left\{ \left\| u(j) - u_r \right\|_\infty^R + \left\| \tilde{x}(j) - \tilde{x}_r \right\|_\infty^{Q_j} + \right. \\ &\left. J_{j+1}^*(\tilde{x}(j+1)) \right\} \\ \text{subj. to } &\begin{cases} \tilde{x}(j+1) = f_{PWA}(\tilde{x}(j), u(j)) \\ \tilde{x}(j+1) \in T_{j+1} \end{cases} \end{aligned} \tag{3}$$

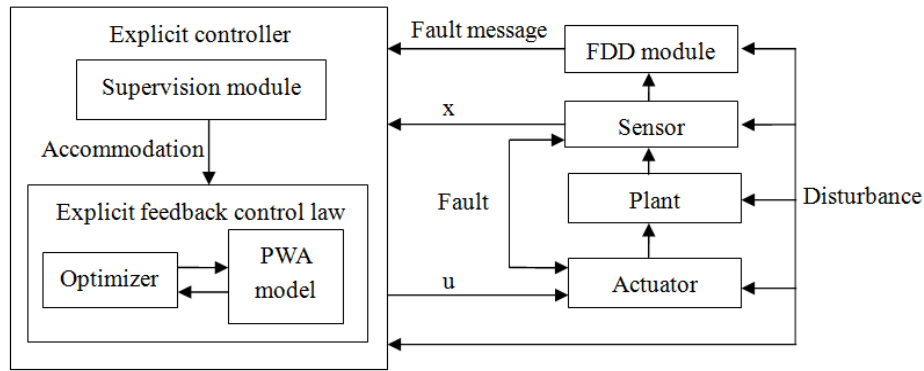


Fig. 3. Structure of the active fault-tolerant control system based on explicit model predictive control approach

Where  $J_N^*(\tilde{x}(N)) = \left\| \left( \tilde{x}(N) - \tilde{x}_r \right) \right\|_{\infty}^{Q_N}$ ,  $T_N = T_{set}$ . For each  $j$  ( $j=1, \dots, N-1$ ),  $T_j = \left\{ \tilde{x}(j) \in \mathbb{R}^{n+l} \mid \exists u(j), f_{PWA}(\tilde{x}(j), u(j)) \in T_{j+1} \right\}$  is the set of all states which make the problem (3) feasible. If we utilize an inverse-order-solving method to solve the above DP problem, for each iteration step, it can be converted into several problems with the form given by the following:

$$J^*(\tilde{x}) = \min_z \left\{ J(z, \tilde{x}) = f^T z \right\}$$

*subj. to*  $Gz \leq E\tilde{x} + W$

Where  $\tilde{x} = \tilde{x}(k)$ .  $f, G, E$  and  $W$  are, respectively, suitable constant matrices easily obtained from  $Q, R$ . It is essentially a multi-parametric linear program if  $\tilde{x}$  is treated as parameters and  $z$  as the optimization vector. According to [24,25], the solutions to the above multi-parametric linear programs have a PWA form:

$$u^*(\tilde{x}(k)) = F_i^k \tilde{x}(k) + G_i^k, \text{ if } \tilde{x}(k) \in P_i^k, i=1, \dots, N^k$$

Where  $P_i^k$  is a polyhedral partition of the set of feasible states  $\tilde{x}(k)$  including system states and fault states and  $N^k$  is the number of  $P_i^k$  at each iteration step  $k=0, \dots, N-1$ . Then, an explicit active fault-tolerant controller is obtained by  $k=0$ . The whole process of designing the controller can be done offline.

For online computation, it just needs to decide the position of the current state in the controller partition and then evaluate the corresponding piecewise affine function. If faults of the system occur, it only requires to change the controller parameters with the fault type which is detected by the FDD module. The architecture of the proposed active fault-tolerant scheme can be depicted in Figure 3. Note that as the system faults which are treated as additional states increase the state dimension, the explicit controller may have a large numbers of partitions and that

can lead to bad real-time performance. Countering this problem, a bounding box search tree method is proposed. The algorithm requires three steps. First, a bounding box search tree is constructed according to [28]. Secondly, traverse the tree from the root node to a leaf node to find partitions possibly containing the current state, and then search among these candidate partitions sequentially to determine the exact partition. Thirdly, evaluate the corresponding piecewise affine function to obtain the optimal control input and apply it to the system. By this way, the speed of the online calculation is significantly improved at the cost of a low additional memory storage demand and a very short pre-computation time.

In the proposed active fault-tolerant control method, the relationship between the states (including intrinsic states and fault states) and the input is explicit, so it doesn't need to repeatedly solve optimization problem online even in the fault condition. It also ensures the closed loop stability via choosing the proper design conditions, such as terminal set, prediction horizon and weight matrices. Moreover, the supervision module is well-placed to be embedded in the explicit controller which makes the whole system more simple and applicable.

#### IV. SIMULATION RESULTS

The CSTR system needs to work at different operating points in order to produce necessary products. In this paper, we choose  $x_{s_1}$  as the operating point and make the system ultimately work at this steady point from an arbitrary initial state even in the condition of actuator or sensor fault by using the proposed active fault-tolerant method.

Here actuator faults mainly cause a change of the coolant temperature range to the range where the actuator is still working. If this kind of fault occurs, the FDD module will pass the new range of the coolant temperature to the controller which subsequently makes a corresponding response to accommodate and recover the faulty system. It supposes the system initial state is  $x_0 = (0.3, 4.5)$ ,  $\theta_1$  is the upper bound of the actuator and  $\theta_2$  is the lower bound. Add  $f = [\theta_1 \ \theta_2]^T$  to the system states as additional states and then establish a fault model. When system runs without faults,  $\theta_1 = 2$  and  $\theta_2 = -2$ , that is to say  $u \in [-2, 2]$ . The simulation results are shown in Figure 4 and Figure 5.

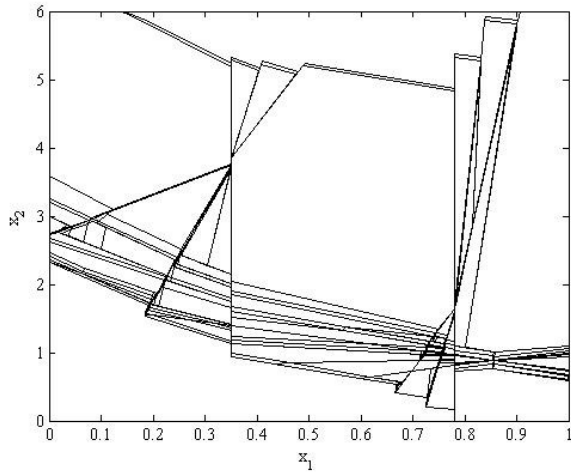


Fig. 4. Projection of the controller partition on  $(x_1, x_2)$  plane cut through  $\theta_1 = 2, \theta_2 = -2$

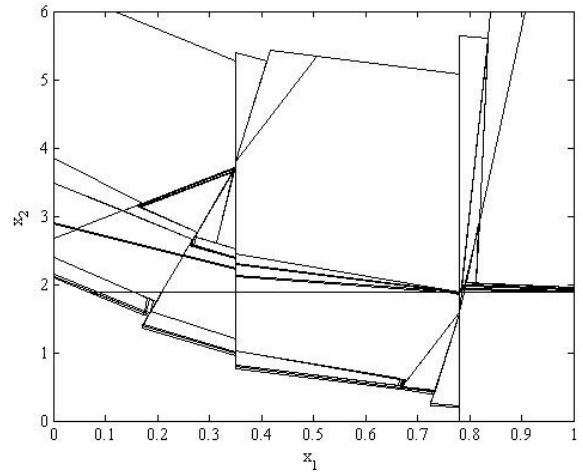


Fig. 7. Projection of the controller partition on  $(x_1, x_2)$  plane cut through  $\theta_1 = 1, \theta_2 = -1$

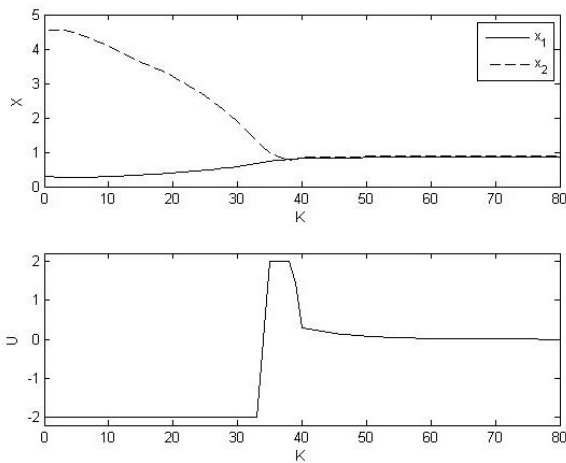


Fig. 5. Evolution of the states and input in the normal condition

If the actuator faults occur during operation, we suppose they are detected by the FDD module at sample time  $k=10, k=20$  and  $k=22$ , which cause the coolant temperature range to change to  $[-1.5, 1.5], [-1, 1]$  and  $[-0.5, 0.5]$  respectively. The simulation results are shown in Figure 6-9.

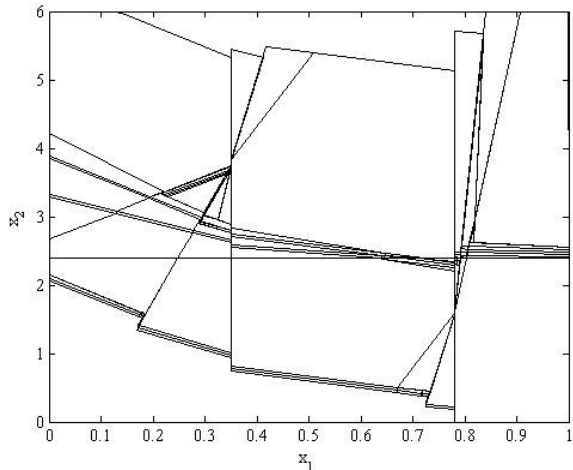


Fig. 6. Projection of the controller partition on  $(x_1, x_2)$  plane cut through  $\theta_1 = 1.5, \theta_2 = -1.5$

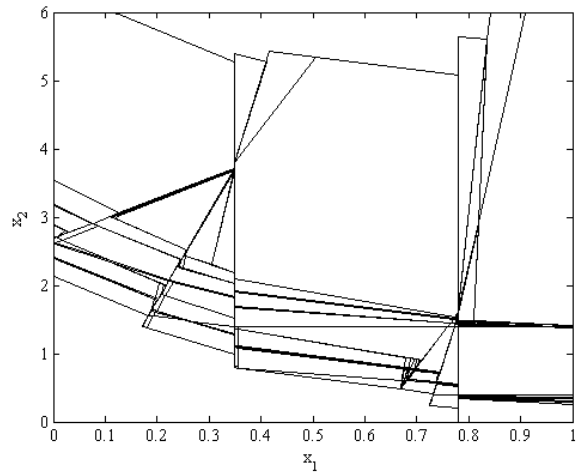


Fig. 8. Projection of the controller partition on  $(x_1, x_2)$  plane cut through  $\theta_1 = 0.5, \theta_2 = -0.5$

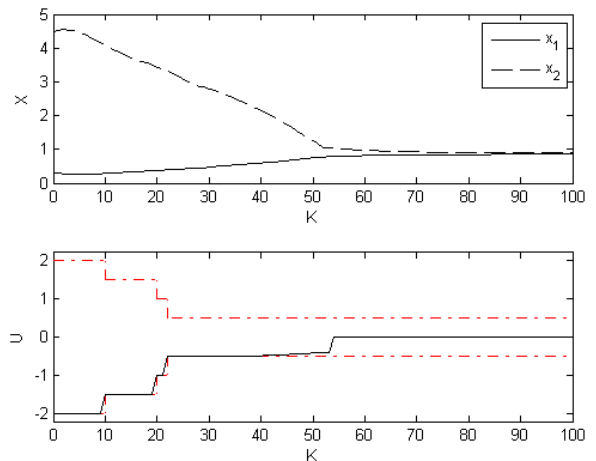


Fig. 9. Evolution of the states and input under the actuator fault condition. The red dash dot line represents real-time constraints of the input during the whole control process.

The explicit active fault-tolerant controller has 6049 polyhedral partitions with 94 different control laws. Figure 4 and Figure 6-8 show projections of the controller partition on two states  $(C_A, T)$  cutting through  $(\theta_1, \theta_2) = (2, -2), (\theta_1, \theta_2) = (1.5, -1.5), (\theta_1, \theta_2) = (1, -1)$  and  $(\theta_1, \theta_2) = (0.5, -0.5)$  respectively. Figure 5 shows the system can

ultimately work at the steady point from the initial state and the whole regulation process is rather fast and smooth. Figure 9 shows the controller can quickly make corresponding remedial actions to correct and recover the system with multiple actuator faults occurrence at a short interval and ensure the control objective is still achieved. In this situation, the whole online control process takes only 3.49 seconds, that is to say, merely needs 0.0349 seconds on average at each sampling time.

The sensor fault is similar to the actuator fault. It mainly causes a change of the system state range to the range where the sensor is still working. We assume the sensor is insensitive to the value below  $\theta_3$  due to the fault. Add  $f = \theta_3$  to the system states as additional states to build a fault model. If the system runs without fault from  $x_0 = (0.3, 2.5)$ , the simulation result is shown in Figure 10(a). While if that kind of sensor fault occurs at  $k = 5$ , which leads to the lower bounds of the states increasing 0.4, the simulation results are shown in Figure 10(b) and Figure 11. It can be seen the system states have reached the desired states without violating constraints. In this situation, the whole online control process takes only 2.51 seconds, that is to say, merely needs 0.0251 seconds on average at each sampling time.

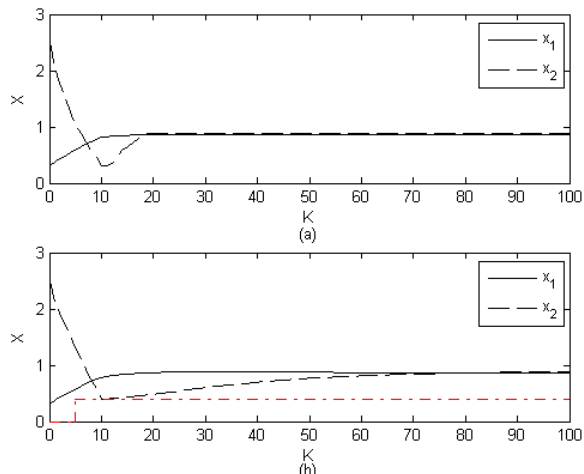


Fig. 10. Evolution of the states under the normal and sensor fault condition. The red dash dot line represents real-time lower bounds of state constraints during the whole control process.

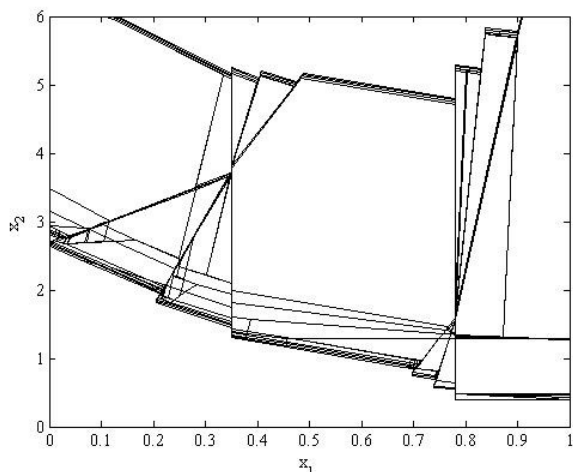


Fig. 11. Projection of the controller partition on  $(x_1, x_2)$  plane cut through  $\theta = 0.4$

## V. CONCLUSION

In this paper, an active fault-tolerant control algorithm for a CSTR system has been researched in detail. System faults are unified in the framework of the system model by treating them as additional states and the controller is designed by using an explicit model predictive control approach. The simulation results show the system states evolve from the initial state to the steady state rapidly and smoothly in both the normal and fault conditions. It demonstrates the effective of the proposed method and also reflects it is excellent in real-time performance. However, as the number of the controller partitions increases with the system state dimension, it may make the optimization problem unfeasible if there are too many considered faults and limits the application of this method. More efforts are required to solve this problem.

## ACKNOWLEDGMENT

The authors thank the reviewers for their valuable comments to improve on this paper. And this work was supported by Shandong Provincial Natural Science Foundation of China (Grant No. 2013ZRE28089).

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