Boundedness Character of a Symmetric System of Max-type Difference Equations

Chang-you Wang, Hao Liu, Rui Li*, Xiao-hong Hu and Ya-bin Shao

***Abstract*—This paper is concerned with the boundedness of the following symmetric system of max-type difference equations**

$$x_{n+1}=\max\{c,\frac{y_n^p}{x_{n-1}^q}\},\quad y_{n+1}=\max\{c,\frac{x_n^p}{y_{n-1}^q}\},\quad n\in N_0,$$

where $N_0=N\bigcup\{0\}$, the parameters $c,p,q\in(0,\infty)$ and the initial conditions x_{-1},x_0,y_{-1},y_0 are arbitrary positive real numbers.

***Index Terms*—max-type system, difference equations, boundedness.**

I. INTRODUCTION

Difference equation appear naturally as discrete analogues and as numerical solutions of differential and delay differential equations, which have been applied in biology, ecology, physics, and so forth (see, [1-7]). Many researchers have studied the asymptotic behavior of the difference equation, for example, in [8, 9] and relevant references cited therein. Recently, the scholars have begun to pay more attention on the studying of so-called max-type difference equations. In the initial study, experts focused on studying the behavior of the following difference equation

$$x_n=\max\{\frac{A_n^{(1)}}{x_{n-1}},\frac{A_n^{(2)}}{x_{n-2}},\cdots,\frac{A_n^{(k)}}{x_{n-k}}\},\quad n\in N_0,\qquad (1)$$

where $k\in N$, $A_n^{(i)}, i=1,2,\cdots,k$, are real sequences (mostly constant or periodic) and the initial values $x_{-1},x_{-2},\cdots,x_{-k}$ are different from zero (see, [10, 11], as well as the references therein).

Elsayed et al. [12] have proved that every positive solution of the following third-order nonautonomous max-type difference equation

$$x_{n+1}=\max\{\frac{A_n}{x_n},x_{n-2}\}\qquad (2)$$

is periodic with period three when A_n is a three-periodic sequence of positive numbers.

In [13], Xiao et al. have shown that every well-defined solution of the following difference equation

$$x_{n+1}=\max\{\frac{\beta}{x_n},x_{n-1}\},\ n\in N_0\qquad (3)$$

is eventually periodic with period two, where the initial conditions x_{-1},x_0 are arbitrary non-zero real numbers and $\beta\in R$.

In 2008, S. Stević [14] proposed some open problem and suggested investigation of positive solutions to the following difference equation

$$x_n=\max\{B_n^{(0)},B_n^{(1)}\frac{x_{n-p_1}^{r_1}}{x_{n-q_1}^{s_1}},B_n^{(2)}\frac{x_{n-p_2}^{r_2}}{x_{n-q_2}^{s_2}},\cdots,B_n^{(k)}\frac{x_{n-p_k}^{r_k}}{x_{n-q_k}^{s_k}}\},\ n\in N_0\qquad (4)$$

where p_i,q_i are natural numbers such that $p_1<p_2<\cdots<p_k$, $q_1<q_2<\cdots<q_k$, $r_i,s_i\in R_+$, $B_n^{(i)}$ is a sequence of positive numbers, $i=1,2,\cdots,k$ and $k\in N$.

As a special case of Equation (4), S. Stević studied the boundedness character of positive solutions to the following max-type difference equation

$$x_n=\max\{A,\frac{x_{n-1}^p}{x_{n-k}^r}\},\quad n\in N_0,\qquad (5)$$

where $k\in N\backslash\{1\}$, the parameters A and r are positive and p is a nonnegative real number (see, [15]).

In view of a natural extension of the model (5), S. Stević continuously investigated the behavior of positive solutions to the following max-type system of differences

$$x_{n+1}=\max\{c,\frac{y_n^p}{x_{n-1}^p}\},\quad y_{n+1}=\max\{c,\frac{x_n^p}{y_{n-1}^p}\},\quad n\in N_0,\qquad (6)$$

where the parameters c and p are positive real numbers. And who proved that all positive solutions of system (6) are bounded when $p\in(0,4)$ and so forth (see [16]). In addition, related research can also be seen in papers [17-20] and the references therein.

In this paper, based on the idea of works [14-16], we study the boundedness character of the following max-type difference equations

$$x_{n+1}=\max\{c,\frac{y_n^p}{x_{n-1}^q}\},y_{n+1}=\max\{c,\frac{x_n^p}{y_{n-1}^q}\},\quad n\in N_0\qquad (7)$$

where $c,p,q\in(0,+\infty)$ and the initial conditions x_{-1},x_0,y_{-1}, y_0 are arbitrary positive real numbers.

II. BOUNDEDNESS CHARACTER OF SOLUTIONS

In this section, we will analyze the boundedness of the

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positive solutions to system (7).

Theorem 1. Assume that $f(\lambda)=\lambda^2-p\lambda+q$ and (a) there is $\lambda_1>1$ such that $f(\lambda_1)=0$, or (b) there is $\lambda_1=\lambda_2=1$ such that $f(\lambda_1)=f(\lambda_2)=0$, then the system (7) has positive unbounded solutions with the positive initial conditions x_{-1}, y_{-1}, x_0, y_0 such that $x_0y_0>x_{-1}y_{-1}>0$.

Proof. Obviously, from (7), we can easily see that

$$x_{n+1}\geq\frac{y_n{}^p}{x_{n-1}{}^q},\quad y_{n+1}\geq\frac{x_n{}^p}{y_{n-1}{}^q}. \tag{8}$$

By taking logarithm in (8), for any $n\in N_0$, we obtain

$$\ln x_{n+1}\geq p\ln y_n-q\ln x_{n-1},\ \ln y_{n+1}\geq p\ln x_n-q\ln y_{n-1}. \tag{9}$$

Moreover, it follows that

$$\ln x_{n+1}y_{n+1}\geq p\ln x_ny_n-q\ln x_{n-1}y_{n-1},\ n\geq -1. \tag{10}$$

Let $z_n=\ln x_ny_n$, then inequality (10) becomes

$$z_{n+1}\geq pz_n-qz_{n-1},\ n\in N_0. \tag{11}$$

By hypothesis (a), we have that $f(\lambda_1)=0$ and $\lambda_1>1$.

Let

$$f_1(\lambda)=\frac{f(\lambda)}{\lambda-\lambda_1}=\lambda+a, \tag{12}$$

then it follows that

$$f(\lambda)=(\lambda+a)(\lambda-\lambda_1). \tag{13}$$

Comparing Eq. (12) with Eq. (13), we can obtain $a=\lambda_1-p$ and $\mathrm{q}=-a\lambda_1$.

Set

$$u_n=z_n+az_{n-1},\ n\in N_0. \tag{14}$$

Then inequation (11) can be written in the following form

$$\begin{aligned} z_{n+1}-pz_n+qz_{n-1}&=z_{n+1}-(\lambda_1-a)z_n+a\lambda_2z_{n-1}\\ &=z_{n+1}+az_n-\lambda_1(z_n+az_{n-1})\\ &=u_{n+1}-\lambda_1u_n\\ &\geq 0. \end{aligned} \tag{15}$$

That is

$$u_{n+1}\geq\lambda_1u_n. \tag{16}$$

Let z_{-1}, z_0 be chosen such that

$$z_0\geq|a||z_{-1}| \tag{17}$$

This, along with (16), yields to

$$u_{n+1}\geq\lambda_1^nu_0,\ \text{and}\ u_0>0. \tag{18}$$

Letting $n\to\infty$ in (18), from assumption $\lambda_1>1$ and $u_0>0$, it follows that

$$u_n=z_n+az_{n-1}\to+\infty\quad\text{as}\quad n\to+\infty. \tag{19}$$

Hence $\{z_n\}_{n\geq-1}$ is unbounded. As $z_n=\ln x_ny_n$, it follows that

$$x_ny_n\to\infty\quad\text{as}\quad n\to\infty, \tag{20}$$

which along with $x_n^2+y_n^2\geq 2x_ny_n$ implies

$$\sqrt{x_n^2+y_n^2}\to+\infty, \tag{21}$$

from which it follows that the sequence $\{(x_n,y_n)\}_{n\geq-1}$ is unbounded.

By hypothesis (b), we have $p=2, q=1$. Then from (8) we get

$$\frac{x_{n+1}}{y_n}\geq\frac{y_n}{x_{n-1}},\quad\frac{y_{n+1}}{x_n}\geq\frac{x_n}{y_{n-1}}. \tag{22}$$

Moreover, by iterative method one has

$$\frac{x_{n+1}y_{n+1}}{y_nx_n}\geq\frac{y_nx_n}{x_{n-1}y_{n-1}}\geq\cdots\geq\frac{y_0x_0}{x_{-1}y_{-1}},\ n\in N_0. \tag{23}$$

And consequently

$$x_ny_n\geq(\frac{y_0x_0}{x_{-1}y_{-1}})^nx_0y_0,\ n\in N_0. \tag{24}$$

If we choose the initial conditions x_{-1}, y_{-1}, x_0, y_0 such that $x_0y_0>x_{-1}y_{-1}>0$, then we obtain (20) and (21), which implies that the sequence $\{(x_n,y_n)\}_{n\geq-1}$ is unbounded. The proof of the theorem is finished.

Next, we study the different cases concerning with the boundedness of positive solutions to the system (7).

Theorem 2. If $c>0$, $p>0$ and $p^2<4q$, then all positive solutions to system (7) are bounded.

Proof. Assume that $(x_n,y_n)_{n\geq-1}$ is a positive solution to system (7). Then the following estimate obviously holds

$$\min\{x_n,y_n\}\geq c,\ n\in N_0. \tag{25}$$

Due to the symmetry between $\{x_n\}$ and $\{y_n\}$, as long as we prove the boundedness of $\{x_n\}$, another sequence $\{y_n\}$ can be proved as well.

From system (7) and iterative method, it follows that

$$x_{n+1}=\max\{c,\frac{y_n^p}{x_{n-1}^q}\}=\max\{c,\frac{c^p}{x_{n-1}^q},\frac{x_{n-1}^{p^2-q}}{y_{n-2}^{pq}}\},n\in N_0. \tag{26}$$

Case1. When $p^2\leq q$, we get

$$x_{n+1}\leq\max\{c,\frac{1}{c^{q-p}},\frac{1}{c^{pq-p^2+q}}\}. \tag{27}$$

Thus, the sequence $\{x_n\}_{n\geq-1}$ is bounded.

Case2. When $p^2>q$, let sequence $\{a_l\}_{l\geq0}$ be defined as follows

$$a_{l+1}=q/(p-a_l),\ a_0=0,\ l\in N_0. \tag{28}$$

From (7), (28) and iterative method, we have

$$\begin{aligned} x_{n+1}&=\max\{c,\frac{y_n^p}{x_{n-1}^q}\}=\max\{c,\frac{c^p}{x_{n-1}^q},\frac{x_{n-1}^{p^2-q}}{y_{n-2}^{pq}}\}\\ &=\max\{c,(\frac{c}{x_{n-1}^{q/p}})^p,(\frac{x_{n-1}}{y_{n-2}^{q/(p-q/p)}})^{(p-q/p)p}\}\\ &=\max\{c,(\frac{c}{x_{n-1}^{q/p}})^p,(\frac{c}{y_{n-2}^{q/(p-q/p)}})^{(p-q/p)p},(\frac{y_{n-2}^{p-q/(p-q/p)}}{x_{n-3}^q})^{(p-q/p)p}\}\\ &=\cdots\cdots\\ &=\max\{c,(\frac{c}{x_{n-1}^{q/p}},(\frac{c}{y_{n-2}^{q/(p-q/p)}},\cdots,(\frac{y_{n-2k}^{p-a_{2k}}}{x_{n-(2k+1)}^q})^{p-a_{2k-1}},\cdots)^{(p-q/p)})^p\}\\ &=\max\{c,(\frac{c}{x_{n-1}^{q/p}},(\frac{c}{y_{n-2}^{q/(p-q/p)}},\cdots,(\frac{c^{p-a_{2k}}}{x_{n-(2k+1)}^q},\frac{x_{n-(2k+1)}^{(p-a_{2k})p-q}}{y_{n-(2k+1)}^q})^{p-a_{2k-1}},\cdots)^{(p-q/p)})^p\}\\ &=\max\{c,(\frac{c}{x_{n-1}^{q/p}},(\frac{c}{y_{n-2}^{q/(p-q/p)}},\cdots,(\frac{x_{n-(2k+1)}^{p-a_{2k+1}}}{y_{n-(2k+2)}^q})^{p-a_{2k}},\cdots)^{(p-q/p)})^p\}. \end{aligned} \tag{29}$$

From the monotonicity of $g(x)=q/(p-x)$ on the interval $(0,p)$ along with the fact $0=a_0<a_1=q/p$, it follows that the sequence $\{a_l\}$ is increasing as far as $a_l\leq p$ for every $l\in \mathrm{N}_0$. Hence, we have $\lim_{l\to+\infty}a_l=x^*$, $x^*\in(0,p]$

and x^* is the solution of the following equation

$$f(x) = x(p-x) - q = 0 . \tag{30}$$

However, the equation (30) has no real roots existing in $(0, p]$ when $p^2 < 4q$, which is contradiction. Hence there is $l_0 \in N_0$ such that $a_{l_0-1} < p$ and $a_{l_0} \geq p$.

If $l_0 = 2k$, using (2.25) in (2.29), it follows that

$$x_{n+1} = \max\{c, (\frac{c}{x_{n-1}^{q/p}}, (\frac{c}{y_{n-2}^{q/(p-q/p)}}, \cdots, (\frac{y_{n-2k}^{p-a_{2k}}}{x_{n-(2k+1)}^{q}})^{p-a_{2k-1}}, \cdots)^{(p-q/p)})^{p}\}$$

$$\leq \max\{c, (\frac{c}{c^{a_1}}, (\frac{c}{c^{a_2}}, \cdots, (\frac{1}{c^{q-p+a_{2k}}})^{p-a_{2k-1}}, \cdots)^{(p-a_1)})^{p}\}. \tag{31}$$

For $n \geq 2k+2$, from which the boundedness of $\{x_n\}_{n\geq -1}$ follows in this case.

If $l_0 = 2k+1$, it follows that

$$x_{n+1} = \max\{c, (\frac{c}{x_{n-1}^{q/p}}, (\frac{c}{y_{n-2}^{q/(p-q/p)}}, \cdots, (\frac{y_{n-2k}^{p-a_{2k+1}}}{x_{n-(2k+2)}^{q}})^{p-a_{2k}}, \cdots)^{(p-q/p)})^{p}\}$$

$$\leq \max\{c, (\frac{c}{c^{a_1}}, (\frac{c}{c^{a_2}}, \cdots, (\frac{1}{c^{q-p+a_{2k+1}}})^{p-a_{2k}}, \cdots)^{(p-a_1)})^{p}\}. \tag{32}$$

For $n \geq 2k+3$, from which the boundedness of $\{x_n\}_{n\geq -1}$ follows in this case.

Combined case 1 $p^2 \leq q$ and case 2 $q < p^2 < 4q$, we obtain that the sequence $\{x_n\}_{n\geq -1}$ is bounded when $p^2 < 4q$. In the same way, we can prove that the sequence $\{y_n\}$ is bounded as well. Hence, every solution to system (7) is bounded when $p^2 < 4q$.

Theorem 3. Assume that $c > 0$, $q > 0$ and $p = 1$, then the solutions of system (7) are bounded.

Proof. Assume that $\{(x_n, y_n)\}$ is any positive solution of system (7) in particular $p = 1$. We can easily know that $x_n \geq c, y_n \geq c$. Therefore, we have

$$x_{n+1} \leq \max\{c, \frac{y_n}{c^q}\},\ y_{n+1} \leq \max\{c, \frac{x_n}{c^q}\}, \quad n \in N_0 . \tag{33}$$

From the above (33), it follows that

$$x_{n+1} \leq \max\{c, \frac{y_n}{c^q}\} \leq \max\{c, \frac{c}{c^q}, \frac{x_{n-1}}{c^{2q}}\}, n \in N_0 . \tag{34}$$

Set

$$z_{n+1} = \max\{c, \frac{c}{c^q}, \frac{z_{n-1}}{c^{2q}}\}, n \in N_0, z_0 = x_0, z_{-1} = x_{-1} . \tag{35}$$

Assume that $\{z_n\}$ is the solution to (35), then z_n is greater than x_n for any n>2 by using (34) and iterative method.

Case 1. $c > 1$.

(a). If $z_{-1} \leq c^{2q+1}$ and $z_0 \leq c^{2q+1}$, from (35), we can obtain that $c^q > 1$ and $\frac{z_{n-1}}{c^{2q}} < c$, so $z_2 = c, z_4 = c, z_6 = c, \cdots$, which implies that $z_{2n} = c$. Moreover, $z_1 = c$, $z_3 = c$, $z_5 = c$, which implies that $z_{2n+1} = c$. Hence, the boundedness of $\{z_n\}_{n\geq -1}$ follows in this case.

(b). If $z_{-1} > c^{2q+1}$ and $z_0 > c^{2q+1}$, from (35), we can obtain that $A < z_4 < z_2 < z_0$. Through iteration, we can get that $\{z_{2n}\}$ is monotonically decreasing. Additionally, $z_n \geq c$ for any $n = 2k, k \in N$, we can obtain that $\{z_{2n}\}$ is bounded. Similarly, $\{z_{2n-1}\}$ is bounded as well. Hence, the boundedness of $\{z_n\}_{n\geq -1}$ follows in this case.

(c). If $z_{-1} \leq c^{2q+1}$ and $z_0 > c^{2q+1}$, from above proof, we can obtain that $z_{2n-1} = c$ and $\{z_{2n}\}$ is monotonically decreasing. Additionally, $z_n \geq c$ for any $n = 2k, k \in N$, we can obtain that $\{z_n\}$ is bounded in this case.

(d). If $z_{-1} > c^{2q+1}$ and $z_0 \leq c^{2q+1}$, from above proof, we can obtain that $\{z_{2n-1}\}$ is monotonically decreasing and $z_{2n} = c$. Additionally, $z_n \geq c$ for any $n = 2k-1, k \in N$, we can get that $\{z_n\}$ is bounded in this case.

Due to the boundedness of $\{z_n\}$ and $x_n \leq z_n$, we can obtain the boundedness of $\{x_n\}$. Similarly, $\{y_n\}$ is bounded as well. Hence, every positive solution to system (7) is bounded.

Case 2. $0 < c \leq 1$.

(a) If $q \geq 1$, in fact $x_n \geq c$, from (7) and iterative method it follows that

$$x_{n+1} = \max\{c, \frac{y_n}{x_{n-1}^q}\} = \max\{c, \frac{c}{x_{n-1}^q}, \frac{x_{n-1}}{x_{n-1}^q y_{n-1}^q}\}$$

$$= \max\{c, \frac{c}{x_{n-1}^q}, \frac{1}{x_{n-1}^{q-1} y_{n-1}^q}\} \leq \max\{c, \frac{c}{c^q}, \frac{1}{c^{2q-1}}\} \tag{36}$$

for $n \in N$, which means that $\{x_n\}$ is bounded.

(b) If $0 < q < 1$, let sequence $\{a_l\}_{l\geq 0}$ be defined as follows

$$a_{l+1} = a_l - b_l, b_{l+1} = qa_l, a_1 = 1-q, b_1 = q, \quad l \in N_0 \tag{37}$$

Thus, from (7) and iterative method we have

$$x_{n+1} = \max\{c, \frac{c}{x_{n-1}^q}, \frac{x_{n-1}^{1-q}}{y_{n-2}^q}\} = \max\{c, \frac{c}{x_{n-1}^q}, \frac{x_{n-1}^{a_1}}{y_{n-2}^{b_1}}\}$$

$$= \max\{c, \frac{c}{x_{n-1}^q}, \frac{c^{a_1}}{y_{n-2}^{b_1}}, \frac{y_{n-2}^{a_1-b_1}}{x_{n-3}^{qa_1}}\}$$

$$= \max\{c, \frac{c}{x_{n-1}^q}, \frac{c^{a_1}}{y_{n-2}^{b_1}}, \frac{c^{a_1-b_1}}{x_{n-3}^{qa_1}}, \frac{x_{n-3}^{a_2-b_2}}{y_{n-4}^{qa_2}}\} \tag{38}$$

$$= \cdots$$

$$= \max\{c, \frac{c}{x_{n-1}^q}, \frac{c^{a_1}}{y_{n-2}^{b_1}}, \cdots, \frac{x_{n-l-1}^{a_l-b_l}}{y_{n-l}^{b_{l-1}}}\},$$

for every $l \in N$.

From (37), we can deduce

$$a_{l+1} - a_l + qa_{l-1} = 0, l \in N . \tag{39}$$

It is easy to see that the general solution of difference equation (39) is

$$a_l = c_1\lambda_1^l + c_2\lambda_2^l, c_1, c_2 \in R \tag{40}$$

where $\lambda_{1,2} = (1 \pm \sqrt{1-4q})/2$. The fact $|\lambda_{1,2}| < 1$ along with (40) implies that the sequence $\lim_{l\to+\infty} a_l = 0$. From this and (37) we get $\lim_{l\to+\infty} b_l = 0$.

Now note that from (38) and iterative method it follows that

$$x_{n+1} \le \max\{c, \frac{c}{x_{n-1}^{q}}, \frac{c^{a_1}}{y_{n-2}^{b_1}}, \cdots, \frac{x_0^{a_n}}{y_{-1}^{b_n}}\} \quad (41)$$

or

$$x_{n+1} \le \max\{c, \frac{c}{x_{n-1}^{q}}, \frac{c^{a_1}}{y_{n-2}^{b_1}}, \cdots, \frac{y_0^{a_n}}{x_{-1}^{b_n}}\} \quad (42)$$

The convergence of sequences $\{a_l\}_{l\ge -1}$ and $\{b_l\}_{l\ge -1}$ along with (41)-(42) implies the boundedness of $\{x_n\}_{n\ge -1}$. Since system (7) is symmetric, the boundedness of $\{x_n\}_{n\ge -1}$ imply the boundedness of $\{y_n\}_{n\ge -1}$, as claimed.

III. Numerical Simulations

In this section, some numerical simulations are given to support our theoretical analysis. When the parameters c, p, q take different values, we have the following difference equations

$$x_{n+1} = \max\{1, \frac{y_n^2}{x_{n-1}}\},\ y_{n+1} = \max\{1, \frac{x_n^2}{y_{n-1}}\},\ n \in N_0, \quad (43)$$

$$x_{n+1} = \max\{0.5, \frac{y_n^{0.5}}{x_{n-1}^2}\},\ y_{n+1} = \max\{0.5, \frac{x_n^{0.5}}{y_{n-1}^2}\},\ n \in N_0, \quad (44)$$

$$x_{n+1} = \max\{1.5, \frac{y_n^2}{x_{n-1}^2}\},\ y_{n+1} = \max\{1.5, \frac{x_n^2}{y_{n-1}^2}\},\ n \in N_0, \quad (45)$$

$$x_{n+1} = \max\{0.8, \frac{y_n}{x_{n-1}^2}\},\ y_{n+1} = \max\{0.8, \frac{x_n}{y_{n-1}^2}\},\ n \in N_0, \quad (46)$$

and

$$x_{n+1} = \max\{1.1, \frac{y_n}{x_{n-1}^{0.125}}\},\ y_{n+1} = \max\{1.1, \frac{x_n}{y_{n-1}^{0.125}}\},\ n \in N_0. \quad (47)$$

By employing Matlab R2013b, we solve the numerical solutions of the above equations, which are shown respectively in the following Figures.

More precisely, the initial conditions of (43) are that $x(-1) = 0.5, x(0) = 0.8, y(-1) = 0.8$ and $y(0) = 1.2$. It is easy to show that the equations (43) satisfy the conditions of Theorem 1. Fig.1 shows that the solutions of the equations (43) are unbounded. The initial conditions of (44) are that $x(-1) = 0.2, x(0) = 0.4, y(-1) = 0.8$ and $y(0) = 0.7$. It is not difficult to find that the equations (44) satisfy the conditions of Theorem 2 and $p^2 \le q$. Fig.2 shows that the solutions of the equations (44) are bounded. The initial conditions of equations (45) are that $x(-1) = 0.5, x(0) = 0.8, y(-1) = 1.2$ and $y(0) = 1.8$. It is obvious that equations (45) satisfy the condition of Theorems 2 and $4q > p^2 > q$. Fig.3 shows that the solutions of the equations (45) are bounded. The initial condition of (46) is that $x(-1) = 1.2, x(0) = 1.4, y(-1) = 1.5$ and $y(0) = 1.2$. It is easy to prove that equations (46) satisfy the conditions of Theorems 3 and $c > 1$. Fig.4 shows that the solutions of the equations (46) are bounded. The initial conditions of (47) are that $x(-1) = 1.5, x(0) = 1.2, y(-1) = 0.5$ and $y(0) = 0.8$. It is easy to find that equations (47) satisfy the conditions of Theorem 3 and $0 < c \le 1$. Fig.5 shows that the solutions of the equations (47) are bounded.

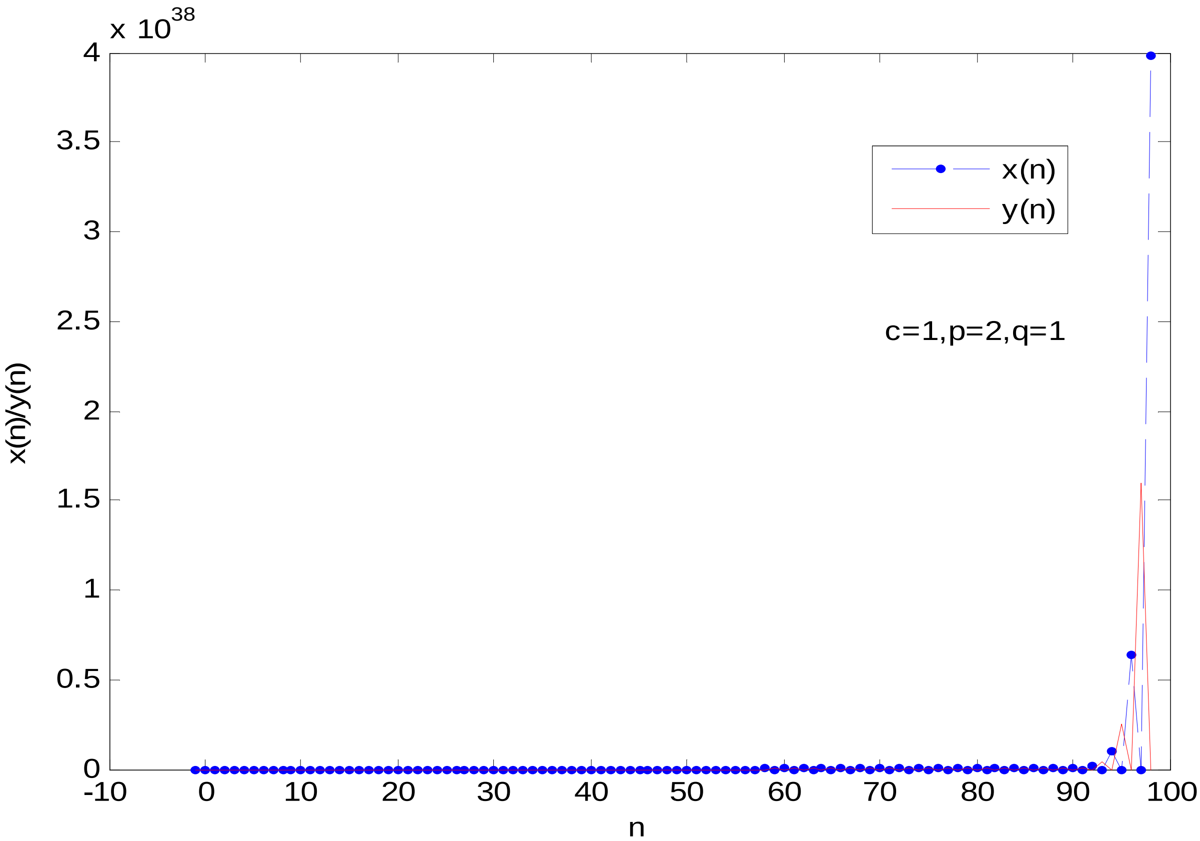

Fig. 1. The solutions of the equation (43) with the initial conditions $x(-1) = 0.5, x(0) = 0.8, y(-1) = 0.8, y(0) = 1.2$

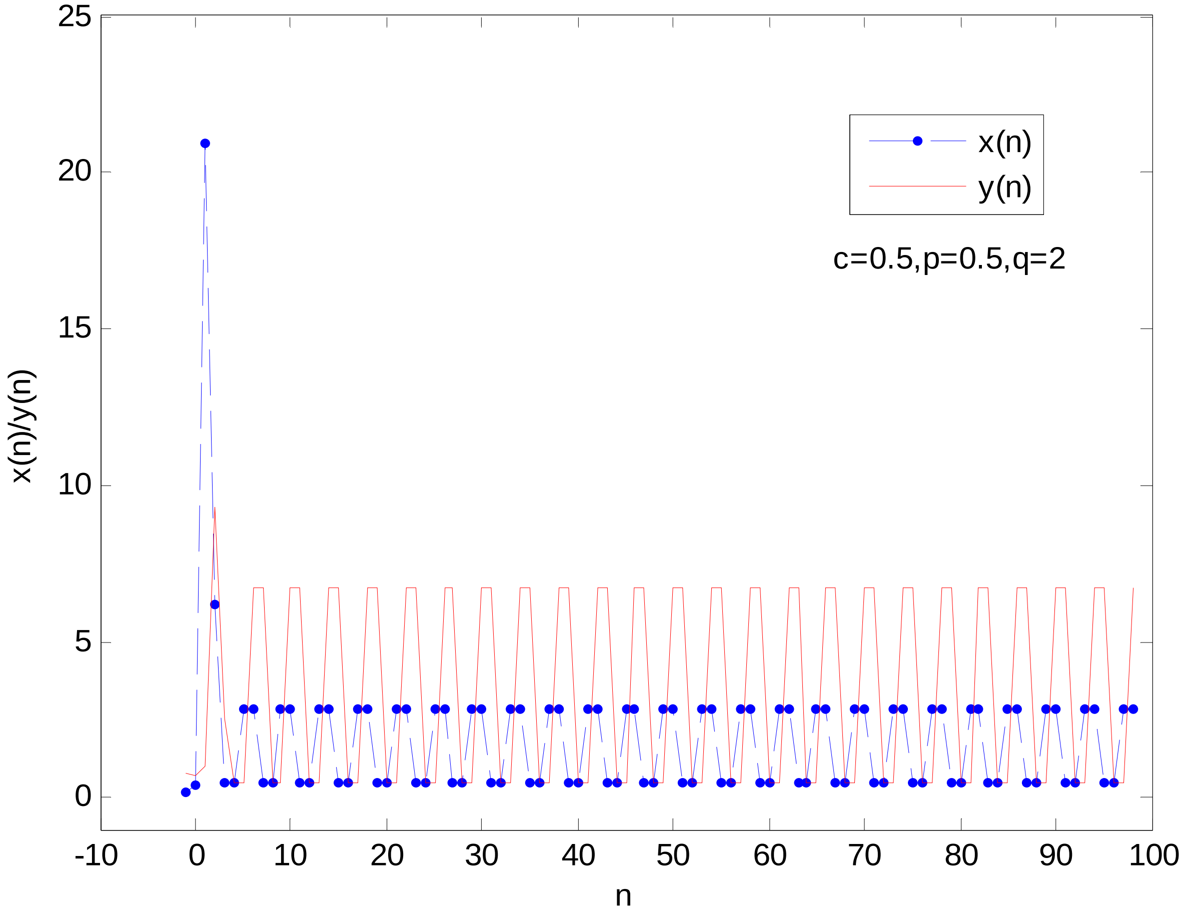

Fig. 2. The solutions of the equation (44) with the initial conditions $x(-1)=0.2, x(0)=0.4, y(-1)=0.8, y(0)=0.7$

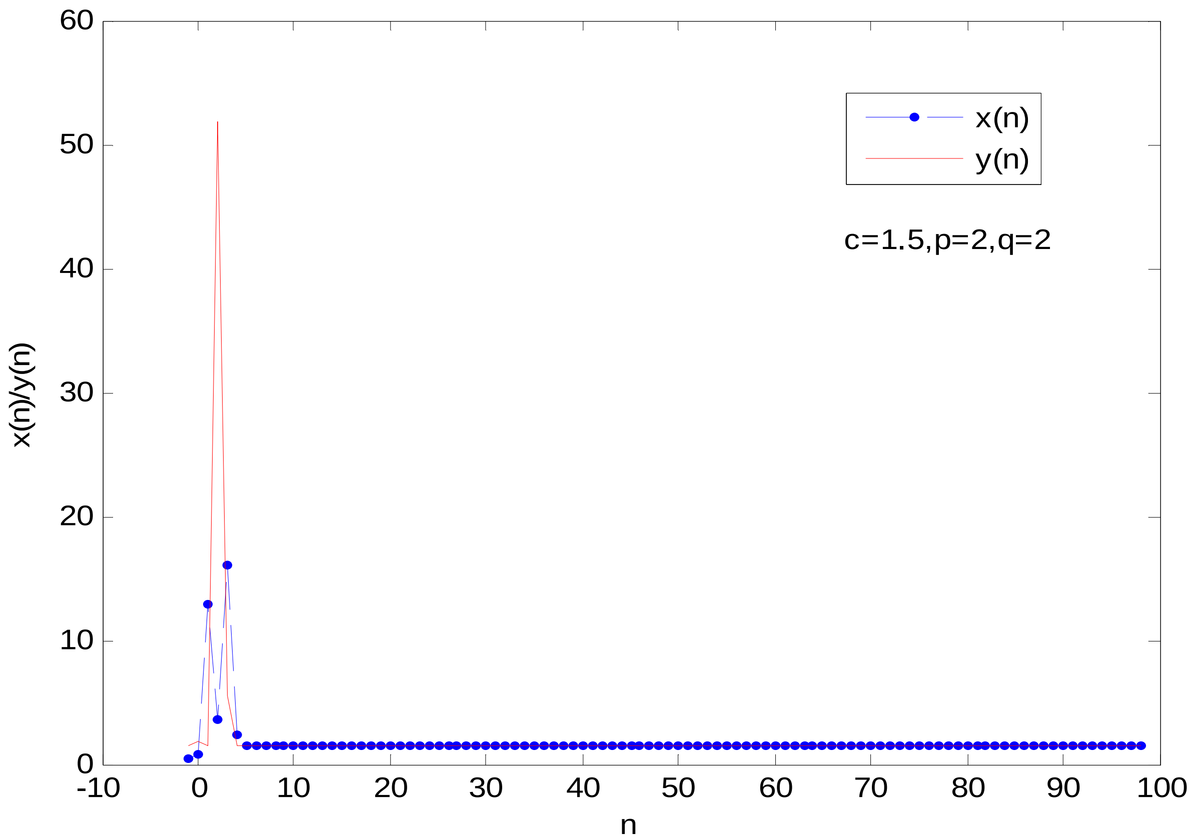

Fig. 3. The solutions of the equation (45) with the initial conditions $x(-1)=0.5, x(0)=0.8, y(-1)=1.2, y(0)=1.8$

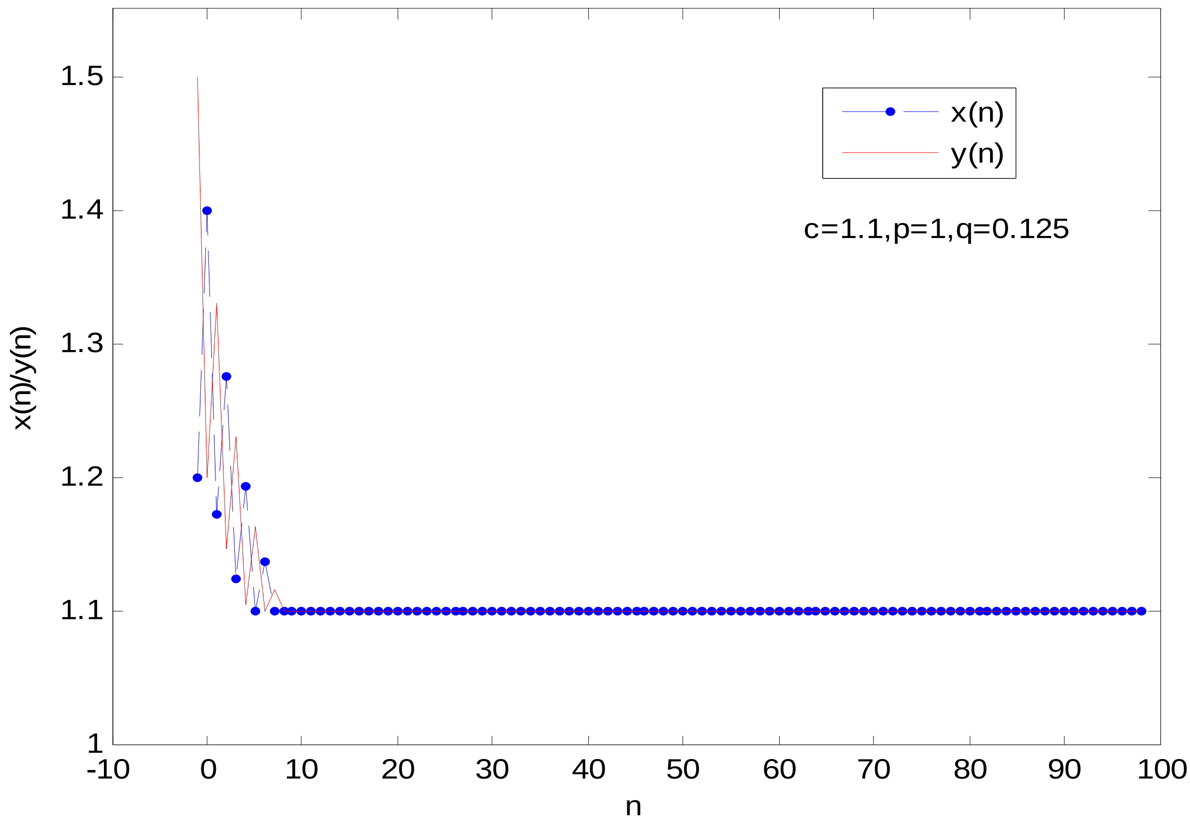

Fig. 4. The solutions of the equation (46) with the initial conditions $x(-1)=1.2$, $x(0)=1.4$, $y(-1)=1.5$, $y(0)=1.2$

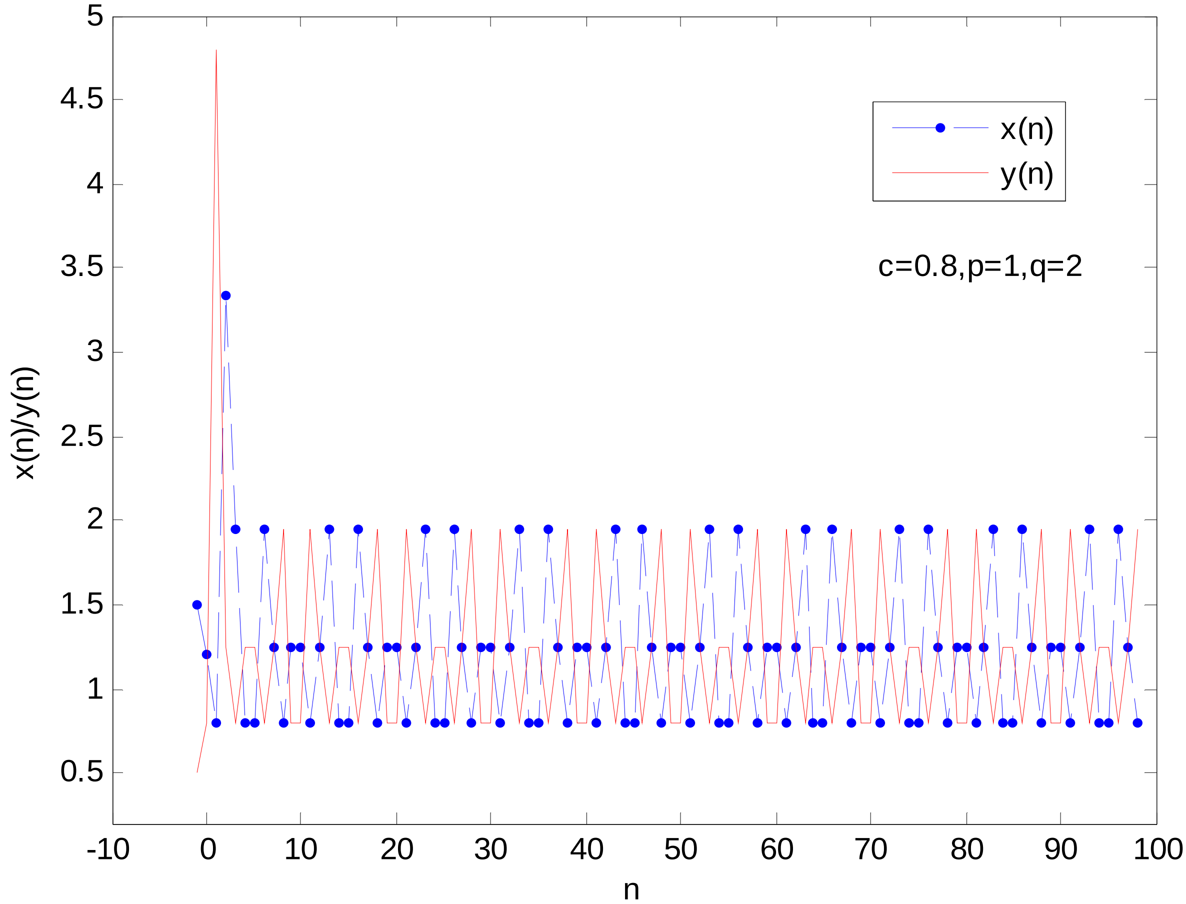

Fig. 5. The solutions of the equation (47) with the initial conditions $x(-1)=1.5, x(0)=1.2$, $y(-1)=0.5$, $y(0)=0.8$

IV CONCLUSIONS

In this paper, we have dealt with the problem of boundedness character for a class of max-type difference system. And we have obtained some sufficient conditions which ensure the boundedness character of the max-type system. The sufficient conditions that we obtained are very simple, which provide flexibility for the application and analysis of max-type difference system. These results generalize and improve some previous works. In addition, we present the use of a new iteration method for symmetric systems of max-type difference equations. This technique is a powerful tool for solving various difference equations and it can be applied to other nonlinear differential equations in mathematical physics. Computations are performed using the software package Matlab R2013b. In particular, some numerical examples are given to show the validity of the obtained theoretic results

REFERENCES

[1] Z. Yan and X. Jia, "Existence of Fractional Stochastic Schrodinger Evolution Equations with Potential and Optimal Controls," IAENG *International Journal of Applied Mathematics*, vol. 46, no.2, pp. 210 -222, 2016

[2] L. Li, and B. Du, "Global Asymptotical Stability in a Stochastic Predator-Prey System With Variable Delays," *IAENG International Journal of Applied Mathematics*, vol. 46, no.2, pp. 241-246, 2016.

[3] D. Jana and E. M. Elsayed, "Interplay between strong Allee effect, harvesting and hydra effect of a single population discrete-time system," *International Journal of Biomathematics*, vol. 9, no. 1, Article ID 1650004, 2016.

[4] M. Kolar, M. Benes, D. Sevcovic, and J. Kratochvil, "Mathematical Model and Computational Studies of Discrete Dislocation Dynamics," *IAENG International Journal of Applied Mathematics*, vol. 45, no.3, pp. 198-207, 2015

[5] J. E. Macias-Diaz, "A positive finite-difference model in the computational simulation of complex biological film models," *Journal of Difference Equations and Applications*, vol. 20, no. 4, pp. 548-569, 2014.

[6] K. Anada and T. Ishiwata, "Some Features for Blow-up Solutions of a Nonlinear Parabolic Equation," *IAENG International Journal of Applied Mathematics*, vol. 45, no.3, pp. 175-182, 2015.

[7] A. Gelisken and C. Cinar, "On the global attractivity of a max-type difference equation," *Discrete Dynamics in Nature and Society*, vol. 2009, Article ID 812674, 5 pages, 2009.

[8] L. X. Hu, W. S. He, and H. M. Xia, "Global asymptotic behavior of a rational difference equation," *Applied Mathematics and Computation*, Vol. 218, no. 15, pp. 7818-7828, 2012.

[9] E. M. Elsayed, "Behavior and Expression of the Solutions of Some Rational Difference Equations," *Journal of Computational Analysis and Applications,* Vol. 15, pp. 73-81, 2013.

[10] E. A. Grove and G. Ladas, *Periodicities in Nonlinear Difference Equations,* Chapman & Hall, CRC Press, 2005.

[11] W.T. Patula, H.D. Voulov, "On a max type recurrence relation with periodic coefficients," *Journal of Difference Equations and Applications,* vol. 10, no. 3, pp. 329-338, 2004.

[12] E. M. Elsayed and B. D. Iricanin, "On a max-type and a min-type difference equation," *Applied Mathematics and Computation*, Vol. 215, no. 2, pp. 608-614, 2009.

[13] Q. Xiao and Q. Shi, "Eventually periodic solutions of a max-type equation," *Mathematical and Computer Modelling*, vol. 57, no. 3-4, pp. 992-996, 2013.

[14] S. Stević, "*Some results on max-type difference equations, The Modelling of Nonlinear Processes and Systems (International Science Conference. Book of abstracts. Moscow)*," Russia, October 14-18, pp. 140, 2008.

[15] S. Stević, "On a nonlinear generalized max-type difference equation," *Journal of Mathematical Analysis and Applications*, Vol. 376, no. 1, pp. 317-328, 2011.

[16] S. Stević, "On a symmetric system of max-type difference equations," *Applied Mathematics and Computation*, Vol. 219, no. 15, pp. 8407-8412, 2013.

[17] F. Sun, "On the asymptotic behavior of a difference equation with maximum," *Discrete Dynamics in Nature and Society*, Vol. 2008, Article ID 243291, 6 pages, 2008.

[18] A. Gelisken and C. Cinar, "On the global attractivity of a max-type difference equation," *Discrete Dynamics in Nature and Society*, vol. 2009, Article ID 812674, 5 pages, 2009.

[19] T. F. Ibrahim and N. Touafek, "Max-type system of difference equations with positive two-periodic sequences," *Mathematical Methods in the Applied Sciences*, Vol. 37, no. 16, pp. 2541-2553, 2014.

[20] M. M. El-Dessoky, "On the periodicity of solutions of max-type difference equation," *Mathematical Methods in the Applied Sciences*, Vol. 38, no. 15, pp. 3295-3307, 2015.