Buyback Contract with Fuzzy Demand and Risk Preference in a Three Level Supply Chain

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***Abstract*—This paper analyzes the buyback contract of a supply chain including one retailer, one distributor and one supplier in a fuzzy decision making environment. The market demand is characterized as a fuzzy variable. To defuzzify the fuzzy number into a crisp one, the weighted possibilistic mean value method is applied, and the risk attitudes of the supply chain members are also considered. The centralized decision-making model and the buyback contract are proposed, and their optimal solutions are also derived. Finally, the impacts of the retail price, risk basic coefficient, and values of the contract parameters are analyzed for illustrating the results of the proposed fuzzy supply chain models with the help of numerical experiments.**

***Index Terms*—supply chain, risk preference, buyback contract, fuzzy demand**

I. Introduction

NOWADAYS, buyback contract has been widely used in practice such as Christmas decorations, seasonal products and in the personal computers industry. In this contract, all firms, which are took part in supply chain, tend to set their order quantities to optimize their total profits of the supply chain system.

In the past ten years, studying the buyback contract with random demand has been investigated by many scholars. For instance, Choi et al. [1] studied the roles of the return policies on the e-marketplace in a two level supply chain. Ding and Chen [2] analyzed a three level supply chain in which the buyback contract was employed to coordinate the chain with one manufacturer, one distributor and one retailer. Chen [3] analyzed the impact of the sharing customer returns information on the buyback contract. Chen and Bell [4] proposed the customer returns policies to coordinate the dual-channel supply chain. Zhao et al. [5] invested the effect of demand uncertainty on the order quantity and wholesale price of the buyback contract, and compared it with the conditions of no buyback policies. Xu et al. [6] proposed a buyback contract for determining the pricing policy, ordering policy and return deadline in a newsboy setting. Some researches also studied the buyback contract in a price-dependent stochastic demand setting, where the demand is a function of retail price in the additive or multiplicative model. For instance, Yao et al. [7] analyzed the effects of price-sensitivity factors on the optimal solutions of the buyback contract and the Stackelberg game was employed to solve the models. Gurnani et al. [8] studied the use of the return policies including partial returns, no returns and full returns. Chen and Bell [9] proposed a return policy for coordinating a decentralized supply chain in this setting. Arcelus et al. [10] also studied the buyback policies in this setting, and they mainly concentrated on the risks of demand uncertain. In addition, Ai et al. [11] proposed a full buyback policy when two supply chains competed with each other in an uncertain demand setting. Wu [12] also used a buyback contract to coordinate the competing supply chains, where the vertical integration model and manufacturer's Stackelberg game were provided to solve the problems. Huang et al. [13] invested a buyback contract for coordinating a chain with one supplier and many competing retailers with a secondary market.

The risk attitudes of the actors were also considered in the buyback contract with random demand. Choi et al. [14] discussed the profit and risk sharing problem by using the mean-variance method under a returns policy. Hsieh and Lu [15] proposed a buyback contract for coordinating a supply chain with one manufacturer and two risk-averse retailers in a random demand environment. Yoo [16] also discussed the risk preference problem in a buyback contract, where they considered the supplier's two different risk attitudes including risk averse and risk neutral.

Recently, fuzzy set theory has been applied to solve the supply chain coordination mechanism problems, where the demands are defined as fuzzy variables. Yu and Jin [17] adopt signed distance method to study the buyback contract, where they considered the demand and the retail price as the triangular fuzzy numbers. Yu et al. [18] also studied the fuzzy newsboy model with return policies in a price-dependent demand environment. Chang and Yeh [19] analyzed the buyback policies of the decentralized and centralized supply chains with fuzzy demand. Sang [20] studied the buyback contract with multiple competing retailers in a fuzzy demand environment. Zhang et al. [21] used the crisp possibilistic mean method to study a two level return contract in a fuzzy random demand environment. Yano et al. [22] proposed the multi-objective fuzzy random linear programming problems based on coefficients of variation. Sang [23] studied a revenue sharing contract with fuzzy demand in a three-echelon supply chain. Yang et al. [24] developed a fuzzy three-echelon inventory model with defective products and rework under credit period.

The works mentioned above studied the fuzzy buyback

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contract in a two level supply chain and considered the actors as risk neutral. In this paper, we extend their works to a three level supply chain, and the risk attitudes of the actors are also considered. Furthermore, we analyze the impact of the retail price, the risk basic coefficient, and the values of contract parameters on the buyback policies.

The paper is organized as follows. In Section II, we briefly described the problem and the notations that will be used in the following sections. In Sections III, we developed the centralized decision-making system and the buyback contract. In Section IV, three numerical examples are given to illustrate the solutions for proposed models. Section V summarizes the work.

II. Problem Descriptions

In this paper, we consider a three level supply chain which consists of a supplier, a distributor and a retailer (see Figure I).

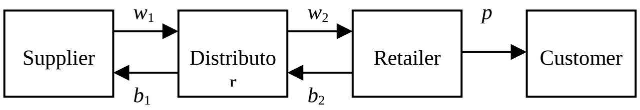

FIGURE I

BUYBACK CONTRACT IN A THREE LEVEL SUPPLY CHAIN

The following notations are used in the models:

p : unit fixed retail price of the market;

w_1 : unit wholesale price offered by the supplier;

w_2 : unit wholesale price offered by the distributor;

b_1 : unit return price of unsold product offered by the supplier;

b_2 : unit return price of unsold product offered by the distributor;

s : unit salvage value of unsold product;

c_1 : unit cost incurred to the supplier;

c_2 : unit cost incurred to the distributor;

c_3 : unit cost incurred to the retailer;

q : The order quantity.

Let $c = c_1 + c_2 + c_3$ be the unit cost incurred to the supply chain system.

For some high-tech products such as PC, it is difficult to predict their accurate demands duce to lack of historical data. In this situation, the demand is usually estimated by the decision maker. In this paper, we considered the demand estimated by the decision maker as a positive trapezoidal fuzzy number with $\tilde{D} = (d_1, d_2, d_3, d_4)$. It means that most possible value of the demand is between d_2 and d_3, the lower bound and upper bound of the demand are d_1 and d_4, respectively. The membership function of the fuzzy demand $\mu_{\tilde{D}}(x)$ is stated as

$$\mu_{\tilde{D}}(x) = \begin{cases} L(x), & d_1 \le x < d_2, \\ 1, & d_2 \le x \le d_3, \\ R(x), & d_3 < x \le d_4, \\ 0, & \textit{otherwise.} \end{cases} \tag{1}$$

Where, the left membership function $L(x) = \dfrac{x - d_1}{d_2 - d_1}$ is increasing with $d_1 \le x < d_2$, and the right membership function $R(x) = \dfrac{d_4 - x}{d_4 - d_3}$ is decreasing with $d_3 \le x < d_4$.

To convert the fuzzy number into a crisp one, Carlsson and Prade [25] proposed a ranking method, namely the weighted possibilistic mean value method as

$$M\left[\tilde{D}\right] = \int_0^1 \left((1-T)L^{-1}(\lambda) + TR^{-1}(\lambda)\right) \mathrm{d}\lambda \tag{2}$$

where $\left[L^{-1}(\lambda), R^{-1}(\lambda)\right]$ is the λ level set of the fuzzy number $\tilde{D}$, and $T\left(0 < T < 1\right)$ reflects the risk attitude of the decision maker. When $T = \frac{1}{2}$, the attitude of the decision maker is risk neutral, When $T < \frac{1}{2}$, the attitude of the decision maker is pessimistic, the lower the value of T, the more risk adverse of the decision maker, and when $T > \frac{1}{2}$, the attitude of the decision maker is optimistic, the higher the value of T, the more risk preference of the decision maker.

To avoid trivial cases, the following assumptions should be met:

$$s + b_2 < b_1,\ w_1 + c_2 < w_2, \text{ and } s + b_i < w_i,\ i = 1, 2.$$

III. Models and Solution Approaches

In this section, we consider a centralized decision-making system and a buyback contract of the supplier, the distributor and the retailer in a fuzzy demand environment.

A. Centralized decision-making system

In a centralized decision-making system, the supplier, the distributor and the retailer cooperate with each other, and their total fuzzy profit can be described as

$$\tilde{\Pi}_{SC} = p\min\left\{q, \tilde{D}\right\} + s\max\left\{q - \tilde{D}, 0\right\} - cq \tag{3}$$

The problem of the supply chain system is to seek the optimal order quantity to maximum the weighted possibilistic mean value of the supply chain's fuzzy profit $M\left(\tilde{\Pi}_{SC}\right)$, which is given by

$$\begin{aligned} \mathrm{Max}_q\, M\left(\tilde{\Pi}_{SC}\right) &= M\left(p\min\left\{q, \tilde{D}\right\} + s\max\left\{q - \tilde{D}, 0\right\} - cq\right) \\ \text{s.t.}\ & d_1 \le q \le d_4 \end{aligned} \tag{4}$$

Since the demand $\tilde{D} = (d_1, d_2, d_3, d_4)$ is a fuzzy variable, then it shows that the optimal order quantity has three conditions, namely $q \in [d_1, d_2)$, $q \in [d_2, d_3]$ and $q \in (d_3, d_4]$.

Condition 1: $q \in [d_1, d_2)$

In this condition, the α cut set of $\min\left\{q, \tilde{D}\right\}$ and $\max\left\{q - \tilde{D}, 0\right\}$ are

$$\left(\min\left\{q, \tilde{D}\right\}\right)_\alpha = \begin{cases} \left[L^{-1}(\alpha), q\right], & 0 < \alpha \le L(q), \\ [q, q], & L(q) < \alpha < 1. \end{cases}$$

$$\left(\max\left\{q - \tilde{D}, 0\right\}\right)_\alpha = \begin{cases} \left[q - L^{-1}(\lambda), q\right], & 0 < \alpha \le L(q), \\ [q, q], & L(q) < \alpha < 1. \end{cases}$$

Then, the λ level set of $\tilde{\Pi}_{SC}$ is

$$\left(\tilde{\Pi}_{SC}\right)_\lambda=\begin{cases}\left[pL^{-1}(\lambda)+s\left(q-L^{-1}(\lambda)\right)-cq,\ pq-cq\right], & 0<\lambda\le L(q),\\ \left[pq-cq,\ pq-cq\right], & L(q)<\lambda<1.\end{cases}$$

Using (2), the weighted possibilistic mean value of the supply chain's fuzzy profit $M\left(\tilde{\Pi}_{SC}\right)$ is

$$\begin{aligned}M\left(\tilde{\Pi}_{SC}\right)&=\int_0^{L(q)}\Big((1-T)\left(pL^{-1}(\lambda)+s\left(q-L^{-1}(\lambda)\right)-cq\right)\\&\quad+T\left(pq-cq\right)\Big)\mathrm{d}\lambda\\&\quad+\int_{L(q)}^1\Big((1-T)(pq-cq)+T\left(pq-cq\right)\Big)\mathrm{d}\lambda\\&=pq-(1-T)(p-s)\int_0^{L(q)}\left(q-L^{-1}(\lambda)\right)\mathrm{d}\lambda-cq\end{aligned}\qquad(5)$$

The first order condition is

$$\frac{\mathrm{d}M\left(\tilde{\Pi}_{SC}\right)}{\mathrm{d}q}=p-(1-T)(p-s)L(q)-c$$

The second order condition is

$$\frac{\mathrm{d}^2M\left(\tilde{\Pi}_{SC}\right)}{\mathrm{d}q^2}=-(1-T)(p-s)L^{'}(q)$$

Since $p>s$, $0<T<1$, and $L(q)$ is increasing with $L^{'}(q)>0$, therefore, $M\left(\tilde{\Pi}_{SC}\right)$ is concave with respect to q. Hence, the first order condition $\frac{\mathrm{d}M\left(\tilde{\Pi}_{SC}\right)}{\mathrm{d}q}=0$ gives

$$L\left(q^*\right)=\frac{p-c}{(1-T)(p-s)}\qquad(6)$$

If $\frac{p-c}{(1-T)(p-s)}<1$, namely, $T(p-s)<c-s$, then

$$q^*=L^{-1}\left(\frac{p-c}{(1-T)(p-s)}\right)\qquad(7)$$

The optimal weighted possibilistic mean value of the supply chain's fuzzy profit $M\left(\tilde{\Pi}_{SC}\right)^*$ in this condition is

$$M\left(\tilde{\Pi}_{SC}\right)^*=(1-T)(p-s)\int_0^{L\left(q^*\right)}L^{-1}(\lambda)\mathrm{d}\lambda\qquad(8)$$

Condition 2: $q\in\left[d_2,d_3\right]$

In this condition, the α cut set of $\min\left\{q,\tilde{D}\right\}$ and $\max\left\{q-\tilde{D},0\right\}$ are

$$\left(\min\left\{q,\tilde{D}\right\}\right)_\alpha=\left[L^{-1}(\alpha),q\right]$$
$$\left(\max\left\{q-\tilde{D},0\right\}\right)_\alpha=\left[q-L^{-1}(\alpha),q\right]$$

Then, the λ level set of $\tilde{\Pi}_{SC}$ is

$$\left(\tilde{\Pi}_{SC}\right)_\lambda=\left[pL^{-1}(\lambda)+s\left(q-L^{-1}(\lambda)\right)-cq,\ pq-cq\right]$$

Using (2), the weighted possibilistic mean value of the supply chain's fuzzy profit $M\left(\tilde{\Pi}_{SC}\right)$ is

$$\begin{aligned}M\left(\tilde{\Pi}_{SC}\right)&=\int_0^1\Big((1-T)\left(pL^{-1}(\lambda)+s\left(q-L^{-1}(\lambda)\right)-cq\right)\\&\quad+T\left(pq-cq\right)\Big)\mathrm{d}\lambda\\&=\left(T(p-s)+s\right)q-cq\\&\quad+(1-T)(p-s)\int_0^1L^{-1}(\lambda)\mathrm{d}\lambda\end{aligned}\qquad(9)$$

The first order condition is

$$\frac{\mathrm{d}M\left(\tilde{\Pi}_{SC}\right)}{\mathrm{d}q}=T(p-s)+s-c$$

When $T(p-s)>c-s$, $M\left(\tilde{\Pi}_{SC}\right)$ is increasing with respect to q, and gets its optimal value at d_3, when $T(p-s)<c-s$, $M\left(\tilde{\Pi}_{SC}\right)$ is decreasing with respect to q, and gets its optimal value at d_2, and when $T(p-s)=c-s$, $M\left(\tilde{\Pi}_{SC}\right)$ obtains its optimal value for any $q^*\in\left[d_2,d_3\right]$.

In this condition, we can conclude that

$$q^*\in\begin{cases}\left\{d_2\right\}, & T(p-s)<c-s,\\ \left[d_2,d_3\right], & T(p-s)=c-s,\\ \left\{d_3\right\}, & T(p-s)>c-s.\end{cases}\qquad(10)$$

Condition 3: $q\in\left(d_3,d_4\right]$

In this condition, the α cut set of $\min\left\{q,\tilde{D}\right\}$ and $\max\left\{q-\tilde{D},0\right\}$ are

$$\left(\min\left\{q,\tilde{D}\right\}\right)_\alpha=\begin{cases}\left[L^{-1}(\alpha),q\right], & 0<\alpha\le R(q),\\ \left[L^{-1}(\alpha),R^{-1}(\alpha)\right], & R(q)<\alpha<1.\end{cases}$$

$$\left(\max\left\{q-\tilde{D},0\right\}\right)_\alpha=\begin{cases}\left[q-L^{-1}(\alpha),q\right], & 0<\alpha\le R(q),\\ \left[q-L^{-1}(\alpha),q-R^{-1}(\alpha)\right], & R(q)<\alpha<1.\end{cases}$$

Then, the λ level set of $\tilde{\Pi}_{SC}$ is

$$\left(\tilde{\Pi}_{SC}\right)_\lambda=\begin{cases}\left[pL^{-1}(\lambda)+s\left(q-L^{-1}(\lambda)\right)-cq,\ pq-cq\right], & 0<\lambda\le R(q),\\ \left[pL^{-1}(\lambda)+s\left(q-L^{-1}(\lambda)\right)-cq,\right.\\ \quad\left.pR^{-1}(\lambda)+s\left(q-R^{-1}(\lambda)\right)-cq\right], & R(q)<\lambda<1.\end{cases}$$

Using (2), the weighted possibilistic mean value of the supply chain's fuzzy profit $M\left(\tilde{\Pi}_{SC}\right)$ is

$$\begin{aligned}M\left(\tilde{\Pi}_{SC}\right)&=\int_0^{R(q)}\Big((1-T)\left(pL^{-1}(\lambda)+s\left(q-L^{-1}(\lambda)\right)-cq\right)\\&\quad+T\left(pq-cq\right)\Big)\mathrm{d}\lambda\\&\quad+\int_{R(q)}^1\Big((1-T)\left(pL^{-1}(\lambda)+s\left(q-L^{-1}(\lambda)\right)-cq\right)\\&\quad+T\left(pR^{-1}(\lambda)+s\left(q-R^{-1}(\lambda)\right)-cq\right)\Big)\mathrm{d}\lambda\\&=T(p-s)\left(\int_{R(q)}^1R^{-1}(\lambda)\mathrm{d}\lambda+qR(q)\right)+sq-cq\\&\quad+(1-T)(p-s)\int_0^1L^{-1}(\lambda)\mathrm{d}\lambda\end{aligned}\qquad(11)$$

The first order condition is

$$\frac{\mathrm{d}M\left(\tilde{\Pi}_{SC}\right)}{\mathrm{d}q}=T(p-s)R(q)+s-c$$

The second order condition is

$$\frac{\mathrm{d}^2 M\left(\tilde{\Pi}_{SC}\right)}{\mathrm{d}q^2} = T\left(p-s\right)R^{'}\left(q\right)$$

Since $p > s$, $T > 0$, and $R(q)$ is decreasing with $R^{'}(q) < 0$, therefore, in this condition $M\left(\tilde{\Pi}_{SC}\right)$ is concave with respect to q. Hence, the first order condition $\frac{\mathrm{d}M\left(\tilde{\Pi}_{SC}\right)}{\mathrm{d}q} = 0$ gives

$$R\left(q^*\right) = \frac{c-s}{T\left(p-s\right)} \tag{12}$$

If $\frac{c-s}{T\left(p-s\right)} < 1$, namely, $T\left(p-s\right) > c-s$, then

$$q^* = R^{-1}\left(\frac{c-s}{T\left(p-s\right)}\right) \tag{13}$$

Therefore, in this condition, $M\left(\tilde{\Pi}_{SC}\right)^*$ is

$$M\left(\tilde{\Pi}_{SC}\right)^* = \left(p-s\right)\left(T\int_{R(q^*)}^{1} R^{-1}\left(\lambda\right)\mathrm{d}\lambda + \left(1-T\right)\int_0^1 L^{-1}\left(\lambda\right)\mathrm{d}\lambda\right) \tag{14}$$

Combining the three conditions, when $T(p-s) < c-s$, we have $M\left(\tilde{\Pi}_{SC}\left(q^*\right)\right) \geq M\left(\tilde{\Pi}_{SC}\left(d_2\right)\right)$, and when $T\left(p-s\right) > c-s$, we have $M\left(\tilde{\Pi}_{SC}\left(d_3\right)\right) \leq M\left(\tilde{\Pi}_{SC}\left(q^*\right)\right)$.

From the above discussions, we can have the following theorem.

Theorem 1. The optimal order quantity q^* is

$$q^* = \begin{cases} L^{-1}\left(\frac{p-c}{(1-T)\left(p-s\right)}\right), & T\left(p-s\right) < c-s, \\ [d_2, d_3], & T\left(p-s\right) = c-s, \\ R^{-1}\left(\frac{c-s}{T\left(p-s\right)}\right), & T\left(p-s\right) > c-s. \end{cases} \tag{15}$$

Remark 1. If $d_2 = d_3$, then the fuzzy demand degenerates to a triangular fuzzy number, and the result in Theorem 1 degenerates to

$$q^* = \begin{cases} L^{-1}\left(\frac{p-c}{(1-T)\left(p-s\right)}\right), & T\left(p-s\right) < c-s, \\ R^{-1}\left(\frac{c-s}{T\left(p-s\right)}\right), & T\left(p-s\right) > c-s. \end{cases} \tag{16}$$

The optimal weighted possibilistic mean value of the supply chain's fuzzy profit $M\left(\tilde{\Pi}_{SC}\right)^*$ is

$$M\left(\tilde{\Pi}_{SC}\right)^* = \begin{cases} (1-T)\left(p-s\right)\int_0^{\frac{p-c}{(1-T)(p-s)}} L^{-1}\left(\lambda\right)\mathrm{d}\lambda, & T\left(p-s\right) < c-s, \\ (1-T)\left(p-s\right)\int_0^1 L^{-1}\left(\lambda\right)\mathrm{d}\lambda, & T\left(p-s\right) = c-s, \\ \left(p-s\right)\left(T\int_{\frac{c-s}{T(p-s)}}^{1} R^{-1}\left(\lambda\right)\mathrm{d}\lambda + (1-T)\int_0^1 L^{-1}\left(\lambda\right)\mathrm{d}\lambda\right), & T\left(p-s\right) > c-s. \end{cases}$$

B. Buyback contract

With a buyback contract, the supplier offers return policy to the distributor, and the distributor offers return policy to the retailer for the unsold products at the end of a single selling season. Therefore, the fuzzy profit for the retailer is

$$\tilde{\Pi}_R = p\min\left\{q, \tilde{D}\right\} + b_2\max\left\{q-\tilde{D}, 0\right\} - \left(w_2 + c_3\right)q \tag{17}$$

The fuzzy profit for the distributor is

$$\tilde{\Pi}_D = \left(w_2 - w_1 - c_2\right)q + b_1\max\left\{q-\tilde{D}, 0\right\} - b_2\max\left\{q-\tilde{D}, 0\right\} \tag{18}$$

The fuzzy profit for the supplier is

$$\tilde{\Pi}_S = \left(w_1 - c_1\right)q + s\max\left\{q-\tilde{D}, 0\right\} - b_1\max\left\{q-\tilde{D}, 0\right\} \tag{19}$$

The problems of the retailer and the distributor are to seek their optimal order quantities to maximum their weighted possibilistic mean value of the fuzzy profit $M\left(\tilde{\Pi}_R\right)$ and $M\left(\tilde{\Pi}_D\right)$, respectively

$$\begin{aligned} \mathrm{Max}_q\ M\left(\tilde{\Pi}_R\right) = M\Big(& p\min\left\{q, \tilde{D}\right\} + b_2\max\left\{q-\tilde{D}, 0\right\} \\ & -\left(w_2 + c_3\right)q\Big) \\ & \mathrm{s.t.}\ d_1 \leq q \leq d_4 \end{aligned} \tag{20}$$

and

$$\begin{aligned} \mathrm{Max}_q\ M\left(\tilde{\Pi}_D\right) = M\Big(& \left(w_2 - w_1 - c_2\right)q + b_1\max\left\{q-\tilde{D}, 0\right\} \\ & -b_2\max\left\{q-\tilde{D}, 0\right\}\Big) \\ & \mathrm{s.t.}\ d_1 \leq q \leq d_4 \end{aligned} \tag{21}$$

Theorem 2. The optimal strategies $\left(b_2^*, w_2^*\right)$ and $\left(b_1^*, w_1^*\right)$ in the buyback contract satisfy the following equations

$$b_2^* = \frac{p-s}{p-c}w_2^* - \frac{p\left(c_1 + c_2\right) - \left(p - c_3\right)s}{p-c} \tag{22}$$

$$b_1^* = \frac{p-s}{p-c}w_1^* - \frac{pc_1 - \left(p - c_2 - c_3\right)s}{p-c} \tag{23}$$

Proof: Three conditions are considered as follows.

Condition 1: $q \in \left[d_1, d_2\right)$

Like previous conditions, the weighted possibilistic mean value of the retailer's fuzzy profit $M\left(\tilde{\Pi}_R\right)$ is

$$\begin{aligned} M\left(\tilde{\Pi}_R\right) = pq - \left(1-T\right)\left(p - b_2\right)\int_0^{L(q)}\left(q - L^{-1}\left(\lambda\right)\right)\mathrm{d}\lambda \\ -\left(w_2 + c_3\right)q \end{aligned} \tag{24}$$

The first order condition is

$$\frac{\mathrm{d}M\left(\tilde{\Pi}_R\right)}{\mathrm{d}q} = p - \left(1-T\right)\left(p - b_2\right)L\left(q\right) - w_2 - c_3$$

The second order condition is

$$\frac{\mathrm{d}^2 M\left(\tilde{\Pi}_R\right)}{\mathrm{d}q^2} = -\left(1-T\right)\left(p - b_2\right)L^{'}\left(q\right)$$

Since $p > b_2$, $0 < T < 1$, and $L(q)$ is increasing with $L^{'}(q) > 0$, therefore, in this condition, $M\left(\tilde{\Pi}_R\right)$ is concave with respect to q. Hence, the first order condition

$\frac{\mathrm{d}M\left(\tilde{\Pi}_R\right)}{\mathrm{d}q}=0$ gives

$$L\left(q^{**}\right)=\frac{p-w_2-c_3}{(1-T)(p-b_2)} \tag{25}$$

In order to obtain coordination of this supply chain, $q^{**}=q^*$ must be hold.

That is

$$\frac{p-w_2-c_3}{(1-T)(p-b_2)}=\frac{p-c}{(1-T)(p-s)} \tag{26}$$

Solving (26), we have

$$b_2^*=\frac{p-s}{p-c}w_2^*-\frac{p(c_1+c_2)-(p-c_3)s}{p-c}$$

The weighted possibilistic mean value of the distributor's fuzzy profit $M\left(\tilde{\Pi}_D\right)$ in this condition is

$$M\left(\tilde{\Pi}_D\right)=(w_2-w_1-c_2)q-(1-T)(b_2-b_1)\int_0^{L(q)}\left(q-L^{-1}(\lambda)\right)\mathrm{d}\lambda \tag{27}$$

The first order condition is

$$\frac{\mathrm{d}M\left(\tilde{\Pi}_D\right)}{\mathrm{d}q}=w_2-w_1-c_2-(1-T)(b_2-b_1)L(q)$$

The second order condition is

$$\frac{\mathrm{d}^2M\left(\tilde{\Pi}_D\right)}{\mathrm{d}q^2}=-(1-T)(b_2-b_1)L'(q)$$

Since $b_2>b_1$, $0<T<1$, and $L(q)$ is increasing with $L'(q)>0$, therefore, in this condition, $M\left(\tilde{\Pi}_D\right)$ is concave with respect to q. Hence, the first order condition $\frac{\mathrm{d}M\left(\tilde{\Pi}_D\right)}{\mathrm{d}q}=0$ gives

$$L\left(q^{**}\right)=\frac{w_2-w_1-c_2}{(1-T)(b_2-b_1)} \tag{28}$$

In order to obtain coordination of this supply chain, $q^*=q^{**}$ must be hold.

That is

$$\frac{p-c}{(1-T)(p-s)}=\frac{w_2-w_1-c_2}{(1-T)(b_2-b_1)}=\frac{p-w_2-c_3}{(1-T)(p-b_2)}$$
$$=\frac{p-w_1-c_2-c_3}{(1-T)(p-b_1)} \tag{29}$$

Solving (29), we have

$$b_1^*=\frac{p-s}{p-c}w_1^*-\frac{pc_1-(p-c_2-c_3)s}{p-c}$$

Condition 2: $q\in[d_2,d_3]$

Like previous conditions, the weighted possibilistic mean value of the retailer's fuzzy profit $M\left(\tilde{\Pi}_R\right)$ is

$$M\left(\tilde{\Pi}_R\right)=\left(T(p-b_2)+b_2\right)q+(1-T)(p-b_2)\int_0^1L^{-1}(\lambda)\mathrm{d}\lambda$$
$$-(w_2+c_3)q \tag{30}$$

The first order condition is

$$\frac{\mathrm{d}M\left(\tilde{\Pi}_R\right)}{\mathrm{d}q}=T(p-b_2)+b_2-w_2-c_3$$

When $T(p-b_2)>w_2+c_3-b_2$, $M\left(\tilde{\Pi}_R\right)$ is increasing with respect to q, and gets its optimal value at d_3, when $T(p-b_2)<w_2+c_3-b_2$, $M\left(\tilde{\Pi}_R\right)$ is decreasing with respect to q, and gets its optimal value at d_2, and when $T(p-b_2)=w_2+c_3-b_2$, $M\left(\tilde{\Pi}_R\right)$ obtains its optimal value for any $q^{**}\in[d_2,d_3]$.

In order to obtain coordination of this supply chain, $q^{**}=q^*$ must be hold.

That is

$$\frac{w_2+c_3-b_2}{T(p-b_2)}=\frac{c-s}{T(p-s)} \tag{31}$$

Solving (31), we have

$$b_2^*=\frac{p-s}{p-c}w_2^*-\frac{p(c_1+c_2)-(p-c_3)s}{p-c}$$

The weighted possibilistic mean value of the distributor's fuzzy profit $M\left(\tilde{\Pi}_D\right)$ in this condition is

$$M\left(\tilde{\Pi}_D\right)=(w_2-w_1-c_2)q-(1-T)(b_2-b_1)q$$
$$+(1-T)(b_2-b_1)\int_0^1L^{-1}(\lambda)\mathrm{d}\lambda \tag{32}$$

The first order condition is

$$\frac{\mathrm{d}M\left(\tilde{\Pi}_D\right)}{\mathrm{d}q}=T(b_2-b_1)+w_2-w_1-c_2+b_1-b_2$$

When $T(b_2-b_1)>w_1-w_2+c_2+b_2-b_1$, $M\left(\tilde{\Pi}_D\right)$ is increasing with respect to q, and gets its optimal value at d_3, when $T(b_2-b_1)<w_1-w_2+c_2+b_2-b_1$, $M\left(\tilde{\Pi}_D\right)$ is decreasing with respect to q, and gets its optimal value at d_2, and when $T(b_2-b_1)=w_1-w_2+c_2+b_2-b_1$, $M\left(\tilde{\Pi}_D\right)$ obtains its optimal value for any $q^{**}\in[d_2,d_3]$.

In order to obtain coordination of this supply chain, $q^*=q^{**}$ must be hold.

That is

$$\frac{c-s}{T(p-s)}=\frac{w_1-w_2+c_2+b_2-b_1}{T(b_2-b_1)}=\frac{w_2+c_3-b_2}{T(p-b_2)}$$
$$=\frac{w_1-b_1+c_2+c_3}{T(p-b_1)} \tag{33}$$

Solving (33), we have

$$b_1^*=\frac{p-s}{p-c}w_1^*-\frac{pc_1-(p-c_2-c_3)s}{p-c}$$

Condition 3: $q\in(d_3,d_4]$

Like previous conditions, the weighted possibilistic mean value of the retailer's fuzzy profit $M\left(\tilde{\Pi}_R\right)$ is

$$M\left(\tilde{\Pi}_R\right)=T\left(p-b_2\right)\left(\int_{R(q)}^{1}R^{-1}\left(\lambda\right)\mathrm{d}\lambda+qR\left(q\right)\right)+b_2q-\left(w_2+c_3\right)q$$

$$+(1-T)\left(p-b_2\right)\int_0^1 L^{-1}\left(\lambda\right)\mathrm{d}\lambda \quad (34)$$

The first order condition is

$$\frac{\mathrm{d}M\left(\tilde{\Pi}_R\right)}{\mathrm{d}q}=T\left(p-b_1\right)R\left(q\right)+b_2-w_2-c_3$$

The second order condition is

$$\frac{\mathrm{d}^2M\left(\tilde{\Pi}_R\right)}{\mathrm{d}q^2}=T\left(p-b_2\right)R^{'}\left(q\right)$$

Since $p>b_2$, $T>0$ and $R(q)$ is decreasing with $R^{'}(q)<0$, therefore, in this condition, $M\left(\tilde{\Pi}_R\right)$ is concave with respect to q. Hence, the first order condition $\frac{\mathrm{d}M\left(\tilde{\Pi}_R\right)}{\mathrm{d}q}=0$ gives

$$R\left(q^{**}\right)=\frac{w_2+c_3-b_2}{T\left(p-b_1\right)}$$

In order to obtain coordination of this supply chain, $q^{**}=q^{*}$ must be hold.

That is

$$\frac{w_2+c_3-b_2}{T\left(p-b_1\right)}=\frac{c-s}{T\left(p-s\right)} \quad (35)$$

Solving (35), we have

$$b_2^{*}=\frac{p-s}{p-c}w_2^{*}-\frac{p\left(c_1+c_2\right)-\left(p-c_3\right)s}{p-c}$$

The weighted possibilistic mean value of the distributor's fuzzy profit $M\left(\tilde{\Pi}_D\right)$ in this condition is

$$M\left(\tilde{\Pi}_D\right)=\left(w_2-w_1-c_2\right)q-\left(b_2-b_1\right)q+T\left(b_2-b_1\right)qR(q)$$

$$+T\left(b_2-b_1\right)\int_{R(q)}^{1}R^{-1}\left(\lambda\right)\mathrm{d}\lambda$$

$$+\left(1-T\right)\left(b_2-b_1\right)\int_0^1 L^{-1}\left(\lambda\right)\mathrm{d}\lambda \quad (36)$$

The first order condition is

$$\frac{\mathrm{d}M\left(\tilde{\Pi}_D\right)}{\mathrm{d}q}=T\left(b_2-b_1\right)R\left(q\right)+w_2-w_1-c_2-b_2+b_1$$

The second order condition is

$$\frac{\mathrm{d}^2M\left(\tilde{\Pi}_D\right)}{\mathrm{d}q^2}=T\left(b_2-b_1\right)R^{'}\left(q\right)$$

Since $b_2>b_1$, $T>0$, and $R(q)$ is decreasing with $R^{'}(q)<0$, therefore, in this condition, $M\left(\tilde{\Pi}_D\right)$ is concave with respect to q. Hence, the first order condition $\frac{\mathrm{d}M\left(\tilde{\Pi}_D\right)}{\mathrm{d}q}=0$ gives

$$R\left(q^{**}\right)=\frac{w_1-w_2+c_2+b_2-b_1}{T\left(b_2-b_1\right)}$$

In order to obtain coordination of this supply chain, $q^{*}=q^{**}$ must be hold.

That is

$$\frac{c-s}{T\left(p-s\right)}=\frac{w_1-w_2+c_2+b_2-b_1}{T\left(b_2-b_1\right)}=\frac{w_2+c_3-b_2}{T\left(p-b_1\right)}=\frac{w_1+c_2+c_3-b_1}{T\left(p-b_2\right)} \quad (37)$$

Solving (37), we have

$$b_1^{*}=\frac{p-s}{p-c}w_1^{*}-\frac{pc_1-\left(p-c_2-c_3\right)s}{p-c}$$

The proof of Theorem 2 is completed.

Theorem 3. For $0<\lambda_1<1$ and $\frac{s+c_3}{c-s}<\lambda_2<1$, the set of buyback contract strategies $\left(b_2^{*},w_2^{*}\right)$ and $\left(b_1^{*},w_1^{*}\right)$ satisfy

$$b_2^{*}=p-\lambda_2\left(p-s\right) \quad (38)$$

$$w_2^{*}=p-c_3-\lambda_2\left(p-c\right) \quad (39)$$

$$b_1^{*}=s+\left(1-\lambda_1\right)\left(1-\lambda_2\right)\left(p-s\right) \quad (40)$$

$$w_1^{*}=c_1+\left(1-\lambda_1\right)\left(1-\lambda_2\right)\left(p-c\right) \quad (41)$$

Proof: Substituting w_2^{*} in (39) into (22), we have

$$b_2^{*}=p-\lambda_2\left(p-s\right)$$

Since $b_2+s<w_2$, we have $\lambda_2>\frac{s+c_3}{c-s}$.

Substituting w_1^{*} in (41) into (23), we have

$$b_1^{*}=s+\left(1-\lambda_1\right)\left(1-\lambda_2\right)\left(p-s\right)$$

For $\lambda_1>0$, $b_1+s<w_1$ and $s+b_2<b_1$.

The proof of Theorem 3 is completed.

Theorem 4. For $0<\lambda_1<1$ and $\frac{s+c_3}{c-s}<\lambda_2<1$, the supply chain actors obtain their optimal weighted possibilistic mean value of fuzzy profits at $\left(b_2^{*},w_2^{*}\right)$ and $\left(b_1^{*},w_1^{*}\right)$ as follows

$$M\left(\tilde{\Pi}_R\right)^{*}=\lambda_2M\left(\tilde{\Pi}_{SC}\right)^{*} \quad (42)$$

$$M\left(\tilde{\Pi}_D\right)^{*}=\lambda_1\left(1-\lambda_2\right)M\left(\tilde{\Pi}_{SC}\right)^{*} \quad (43)$$

$$M\left(\tilde{\Pi}_S\right)^{*}=\left(1-\lambda_1\right)\left(1-\lambda_2\right)M\left(\tilde{\Pi}_{SC}\right)^{*} \quad (44)$$

Proof: Condition 1:

Substituting b_2^{*}, w_2^{*} and $L\left(q^{*}\right)$ in (6) into (24), we can get the optimal weighted possibilistic mean value of the retailer's fuzzy profit $M\left(\tilde{\Pi}_R\right)^{*}$ as

$$M\left(\tilde{\Pi}_R\right)^{*}=\lambda_2\left(1-T\right)\left(p-s\right)\int_0^{\frac{p-c}{(1-T)(p-s)}}L^{-1}\left(\lambda\right)\mathrm{d}\lambda$$

$$=\lambda_2M\left(\tilde{\Pi}_{SC}\right)^{*}$$

Substituting b_2^{*}, w_2^{*}, b_1^{*}, w_1^{*} and $L\left(q^{*}\right)$ into (27), we can get the optimal weighted possibilistic mean value of the distributor's fuzzy profit $M\left(\tilde{\Pi}_D\right)^{*}$ as

$$M\left(\tilde{\Pi}_D\right)^{*}=\lambda_1\left(1-\lambda_2\right)\left(1-T\right)\left(p-s\right)\int_0^{\frac{p-c}{(1-T)(p-s)}}L^{-1}\left(\lambda\right)\mathrm{d}\lambda$$

$$=\lambda_1\left(1-\lambda_2\right)M\left(\tilde{\Pi}_{SC}\right)^{*}$$

Then, the optimal weighted possibilistic mean value of the supplier's fuzzy profit $M\left(\tilde{\Pi}_S\right)^{*}$ is given as

$$M\left(\tilde{\Pi}_S\right)^* = M\left(\tilde{\Pi}_{SC}\right)^* - M\left(\tilde{\Pi}_R\right)^* - M\left(\tilde{\Pi}_D\right)^*$$
$$= (1-\lambda_1)(1-\lambda_2) M\left(\tilde{\Pi}_{SC}\right)^*$$

Condition 2:

Substituting b_2^*, w_2^* and $T(p-s) = c-s$ into (30), we can get the optimal weighted possibilistic mean value of the retailer's fuzzy profit $M\left(\tilde{\Pi}_R\right)^*$ as

$$M\left(\tilde{\Pi}_R\right)^* = \lambda_2(1-T)(p-s)\int_0^1 L^{-1}(\lambda)\,\mathrm{d}\lambda = \lambda_2 M\left(\tilde{\Pi}_{SC}\right)^*$$

Substituting b_2^*, w_2^*, b_1^*, w_1^* and $T(p-s) = c-s$ into (32), we can get the optimal weighted possibilistic mean value of the distributor's fuzzy profit $M\left(\tilde{\Pi}_D\right)^*$ as

$$M\left(\tilde{\Pi}_D\right)^* = \lambda_1(1-\lambda_2)(1-T)(p-s)\int_0^1 L^{-1}(\lambda)\,\mathrm{d}\lambda$$
$$= \lambda_1(1-\lambda_2) M\left(\tilde{\Pi}_{SC}\right)^*$$

Then, the optimal weighted possibilistic mean value of the supplier's fuzzy profit $M\left(\tilde{\Pi}_S\right)^*$ in this condition is given as

$$M\left(\tilde{\Pi}_S\right)^* = M\left(\tilde{\Pi}_{SC}\right)^* - M\left(\tilde{\Pi}_R\right)^* - M\left(\tilde{\Pi}_D\right)^*$$
$$= (1-\lambda_1)(1-\lambda_2) M\left(\tilde{\Pi}_{SC}\right)^*$$

Condition 3:

Substituting b_2^*, w_2^* and $R\left(q^*\right)$ in (12) into (34), we can get the optimal weighted possibilistic mean value of the retailer's fuzzy profit $M\left(\tilde{\Pi}_R\right)^*$ as

$$M\left(\tilde{\Pi}_R\right)^* = \lambda_2(p-s)\left(T\int_{\frac{c-s}{T(p-s)}}^1 R^{-1}(\lambda)\,\mathrm{d}\lambda + (1-T)\int_0^1 L^{-1}(\lambda)\,\mathrm{d}\lambda\right)$$
$$= \lambda_2 M\left(\tilde{\Pi}_{SC}\right)^*$$

Substituting b_2^*, w_2^*, b_1^*, w_1^* and $R\left(q^*\right)$ into (36), we can get the optimal weighted possibilistic mean value of the distributor's fuzzy profit $M\left(\tilde{\Pi}_D\right)^*$ as

$$M\left(\tilde{\Pi}_D\right)^* = \lambda_1(1-\lambda_2)(p-s)\left(T\int_{\frac{c-s}{T(p-s)}}^1 R^{-1}(\lambda)\,\mathrm{d}\lambda + (1-T)\int_0^1 L^{-1}(\lambda)\,\mathrm{d}\lambda\right)$$
$$= \lambda_1(1-\lambda_2) M\left(\tilde{\Pi}_{SC}\right)^*$$

Then, the optimal weighted possibilistic mean value of the supplier's fuzzy profit $M\left(\tilde{\Pi}_S\right)^*$ in this condition is given as

$$M\left(\tilde{\Pi}_S\right)^* = M\left(\tilde{\Pi}_{SC}\right)^* - M\left(\tilde{\Pi}_R\right)^* - M\left(\tilde{\Pi}_D\right)^*$$
$$= (1-\lambda_1)(1-\lambda_2) M\left(\tilde{\Pi}_{SC}\right)^*$$

The proof of Theorem 4 is completed.

IV. Numerical Examples

In this section, we provide three numerical examples to show the effects of retail price p, the risk basic coefficient T, and the values of parameters λ_1 and λ_2 on the optimal policies. Let $c_1 = 50$, $c_2 = 15$, $c_3 = 10$ and $s = 15$. We further assume that the most possible value of the market demand is between 200 and 300, the demand is no less than 100 and no more than 400, that is $\tilde{D} = (100, 200, 300, 400)$.

From Theorem 4, we have $0.42 < \lambda_2 < 1$.

Discussion A

In this subsection, we discuss the effect of retail price p on the optimal policies in the buyback contract. Let $\lambda_1 = 0.40$ and $\lambda_2 = 0.50$. The optimal solutions in the buyback contract are given in Tables I and II.

TABLE I
THE OPTIMAL PRICES IN BUYBACK CONTRACT WITH DIFFERENT p

T	p	q^*	b_1^*	w_1^*	b_2^*	w_2^*
0.40	155	195	57.0	74.0	85.0	105.0
	160	198	58.5	75.5	87.5	107.5
	165	[200,300]	60.0	77.0	90.0	110.0
	170	303	61.5	78.5	92.5	112.5
	175	306	63.0	80.0	95.0	115.0
0.50	125	191	48.0	65.0	70.0	90.0
	130	196	49.5	66.5	72.5	92.5
	135	[200,300]	51.0	68.0	75.0	95.0
	140	304	52.5	69.5	77.5	97.5
	145	308	54.0	71.0	80.0	100.0
0.60	105	183	42.0	59.0	60.0	80.0
	110	192	43.5	60.5	62.5	83.5
	115	[200,300]	45.0	62.0	65.0	85.0
	120	305	46.5	63.5	67.5	87.5
	125	309	48.0	65.0	70.0	90.0

TABLE II
THE OPTIMAL PROFITS OF THE MEMBERS WITH DIFFERENT p

T	p	$M\left(\tilde{\Pi}_R\right)^*$	$M\left(\tilde{\Pi}_D\right)^*$	$M\left(\tilde{\Pi}_S\right)^*$
0.40	155	5905	2362	3543
	160	6326	2530	3796
	165	6750	2700	4050
	170	7277	1455	4366
	175	7806	1561	4684
0.50	125	3636	1455	2182
	130	4065	1626	2439
	135	4500	1800	2700
	140	5065	2026	3039
	145	5635	2254	3381
0.60	105	2125	850	1275
	110	2556	1022	1534
	115	3000	1200	1800
	120	3604	1441	2162
	125	4214	1685	2528

From Tables I and II, we can see that:

(1) The optimal order quantity q^* increases as the retail price p increases. Especially, in this numerical example, when $(T,p)=(0.40,165)$, $(T,p)=(0.50,135)$ and $(T,p)=(0.40,115)$, the optimal order quantity q^* can be any values between 200 and 300.

(2) Increasing retail price p will increase the wholesale prices w_1^* and w_2^* , the return prices b_1^* and b_2^* , and the optimal weighted possibilistic mean value of the supply chain actor's fuzzy profit. It indicates that an increase in retail price results in an increase in order quantity. This results in an increase in the supply chain actor's profit.

Discussion B

In this subsection, we discuss the effect of the risk basic coefficient T on the optimal policies in the buyback contract. Let $p=135$, $\lambda_1=0.40$ and $\lambda_2=0.50$. We can have $b_1^*=51$, $w_1^*=68$, $b_2^*=75$ and $w_2^*=95$. The other optimal solutions in the buyback contract are given in Table III.

TABLE III
THE OPTIMAL SOLUTIONS IN BUYBACK CONTRACT WITH DIFFERENT T

T	q^*	$M(\tilde{\Pi}_R)^*$	$M(\tilde{\Pi}_D)^*$	$M(\tilde{\Pi}_S)^*$
0.30	171	4071	1629	2443
0.35	177	4154	1662	2492
0.40	183	4250	1700	2550
0.45	191	4364	1745	2618
0.50	[200,300]	4500	1800	2700
0.55	309	4964	1985	2978
0.60	317	5450	2180	3270
0.65	323	5954	2382	3572
0.70	329	6471	2589	3883

We can see that:

(3) The optimal order quantity q^* increases as the risk basic coefficient T increases, and can be any values between 200 and 300, when $T=0.50$ in this case.

(4) The change of the risk basic coefficient T will not impact on the wholesale prices w_1^* and w_2^*, and the return prices b_1^* and b_2^*.

(5) When the risk basic coefficient T increases, the optimal weighted possibilistic mean values of the fuzzy profits for all actors will increase. This is because the risk basic coefficient T reflects the risk attitude of the supply chain actors. The more responsibility of risk the more weighted possibilistic mean values of the fuzzy profits they can obtain.

Discussion C

In this subsection, we discuss the effects of the values of parameters λ_1 and λ_2 on the optimal policies in the buyback contract. Let $p=135$ and $T=0.40$. The optimal solutions in the buyback contract are given in Tables IV and V.

TABLE IV
THE OPTIMAL PRICES IN BUYBACK CONTRACT WITH DIFFERENT λ_1 AND λ_2

(λ_1,λ_2)	q^*	b_1^*	w_1^*	b_2^*	w_2^*
(0.40, 0.45)	183	54.6	69.8	81.0	98.0
(0.40, 0.50)	183	51.0	68.0	75.0	95.0
(0.40, 0.55)	183	47.4	66.2	69.0	92.0
(0.40, 0.60)	183	43.8	64.4	63.0	89.0
(0.50, 0.50)	183	45.0	65.0	75.0	95.0
(0.60, 0.50)	183	39.0	62.0	75.0	95.0
(0.70, 0.50)	183	33.0	59.0	75.0	95.0
(0.80, 0.50)	183	27.0	56.0	75.0	95.0

TABLE V
THE OPTIMAL PROFITS IN BUYBACK CONTRACT WITH DIFFERENT λ_1 AND λ_2

(λ_1,λ_2)	$M(\tilde{\Pi}_R)^*$	$M(\tilde{\Pi}_D)^*$	$M(\tilde{\Pi}_S)^*$
(0.40, 0.45)	3825	1870	2805
(0.40, 0.50)	4250	1700	2550
(0.40, 0.55)	4675	1530	2295
(0.40, 0.60)	5100	1360	2040
(0.50, 0.50)	4250	2125	2125
(0.60, 0.50)	4250	2550	1700
(0.70, 0.50)	4250	2975	1275
(0.80, 0.50)	4250	3400	850

From Tables IV and V, we can see that:

(6) The change of the values of parameters λ_1 and λ_2 will not impact on the optimal order quantity q^* .

(7) The wholesale prices w_1^* and w_2^* , and the return prices b_1^* and b_2^* will decrease with the increasing of the value of parameter λ_2 , when λ_1 is fixed. With the increasing of λ_1 , the wholesale price w_1^* and the return price b_1^* will decrease, and the wholesale price w_2^* and the return price b_2^* will not vary, when λ_2 is fixed.

(8) With the increasing of λ_2 , the optimal weighted possibilistic mean value of the fuzzy profit for the retailer will increase, and the optimal weighted possibilistic mean values of the fuzzy profits for the distributor and the supplier will decrease, when λ_1 is fixed. Therefore, for the retailer, he should seek as high value of parameter λ_2 as possible. Conversely, for the distributor or the supplier, he should seek as low value of parameter λ_2 as possible. With the increasing of λ_1 , the optimal weighted possibilistic mean value of the fuzzy profit for the retailer will not vary, when λ_2 is fixed. The optimal weighted possibilistic mean value of the fuzzy profit for the distributor will increase as λ_1 increases. Therefore, for the distributor, he should seek as high value of parameter λ_1 as possible. The optimal weighted possibilistic mean value of the fuzzy profit for the supplier will decrease as λ_1 increases. Therefore, for the supplier, he

should seek as low value of parameter λ_1 as possible.

V. Conclusions

This article deals with the buyback contract with fuzzy demand in a three level supply chain, where the risk attitudes of the actors are considered. For examining the performance of supply chain members in the models, the weighted possibilistic mean value method is used to solve fuzzy models. We find that the change of the risk basic coefficient does not impact on the wholesale prices and return prices. The optimal weighted possibilistic mean value of the fuzzy profit for the supply chain actors vary with the changing of the risk basic coefficient and the values of contract parameters. One limitation of this article is that we only consider one supplier, one distributor and one retailer. Therefore, one possible extension work is to study the buyback contract with multiple competing retailers, distributors or suppliers in a fuzzy decision making environment. The other limitation is that the market demand of the supply chain models is considered as a trapezoidal fuzzy number. In fact, the membership function of the fuzzy number can be nonlinear, one can consider the case the demand is a fuzzy random variable.

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