Two-echelon Price Competition with the Choice of Manufacturer's Direct Channel and Retailer's Store Brand

Zonghong Cao[1*], Ju Zhao[2], Yongwu Zhou[3], Chuangwen Li[3]

Abstract—**This paper considers a choice game that whether a manufacturer runs a direct channel (*d*-channel) and whether a retailer should respond by introducing a store brand (SB) in a two-echelon supply chain in which a dominant manufacturer sells his national brand product (NB) through a retailer. The results show that (i) the two channel competition and the brand competition can weaken the negative effects of double marginalization; (ii) when the operating costs for the *d*-channel and the SB are small, the optimal stategy is to introduce the *d*-channel and the SB, and a win-win outcome is achieved, and when they are relatively high, it is contrary; (iii) as the leader, the manufacturer has a first-mover advantage to maximize his profit when the operating costs are medium. The sensitivity analyses indicate that the manufacturer can benefit from the fierce channel competition, whereas the retailer prefers to the fierce brand competition.**

Index Terms—**supply chain, pricing, direct channel, store-brand, equilibrium**

I. Introduction

A. Motivation

With the rapid development of e-commerce, many manufacturers such as IBM, Cisco and Nike have opened their own *d*-channels besides traditional *r*-channels (Kumar and Ruan 2006[1], Wang et al. 2016[2]). When a *d*-channel is established, opportunities and threats coexist (Choi 2003[3]). From manufacturers' perspectives, running a *d*-channel may reduce the dependence on *r*-channels and enhance their bargaining power. However, the presence of the *d*-channel may intensify the competition between manufactures and retailers, sometimes deteriorating retailers (Chiang et al. 2003[4], Seifert et al. 2006[5]). This may result in retailers' counterattack, including the improvement of service level and introduction of the SB product, *etc.*. Among these measures, introducing the SB product is a prevailing strategy. The latest available data show that the SB now account for at least 30% of all packaged food products sold in Europe (Nielsen/PLMA, 2014[6]). Many empirical researches have shown that introducing the SB tends to alleviate the retailer's dependence on the NB product, increase the demand of the *r*-channel and improve customer loyalty to the retailer (Kadiyali et al. 2000[7], Pauwels and Srinivasan 2004[8], Hansen et al. 2006[9], Marta and Javier 2012[10]). Thus, in reality, *d*-channel vs. *r*-channel competition and NB vs. SB competition may occur simultaneously in a supply chain. Since advantages and disadvantages of running dual channel or selling both NB and SB products are coexistent, several natural questions arise. Whether or under what conditions should a manufacturer choose to run *d*-channel? Whether or under what conditions should a retailer introduce SB product? How should a manufacturer and a retailer determine their pricing policies when *d*-channel and SB product occur in a supply chain? To the best of our knowledge, little literature answers these questions.

This paper will discuss a two-echelon supply chain, where one dominant manufacturer with the option of running a *d*-channel sells a NB through one retailer, who has the option of introducing a SB. We will investigate the two partners' equilibrium options and their corresponding pricing policies.

B. Literature Review

The earlier works related to this paper mainly include two categories, which involves channel competition and brand competition.

With the emergence of e-commerce, the channel competition issues have gained increasing attention from academy. Rhee and Park (2000)[11] developed a hybrid channel model in which they divide consumers into two segments: a price sensitive segment and a service sensitive segment. They indicated that the hybrid channel is optimal when the segments are similar in their valuations of the retail service. Chiang et al. (2003)[4] considered the effect of the *d*-channel on the pricing strategies, the sales, the profits of a vertically integrated firms, and customer channel preference. They assumed that customer's acceptance of *d*-channel is homogeneous, and showed that the *d*-channel could enhance the manufacturer's negotiation power. Kumar and Ruan (2006)[1] considered a dual channel model in which consumers are divided into two segments: manufacturer loyal and retailer loyal. They also presented that the manufacturer can benefit from a *d*-channel. Using the same demand function as in [1], Cai et al. (2009)[12] evaluated the impact of price discount contracts and pricing schemes on the dual channel supply chain. Xu et al. (2013)[13] noted that customers preferred dual channels that offered them more shopping choices and experiences, and this trend forced the manufacturer to introduce a *d*-channel as a necessary strategy. Shang and Yang (2015)[14] applied the profit-sharing contract to coordinate a dual channel supply chain and examined the

Manuscript received Mar 11, 2016; revised May 12, 2016. This paper was supported by Humanities and Social Science Foundation of the Ministry of Education under Grant Nos. 16YJA630003, 16YJC630164 and 15YJC630053, and National Natural Science Foundation of China under Grant Nos. 71201044 and 91546108.

1*. School of Natural Sciences, Anhui Agricultural University, Hefei, 230036; Phone:(+86)0551-65786164; E-mail: (caozh666@sina.com).

2. School of Management, Hefei University of Technology, Hefei, 230009;

3. School of Business Administration, South China University of Technology, Guangzhou, Guangdong 510640.

selection of profit-sharing parameters and the allocation of extra system profit. Matsui (2016)[15] studied an asymmetric product distribution strategy for a manufacturer that uses dual channel supply chain. More examples falling into this category can be found in the review article by Tsay and Agrawal (2004)[16].

Brand competition includes two streams. One stream is empirical studies on SB products. These empirical studies mainly focus on SB product introduction strategies for retailers, prevention strategies for NB manufacturers, and the role of SB product in channel relations[7]-[10]. The other stream discusses competitive pricing issues between NB and SB by mathematical modeling. Narasimhan and Wilcox (1998)[17] analyzed the impact of SB product on equilibrium pricing strategies and corresponding profits. Their research results showed that SB product introduction shifts some surplus from the manufacturer not only to the retailer but also to consumers. Groznik and Heese (2010)[18] and Choi and Fredj (2013)[19] studied pricing strategies between two r-channels with an endogenous manufacturer, where the manufacturer sells a NB product through two competing retailers, and each retailer has the option of introducing SB product. Ru et al. (2015)[20] showed that a SB may benefit the manufacturer when the manufacturer and the retailer play a retailer-Stackelberg game. Kurata et al. (2007)[21] analyzed channel pricing, where an NB is distributed through both a d-channel and a r-channel but SB is only distributed through a r-channel. In a Nash pricing game frame, they focused on channel competition and coordination issues. The results indicated that wholesale price failed to coordinate the supply chain, but an appropriate combination of markup and markdown prices can coordinate it and achieve a win-win outcome for each channel. Amrouche and Yan (2012)[22] proposed a model by implementing a d-channel for NB competing against SB. They discussed the impact of introducing the SB and implementing a d-channel on two the players' profits in three cases: NB product was sold solely through a r-channel, the SB was introduced by the retailer, and the manufacturer opened a d-channel. Different from [21] and [22], this paper establishes a choice game model, that is, we focus on whether or under what conditions the manufacturer and the retailer should introduce d-channel and SB product, respectively. In addition, the paper discusses the effect of the operating cost difference between the NB at the d-channel and the SB. This paper also investigates pricing policies, and profits of the two players and the whole chain.

The main contributions of this paper include three aspects. First, while most related papers discussed pricing under either channel competition or brand competition, this paper considers a choice game of the manufacturer and the retailer, i.e., whether to introduce the d-channel and the SB. Second, differentiated from Nash pricing game frame in [21], we discuss pricing game under a manufacturer-Stackelberg framework. Finally, different from [22], this paper specifies the conditions under which the manufacturer and the retailer should introduce the d-channel and the SB.

II. Problem Formulation and Basic Model

Consider a two-echelon supply chain consisting of a dominant manufacturer (he) and a retailer (she). The manufacturer produces a NB at c_1/unit and sells it at w_1/unit to the retailer, who sells it at p_1/unit to consumers. Now the manufacturer and the retailer may decide whether to run a d-channel and to introduce a SB respectively. Suppose that the manufacturer runs a d-channel, the NB's unit cost in the d-channel, c_0, will be no less than its unit production cost, *i.e.*, $c_0 \geq c_1$, because running the d-channel involves extra charge such as channel building and managing cost. For brevity, we also refer to c_0 as the unit operating cost in the d-channel. Likewise, c_2 is the unit operating cost of the SB, which includes its unit production cost. p_0 represents the NB's price in the d-channel and p_2 is the SB's retail price.

We now design the choice game between the manufacturer and the retailer unfolding in three stages. In the first stage, the manufacturer decides whether to or not to run the d-channel, and the retailer decides whether or not to complement the SB with the NB. Second, the manufacturer sets the wholesale price w_1 for the NB and the online price p_0 when he decides to sell direct. Finally, knowing the manufacturer's decision, the retailer sets the NB's price p_1 at the r-channel and the SB's price p_2 if she offers the SB. There are four subgames in the model as follows: the r-channel providing only the NB (Case 1), and the r-channel providing the SB other than the NB (Case 2), introducing d-channel except the r-channel providing the NB (Case 3), r-channel introducing the SB by the retailer and d-channel being run by the NB manufacturer (Case 4). The choice game structure is illustrated in Fig. 1.

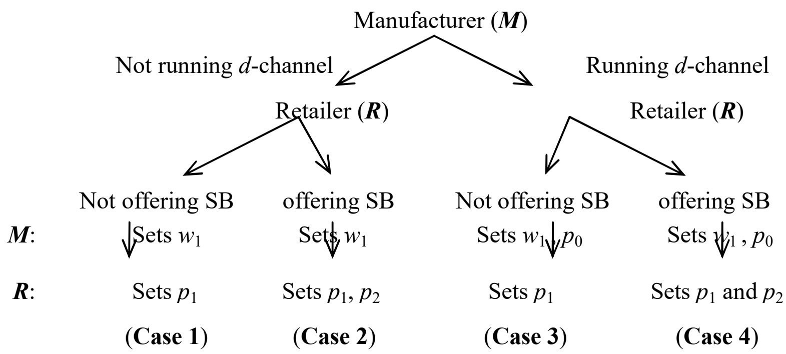

Fig. 1 The choice game structure

Comparing with [22], Case 3 discussed in this paper was not considered in [22]. They discussed the effect of the quality difference between NB and SB, whereas we consider the impact of the unit operating cost difference between the NB in the d-channel and the SB. Additionally, we will focus the game's equilibrium outcome in Section 4, which was not discussed in [22].

Consistent with [1], assume that consumers consist of two groups: *brand* loyal and *store* loyal. The *brand* loyal consumers only purchase the NB from either the *d*-channel or the *r*-channel, whereas the *store* loyal consumers buy either the NB or the SB only from the specific retailer. The *brand* loyal consumers have a strong preference for the NB and will never consider buying a different brand and the segment of size is ε_M. However, the *store* loyal consumers are not loyal to any specific brand and the segment of size is ε_R. Relative to the *brand* loyal consumers, the *store* loyal consumers maybe be viewed as those who are less informed about the products in the specific category. They have a need to touch and feel the product before purchasing. Consequently, consumers of this type will never consider buying the NB through the *d*-channel.

The demand for each product and profit of each player in the four cases are summarized as follows.

Case 1: only the NB is offered in the *r*-channel. The demand is linear in its retail price p_1, shown as follows

$$D=(\varepsilon_M+\varepsilon_R)(1-\beta p_1). \quad (1)$$

where β measures the effect of retail price on the demand.

The linear demand function is widely applied in price competition literature (*e.g.*, [12], [22]) since it is tractable and enables closed-form solutions.

The profits in Case 1 are given as follows

$$\Pi_M(w_1)=(\varepsilon_M+\varepsilon_R)(1-\beta p_1)(w_1-c_1). \quad (2)$$

$$\Pi_R(p_1)=(\varepsilon_M+\varepsilon_R)(1-\beta p_1)(p_1-w_1). \quad (3)$$

Case 2: the SB and the NB are offered through the *r*-channel. If the SB is offered, a fraction of *store* loyal consumers will shift from the NB's ones. As noted earlier *store* loyal consumers fulfill all their purchasing needs only in the *r*-channel, the retailer could influence the purchasing decision of *store* loyal consumers. To capture this feature, we assume that the fraction of *store* loyal consumers that purchase the SB depends on the level of sales effort (*e.g.*, advertisement and shelf space) that the retailer allocates to the SB. We assume that the total level of sales effort to SB and NB is normalized for simplicity to 1, and λ_2 represents the NB's sales effort level. Correspondingly, the SB's sales effort is $1-\lambda_2$. The baseline demand for NB in the *r*-channel is equal to $\varepsilon_M+\lambda_2\varepsilon_R$, and the baseline demand for SB is equal to $(1-\lambda_2)\varepsilon_R$. The demand for NB through the *r*-channel (denoted by D_1) and for the SB (denoted by D_2) and the profits of the two partners are given as follows.

$$D_1=(\varepsilon_M+\lambda_2\varepsilon_R)(1-\beta p_1)+\eta(p_2-p_1). \quad (4)$$

$$D_2=(1-\lambda_2)\varepsilon_R(1-\beta p_2)+\eta(p_1-p_2). \quad (5)$$

$$\Pi_M(w_1)=[(\varepsilon_M+\lambda_2\varepsilon_R)(1-\beta p_1)+\eta(p_2-p_1)](w_1-c_1). \quad (6)$$

$$\Pi_R(p_1,p_2)=[(\varepsilon_M+\lambda_2\varepsilon_R)(1-\beta p_1)+\eta(p_2-p_1)](p_1-w_1) +[(1-\lambda_2)\varepsilon_R(1-\beta p_2)+\eta(p_1-p_2)](p_2-c_2). \quad (7)$$

Where η is the competition intensity between NB and SB.

Case 3: the manufacturer runs the *d*-channel and the retailer only sells the NB. If the manufacturer runs a *d*-channel, a fraction of *brand* loyal consumers shift their purchases online. *Brand* loyal consumers switch from the *r*-channel to the *d*-channel due to the convenience that online shopping affords, and/or their expensive shopping (transportation) costs and/or their price sensitivities to the price. Let λ_1 represent the initial ratio of the *brand* loyal consumers who buy the NB from the retailer to all *brand* loyal consumers, i.e., the baseline demand for the NB product in the *d*-channel is equal to $(1-\lambda_1)\varepsilon_M$, and the total demand for the NB in the *r*-channel is equal to $\lambda_1\varepsilon_M+\varepsilon_R$. The demand for the NB through the *d*-channel (denoted by D_0) and the demand for the NB through the retail channel (denoted by D_1) are

$$D_0=(1-\lambda_1)\varepsilon_M(1-\beta p_0)+\gamma(p_1-p_0). \quad (8)$$

$$D_1=(\lambda_1\varepsilon_M+\varepsilon_R)(1-\beta p_1)+\gamma(p_0-p_1). \quad (9)$$

Both members' profits are

$$\Pi_M(w_1,p_0)=[(1-\lambda_1)\varepsilon_M(1-\beta p_0)+\gamma(p_1-p_0)](p_0-c_0) +[(\lambda_1\varepsilon_M+\varepsilon_R)(1-\beta p_1)+\gamma(p_0-p_1)](w_1-c_1), \quad (10)$$

$$\Pi_R(p_1)=[(\lambda_1\varepsilon_M+\varepsilon_R)(1-\beta p_1)+\gamma(p_0-p_1)](p_1-w_1). \quad (11)$$

Where γ is viewed as the channel competition intensity between the *d*-channel and the *r*-channel.

Case 4: the manufacturer runs the *d*-channel and the retailer offers the SB. According to Case 2 and Case 3, we assume that these demands are linearly dependent on the sales prices, which are given as follows:

(*a*) the *brand* loyal demand for NB in *d*-channel is
$D_0=(1-\lambda_1)\varepsilon_M(1-\beta p_0)+\gamma(p_1-p_0)$,

(*b*) the *brand* loyal demand for NB in *r*-channel is
$D_{10}=\lambda_1\varepsilon_M(1-\beta p_1)+\gamma(p_0-p_1)$,

(*c*) the *store* loyal demand for NB in *r*-channel is
$D_{12}=\lambda_2\varepsilon_R(1-\beta p_1)+\eta(p_2-p_1)$,

(*d*) the *store* loyal demand for SB in *r*-channel is
$D_2=(1-\lambda_2)\varepsilon_R(1-\beta p_2)+\eta(p_1-p_2)$.

The profits of the two sides are given as follows:

$$\Pi_M(w, p_0)=D_0(p_0-c_0)+(D_{10}+D_{12})(w-c_1) =(a_0-b_0p_0+\gamma p_1)(p_0-c_0)+(a_1-b_1p_1+\gamma p_0+\eta p_2)(w-c_1). \quad (12)$$

$$\Pi_R(p_1, p_2)=(D_{10}+D_{12})(p_1-w)+D_2(p_2-c_2) =(a_1-b_1p_1+\gamma p_0+\eta p_2)(p_1-w)+(a_2-b_2p_2+\eta p_1)(p_2-c_2). \quad (13)$$

where $a_0=(1-\lambda_1)\varepsilon_M$, $b_0=\beta a_0+\gamma$, $a_1=\lambda_1\varepsilon_M+\lambda_2\varepsilon_R$, $b_1=\beta a_1+\gamma+\eta$, $a_2=(1-\lambda_2)\varepsilon_R$ and $b_2=\beta a_2+\eta$.

Due to $D_0+D_{10}+D_{12}+D_2=(1-\lambda_1)\varepsilon_M(1-\beta p_0)+(\lambda_1\varepsilon_M+\lambda_2\varepsilon_R)(1-\beta p_1)+(1-\lambda_2)\varepsilon_R(1-\beta p_2)$, the total demand is not affected by γ and η. This implies that a change in intensities of both channel competition and brand competition do not lead to any variation in the aggregate demand.

III. Two Members' Decisions in Each Case

Case 1: only NB available through the *r*-channel

As a benchmark, we develop a basic model where neither the manufacturer runs the *d*-channel nor the retailer introduces SB (denoted by superscript "*n*"). As the leader, the manufacturer first declares w_1, the retailer then decides her retail price p_1. From (2) and (3), one can derive by backward induction that the prices and the profits are

$$w_1^n=c_1+(1-\beta c_1)/(2\beta),$$
$$p_1^n = c_1+3(1-\beta c_1)/(4\beta),$$
$$\Pi_M^n=(\varepsilon_M+\varepsilon_R)(1-\beta c_1)^2/(8\beta),$$
$$\Pi_R^n=(\varepsilon_M+\varepsilon_R)(1-\beta c_1)^2/(16\beta). \quad (14)$$

Case 2: only introducing the SB

In this setting, only the retailer introduces the SB, and the profits for both sides are respectively given in (6) and (7). One can derive two members' optimal pricing strategies (denoted by superscript "*s*"). Lemma 1 gives the optimal pricing strategies for both members.

Lemma 1. Define $m=b_0+b_1-2\gamma$, if only the retailer sells the SB, then the optimal pricing strategies are given by

$$p_1^s= c_1+3(1-\beta c_1)/(4\beta)-\eta(1-\beta c_2)/(4\beta m),$$
$$p_2^s= c_2+(1-\beta c_2)/(2\beta),$$
$$w_1^s= c_1+(1-\beta c_1)/(2\beta)-\eta(1-\beta c_2)/(2\beta m).$$

All proofs are provided in Appendix. From Lemma 1, one can easily derive that the market demands of the two products and the profits of two sides are given as follows:

$D_1^s= m(1-\beta c_1)/(4\beta)-\eta(1-\beta c_2)/(4\beta)$,

$D_2^s= (2b_2m-\eta^2)(1-\beta c_2)/(4\beta m)-\eta(1-\beta c_1)/(4\beta)$,

$\Pi_M{}^s=[m(1-\beta c_1)-\eta(1-\beta c_2)]^2/(8\beta^2 m)$,

$\Pi_R{}^s=(4b_2m-3\eta^2)(1-\beta c_2)^2/(16\beta^2 m)-\eta(1-\beta c_1)(1-\beta c_2)/(8\beta^2)$
$\quad +m(1-\beta c_1)^2/(16\beta^2)$.

The differences of the wholesale prices and the retail prices between Case 1 and Case 2 are as follows:

$w_1^s-w_1^n= -\eta(1-\beta c_2)/(2\beta m)<0$,

$p_1^s-p_1^n= -\eta(1-\beta c_2)/(4\beta m)<0$.

The wholesale price and the retail price of the NB in Case 2 are lower than that in Case 1. This implies that introducing the SB induces the decreasing of the wholesale price and the retail price for NB. As a result, the manufacturer's profit margin for the NB is reducing. However, an interesting phenomenon is that the retailer's profit margin for NB is increasing due to $(p_1^s- w_1^s)-(p_1^n-w_1^n)=\eta(1-\beta c_2)/(4\beta m)>0$.

Table 1 Sensitivity of pricing strategies to the parameters in Case 2

	c_2	η	λ_2	ε_M	ε_R
w_1^s	+	-	+	+	+
p_2^d	+	0	0	0	0
p_1^d	+	-	+	+	+

The sensitivity of the pricing strategies to the parameters is listed in Table 1. When c_2 increases, the SB's price increases. The higher the SB's cost, the weaker the SB's competition with the NB will be. Correspondingly, the manufacturer increases the wholesale price and the retailer increases the retail price. The higher the competition intensity (η) between the NB and the SB, the lower the wholesale price and the retail price is. When the *brand* loyal consumers (ε_M), or the *store* loyal consumers (ε_R) or the initial ratio of the *store* loyal consumers who prefer to buy the NB (λ_2) increase, the NB's baseline demand increases and, hence, the manufacturer increases the wholesale price and the retailer will increase the NB's retail price.

Theorem 1 gives the condition under which the retailer is willing to introduce the SB, and the impact of introducing the SB on the profits for the manufacture, the retailer and the whole supply chain.

Theorem 1. (1) if $c_2\in[c_2^{s\text{-min}}, c_2^{s\text{-}n}]$, the retailer is willing to introduce the SB;

(2) if $c_2\in[c_2^{s\text{-min}}, c_2^{s\text{-}n}]$, then $\Pi_R{}^s\geq\Pi_R{}^n$ and $\Pi_M{}^s\leq\Pi_M{}^n$;

(3) if $c_2\in[c_2^{s\text{-min}}, c_2^{C\text{-}s})$, then $\Pi_C{}^s=\Pi_R{}^s+\Pi_M{}^s>\Pi_C{}^n=\Pi_R{}^n+\Pi_M{}^n$, and if $c_2\in[c_2^{C\text{-}s}, c_2^{s\text{-}n}]$, then $\Pi_C{}^s\leq\Pi_C{}^n$, where

$c_2^{s\text{-min}} =\max[(\eta-m(1-\beta c_1))/(\beta\eta),0]$;

$c_2^{s\text{-}n} = \min\{1/\beta-m\eta(1-\beta c_1)/[\beta(2b_2m-\eta^2)],\ 1/\beta-[m\eta+\surd(m^2\eta^2$
$\quad -m(b_2-2\eta)(4b_2m-3\eta^2)](1-\beta c_1)/[\beta(4b_2m-\eta^2)]\}$;

$c_2^{C\text{-}s} =1/\beta-[3m\eta(1-\beta c_1)$
$\quad +\surd(9m^2\eta^2-3(4b_2m-\eta^2)(b_2-2\eta))](1-\beta c_1)/[\beta(4b_2m-\eta^2)]$.

From Theorem 1, we obtain that the retailer can benefit from the introduction of the SB, whereas the manufacturer will be worse off in the presence of the SB. Besides, the brand competition between the two partners benefits to the supply chain only if the unit cost of SB is low, i.e., $c_2<c_2^{C\text{-}s}$, correspondingly, Pareto improving is achieved, whereas the brand competition harms the supply chain if $c_2>c_2^{C\text{-}s}$.

Case 3: only running introducing the *d*-channel

In this case, only the manufacturer runs the *d*-channel, and the profits of both sides are given in (10) and (11). Two members' optimal pricing strategies are shown in Lemma 2 (denoted by superscript "*d*").

Lemma 2. Define $k=b_1+b_2-2\eta$, if the manufacturer runs the *d*-channel, then the optimal pricing strategies are given by

$p_1^d= c_1+3(1-\beta c_1)/(4\beta)-\gamma(1-\beta c_0)/(4\beta k)$,

$w_1^d= c_1+(1-\beta c_1)/(2\beta)$,

$p_0^d=c_0+(1-\beta c_0)/(2\beta)$.

It is easy to have $p_0^d-w_1^d=(c_0-c_1)/2\geq0$ due to $c_0\geq c_1$. This means that when the manufacturer runs the *d*-channel, the manufacturer's online price is no less than his wholesale price offered to the retailer, which means that the retailer will not purchase the NB from the *d*-channel.

From Lemma 2, one easily derives that the demands and the profits of two members are respectively given by

$D_0^d =(2b_0k-\gamma^2)(1-\beta c_0)/(4\beta k)-\gamma(1-\beta c_1)/(4\beta)$,

$D_1^d =k(1-\beta c_1)/(4\beta)-\gamma(1-\beta c_0)/(4\beta)$,

$\Pi_M{}^d=(2b_0k-\gamma^2)(1-\beta c_0)^2/(8\beta^2k)-\gamma(1-\beta c_0)(1-\beta c_1)/(4\beta^2)$
$\quad +k(1-\beta c_1)^2/(8\beta^2)$,

$\Pi_R{}^d=[k(1-\beta c_1)-\gamma(1-\beta c_0)]^2/(16\beta^2k)$.

Comparing the wholesale price and the retail price between Case 1 and Case 3 leads to:

$w_1^d-w_1^n=0$,

$p_1^d-p_1^n=-\gamma(1-\beta c_0)/(4\beta k)<0$.

Note that the wholesale price for the NB in Case 1 and Case 3 is equal, whereas the retail price in the *r*-channel in Case 3 is lower than the one in Case 1. This implies that running the *d*-channel forces the retailer to decrease the retail price. Correspondingly, her profit margin is decreasing. This indicates that the channel competition harms the retailer, benefits consumers, and weakens the negative effects of double marginalization caused by high retail price.

Table 2 lies in the sensitivity of the pricing strategies in Case 3 with respect to the parameters. The direct price and the retail price for the NB increase with the operating cost of *d*-channel (c_0). If the channel competition intensity (γ) between the *d*-channel and the *r*-channel increases, the retailer has to decrease the NB's price. When the *brand* loyal consumers (ε_M), or the *store* loyal consumers (ε_R) or the initial ratio of the *brand* loyal consumers who prefer to buy the NB from the retailer (λ_1) increase, the baseline demand for the NB at the *r*-channel increase and, hence, the retailer will increase the NB's retail price.

Table 2 Sensitivity of pricing strategies to the parameters in Case 3

	c_0	γ	λ_1	ε_M	ε_R
w_1^d	0	0	0	0	0
p_0^d	+	0	0	0	0
p_1^d	+	-	+	+	+

The specified condition under which the manufacturer is willing to run the *d*-channel is discussed as follows.

Theorem 2. (1) if $c_0\in[c_1, c_0^{d\text{-}n}]$, the manufacturer is willing to run a *d*-channel;

(2) if $c_0\in[c_1, c_0^{d\text{-}n}]$, then $\Pi_M{}^d\geq\Pi_M{}^n$ and $\Pi_R{}^d\leq\Pi_R{}^n$;

(3) if $c_0\in[c_1, c_0^{C\text{-}d})$, then $\Pi_C{}^d>\Pi_C{}^n$, and if $c_0\in[c_0^{C\text{-}d}, c_0^{d\text{-}n}]$, then $\Pi_C{}^d<\Pi_C{}^n$, where

$c_0^{s\text{-max}}= \min\{1/\beta -\gamma k(1-\beta c_1)/[\beta(2b_0k-\gamma^2)],\ 1/\beta-[\gamma k+\surd(k^2\gamma^2$
$\quad -k(b_0-2\gamma)(2b_0k-\gamma^2)](1-\beta c_1)/[\beta(4b_0k-\gamma^{-2})]\}$;

$c_0^{C\text{-}d}=1/\beta-[3\ k\gamma(1-\beta c_1)$
$\quad +\surd(9k^2\gamma^2-3(4b_0k-\gamma^2)(b_0-2\gamma))](1-\beta c_1)/[\beta(4b_0k-\gamma^2)]$.

From Theorem 2, we can obtain that the manufacturer can benefit from running the *d*-channel, whereas the retailer will be worse off from running the d-channel. Besides, the channel competition between the two partners benefits to the supply chain only if the unit cost in the d-channel is low than $c_0^{C\text{-}d}$, i.e., $c_0<c_0^{C\text{-}d}$, correspondingly, Pareto improving is achieved.

Case 4: running the *d*-channel and introducing the SB

If the manufacturer runs the d-channel and the retailer introduces the SB, the profits are given in (12) and (13). One can derive two members' optimal pricing strategies (denoted by the superscript "ds"). Lemma 3 gives the optimal pricing strategies for both sides.

Lemma 3. Define $n=(2b_1b_2-\eta^2)(b_0b_1-\gamma^2)-\eta^2b_0b_1$, if the d-channel and the SB occur simultaneously, the optimal pricing strategies for both members are given by

$p_1^{ds}= c_1+3(1-\beta c_1)/(4\beta)-\gamma b_2(1-\beta c_0)/[4\beta(b_1b_2-\eta^2)] -\eta b_0(b_1b_2-\eta^2)(1-\beta c_2)/(2\beta n)$,

$p_2^{ds}= c_2+(1-\beta c_2)/(2\beta)-\gamma\eta(1-\beta c_0)/[4\beta(b_1b_2-\eta^2)] -\gamma^2\eta^2(1-\beta c_2)/(4\beta n)$,

$w_1^{ds}= c_1+(1-\beta c_1)/(2\beta)-\eta[2b_0(b_1b_2-\eta^2)-\gamma^2b_2](1-\beta c_2)/(2\beta n)$,

$p_0^{ds}= c_0+(1-\beta c_0)/(2\beta)-\gamma\eta(b_1b_2-\eta^2)(1-\beta c_2)/(2\beta n)$.

The optimal pricing policies will lead to the demands of two products in the two channels below.

$D_0^{ds}= [2b_0(b_1b_2-\eta^2)- \gamma^2b_2](1-\beta c_0)/[4\beta(b_1b_2-\eta^2)] -\gamma(1-\beta c_1)/(4\beta)$;

$D_1^{ds}= [b_1(1-\beta c_1)-\gamma(1-\beta c_0)-\eta(1-\beta c_2)]/(4\beta)$;

$D_2^{ds}=b_2(1-\beta c_2)/(4\beta)+(b_1b_2-\eta^2)(b_0b_1b_2-\gamma^2b_2-\eta^2b_0)(1-\beta c_2) /(2\beta n)-\eta(1-\beta c_1)/(4\beta)$;

Since $c_0\geq c_1$, it is obvious to have $p_0^{ds}-w_1^{ds}\geq 0$. That is to say, when the manufacturer runs the d-channel and the retailer introduces the SB as well, the manufacturer's online price in the d-channel is no less than his wholesale price offered to the retailer and, hence, the retailer will not purchase the NB product from the d-channel.

From the above analyses, we can derive Lemma 4.

Lemma 4. (1) $p_1^{ds}\leq p_1^{s}\leq p_1^{n}$, $p_1^{ds}\leq p_1^{d}\leq p_1^{n}$, $p_0^{ds}\leq p_0^{d}$ and $p_2^{ds}\leq p_2^{s}$;

(2) $\dfrac{\partial\Pi_M^{\ d}}{\partial c_0}<0, \dfrac{\partial\Pi_M^{\ ds}}{\partial c_0}<0, \dfrac{\partial\Pi_M^{\ s}}{\partial c_2}>0, \dfrac{\partial\Pi_M^{\ ds}}{\partial c_2}>0;$

(3) $\dfrac{\partial\Pi_R^{\ s}}{\partial c_2}<0, \dfrac{\partial\Pi_R^{\ ds}}{\partial c_2}<0, \dfrac{\partial\Pi_R^{\ d}}{\partial c_0}>0, \dfrac{\partial\Pi_R^{\ ds}}{\partial c_0}>0.$

Lemma 4 indicates that the channel and the brand competition induce the decreasing of retail prices. This means that the competition benefits consumers, increases the total demand and, hence, weakens the negative effects of double marginalization. From Lemma 4, one can also observe that if the manufacturer runs the d-channel, his profit will decrease but the retailer's will increase as the unit operating cost in the d-channel increases. Likewise, if the retailer introduces the SB, her profit will decrease but the manufacturer's profit will increase as the unit purchasing cost of the SB increases.

We will discuss under what condition the manufacturer will run the d-channel given that the retailer has introduced the SB, and under what condition the retailer will introduce the SB given that the manufacturer has run the d-channel.

Suppose that the retailer has introduced the SB (i.e., the parameter c_2 is given). In that case, if the manufacturer wants to run a d-channel, a fundamental condition is to assure $D_0^{ds}\geq 0$, which is equivalent to

$$c_0\leq\frac{1}{\beta}-\frac{\gamma(b_1b_2-\eta^2)(1-\beta c_1)}{\beta[2b_0(b_1b_2-\eta^2)-\gamma^2b_2]}\equiv c_0^{\ ds-\max}.$$

Besides, it is also necessary to pledge the manufacturer's profit increment incurred by running the d-channel no less than zero. Denote this increment by $\triangle\Pi_M^{\ ds\text{-}s}(c_0|c_2)$, then

$$\begin{aligned}\triangle\Pi_M^{\ ds-s}(c_0\mid c_2)&=\frac{\left(2b_0(b_1b_2-\eta^2)-b_2\gamma^2\right)}{8\beta^2(b_1b_2-\eta^2)}(1-\beta c_0)^2\\&-\frac{\gamma}{4\beta^2}(1-\beta c_1)(1-\beta c_0)-\left(\frac{b_0-2\gamma}{8\beta^2}\right)(1-\beta c_1)^2\\&+\left[\left(2(b_1b_2-\eta^2)b_0-b_2\gamma^2\right)m-n\right]\frac{\eta^2}{8\beta^2nm}(1-\beta c_2)^2.\end{aligned}$$

$\Pi_M^{\ s}$ is constant with respect to c_0, and $\Pi_M^{\ ds}$ decreased with c_0 according to Lemma 4. Thus, $\triangle\Pi_M^{\ ds\text{-}s}(c_0|c_2)$ decreases with c_0. Hence, if $\triangle\Pi_M^{\ ds\text{-}s}(c_0^{\ ds}|c_2)\geq 0$, then $c_0^{\ ds\text{-max}}$ is the maximal unit operating cost (denoted by $c_0^{\ ds\text{-}s}$) that the manufacturer can run the d-channel. Otherwise, if $\triangle\Pi_M^{\ ds\text{-}s}(c_1|c_2)>0$, then the equation $\triangle\Pi_M^{\ ds\text{-}s}(c_0|c_2)=0$ will be a unique root (denoted by $c_0^{\ 1}(c_2)$) in the interval (c_1, $c_0^{\ ds\text{-max}}$), whereas $\triangle\Pi_M^{\ ds\text{-}s}(c_1|c_2)\leq 0$ is more beneficial for the manufacturer not to run the d-channel. To sum up, given that the retailer has introduced the SB, the condition that the manufacturer should run the d-channel is that his unit operating cost in the d-channel does not exceed $c_0^{\ ds\text{-s}}$, where

$$c_0^{\ ds-s}=\begin{cases}c_0^{\ ds-\max}, & \Delta\Pi_M^{\ ds-s}\left(c_0^{\ ds-\max}\mid c_2\right)\geq 0,\\ c_0^{\ 1}\left(c_2\right), & \Delta\Pi_M^{\ ds-s}\left(c_1\mid c_2\right)>0>\Delta\Pi_M^{\ ds-s}\left(c_0^{\ ds-\max}\mid c_2\right).\end{cases}$$

Given that the manufacturer has run the d-channel, a fundamental condition for the retailer selling the SB is to have $D_2^{ds}\geq 0$, which is equivalent to

$$c_2\leq\frac{1}{\beta}-\frac{\eta nb_1\left(1-\beta c_1\right)}{\beta\left(nb_1b_2+(b_1b_2-\eta^2)(n-\gamma^2\eta^2)\right)}\equiv c_2^{\ ds-\max}.$$

, i.e., $c_0^{\ ds\text{-}s}=c_0^{\ ds\text{-max}}$;

Additionally, it is also natural to have the retailer's profit increment incurred by selling the SB product no less than zero. Denote this profit increment of the retailer by $\triangle\Pi_R^{\ ds\text{-}d}(c_2|c_0)$, then

$$\begin{aligned}\Delta\Pi_R^{\ ds-d}\left(c_2\mid c_0\right)&=\left(\frac{n^2b_1b_2+(b_1b_2-\eta^2)(n-\gamma^2\eta^2)(3n-\gamma^2\eta^2)}{16\beta^2n^2b_1}\right)(1-\beta c_2)^2\\&-\frac{\eta\gamma(n-\gamma^2\eta^2)}{8\beta^2nb_1}(1-\beta c_0)(1-\beta c_2)-\frac{\eta}{8\beta^2}(1-\beta c_1)(1-\beta c_2)\\&-\frac{(b_2-2\eta)}{16\beta^2}(1-\beta c_1)^2+\frac{\gamma^2(b_2-\eta)^2}{16\beta^2k(b_1b_2-\eta^2)}(1-\beta c_0)^2.\end{aligned}$$

Noting that $\Pi_R^{\ d}$ is constant with respect to c_2 and $\Pi_R^{\ ds}$ is a decreasing function of c_2 due to Lemma 4, we have that $\triangle\Pi_R^{\ ds\text{-}d}(c_2|c_0)$ is a decreasing function of c_2. Therefore, if $\triangle\Pi_R^{\ ds\text{-}d}(c_2^{\ ds\text{-max}}|c_0)\geq 0$, then $c_2^{\ ds\text{-max}}$ will be the maximal unit operating cost that the retailer can introduce the SB, otherwise, the equation $\triangle\Pi_R^{\ ds\text{-}d}(c_2|c_0)=0c_2^{\ ds\text{-}d}$ will be a unique root (denoted by $c_2^{\ 1}(c_0)$) in (c_1, $c_2^{\ ds\text{-max}}$) because $\triangle\Pi_R^{\ ds\text{-}d}(c_1|c_0)>0$, which will be derived in Appendix. Summing up the above analysis, we have that, given that the manufacturer has run the d-channel, the condition that the retailer introduce the SB is that her unit operating cost for the SB does not exceed $c_2^{\ ds\text{-}d}$, where

$$c_0^{\ ds-d}=\begin{cases}c_2^{\ ds-\max}, \Delta\Pi_R^{\ ds-d}\left(c_2^{\ ds}\mid c_0\right)\geq 0,\\ c_2^{\ 1}\left(c_0\right), \Delta\Pi_R^{\ ds-d}\left(c_2^{\ ds}\mid c_0\right)<0.\end{cases}$$

However, it is difficult to discuss theoretically the impact of the SB and running the d-channel on the whole supply chain's profit. We use a numerical example to illustrate how the parameters c_0 and c_2 affect the supply chain's profit.

Example: Assume that ε_R=50, ε_M=60, β=0.05, γ=0.7, η=0.5, λ_1=0.5, λ_2=0.6 and c_1=4.

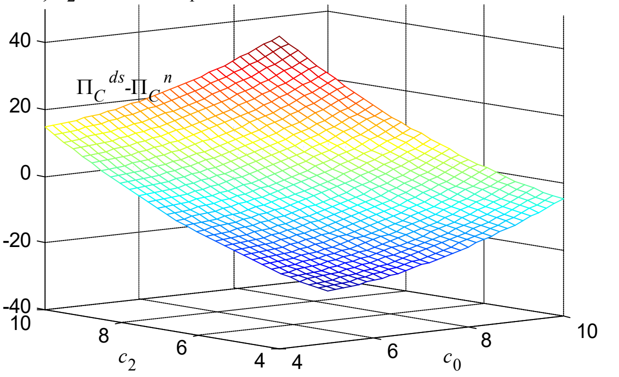

Fig.2 The profit increment by running d-channel and SB

Fig. 2 shows that the profit difference between Case 4 and Case 1 varies with the parameters c_0 and c_2. From Fig. 2, we can conclude that when with both c_0 and c_2 are low, introducing SB and running a d-channel can achieve the whole chain's performance, and Pareto improving, which is consistent with that discusses in Case 2 and Case 3.

IV. BOTH MEMBERS' CHOICE GAME

Section 3 has shown two members' option behaviors given the action of their individual adversary. In this section, we will discuss their Nash equilibrium options.

In the mS framework, the manufacturer first determines whether to run a d-channel and he has two possible strategies: "Yes" and "No" (denoted by {Y} and {N} for brevity, respectively). "{Y}/{N}" means "running/not running a d-channel". As a follower, the retailer's action depends on the manufacturer's strategy. Thus, she has four possible policies: {Y, Y}, {Y, N}, {N, Y} and {N, N}. {Y, N} represents that the retailer introduces SB if the manufacturer runs the d-channel, and introduce SB without the d-channel. {N, Y} is contrary to {Y, N}. {Y, Y} means that the retailer always introduces the SB whether or not the manufacturer runs the d-channel. {N, N} is contrary to {Y, Y}. Thus, the choice game between the two partners has eight possible strategy profiles: ({Y}, {Y, Y}), ({Y}, {Y, N}), ({Y}, {N, Y}), ({Y}, {N, N}), ({N}, {Y, Y}), ({N}, {Y, N}), ({N}, {N, Y}) and ({N}, {N, N}). In each strategy profiles, the first element is the manufacturer's action and the second one is the retailer's action. The two strategies ({Y}, {Y, Y}) and ({Y}, {Y, N}) induce the same outcome (Y, Y) i.e., the manufacturer runs the d-channel and the retailer introduces the SB. Therefore, there are four possible equilibrium outcomes: (Y, Y), (Y, N), (N, Y) and (N, N). Table 3 shows the payoff matrix of two players.

Table 3 Payoff matrix of the two players

		Retailer			
		{Y,Y}	{Y,N}	{N,Y}	{N,N}
Manu.	{Y}	(Π_M^{ds}, Π_R^{ds})	(Π_M^{ds}, Π_R^{ds})	(Π_M^{d}, Π_R^{d})	(Π_M^{d}, Π_R^{d})
	{N}	(Π_M^{s}, Π_R^{s})	(Π_M^{*}, Π_R^{*})	(Π_M^{s}, Π_R^{s})	(Π_M^{*}, Π_R^{*})

From Table 3 and the analyses in Sections 3, we can derive the Nash equilibrium options according to the parameters c_0 and c_2, which are shown as follows.

(1) If $c_2\leq \min\{c_2^{s\text{-}n}, c_2^{ds\text{-}d}\}$, then the retailer's strategy is {Y, Y}. In such a case, if the manufacturer introduces the d-channel, his profit is Π_M^{ds}; otherwise, his profit is Π_M^{s}. Thus, the manufacturer chooses the d-channel if and only if $\Pi_M^{ds}>\Pi_M^{s}$, i.e., $c_0<c_0^{ds\text{-}s}$. From the above analysis, the Nash equilibrium is (Y, Y) for $c_0<c_0^{ds\text{-}s}$ and $c_2\leq\min\{c_2^{s\text{-}n},c_2^{ds\text{-}d}\}$, and (N, Y) for $c_0\geq c_0^{ds\text{-}s}$ and $c_2\leq \min\{c_2^{s\text{-}n},c_2^{ds\text{-}d}\}$.

(2) If $c_2^{s\text{-}n}<c_2\leq c_2^{ds\text{-}d}$, the retailer's strategy is {Y, N}. This means that she will introduce the SB if the manufacturer runs the d-channel, but she will not introduce the SB if the manufacturer does not sell online. In such a case, if the manufacturer introduces the d-channel, his profit is Π_M^{ds}; otherwise, his profit is Π_M^{n}. Thus, the manufacturer introduces the d-channel only if $\Pi_M^{ds}>\Pi_M^{n}$. Obviously, Π_M^{n} is constant with respect to c_0, and Π_M^{ds} is a decreasing function of c_0 according to Lemma 4. We have that $\triangle\Pi_M^{ds\text{-}n}(c_0|c_2)=\Pi_M^{ds}$-$\Pi_M^{n}$ is a decreasing function of c_0. Thus, if $\triangle\Pi_M^{ds\text{-}n}(c_0^{ds\text{-}\max}|c_2)\geq0$, then $c_0^{ds\text{-}\max}$ will be the maximal unit operating cost that the manufacturer runs the d-channel, otherwise, the equation $\triangle\Pi_M^{ds\text{-}n}(c_0|c_2)$=0 will be a unique root (denoted by $c_0^{(2)}(c_2)$). To sum up, the Nash equilibrium is (Y, Y) for $c_0<c_0^{ds\text{-}n}$ and $c_2^{s\text{-}n}<c_2\leq c_2^{ds\text{-}d}$, and (N, N) for $c_0\geq c_0^{ds\text{-}n}$ and $c_2^{s\text{-}n}<c_2\leq c_2^{ds\text{-}d}$, where

$$c_0^{ds-n}=\begin{cases} c_0^{ds-\max}, & \Delta\Pi_M^{ds-n}(c_0^{ds}\,|\,c_2)\geq 0\,, \\ c_0^{(2)}(c_2), & \Delta\Pi_M^{ds-n}(c_1\,|\,c_2)>0>\Delta\Pi_M^{ds-n}(c_0^{ds}\,|\,c_2). \end{cases}$$

(3) If $c_2^{ds\text{-}d}<c_2\leq c_2^{s\text{-}n}$, the retailer's strategy is {N, Y}. In such a case, if the manufacturer introduces the d-channel, his profit is Π_M^{d}, otherwise, his profit is Π_M^{s}. Thus, the manufacturer is willing to introduce the d-channel only if $\Pi_M^{d}>\Pi_M^{s}$. Π_M^{s} is constant with respect to c_0, and Π_M^{d} is a decreasing function of c_0 in Case 2. Thus $\triangle\Pi_M^{d\text{-}s}(c_0)$ =Π_M^{d}-Π_M^{s} is a decreasing function of c_0. Therefore, if $\triangle\Pi_R^{d\text{-}s}(c_0^{d\text{-}\max})\geq0$, then $c_0^{d\text{-}\max}$ will be the maximal unit operating cost that the manufacturer can introduce the d-channel, otherwise, the equation $\triangle\Pi_M^{d\text{-}s}(c_0)$=0 will be a unique root (denoted by $c_0^{(2)}$) for $c_0\in(c_1, c_0^{d\text{-}\max})$. To sum up, we have that, the Nash equilibrium is (Y, N) for $c_0<c_0^{d\text{-}s}$ and $c_2^{ds\text{-}d}<c_2\leq c_2^{s\text{-}n}$, and (N, N) for $c_0\geq c_0^{d\text{-}s}$ and $c_2^{ds\text{-}d}<c_2\leq c_2^{s\text{-}n}$,

$$\text{where } c_0^{d-s}=\begin{cases} c_0^{d-\max}, & \left(m-\sqrt{\dfrac{2(b_0k-\gamma^2)}{\left(2b_0k-\gamma^2\right)}mk}\right)(1-\beta c_1)<\eta(1-\beta c_2) \\ c_0^{(2)}, & \textit{otherwise} \end{cases}$$

$$\text{and } c_0^{(2)}=\frac{1}{\beta}-\frac{\gamma km(1-\beta c_1)}{\beta\left(2b_0k-\gamma^2\right)m}$$
$$-\frac{\sqrt{\left(2b_0k-\gamma^2\right)km[m(1-\beta c_1)-\eta(1-\beta c_2)]^2-2\left(b_0k-\gamma^2\right)k^2m^2(1-\beta c_1)^2}}{\beta\left(2b_0k-\gamma^2\right)m}.$$

(4) If $c_2>\max\{c_2^{s\text{-}n}, c_2^{ds\text{-}d}\}$, the retailer's strategy is {N, N}. This means that if the manufacturer introduces the d-channel, his profit is Π_M^{d}, otherwise, his profit is Π_M^{n}. Thus, the manufacturer chooses the d-channel if and only if $\Pi_M^{d}>\Pi_M^{n}$, i.e., $c_0<c_0^{d\text{-}n}$. Consequently, the Nash equilibrium is (Y, N) for $c_0<c_0^{d\text{-}n}$ and $c_2>\max\{c_2^{s\text{-}n}, c_2^{ds\text{-}d}\}$, and (N, N) for $c_0\geq c_0^{d\text{-}n}$ and $c_2>\max\{c_2^{s\text{-}n}, c_2^{ds\text{-}d}\}$.

The above analyses show that the equilibrium options are dependent on the unit operating cost (c_0) of the d-channel and the SB's unit operating cost (c_2). We can conclude that the retailer's optimal option is to introduce the SB if c_2 is relatively low, whereas she does not introduce the SB if it is relatively high. If c_2 is medium, the retailer's optimal policy depends on the manufacturer's decision. Thus the manufacturer has the advantage of making the first move, and it is possible that he can choose the policy which maximizes his profit, but may be harmful to the retailer. Besides, if c_0 and c_2 are very low (i.e., $c_0<c_0^{ds\text{-}s}$ and $c_2\leq \min\{c_2^{s\text{-}n}, c_2^{ds\text{-}d}\}$), the optimal choice is (Y, Y), whereas if c_0 and c_2 are very high (i.e., $c_0>c_0^{d\text{-}n}$ and $c_2>\max\{c_2^{s\text{-}n}, c_2^{ds\text{-}d}\}$), the choice game's outcome is (N, N). This is consistent with our intuition.

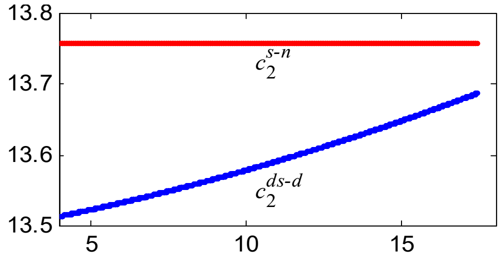

Fig. 3.1 Variation of $c_2^{ds\text{-}d}$ and $c_2^{s\text{-}n}$ with c_0

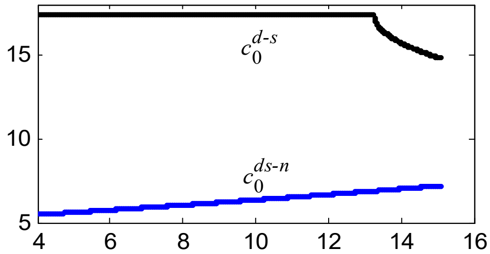

Fig. 3.2 Variation of $c_0^{d\text{-}s}$ and $c_0^{ds\text{-}n}$ with c_2

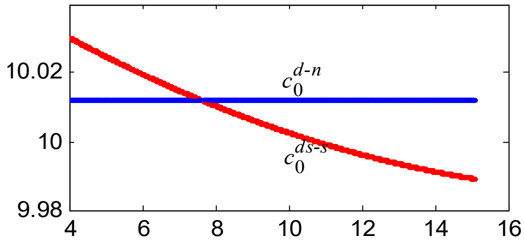

Fig. 3.3 Variation of $c_0^{ds\text{-}s}$ and $c_0^{d\text{-}n}$ with c_2

We use numerical examples to further illustrate how the two players determine their equilibrium options. The value of these parameters is also consistent with that in the above example in Case 4, Section 3.

Fig. 3.1 shows how $c_2^{s\text{-}n}$ and $c_0^{ds\text{-}d}$ change with the parameter c_0, and the horizontal axis represents c_0. From Fig. 3.1, we see that $c_2^{s\text{-}n}>c_2^{ds\text{-}d}$. Fig. 3.2 and Fig. 3.3 indicate how $c_0^{d\text{-}s}$, $c_0^{ds\text{-}n}$, $c_0^{ds\text{-}s}$ and $c_0^{d\text{-}n}$ change with parameter c_2, respectively, and the horizontal axis represents parameter c_2. If the manufacturer runs the d-channel, the retailer introduces the SB only if $c_2<c_2^{ds\text{-}d}$; otherwise, the retailer introduces SB only if $c_2<c_2^{s\text{-}n}$. This means that running the d-channel may decrease the possibility of introducing SB.

According to the above analyses and Figs. 3.1-3.3, we can induce the equilibrium outcome for the two players, which is shown in Fig. 4. From Fig. 4, we can observe that the optimal equilibrium outcome is (Y, Y) if $c_0<c_0^{ds\text{-}s}$ and $c_2<c_2^{ds\text{-}d}<c_2^{s\text{-}n}$, and (N, Y) if $c_0>c_0^{ds\text{-}s}$ and $c_2<c_2^{ds\text{-}d}<c_2^{s\text{-}n}$. This is because that $c_2<c_2^{ds\text{-}d}<c_2^{s\text{-}n}$ induces the retailer's optimal choice {Y, Y}, which means that the manufacturer's choice does not affect the retailer's strategy. Given the retailer's optimal strategy, the manufacturer should choose to run the d-channel if $c_0<c_0^{ds\text{-}s}$, and not to run the d-channel if $c_0>c_0^{ds\text{-}s}$. Similarly, the equilibrium outcome is (Y, N) if $c_0<c_0^{d\text{-}s}$ and $c_2^{ds\text{-}d}<c_2<c_2^{s\text{-}n}$, and (N, Y) if $c_0>c_0^{d\text{-}s}$ and $c_2^{ds\text{-}d}<c_2<c_2^{s\text{-}n}$. The outcome is (Y, N) if $c_0<c_0^{d\text{-}n}$ and $c_2^{ds\text{-}d}<c_2^{s\text{-}n}<c_2$, and (N, N) if $c_0>c_0^{d\text{-}n}$ and $c_2^{ds\text{-}d}<c_2^{s\text{-}n}<c_2$. From Fig. 4, we maybe conclude that the equilibrium outcome is (Y, Y) for low c_0 and low c_2, and (N, N) for high c_0 and high c_2, and (N, Y) for high c_0 and low c_2, and (Y, N) for low c_0 and high c_2. However, for medium c_0 and medium c_2, the equilibrium outcome is (Y, N). In such a case, the retailer's choice will depend on the manufacturer's strategy. As the choice game's leader, the manufacturer has the advantage of the first move, and he prefers to run the d-channel.

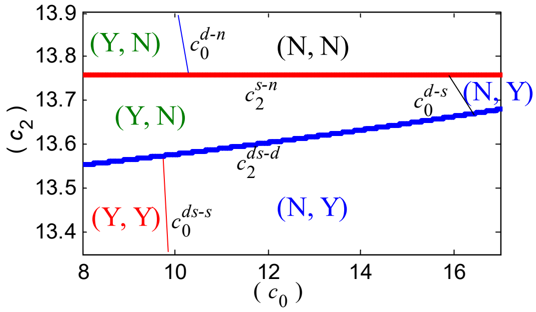

Fig. 4 The choice game's equilibrium outcome

We will discuss how the parameters c_0 and c_2 influence the players' profit increments derived from their optimal options. Let Π_R^*, Π_M^* and $\Pi_C^*(=\Pi_R^*+\Pi_M^*)$ be the retailer's profit, the manufacturer's profit and the supply chain profit under the optimal strategies, respectively. For example, from Fig. 4, the optimal equilibrium outcome is (Y, Y) if c_0=7 and c_2=5. In such a case, we have that $\Pi_R^*=\Pi_R^{ds}$, $\Pi_M^*=\Pi_M^{ds}$ and $\Pi_C^*=\Pi_C^{ds}=\Pi_R^{ds}+\Pi_M^{ds}$.

Fig. 5 shows how the profit increments vary with c_0 given c_2=5, c_2=13.7 and c_2=15, respectively, and the horizontal axis represents parameter c_0. From Fig. 5, we can observe that the increase of c_0 is always adverse to the manufacturer but beneficial to the retailer, whereas the increase of c_2 is always harmful to the retailer but profitable to the manufacturer. This means that the manufacturer (the retailer) should lower the unit operating cost c_0 (c_2) to enhance competitive power in introducing the d-channel (the SB). From supply chain perspective, the increase of c_0 (c_2) is always adverse to the whole chain. Another observation from Fig. 5 is that when both c_0 and c_2 are relatively smaller (say, $c_0\leq 6$ and c_2=5 in the leftmost part of Fig. 5), the three profit increments Π_R*-Π_R^n, Π_M*-Π_M^n and Π_C*-Π_C^n all remain non-negative. This indicates that when the operating costs of both the d-channel and the SB are not very high, the two parties' game of introducing the d-channel and the SB product leads to an increase of both players' profits and the whole channel profit as well. This also implies that the two parties' competition on introducing the d-channel and the SB can eliminate the negative effects of double marginalization, which is further verified by the fact that the competition induces lower pricing of both the manufacturer and the retailer (shown in Lemma 4).

In order to illustrate how parameters ε_M, ε_R, λ_1, λ_2, γ and η influence the two parties' profits, respectively, we present the sensitivity analysis of the two parties' profits with respect to these parameters. The initial parameter values are those assumed in Example except for c_0=7 and c_2=5. We carry out the analysis by increasing the value of one single parameter by -50% up to 50% while holding all the other parameters constant. Fig. 6 shows that how the changes of ε_M, ε_R, λ_1, λ_2, γ and η influence two parties' profits.

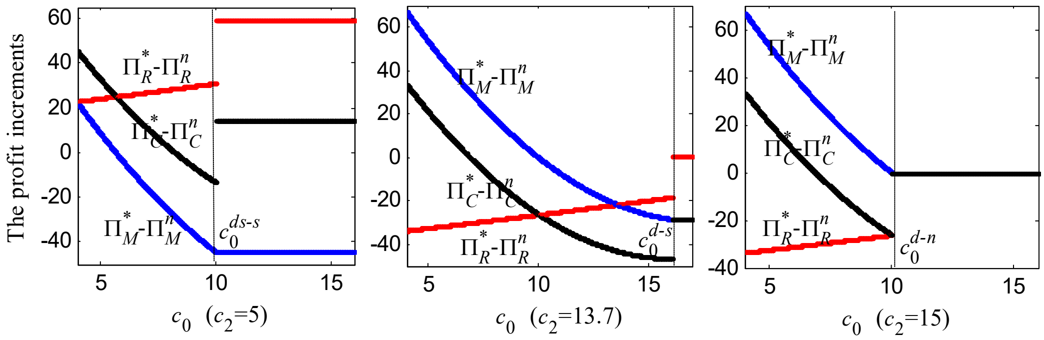

Fig. 5 The impact of c_0 and c_2 on profit increments

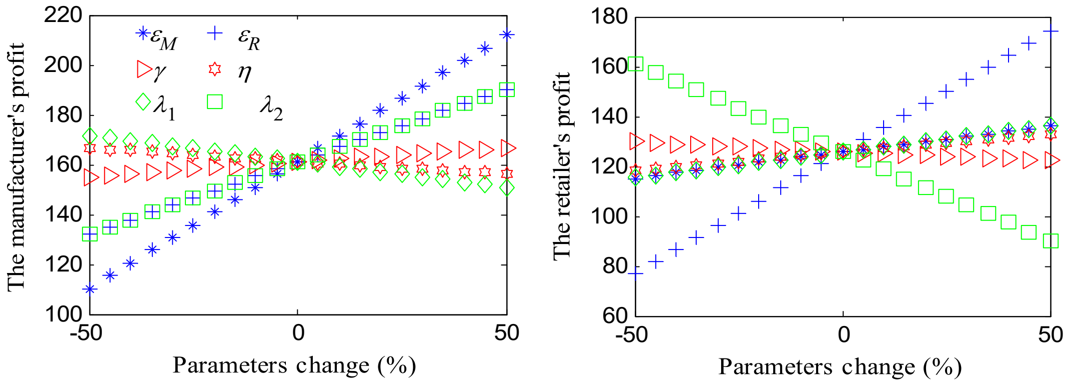

Fig. 6 Impact of ε_M , ε_R , γ , η, $\lambda_1$and λ_2 on two players' profits

Fig.6 gives the following conclusions.

(1) The manufacturer's/retailer's profit increases as either ε_M or ε_R increases. That is, the increase of the market size for manufacturer loyalty or retailer loyalty is beneficial to both the retailer and the manufacturer, which is explained by the fact that the increase of ε_M or ε_R leads to the arising of the total market demand.

(2) The retailer's profit increases as parameters λ_1 and η increase but decreases as λ_2 and γ increase, whereas the manufacturer's profit decreases as parameters λ_1 and η increase but increases as λ_2 and γ increase. This indicates that (i) the higher the proportion of the *brand* loyal consumers who buy NB from the *r*-channel, the more beneficial to the retailer but the more harmful to the manufacturer; (ii) the greater the proportion of the *store* loyal consumers who prefer the NB product, the more profitable to the manufacturer but the more harmful to the retailer; (iii) the fiercer the channel competition, the more beneficial to the manufacturer but the more harmful to the retailer, whereas it is just reverse for the brand competition. The former two points are consistent with our expectation. They imply that when the manufacturer competes for both channel and brand with the retailer, the manufacturer should strive to expand *d*-channel loyalty but also attract more *store* loyal customers buying the NB product, whereas the retailer should build up the SB loyalty and attract more *brand* loyal customers buying from the *r*-channel. The last one implies that the manufacturer should strengthen channel competition whereas the retailer should increase the brand competition.

V. Discussion and Implications

A. Theoretical contributions

The prior literature mainly focused on dual-channel supply chains (Kumar and Ruan, 2006[1], Wang et al. 2016[2], Chiang et al. (2003)[4]), ignoring the introduction of the SB, whereas, most of the literature on the SB ignored the channel competition between the *r*-channel and the *d*-channel. This study has concentrated on a choice game in which whether a manufacturer runs a *d*-channel and whether a retailer should respond by introducing the SB, thereby enriching literature in this area. The theoretical contributions of this paper are shown as follows.

Corroborating the studies (Amrouche and Yan (2012)[22]; Shang and Yang (2015)[14]; Kurata et al. (2007)[21]), running the *d*-channel and introducing the SB has been found to cause the competition between the manufacturer and the retailer, and to exert significant influence on pricing strategies for the two partners as well. It reaffirms the argument that the competition forces the firms to reduce pricing. It is therefore not surprising to see that the competition between the firms can weaken the negative effects of double marginalization on the overall supply chain profit, and increase more consumer surpluses.

With the absence of the SB, running a *d*-channel benefits the manufacturer (Kumar and Ruan, 2006[1], Cai et al. (2009)[12]). Similarly, Without running a *d*-channel, the retailer can be better off through introducing SBs (Pauwels and Srinivasan, 2004[8], Narasimhan and Wilcox (1998)[17]). Our study suggests that the operating cost in the *d*-channel and the SB operating cost have been found to exert a significant influence on the strategies and the profits for the manufacturer and the retailer. Specifically, when the *d*-channel's operating costs and the SB's operating costs are small, running a *d*-channel and introducing *a* SB can benefits the manufacturer and the retailer, whereas when the costs are high, it is contrary.

B. Implications for practice

With economic globalization and rapid development of

Internet, interest in establishing a *d*-channel has grown explosively among manufactures in recent years. However, opportunities and threats exist when the *d*-channel is established in addition to existing *r*-channels. Meanwhile, in consumer goods retailing, the SB is threatening the NB. This study has focused on channel selection and brand choice between the manufacturer and the retailer. The findings provide recommendations to the two partners in decision-making.

Both of the manufacturer and the retailer should strive to expand the market and increase the market size. For the manufacturer, he should strive to expand *d*-channel loyalty to increases brand loyal customers, and strengthen the channel competition between *d*-channel and *r*-channel. However, for the retailer, our finding suggests that the retailer should strive to expand store loyalty to increases *r*-channel loyal customers, and strengthen the brand competition between the NB and the SB.

VI. Conclusions

This paper discusses both channel competition and brand competition issues in a two-echelon supply chain, where a dominant manufacturer sells a NB through a retailer. Besides the *r*-channel, the manufacturer may select a *d*-channel. Likewise, the retailer may choose a SB. We study the two partners' optimal strategies. The results show that (i) the competition between channels or brands can weaken the negative effects of double marginalization, and (ii) when the operating costs for the *d*-channel and the SB are relatively low, a win-win equilibrium outcome can be achieved, which is not the case when the operating costs are relatively high. The sensitivity analyses reveal that the retailer should build up the product loyalty for the SB and increase the substitutability between the SB and the NB, whereas the manufacturer should build up the loyalty for *d*-channel and decrease the substitutability between the SB and the NB, which can be achieved by increasing product differentiation.

Future research may include two aspects. First, this paper only investigated the aspect of price. It would be interesting to introduce other marketing elements in the model, such as advertising, and shelf-space allocation. Second, this model considers deterministic demand and symmetric information. Stochastic demand and information asymmetry are challenging and interesting.

Appendix

Proof of Lemma 1. From (6), we have Π_R is a concave function with respect to (p_1, p_2). Solving the first-order conditions gives

$$p_1^s=\frac{1+\beta w_1}{2\beta},\ p_2^s=\frac{1+\beta c_2}{2\beta}. \qquad (A.1)$$

Substituting (A.1) to (7) gives the manufacturer's profit as:

$$\Pi_M(w_1)=\frac{m(1-\beta w_1)-\eta(1-\beta c_2)}{2\beta}(w_1-c_1). \qquad (A.2)$$

From (A.2), one easily derives that $d\Pi_M(w_1)/dw_1=-m<0$. Thus

$$w_1^s= c_1+(1-\beta c_1)/(2\beta)-\eta(1-\beta c_2)/(2\beta m). \qquad (A.3)$$

Substituting (A.3) to (A.1) gives p_1^s, w_1^s and p_2^s.

Proof of Theorem 1. (1) One easily derives that $D_1^s\geq 0$ and $D_2^s\geq 0$ are equivalent to $\max[(\eta-m(1-\beta c_1))/(\beta\eta),0]\equiv c_2^{s\text{-min}}\leq c_2\leq c_2^{s\text{-max}}\equiv 1/\beta-\eta m(1-\beta c_1)/[\beta(2b_2m-\eta^2)]$. It implies that the retailer or the manufacturer or will exit when $c_2\notin[c_2^{s\text{-min}}, c_2^{s\text{-max}}]$. We do not consider the situation where any of the two members exits. Hence, we restrict $c_2\in[c_2^{s\text{-min}}, c_2^{s\text{-max}}]$. For $c_2\in[c_2^{s\text{-min}}, c_2^{s\text{-max}}]$, the retailer's profit increment is given by

$$\triangle\Pi_R^{s\text{-}n}(c_2)=[(4b_2m-3\eta^2)(1-\beta c_2)^2-2m\eta(1-\beta c_1)(1-\beta c_2)-m(b_2-2\eta)(1-\beta c_1)^2]/(16\beta^2 m).$$

The retailer will introduce the SB as long as $\triangle\Pi_R^{s\text{-}n}(c_2)>0$.

Since $d\triangle\Pi_R^{s\text{-}n}(c_2)/dc_2=-D_2^s<0$ for $c_2^{s\text{-min}}\leq c_2\leq c_2^{s\text{-max}}$, $\triangle\Pi_M^{s\text{-}n}(c_2)$ is decreasing with c_2. $(2b_2m-\eta^2)(b_2-2\eta)+m^2\eta^4<0$ derives $\triangle\Pi_R^{s\text{-}n}(c_2^{s\text{-max}})>0$. Hence, the monotonicity of $\triangle\Pi_M^{s\text{-}n}(c_2)$ means that $\triangle\Pi_R^s(c_2)>0$ keeps in $[c_2^{s\text{-min}}, c_2^{s\text{-max}}]$. If $(2b_2m-\eta^2)^2(b_2-2\eta)+m^2\eta^4\geq 0$, it is clear to have $\triangle\Pi_R^{s\text{-}n}(c_2^{s\text{-max}})\leq 0$. In that case, due to $c_2^{s\text{-min}}<c_1$, $\triangle\Pi_R^{s\text{-}n}(c_1)=(b_2m-\eta^2)(1-\beta c_1)^2/(8\beta^2m)>0$ and the monotonicity of $\triangle\Pi_R^{s\text{-}n}(c_2)$, the equation $\triangle\Pi_R^{s\text{-}n}(c_2)=0$ has a unique zero-point in $[c_2^{s\text{-min}}, c_2^{s\text{-max}}]$. Solving $\triangle\Pi_R^{s\text{-}n}(c_2)=0$ will give $c_2^{(1)}$ as the unique zero-point in the interval $[c_2^{s\text{-min}}, c_2^{s\text{-max}}]$. Therefore, $\triangle\Pi_R^{s\text{-}n}(c_2)>0$ will hold in the interval $[c_2^{s\text{-min}}, c_2^{(1)}]$, where

$$c_2^{(1)}=1/\beta-[m\eta+\sqrt{(m^2\eta^2-m(b_2-2\eta)(4b_2m-3\eta^2)}](1-\beta c_1)/[\beta(4b_2m-\eta^2)]\}.$$

Define $c_2^{s\text{-}n}=\min\{c_2^{s\text{-max}}, c_2^{(1)}\}$, then the retailer introduces the SB if and only if $c_2\in[c_2^{s\text{-min}}, c_2^{s\text{-max}}]$.

(2) For $c_2\in[c_2^{s\text{-min}}, c_2^{s\text{-}n}]$, then $\Pi_R^s>\Pi_R^n$. The manufacturer's profit increment is

$$\triangle\Pi_M^{s\text{-}n}(c_2)=[m(1-\beta c_1)-\eta(1-\beta c_2)]^2/(8\beta^2m)-(m+b_2-2\eta)(1-\beta c_1)^2/(8\beta^2).$$

Due to $d\triangle\Pi_M^{s\text{-}n}(c_2)/dc_2=\eta D_1^s/(2m)>0$, $\triangle\Pi_M^{s\text{-}n}(c_2)$ is an increasing function of c_2 in $[c_2^{s\text{-min}}, c_2^s]$. Since $c_2^{s\text{-}n}\leq c_2^{s\text{-max}}$, for $c_2^{s\text{-min}}\leq c_2\leq c_2^{s\text{-}n}$ we have

$$\begin{aligned}\triangle\Pi_M^{s\text{-}n}(c_2)&\leq\triangle\Pi_M^{s\text{-}n}(c_2^{s\text{-}n})\\ &\leq\triangle\Pi_M^{s\text{-}n}(c_2^{s\text{-max}})\\ &=-\frac{[4m(b_2m-\eta^2)(b_2-\eta)^2+\eta^4(m+b_2-2\eta)](1-\beta c_1)}{8\beta^2(2b_2m-\eta^2)^2}<0.\end{aligned}$$

(3) The chain's profit increment is given by

$$\triangle\Pi_C^{s\text{-}n}(c_2)=(4b_2m-3\eta^2)(1-\beta c_2)^2/(16\beta^2m)-3\eta(1-\beta c_1)(1-\beta c_2)/(8\beta^2)-3(b_2-2\eta)(1-\beta c_1)^2/(16\beta^2).$$

$\triangle\Pi_C^{s\text{-}n}(c_1)=(b_2m-\eta^2)(1-\beta c_1)^2/(16\beta^2m)>0$, and $\triangle\Pi_C^{s\text{-}n}(c_2^{s\text{-}n})<0$ (due to $\Pi_R^s(c_2^{s\text{-}n})=\Pi_R^n$ and $\Pi_M^s(c_2^{s\text{-}n})<\Pi_M^n$). Thus, $\triangle\Pi_C^{s\text{-}n}(c_2)$ has the unique zero-point in $(c_1, c_2^{s\text{-max}})$ because $\triangle\Pi_C^{s\text{-}n}(c_2)$ is a quadratic function on c_2. Solving $\triangle\Pi_C^{s\text{-}n}(c_2)=0$ gives the unique zero-point $c_2^{C\text{-}s}$, of $\triangle\Pi_C^{s\text{-}n}(c_2)$ in the interval $(c_1, c_2^{s\text{-max}})$ as

$$c_2^{C\text{-}s}=1/\beta-[3m\eta(1-\beta c_1)+\sqrt{(9m^2\eta^2-3(4b_2m-\eta^2)m(b_2-2\eta))}](1-\beta c_1)/[\beta(4b_2m-\eta^2)].$$

Thus, it derive that $\triangle\Pi_C^{s\text{-}n}(c_2)>0$ for $c_2\in(c_2^{s\text{-min}}, c_2^{C\text{-}s})$ and $\triangle\Pi_C^{s\text{-}n}(c_2)<0$ for $c_2\in(c_2^{C\text{-}s}, c_2^{s\text{-}n})$.

Proof of Lemma 2. For any w_1 and p_0, one easily derives from (11) that the second-order derivatives of the retailer's profit are given by $d^2\Pi_R/dp_1^2=-2(b_1+b_2-2\eta)<0$. Thus, the optimal price is

$$p_1^d=\frac{1}{\beta}-\frac{\gamma(1-\beta p_0)+(b_1+b_2-2\eta)(1-\beta w_1)}{2\beta(b_1+b_1-2\eta)}. \qquad (A.4)$$

Second, substituting (A.4) into (10) gives

$$\Pi_M(w_1,p_0)=\frac{1}{2\beta k}\left((2b_0k-\gamma^2)(1-\beta p_0)-\gamma k(1-\beta w_1)\right)(p_0-c_0)+\frac{1}{2\beta}\left(k(1-\beta w_1)-\gamma(1-\beta p_0)\right)(w_1-c_1). \qquad (A.5)$$

Where $k=b_1+b_2-2\eta$. From (A.5), it is easy to obtain Π_M is a concave function with respect to (w_1, p_0). Hence, solving equations $\partial\Pi_M/\partial w_1=0$ and $\partial\Pi_M/\partial p_0=0$ gives

$$p_0^d=\frac{1+\beta c_0}{2\beta},\ w_1^d=\frac{1+\beta c_1}{2\beta}. \qquad (A.6)$$

Substituting (A.6) into (A.4) will yield Lemma 2.

Proof of Theorem 2. (1) $c_0\geq c_1$ derives $p_1^d-w_1^d>0$ and $D_1^d>0$. To assure $D_0^d\geq 0$, it is necessary to have $c_0\leq c_0^{d\text{-max}}=1/\beta-\gamma k(1-\beta c_1)/[\beta(2b_0k-\gamma^2)]$. Thus, we restrict c_0 in the interval $[c_1, c_0^{d\text{-max}}]$.

The manufacturer's profit increment is given as follows

$$\triangle\Pi_M^{d\text{-}n}(c_0)=[(2b_0k-\gamma^2)(1-\beta c_0)^2-2\gamma k(1-\beta c_0)(1-\beta c_1)-k(b_0-2\gamma)(1-\beta c_1)^2]/(8\beta^2k).$$

$d\triangle\Pi_M^{d\text{-}n}(c_0)/dc_0=-[D_0^d/2+(b_0k-\gamma^2)(1-\beta c_0)/(4\beta k)]<0$ implies that $\triangle\Pi_M^{d\text{-}n}(c_0)$ decreases on c_0 in the interval $[c_1, c_0^{d\text{-max}}]$. Due to $\triangle\Pi_M^{d\text{-}s}(c_1)=(b_0k-\gamma^2)(1-\beta c_1)^2/(8\beta^2k)>0$, we have (i) if $(2b_0k-\gamma^2)(b_0-2\gamma)+\gamma^2k\le 0$, then $\triangle\Pi_M^d(c_0^{d\text{-max}})>0$, thus $\triangle\Pi_M^{d\text{-}n}(c_0)>0$ will keep for $c_0\in[c_1, c_0^{d\text{-max}}]$, (ii) if $(2b_0k-\gamma^2)(b_0-2\gamma)+\gamma^2k\le 0$, then $\triangle\Pi_M^{d\text{-}n}(c_0)$ has the unique zero-point in the interval $[c_1, c_0^{d\text{-max}}]$. Solving the equation $\triangle\Pi_M^{d\text{-}n}(c_0)=0$ gives the unique zero-point as

$$c_0=1/\beta-[\gamma k+\sqrt{(k^2\gamma^2-k(b_0-2\gamma)(2b_0k-\gamma^2)}](1-\beta c_1)/[\beta(4b_0k-\gamma\text{-}^2)].$$

Thus, define

$$c_0^{d\text{-}n}=\min\{1/\beta-\gamma k(1-\beta c_1)/[\beta(2b_0k-\gamma^2)],\ 1/\beta-[\gamma k+\sqrt{(k^2\gamma^2-k(b_0-2\gamma)(2b_0k-\gamma^2)}](1-\beta c_1)/[\beta(4b_0k-\gamma\text{-}^2)]\},$$

then $c_0\in[c_0^{d\text{-min}}, c_0^{d\text{-}n}]$ can derive $\triangle\Pi_M^{d\text{-}n}(c_0)\ge 0$, i.e., the manufacturer is willing to run the *d*-channel.

(2) If $c_0\in[c_1, c_0^{d\text{-}n}]$, $\Pi_M^d>\Pi_M^n$. The retailer's profit increment is

$$\triangle\Pi_R^{d\text{-}n}(c_0)=\{[k(1-\beta c_1)-\gamma(1-\beta c_0)]^2-k(k+b_0-2\gamma)(1-\beta c_1)^2\}/(16\beta^2k).$$

Since $d\triangle\Pi_R^{d\text{-}n}(c_0)/dc_0=\gamma D_1^d/k>0$ in $[c_1, c_0^{d\text{-max}}]$, $\triangle\Pi_R^{d\text{-}n}(c_0)$ is increasing with respect to c_0 in $[c_1, c_0^{d\text{-max}}]$. Noticing that $c_0^{d\text{-}n}\le c_0^{d\text{-max}}$, we have

$$\Delta\Pi_R^{d-n}(c_0^d)\le\Delta\Pi_R^{d-n}(c_0^{d-\max})=-\frac{[4k(b_0k-\gamma^2)(b_0-\gamma)^2+\gamma^4(k+b_0-2\gamma)](1-\beta c_1)^2}{8\beta^2(2b_0k-\gamma^2)^2}<0.$$

(3) $\triangle\Pi_C^{d\text{-}n}(c_0)=(4b_0k-\gamma^2)(1-\beta c_0)^2/(16\beta^2k)$

$$=-6\gamma(1-\beta c_0)(1-\beta c_1)/(16\beta^2)-3(b_0-2\gamma)(1-\beta c_1)^2/(16\beta^2).$$

$\triangle\Pi_C^{d\text{-}n}(c_1)=(b_0k-\gamma^2)(1-\beta c_1)^2/(16\beta^2k)>0$, and $\triangle\Pi_C^{d\text{-}n}(c_0^{d\text{-}n})<0$ (due to $\Pi_M^d(c_0^{d\text{-}n})=\Pi_M^n$ and $\Pi_R^d(c_0^{d\text{-}n})<\Pi_R^n$). Thus, $\triangle\Pi_C^{d\text{-}n}(c_0)$ has the unique zero-point in $(c_1, c_0^{d\text{-max}})$ because $\triangle\Pi_C^{d\text{-}n}(c_0)$ is a quadratic function on c_0. Sloving $\triangle\Pi_C^{d\text{-}n}(c_0)=0$ gives the unique zero-point in $(c_1, c_0^{d\text{-}n})$ as

$$c_0=1/\beta-[3k\gamma(1-\beta c_1)+\sqrt{(9k^2\gamma^2-3(4b_0k-\gamma^2)k(b_0-2\gamma))}](1-\beta c_1)/[\beta(4b_0k-\gamma^2)]\equiv c_0^{C\text{-}d}.$$

Thus, we can derive that $\triangle\Pi_C^{d\text{-}n}(c_0)>0$ for $c_0\in(c_1, c_0^{C\text{-}d})$ and $\triangle\Pi_C^{d\text{-}n}(c_0)<0$ for $c_0\in(c_0^{C\text{-}d}, c_0^{d\text{-}n})$.

Proof of Lemma 3. For any given w_1 and p_0, it derives from (10) that Π_R is a concave function with respect to p_1 and p_2 through the sign of the second-order partial derivatives. Thus

$$p_1=\frac{1}{\beta}-\frac{b_2\gamma(1-\beta p_0)+(b_1b_2-\eta^2)(1-\beta w_1)}{2\beta(b_1b_2-\eta^2)},$$

$$p_2=\frac{1}{\beta}-\frac{\eta\gamma(1-\beta p_0)+(b_1b_2-\eta^2)(1-\beta c_2)}{2\beta(b_1b_2-\eta^2)}. \qquad (A.7)$$

Substituting (A.7) to (11) gives the retailer's profit as:

$$\Pi_M=\frac{[2(b_1b_2-\eta^2)b_0-b_2\gamma^2](1-\beta p_0)-\gamma(b_1b_2-\eta^2)(1-\beta w_1)}{2\beta(b_1b_2-\eta^2)}\cdot(p_0-c_0)+\frac{[b_1((1-\beta w_1)-\gamma(1-\beta p_0)-\eta(1-\beta c_2)]}{2\beta}(w_1-c_1). \qquad (A.8)$$

From (A.8), one derives Π_M is a concave function with w_1 and p_0. Define $n=b_1[2(b_0b_1-\gamma^2)b_2-\eta^2b_0]-\eta^2(b_0b_1-\gamma^2)$, then $n>2b_1(b_0b_1b_2-\gamma^2b_2-\eta^2b_0)>2b_1[b_0(\gamma+\eta)b_2-\gamma^2b_2-\eta^2b_0]>0$. Thus, solving the equation $\partial\Pi_M/\partial w_1=0$ and $\partial\Pi_M/\partial p_0=0$ gives

$$w_1^{ds}=c_1+\frac{n(1-\beta c_1)\text{-}\eta[2b_0(b_1b_2-\eta^2)-\gamma^2b_2](1-\beta c_2)}{2\beta n}, \qquad (A.9)$$

$$p_0^{ds}=c_0+\frac{(1-\beta c_0)-\gamma\eta(b_1b_2-\eta^2)(1-\beta c_2)}{2\beta n}.$$

Substituting (A.9) to (A.7) gives p_1^{ds}, p_2^{ds}, w_1^{ds} and p_0^{ds}.

Proof of Lemma 4. From Lemma 1, 2 and 3, we have

(1) $p_1^*-p_1^d=\gamma(1-\beta c_0)/(4\beta k)>0$;

$p_1^*-p_1^s=\eta(1-\beta c_2)/(4\beta m)>0$,

$p_1^d-p_1^{ds}=(b_2-\eta)^2\gamma(1-\beta c_0))/[4\beta(b_1b_2-\eta^2)k]+\eta b_0(b_1b_2-\eta^2)(1-\beta c_2)/(2\beta n)>0$,

$p_1^s-p_1^{ds}=(2(b_1b_2-\eta^2)(b_0-\gamma)^2+\gamma^2\eta^2)(1-\beta c_2)/(4\beta mn)+\gamma b_2(1-\beta c_0)/[4\beta(b_1b_2-\gamma^2)]>0$,

$w_1^s-w_1^{ds}=c_1+(1-\beta c_1)/(2\beta)-\eta(1-\beta c_2)/(2\beta m)$.

$w_1^d-w_1^{ds}=\eta[2b_0(b_1b_2-\eta^2)-\gamma^2b_2](1-\beta c_2)/(2\beta n)>0$

(2) $\partial\Pi_M^d/\partial c_0=-D_0^d<0$;

$\partial\Pi_M^s/\partial c_2=\eta[m(1-\beta c_1)-\eta(1-\beta c_2)]/(4\beta m)=\eta D_1^s/m>0$;

$\partial\Pi_M^{ds}/\partial c_2=\eta[(1-\beta c_1)/(4\beta)-\eta[2b_0(b_1b_2-\eta^2)-\gamma^2b_2](1-\beta c_2)/(4\beta n)]=2\eta(w_1^{ds}-c_1)>0$;

$\partial\Pi_M^{ds}/\partial c_0=-D_0^{ds}<0$;

(3) $\partial\Pi_R^d/\partial c_0=\gamma D_1^d/2>0$;

$\partial\Pi_R^s/\partial c_2=-[(4b_2m-3\eta^2)(1-\beta c_2)/(8\beta m)-\eta(1-\beta c_1)/(8\beta)]=-D_2^s/2-(b_2m-\eta^2)(1-\beta c_2)/(4\beta m)<0$.

$\partial\Pi_R^{ds}/\partial c_0=\gamma(p_1^{ds}-w_1^{ds})/2>0$;

$\partial\Pi_R^{ds}/\partial c_2=-D_2^{ds}/2-(b_1b_2-\eta^2)(n-\gamma^2\eta^2)(p_2^{ds}-c_2)/(2nb_1)<0$.

Derivation of $\triangle\Pi_R^{ds}(c_1|c_0)>0$.

$$\Delta\Pi_R^{ds}(c_1\mid c_0)>\Delta\Pi_R^{ds}(c_1\mid c_1)=\left(\frac{(b_1b_2-\eta^2)(n-\gamma^2\eta^2)(3n-\gamma^2\eta^2)}{16\beta^2n^2b_1}-\frac{\eta\gamma(n-\gamma^2\eta^2)}{8\beta^2nb_1}+\frac{\gamma^2(b_2-\eta)^2}{16\beta^2k(b_1b_2-\eta^2)}\right)(1-\beta c_1)^2$$

$$>\left((b_1b_2-\eta^2)(3n-\gamma^2\eta^2)-2n\eta\gamma\right)\frac{(n-\gamma^2\eta^2)(1-\beta c_1)^2}{16\beta^2n^2b}$$

$$>\left(((\eta+\gamma)\eta-\eta^2)2n-2n\eta\gamma\right)\frac{(n-\gamma^2\eta^2)(1-\beta c_1)^2}{16\beta^2n^2b}>0.$$

ACKNOWLEDGMENT

The authors thank the editor and anonymous reviewers of the paper. Their constructive suggestions and comments have considerably improved the quality of the paper.

REFERENCES

[1] Kumar N, Ruan R, "On manufacturers complementing the traditional retail channel with a direct online channel", *Quantitative Marketing and Economics*, 4(3), 289-323, 2006.

[2] Wang W, Li G, Cheng T C E, Channel selection in a supply chain with a multi-channel retailer: The role of channel operating costs, *International Journal of Production Economics*, 173, 54-65, 2016.

[3] Choi S C, "Expanding to direct channel: Market coverages as entry barrier", *Journal of Interactive Marketing*, 17(1), 25-40, 2003.

[4] Chiang W K, Chhajed D, Hess J D, "Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design", *Management Science*, 49, 1-20, 2003.

[5] Seifert R W, Thonemann U W, Sieke M A, Integrating direct and indirect sales channels under decentralized decision-making, *International Journal of Production Economics*,103(1), 209-229. 2006.

[6] Nielsen/PLMA, Annual International Private Label Yearbook, 2014.

[7] Kadiyali V, Chintagunta P, Vilcassim N, "Manufacturer-retailer channel interaction and implications for channel power: An empirical investigation of pricing in a local market", *Marketing Science*, 19(2), 127-148, 2000.

[8] Pauwels K, Srinivasan S, "Who benefits from store brand entry?" *Marketing Science*, 23(3), 364-390, 2004.

[9] Hansen K, Singh V, Chintagunta P, "Understanding store-brand purchase behavior across categories", *Marketing Science*, 25(1), 75-90,2006

[10] Marta A U, Javier C, "Private Labels and National Brands across Online and Offline Channels", *Management Decision*, 50(10):32-45, 2012.

[11] Rhee B, Park s, Online store as a new direct channel and emerging hybrid channel system, Working Paper, The Hong Kong University of Science & Technology, 2000.

[12] Cai G S, Zhang Z, Zhang M, "Game theoretical perspectives on dual channel supply chain competition with price discounts and pricing schemes", *International Journal of Production Economics*, 117, 80-96, 2009.

[13] Xu X, Meng Z, Shen R, A cooperation model based on CVaR measure for a two-stage supply chain, *International Journal of System Science*, 46(10), 1-9, 2013.

[14] Shang W F, Yang L, "Contract negotiation and risk preferences in dual channel supply chain coordination", *International Journal of Production Research*, 53(16):4836-4857,2015.

[15] Matsui K, "Asymmetric product distribution between symmetric manufacturers using dual-channel supply chains", *European Journal of Operational Research*, 248(2):646-657, 2016.

[16] Tsay A, Agrawal N, "Channel conflict and coordination in the e-commerce age", *Production and Operations Management*, 13(1), 93-110, 2004.

[17] Narasimhan C, Wilcox R T, "Private labels and the channel relationship: A cross-category analysis", *The Journal of Business*, 71(4), 573-600, 1998.
[18] Groznik A, Heese H S, "Supply chain interactions due to store-brand introductions: The impact of retail competition", *European Journal of Operational Research*, 203, 575-582, 2010.
[19] Choi S C, Karima F, "Price competition and store competition: Store brands vs. national brand", *European Journal of Operational Research*, 225(1):166-178, 2013.
[20] Ru J, Shi R X, Zhang J, "Does a Store Brand Always Hurt the Manufacturer of a Competing National Brand?", *Production and Operations Management*, 24(2):272-286, 2015.
[21] Kurata H, Yao D O, Liu J J, "Pricing policies under direct vs. indirect channel competition and national vs. store brand competition", *European Journal of Operational Research*, 180, 262-281, 2007.
[22] Amrouche N, Yan R L, "Implementing online store for national brand competing against private label", *Journal of Business Research*, 65, 325-332, 2012.