Application and Performance Comparison of Biogeography-based Optimization Algorithm on Unconstrained Function Optimization Problem

Jie-Sheng Wang, and Jiang-Di Song

Abstract—**Biogeography-based optimization (BBO) algorithm realizes the information circulation and sharing by the species migration among habitats and achieves the global by improving the adaptability of habitats. Based on the population adaptive migration mechanism of BBO algorithm, the unconstrained function optimization problem is solved with six species migration models. For performance comparison, the ant colony optimization (ACO) algorithm, the differential evolution (DE) algorithm and the particle swarm optimization (PSO) algorithm are adopted to solving six unconstrained functions optimization problems. Simulation results show that the convergence speed, optimization accuracy and solution uniformity of BBO algorithm have been improved significantly, and BBO algorithm is more efficient to solve the unconstrained function optimization problem.**

Index Terms—**biogeography-based optimization algorithm, function optimization, performance comparison**

I. Introduction

THE nature of function optimization problem is to find the optimal solution of an objective function through iterative [1]. The function features are usually described as continuous, discrete, linear, non-linear, convex function, etc. In that the constraint function optimization problem can be converted into unconstrained problem by using the designed special operators and penalty functions to make solution always feasible, the unconstrained function optimization problem is the main research focus. The swarm intelligent optimization algorithms [2] are a kind of random search algorithm to simulate the biological population evolution and evolution, which solves the complex global optimization problems through individual cooperation and competition between species, and is applied in many fields, such as multi-objective optimization, data mining, network routing, signal processing, pattern recognition, etc. The typical swarm intelligence optimization algorithms include Ant Colony Optimization (ACO) algorithm [3], Genetic Algorithm (GA) [4], Bat Algorithm (BA) [5], Artificial Bee Colony (ABC) algorithm [6], etc.

Artificial bee colony (ABC) algorithm is inspired by the foraging behavior of honey bee swarm. Inspired by PSO, an improved artificial bee colony (ABC) algorithm called gbest-guided ABC (GABC) algorithm was proposed by incorporating the information of global best (gbest) solution into the solution search equation to improve the exploitation [7]. In that ABC is good at exploration but poor at exploitation, and its convergence speed is also an issue in some cases, an improved ABC algorithm called I-ABC was proposed, where inertia weight and acceleration coefficients are introduced to modify the search process [8]. A new modified genetic algorithm with adaptive elitist-population strategies was proposed for multimodal function optimization, which is based on the concept of adaptively adjusting the population size according to the individuals' dissimilarity and a novel direction dependent elitist genetic operator [9]. A hybrid niching algorithm based on the PSO was proposed to deal with multimodal function optimization problems [10], where the recombination-replacement crowding strategy that works on the archive population is introduced to improve the exploration capability. An ensemble of differential evolution algorithms employing the variable parameter search and two distinct mutation strategies in the ensemble was proposed to solve real-parameter constrained optimization problems, which was tested using benchmark instances [11]. An improved fruit fly optimization (IFFO) algorithm was proposed for solving continuous function optimization problems. A new control parameter is introduced to tune the search scope around its swarm location adaptively and a new solution generating method is developed to enhance accuracy and convergence rate of the algorithm [12]. Cuckoo search algorithm which reproduces the breeding strategy of the best known brood parasitic bird, the cuckoos has demonstrated its superiority in obtaining the global solution for numerical optimization problems. An improved cuckoo search algorithm with adaptive step size adjustment is introduced and its feasibility on a variety of benchmarks is validated [13].

Biogeography-based optimization (BBO) Algorithm is put forward by Simon in 2008 [14-15], whose basic idea is the

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species migration to complete the information flows among habitats. By adjusting immigration rate, emigration rate, migration topology, migration interval and migration strategies, the information sharing is realized in the migration process in order to improve the suitability of habitats and obtain the optimal solution [16]. BBO algorithm has been successfully applied in economic load assignment [17], combinatorial optimization [18], power distribution of wireless sensor network [19], function optimization [20], etc. In this paper, based on the population adaptive migration mechanism of BBO algorithm, the unconstrained function optimization problem is solved with six species migration models. The remainder of this paper is organized as follows. In Section 2, the BBO algorithm is introduced. The simulation experiments and the analysis of the results are discussed in detail in Section 3. The concluding remarks are presented in the last section.

II. Biogeography-based Optimization Algorithm

A. Biogeography

BBO algorithm [14] is derived from biogeography, whose main contents is to establish mathematical models for a series of events include residence, migration routes, production of new species, and extinction of species in nature. Figure 1 introduces this migration relationship among habitants.

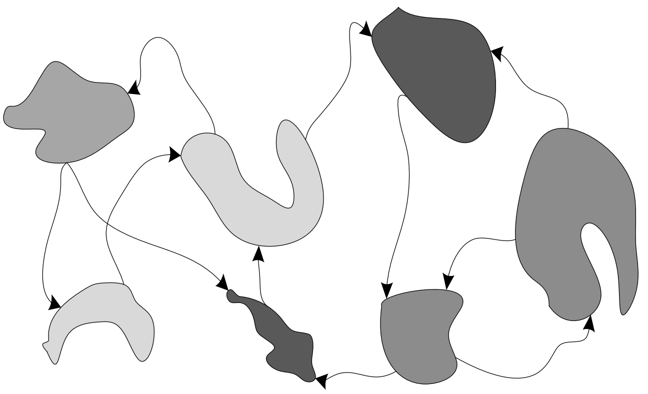


Fig. 1. Multi-habitats in biological geography

In addition to the relationship among these islands, each island has its own factors and survival indicators. For those islands suitable for breeding populations, a higher habitat suitable index (HSI) is obtained. The un-isolated index variables affecting HIS are named as independent habitat variable. However, when the HSI is high, the population on the corresponding island is more crowded, many populations will migrate to other neighboring islands to multiply. Meanwhile, there are other populations from other low HSI islands migrating into this island with higher HSI. The immigration rate of the island with lower HIS is higher than the island with higher HIS. The habitat migration operator in BBO algorithm is set up based on probability theory to realize the information sharing among each individual in the population. Each individual has its emigration rate μ and immigration rate λ for controlling the moving probability of individuals.

B. Mathematical model of biogeography

The species migratory model of monomer HIS shown in figure 2[14] is described as follows. Based on emigration rate μ and immigration rate λ, the function of the number of species on the island is established. Is can be seen from Figure 2, the larger the number of species, the larger the emigration rate. When the number of species reached maximum number of species capacity $S_{\max}$, the emigration rate reaches its maximum value E. On the other hand, when the number of species on the island is 0, the immigration rate is the largest value I. The equilibrium point of the number of species on this island is S_0, where the immigration rate equals to the emigration rate.

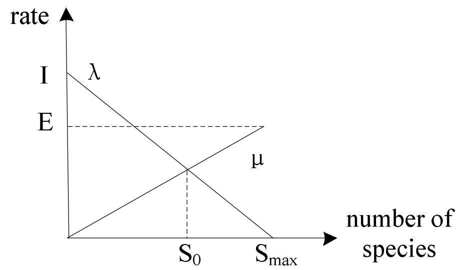


Fig. 2. Species migration model of single island

In BBO algorithm, an island has S species, whose probability is P_s. P_s changes within the time $[t, t+\Delta t]$ described as follows.

$$P_s(t+\Delta t) = P_s(t)(1-\lambda_s \Delta t - \mu_s \Delta t) + P_{s-1}\lambda_{s-1}\Delta t + P_s \mu_{s+1}\Delta t \quad (1)$$

When the number of species of the island is S, the emigration rate is μ_s ,= and the immigration rate is λ_s. Suppose Eq. (1) is established, the number of species is S at the time $t+\Delta t$. Aiming at this situation, it must satisfy at least one of the following conditions.

(1) At the time t, there are S species in this island. At the time $t+\Delta t$, no species emigrate and immigrate.

(2) At the time t, there are $S+1$ species in this island. At the time $t+\Delta t$, there is a specie to emigrate.

(3) At the time t, there are $S-1$ species in this island. At the time $t+\Delta t$, there is a specie to immigrate.

If Δt is small enough, the probability of emigration/immigration can be neglected for this species. Define $n = S_{\max}$ and $P_s(S = 0, 1, \cdots, n)$. Then equation $P = [P_0, P_1, \cdots, P_n]^T$ can be represented as a matrix:

$$P = AP \quad (2)$$

where A is given in the Eq. (4).

$$p = \begin{cases} -(\mu_s + \lambda_s)P_s + \mu_{s+1}P_{s+1}, S = 0 \\ -(\mu_s + \lambda_s)P_s + \mu_{s-1}P_{s-1} + \mu_{s+1}P_{s+1}, 1 \le S < S_{\max} - 1 \\ -(\mu_s + \lambda_s)P_s + \mu_{s-1}P_{s-1}, S = S_{\max} \end{cases} \quad (3)$$

$$A = E\begin{bmatrix} -(\mu_0+\lambda_0) & \mu & 0 & \cdots & 0 \\ \lambda_0 & -(\mu_1+\lambda_1) & \mu_2 & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \lambda_{n-2} & -(\mu_{n-1}+\lambda_{n-1}) & \mu_n \\ 0 & \cdots & 0 & \lambda_{n-1} & -(\mu_n+\lambda_n) \end{bmatrix} \tag{4}$$

In order to facilitate our research, Figure 2 can be changed to Figure 3 assumed $E = I$.

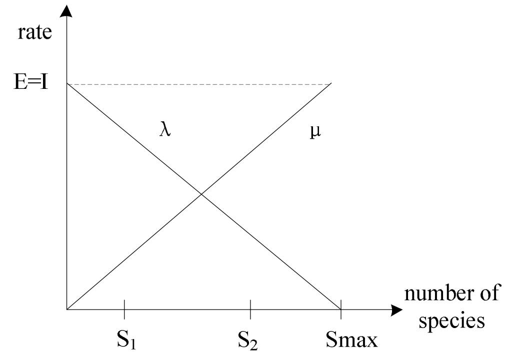


Fig. 3. Simplified species migration model of single island

$$\mu_k = \frac{E_k}{n} \tag{5}$$

$$\lambda_k = I\left(1-\frac{k}{n}\right) \tag{6}$$

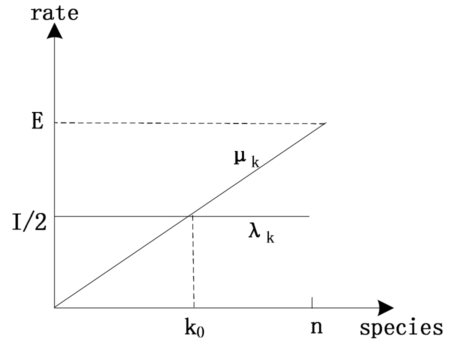
where $n = S_{\max}$ and k equals the number of species. Eq. (4) can be further changed as:

$$A = E\begin{bmatrix} -1 & \frac{1}{n} & 0 & \cdots & 0 \\ \frac{n}{n} & -n & \frac{2}{n} & \ddots & \cdots \\ \cdots & \ddots & \ddots & \ddots & \cdots \\ \cdots & \cdots & \frac{2}{n} & -1 & \frac{n}{n} \\ 0 & \cdots & 0 & \frac{1}{n} & -1 \end{bmatrix} = EA' \tag{7}$$

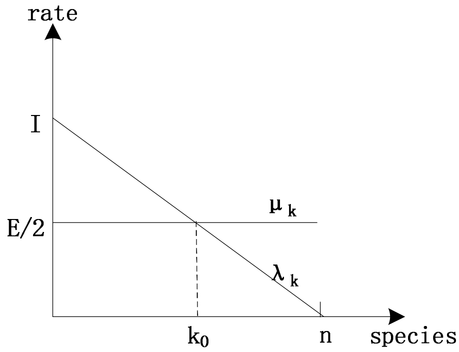
The probability to accommodate the number of species of each island is given by the following formula:

$$P_k = \begin{cases} P_0 = \dfrac{1}{1+\sum\limits_{l=1}^{n}\dfrac{\lambda_0\lambda_1\cdots\lambda_{l-1}}{\mu_1\mu_2\cdots\mu_l}} & ,k=0 \\ P_k = \dfrac{\lambda_0\lambda_1\cdots\lambda_{k-1}}{\mu_1\mu_2\cdots\mu_k\left(1+\sum\limits_{l=1}^{n}\dfrac{\lambda_0\lambda_1\cdots\lambda_{l-1}}{\mu_1\mu_2\cdots\mu_l}\right)} & ,1\le k\le n \end{cases} \tag{8}$$

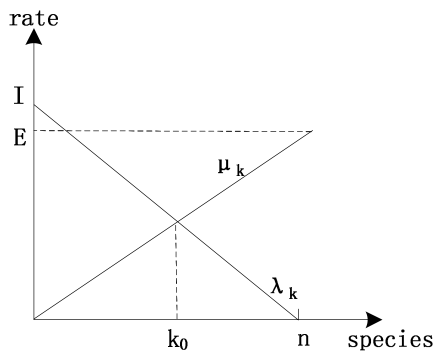
If each island (solution) has the same species migratory curve, that is to say S_2 represents a solution with higher HIS and S_1 represents a solution with lower HIS. The emigration rate of S_1 is lower than the corresponding value of S_2, while the immigration rate of S_1 is higher than the corresponding value of S_2. Through the mobility of each solution, it can make the information sharing among islands. Six species migration models of BBO algorithm are shown in Figure 4.



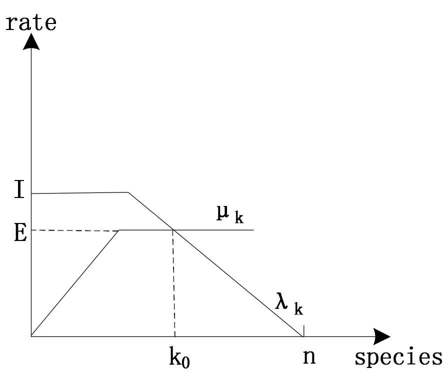
(a) Model 1



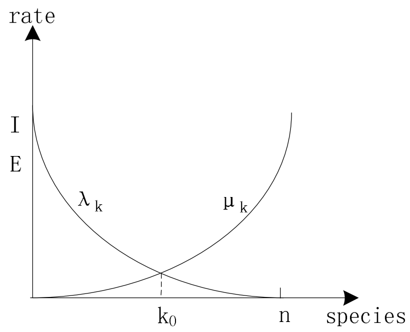
(b) Model 2



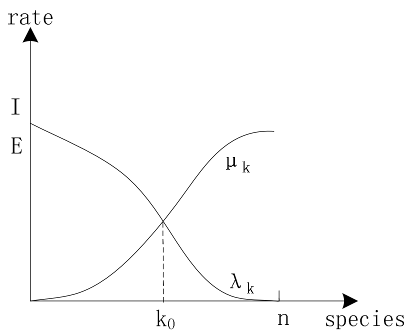
(c) Model 3



(d) Model 4



(e) Model 5



(f) Model 6

Fig. 4. Six species migration models of BBO algorithm

C. *BBO algorithm*

The BBO algorithm is a method composed by n habitats with D -dimension SIV fitness vector. H_i represents the fitness value of the habitat i. By comparing the habitat values of H_i with $S_{\max}$, the number of all species are denoted as n. Then the rest habitat population S_i is realized the successive reduction i according to H_i from good to bad, that is to say $S_i = S_{\max} - i$ ($i = 1, 2, \cdots, n$). By the above calculatation, the emigration rate μ and immigration rate λ of H_i in the different migration model can be obtained. Also the species contained probability $P(K_i)$ of H_i can be calculated.

$$M_S = M_{\max} \cdot (1 - \frac{P_S}{P_{\max}}) \tag{9}$$

So the mutation rate M_i of each H_i is obtained. The global variable is composed of the maximum emigration rate E, the immigration rate I, the mutation rate $M_{\max}$, the individual reservations elitist number Z and the global mobility rate P_{mod} of species.

The flowchart of BBO algorithm is shown in Figure 5. The algorithm procedure is described as follows.

Step 1: Initialize the parameters of BBO algorithm and the H_i vector of any habitat.

Step 2: For different suitability H_i, sort the habitats from good to bad. Generally the update rate of habitats $i = 1$.

Step 3: By comparison, judge whether the desired optimum is satisfied or not. If it is satisfied, the optimum is output and algorithm procedure is terminated. Otherwise, turn to Step 4.

Step 4: Suppose the maximum number of a specie in a habitat $S_{\max} = n$. Then by means of $S_i = S_{\max} - i$ ($i = 1, 2, \cdots, n$), the populations value S_i of habitat i is obtained, which is further brought into the migration model to obtain its λ_i and μ_i.

Step 5: After the cyclic operation of P_{mod}, whether i has entered into the immigration pattern (the number n of i is defined as the number of cycles) can be determined. If habitat i is carried out the immigration operation, the habitat immigration rate λ_i (the dimension D of SIV as the number of cycles) is used to judge its characterized component SIV_{ij} whether to be immigrated or not. If SIV_{ij} is implemented with the immigration, then, through its emigration rate μ_m ($m = 1, 2, \cdots, n$, $m \neq i$) it can be performed by selecting, and then the feature component SIV_{ij} of the i is replaced by a component of selected m.

Step 6: By calculating M_i of the corresponding habitat, the related variable of the Habitat i is judged to see whether the mutation has occurred. The results are compared and turn to Step 2.

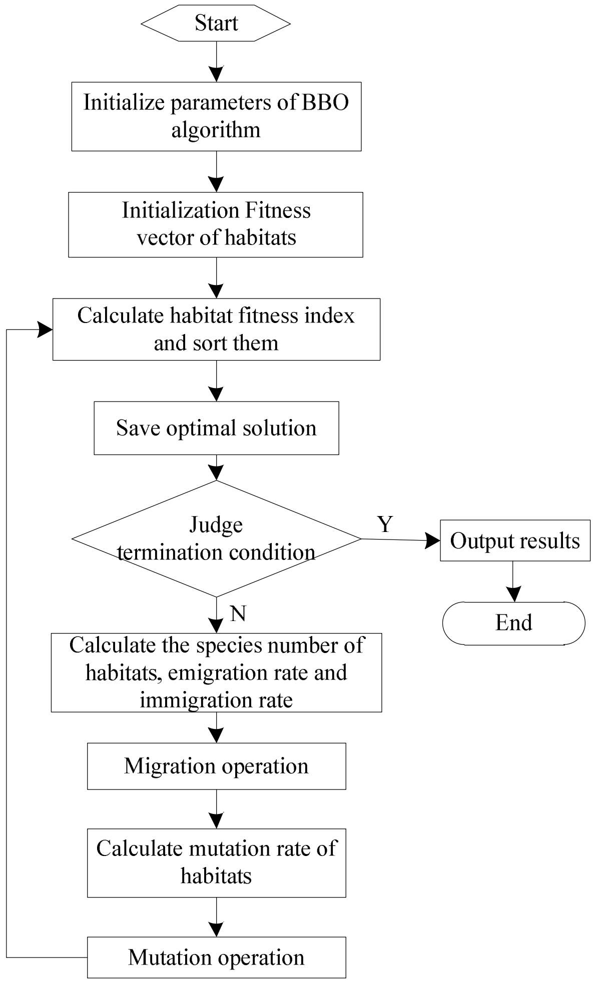


Fig. 5. Flowchart of BBO algorithm

III. Simulation Experiments

In this paper, ant colony optimization (ACO) algorithm, differential evolution (DE) algorithm and particle swarm optimization (PSO) algorithm are chosen for realizing the performance comparison with six Benchmark functions (Ackley function, Griewank function, Rastrigin function, Sphere function, Step function and Quartic function). Their n input parameters are defined as $x_i (i = 1, 2, \ldots, n)$. The global minimum point is $x^* = (0, \ldots, 0)$ and the minimum of the objective function is $f(x^*) = 0$. These six testing functions are described as follows.

In this paper, the parameters of each optimization algorithm are roughly adjusted in order to obtain better performance, but the parameters of any algorithm are not fine-tuned specially. The adopted parameters are described as follows. In ACO algorithm, the pheromone evaporation coefficient is 0.9 and the quality factor of the pheromone is 1. In DE algorithm, the weight factor and the cross-constant are 0.5. In PSO algorithm, the inertia constant is 0.3, the cognitive constant is 1 and the particle swarm interaction constant is 1. In BBO algorithm, the habitat update probability is 1, the mobility range of each generation is [0, 1], the step length of probability integral calculation, the maximum immigration and the emigration rate are all 1. For each algorithm, the population size is 50 and the maximum iteration number is 50.

The simulation results are shown in Figure 6-11. The statistics results are shown in Table 1 and Table 2.

(1) Ackley function: $f_1(x) = 20 + e - 20e^{\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right)} - e^{\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)}, -32 \le x_i \le 32, \min(f_1) = 0.$

(2) Griewank function: $f_2(x) = \frac{1}{4000}\left(\sum_{i=1}^{n}(x_i^2)\right) - \left(\prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}})\right) + 1, -600 \le x_i \le 600, \min(f_2) = 0.$

(3) Rastrigin function: $f_3(x) = \sum_{i=1}^{n}(x_i^2 - 10\cos(2\pi x_i) + 10), -15 \le x_i \le 15, \min(f_3) = 0.$

(4) Sphere function: $f_4(x) = \sum_{i=1}^{n} x_i^2, -100 \le x_i \le 100, \min(f_4) = 0$

(5) Step function: $f_5(x) = \sum_{i=1}^{n}(|x_i + 0.5|)^2, -100 \le x_i \le 100, \min(f_5) = 0$

(6) Quartic function: $f_7(x) = \sum_{i=1}^{n} ix_i^4 + random[0,1), -1.28 \le x_i \le 1.28, \min(f_7) = 0$

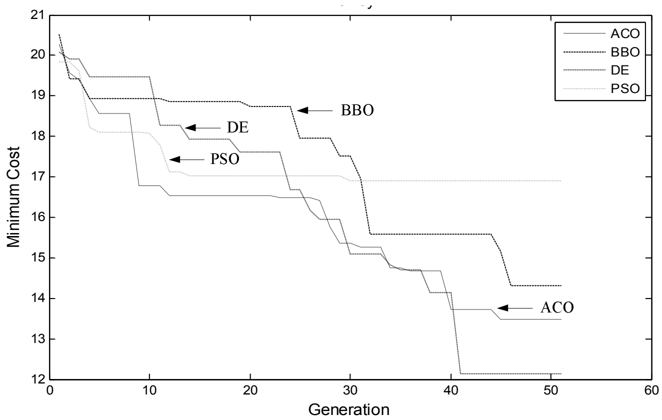


Fig. 6. Simulation results for Ackley function

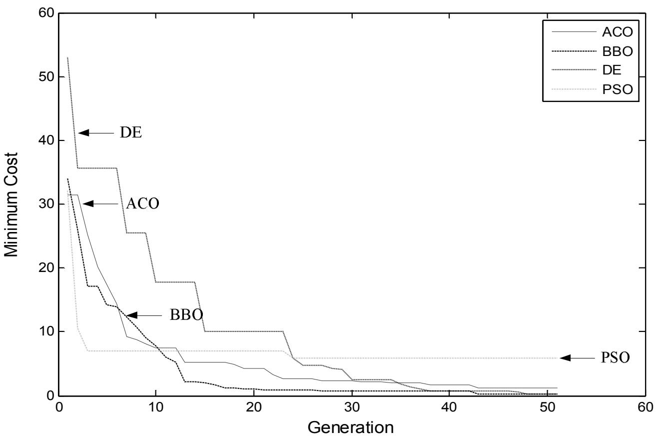


Fig. 8. Simulation results for Quartic function

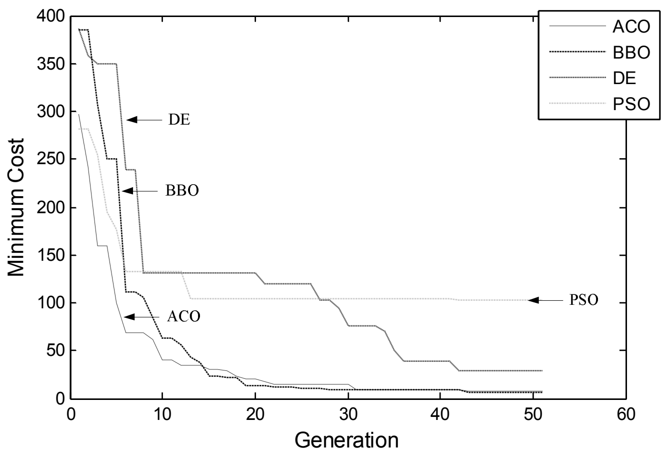


Fig. 7. Simulation results for Griewank function

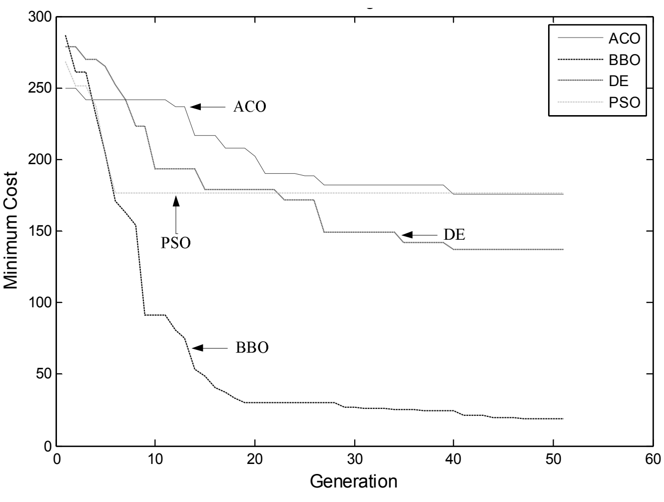


Fig. 9. Simulation results for Rastrigin function

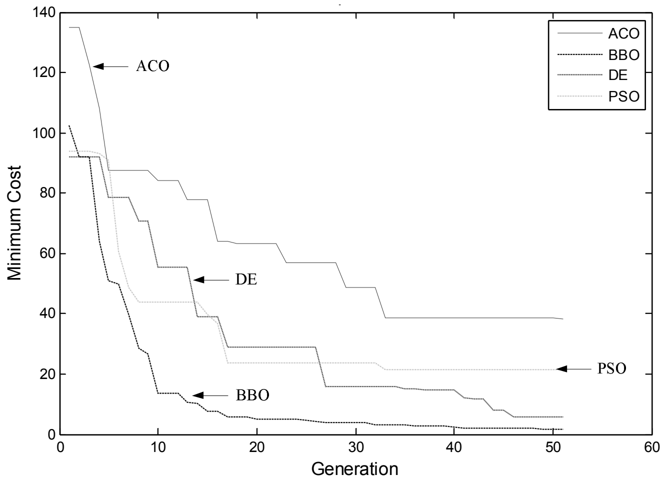


Fig. 10. Simulation results for Sphere function

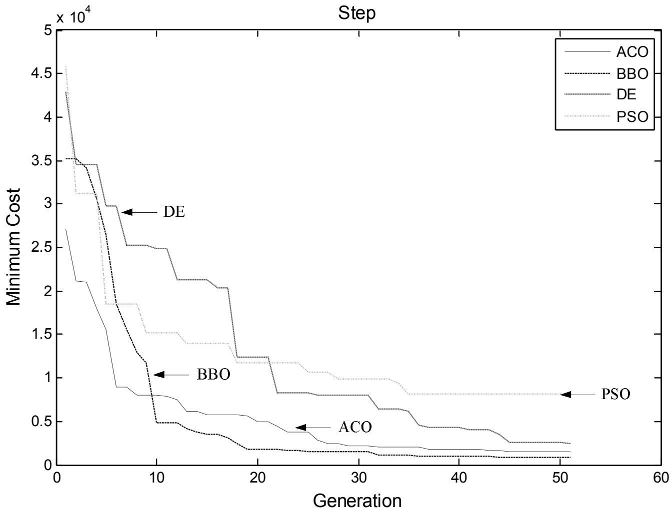


Fig. 11. Simulation results for Step function

TABLE 1. OPTIMAL SOLUTION UNDER FOUR ALGORITHMS

Function	ACO	BBO	DE	PSO
Ackley	13.4783	14.3126	16.901	12.1444
Griewank	7.6377	5.9489	28.9109	102.6348
Rastrigin	175.6749	18.51	137.1462	176.2998
Sphere	38.3087	1.8444	5.7803	21.5655
Step	1514	812	2445	8121
Quartic	1.1613	0.26099	0.27211	5.8301

TABLE 2. AVERAGE FUNCTION VALUES UNDER FOUR ALGORITHMS

Function	ACO	BBO	DE	PSO
Ackley	20.1244	20.2531	20.0362	14.0955
Griewank	116.1742	17.1637	49.235	314.7486
Rastrigin	359.658	28.4341	187.2935	302.6841
Sphere	165.911	5.6114	12.4625	99.4257
Step	16018.68	1807.16	4298.18	40422
Quartic	29.3765	2.2185	1.5771	150.0125

It can be seen form Figure 6-11, the BBO algorithm is behaved very well compared with other three swarm intelligent algorithms in five benchmark functions in addition to seem slightly less in Ackley function. The iteration number of BBO algorithm is about 20 times to be leveled to the minimum, which proves that the algorithm is more rapid and efficient in global function optimization problem. Seen form the simulation results, the optimal solution and the average value of the BBO algorithm for five benchmark functions are both at the forefront position. For Ackley function, it is slightly inferior to ACO and DE algorithm. The simulation results may indicate that BBO algorithm is the most effective in terms of solving function optimization problems, the DE and ACO algorithm secondly, the PSO algorithm has worst performance.

IV. CONCLUSION

In that the current optimization algorithms have poor convergence in solving function optimization problems, the BBO algorithm is used to solve the function optimization problem. The simulation experiments are carried out with multiple test functions to verify it's optimization performance. Simulation results show that BBO algorithm can solve function optimization problems more efficiently. The convergence speed and optimization accuracy are higher than other intelligent optimization algorithms. These results also indicate deep-seated that the biogeography mechanism is the law of nature in the formation of long-term evolution and it is unique and effective in handling all kinds of optimization problems.

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