Global Exponential Stabilization for Some Impulsive T-S Fuzzy Systems with Uncertainties

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Abstract—**This paper is concerned with the global uniform exponential stability for two classes of impulsive Takagi-Sugeno (T-S) fuzzy systems with uncertainties. We first introduce impulses into each subsystem of the T-S fuzzy IF-THEN rules and then present a unified T-S impulsive fuzzy model with uncertainties for control. Based on the new model, a new control approach for the stability analysis is proposed. This approach allows the computation of the bound which characterizes the exponential rate of convergence of the solutions under some assumptions on the perturbed term. The common quadratic Lyapunov function and parallel distributed compensation controller are used to show the exponential stability of solutions of the impulsive T-S fuzzy systems with uncertainties, provided that the uncertainties are supposed uniformly bounded by a known function. A numerical example is given to illustrate the applicability of the theoretical results.**

Index Terms—**T-S fuzzy system; Impulse; Uncertainty; Feedback controller; Exponential stabilization.**

I. INTRODUCTION

THE dynamics of many evolving processes are subject to abrupt changes such as shocks, harvesting and natural disasters. These phenomena involve short term perturbations from continuous and smooth dynamics, whose duration is negligible in comparison with the duration of an entire evolution. In models involving such perturbations, it is natural to assume that these perturbations act instantaneously or in the form of impulses.

Fuzzy systems in the form of the Takagi-Sugeno (T-S) model [1] have attracted rapidly growing interest in recent years. T-S fuzzy systems are nonlinear systems described by a set of IF-THEN rules. It has shown that the T-S model can give an effective way to represent complex nonlinear systems by some simple local linear dynamic systems with their linguistic description, see [2-14].

Within the control theory it is necessary to study the stationary states of systems with fuzzy control. A lot of such systems are described by T-S models and can be mathematically formalized as a locally-linear system of differential equations. However, there are many systems that cannot endure the continuous effects of control inputs but impulsive control which instantly changes the system's state is often an effective way to solve the stability problem for such systems. Therefore, the theory of impulsive differential equations has been the subject of many investigations. The T-S fuzzy model based impulsive control of chaotic systems and impulsive synchronization for T-S fuzzy model have been investigated; see, for example, [15,16]. But the existing impulsive control theory lacks the unified way for dealing with different nonlinear systems. It is important to point out that combining impulsive control with fuzzy modeling for uncertain system has been rarely discussed until now.

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Moreover, the results in [15,16] require both the continuous dynamics and discrete dynamics of the impulsive systems are stable/stabilizable. In this paper, we shall investigate the control problem for more general impulsive systems. Different from [15,16], we shall divide the impulsive systems into three classes: the systems with stable/stabilizable continuous dynamics and unstable/unstabilizable discrete dynamics; the systems with unstable/unstabilizable continuous dynamics and stable/stabilizable discrete dynamics; the systems that both the continuous-time dynamics and the discrete-time dynamics are stable/stabilizable. The first class of impulsive systems corresponds to the case that the continuous dynamics are subject to impulsive perturbations. The second class of impulsive systems corresponds to the case that the impulses are employed to stabilize the unstable continuous dynamics.

It is note that the standard methods from the control for continuous systems are not suitable to the control for the first and the second class of impulsive systems. Some new analysis technique will be employed to derive the sufficient conditions for the control of each class of impulsive systems.

This rest of this paper is organized as follows: Section 2 reviews the conventional T-S fuzzy model and issues about stability. Section 3 presents the global uniform exponential stability for impulsive T-S fuzzy uncertain systems at the origin where some controllers are constructed to ensure the exponential stability. Section 4 provides an example to demonstrate the applicability of the proposed approach. Finally, a conclusion is made in section 5.

II. T-S FUZZY CONTROL SYSTEM

The T-S fuzzy model with impulses is given by:
Plant Rule i:

If $\{z_1(t)$ is $F_{i1}\}$ and $\{z_2(t)$ is $F_{i2}\}$ $\cdots$ and $\{z_p(t)$ is $F_{ip}\}$, then

$$\begin{cases} \dot{x} = A_i x + B_i u + f_i(t,x), \ i = 1,2,\cdots,r, t \neq t_n, \\ \Delta x = C_n x, t = t_n, \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the control input vector; $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are constant matrix; $f_i(i = 1,2,\cdots,r)$ are time-varying functions, represent the uncertainties of each fuzzy subsystem, r is the number of fuzzy rules; $F_{ik}(k = 1,2,\cdots,p)$ is the fuzzy set; $z(t) = (z_1(t),\cdots,z_p(t))^T$ is the premise variable vector associated with the system states and inputs; $\Delta x(t_n) = x(t_n^+) - x(t_n^-)$ is the impulse at the moment t_n, and $x(t_n^-) = x(t_n)$, $\{t_n\}$ is a sequence of real number such that $0 < t_1 < t_2 < \cdots < t_n \to +\infty$ as $n \to +\infty$; C_n is a constant matrix.

By the center of gravity defuzzification method, it follows from the first equation of fuzzy system (1) that

$$\dot{x} = \frac{\sum_{i=1}^{r} \omega_i(z)(A_i x + B_i u + f_i(t,x))}{\sum_{i=1}^{r} \omega_i(z)},$$

where $\omega_i(z) = \Pi_{k=1}^{p} F_{ik}(z_k)$ and $F_{ik}(z_k)$ denotes the grade of the membership function F_{ik}, corresponding to $z_k(t)$.

Let

$$\eta_i = \frac{\omega_i(z)}{\sum_{i=1}^{r} \omega_i(z)},$$

then the fuzzy system (1) has the state-space form

$$\begin{cases} \dot{x} = \sum_{i=1}^{r} \eta_i (A_i x + B_i u + f_i(t,x)), t \neq t_n, \\ \Delta x = C_n x, t = t_n. \end{cases} \quad (2)$$

Clearly, $\sum_{i=1}^{r} \eta_i = 1$ and $\eta_i \geq 0$ for $i = 1, 2, \cdots, r$.

Throughout this paper, the pairs (A_i, B_i), $i = 1, 2, \cdots, r$ are controllable, that is, the nominal fuzzy system is locally controllable. Based on this assumption, a state feedback control gain K_i can be obtained by pole placement design or Ackerman's formula, such that each local dynamics is stably controlled. The representation of the global control input matrix, denoted by B, is in the form

$$B = \sum_{i=1}^{r} \eta_i B_i.$$

This means that the global control input matrix dominates the control performance. The design of the fuzzy controller can be taken as a linear state feedback control for system (1) which can be defined as:

Plant Rule i:

If $\{z_1(t)$ is $F_{i1}\}$ and $\{z_2(t)$ is $F_{i2}\}$ $\cdots$ and $\{z_p(t)$ is $F_{ip}\}$, then

$$u(t) = K_i x(t), \; i = 1, 2, \cdots, r,$$

where K_i is the local state feedback gain.

Consequently, the defuzzified result is

$$u(t) = \sum_{i=1}^{r} \eta_i K_i x(t).$$

III. Control of fuzzy systems

Assume that the functions $f_i (i = 1, 2, \cdots, r)$ in system (1) satisfy

(H_1) $\|f_i(t,x)\| \leq \rho_i(x)\|x\|$, $\forall t \geq t_0, x \in \mathbb{R}^n$, $i = 1, 2, \cdots, r$, where ρ_i are some nonnegative continuous functions with $\rho_i(t_0) = 0$. Let $\rho : \mathbb{R}^n \to \mathbb{R}^+$,

$$\rho(x) = \left[\sum_{i=1}^{r} \rho_i^2(x)\right]^{\frac{1}{2}},$$

then ρ is a bounded positive continuous function, and $\rho(t_0) = 0$.

Applying the fuzzy controller

$$u(t) = \sum_{j=1}^{r} \eta_j K_j x(t) \quad (3)$$

to system (2), then the closed-loop system is given by

$$\begin{cases} \dot{x} = \sum_{i=1}^{r} \eta_i \Big(A_i x + B_i \sum_{j=1}^{r} \eta_j K_j x \Big) \\ \quad + \sum_{i=1}^{r} \eta_i f_i(t,x), \\ \Delta x = C_n x, t = t_n. \end{cases} \quad (4)$$

Since $\sum_{j=1}^{r} \eta_j = 1$, then $\sum_{i=1}^{r} \eta_i A_i x = \sum_{i=1}^{r} \sum_{j=1}^{r} \eta_i \eta_j A_i x$. Thus,

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{r} \sum_{j=1}^{r} \eta_i \eta_j (A_i + B_i K_j) x + \sum_{i=1}^{r} \eta_i f_i(t,x) \\ &= \sum_{i=1}^{r} \eta_i^2 G_{ii} x + 2 \sum_{i<j}^{r} \eta_i \eta_j G_{ij} x + \sum_{i=1}^{r} \eta_i f_i(t,x), \end{aligned}$$

where

$$\begin{aligned} G_{ii} &= A_i + B_i K_i, \\ G_{ij} &= \frac{1}{2}\Big(A_i + B_i K_j + A_j + B_j K_i \Big). \end{aligned}$$

The controller synthesis initially considers the stability of the local fuzzy dynamics. That is, the stable feedback gains are determined for each subsystem.

(H_2) Suppose that there exists a symmetric and positive definite matrix P, and some matrices $K_i, i = 1, 2, \cdots, r$, such that the following stability conditions

$$G_{ii}^T P + P G_{ii} < -Q_i, i = 1, 2, \cdots, r$$

hold, where Q_i is a positive definite matrix.

Based on (H_2), each subsystem is locally controllable and a stable feedback gain is obtained.

Theorem 1. *Assume that $(H_1) - (H_2)$ hold. If the function $\rho(x)$ satisfies*

$$\begin{aligned} \rho(x) < & \frac{1}{2\|P\|} \frac{1}{(\sum_{i=1}^{r} \eta_i^2)^{\frac{1}{2}}} \Big(\inf_{i=1,\cdots,r} \lambda_{\min}(Q_i) \sum_{i=1}^{r} \eta_i^2 \\ & -2\mu - \frac{\ln \zeta}{\beta} \lambda_{\max}(P) \Big) \end{aligned} \quad (5)$$

with

$$\mu < \frac{1}{2} \inf_{i=1,\cdots,r} \lambda_{\min}(Q_i) \sum_{i=1}^{r} \eta_i^2,$$

where $\beta > 0$ is a constant, $\zeta = \frac{\tilde{\lambda}}{\lambda_{\min}(P)}$, $\tilde{\lambda} = \max\{\lambda_1, \lambda_2, \cdots, \lambda_n\} > 0$, and $\lambda_1, \lambda_2, \cdots, \lambda_n$ be the eigenvalues of $(I + C_n)^T P (I + C_n)$, then

(i) If $\zeta > 1$, and $\inf_n \{t_n - t_{n-1}\} \geq \beta > 0$, then the closed-loop system (4) is globally uniformly exponentially stable;

(ii) If $\zeta \in (0, 1)$, and $0 < \sup_n \{t_n - t_{n-1}\} \leq \beta$, then the closed-loop system (4) is globally uniformly exponentially stable;

(iii) *If $\zeta = 1$, then the closed-loop system (4) is globally uniformly exponentially stable.*

Proof: Let $V(t) = x^T Px$. Calculate the derivative of $V(t)$ along the trajectories of the closed-loop system (4) with $t \in (t_n, t_{n+1})$, we have

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{r} \eta_i^2 x^T (G_{ii}^T P + PG_{ii})x \\ &\quad + 2\sum_{i<j}^{r} \eta_i \eta_j x^T (G_{ij}^T P + PG_{ij})x \\ &\quad + 2x^T P \sum_{i=1}^{r} \eta_i f_i(t,x).\end{aligned}$$

Since each matrix $(G_{ij}^T P + PG_{ij}), 1 \le i < j \le r$, is symmetric, then

$$\begin{aligned}&\lambda_{\min}(G_{ij}^T P + PG_{ij})||x||^2 \\ &\le x^T (G_{ij}^T P + PG_{ij})x \le \lambda_{\max}(G_{ij}^T P + PG_{ij})||x||^2,\end{aligned}$$

where $\lambda_{\min}(\cdot)$(resp. $\lambda_{\max}(.)$) denotes the smallest (resp. the largest) eigenvalue of the matrix.

Let $\mu = \max_{i,j} \lambda_{\max}(G_{ij}^T P + PG_{ij}), 1 \le i < j \le r$, then

$$\sum_{i<j}^{r} \eta_i \eta_j x^T (G_{ij}^T P + PG_{ij})x \le \mu ||x||^2.$$

In fact,

$$\begin{aligned}&\sum_{i<j}^{r} \eta_i \eta_j x^T (G_{ij}^T P + PG_{ij})x \\ \le\ &\sum_{i<j}^{r} \eta_i \eta_j \lambda_{\max}(G_{ij}^T P + PG_{ij})||x||^2 \\ \le\ &\sum_{i<j}^{r} \eta_i \eta_j \max_{i,j} \lambda_{\max}(G_{ij}^T P + PG_{ij})||x||^2 \\ \le\ &\mu \sum_{i<j}^{r} \eta_i \eta_j ||x||^2 \le \mu ||x||^2.\end{aligned}$$

From the above analysis and (H_2), we have

$$\begin{aligned}\dot{V}(t) &\le -\sum_{i=1}^{r} \eta_i^2 \lambda_{\min}(Q_i)||x||^2 + 2\mu||x||^2 \\ &\quad + 2||P|| \sum_{i=1}^{r} \eta_i \rho_i(x)||x||^2.\end{aligned}$$

By using Cauchy-Schwartz inequality, then

$$\begin{aligned}\dot{V}(t) &\le -\sum_{i=1}^{r} \eta_i^2 \lambda_{\min}(Q_i)||x||^2 + 2\mu||x||^2 \\ &\quad + 2||P||[\sum_{i=1}^{r} \eta_i^2]^{\frac{1}{2}} [\sum_{i=1}^{r} \rho_i^2(x)]^{\frac{1}{2}} ||x||^2 \\ &\le \Big(-\inf_{i=1,\cdots,r} \lambda_{\min}(Q_i) \sum_{i=1}^{r} \eta_i^2 + 2\mu \\ &\quad + 2||P||\rho(x)[\sum_{i=1}^{r} \eta_i^2]^{\frac{1}{2}} \Big) ||x||^2.\end{aligned}$$

Since $\rho(x)$ satisfies (5), then there exists a sufficient small positive constant $\sigma > 0$ such that

$$\dot{V}(t) \le -(\frac{\ln \zeta}{\beta} + \sigma)\lambda_{\max}(P)||x||^2.$$

Moreover, $V(t) = x^T Px \le \lambda_{\max}(P)||x||^2$, then

$$\dot{V}(t) \le -\xi V(t), \tag{6}$$

where $\xi = \frac{\ln \zeta}{\beta} + \sigma$. Let $t \in (t_n, t_{n+1}]$, from (6), we have

$$V(t) \le V(t_n^+) e^{-\xi(t-t_n)}, t \in (t_n, t_{n+1}]. \tag{7}$$

On the other hand, if $t = t_n, n = 1, 2, \cdots$, then $x(t_n^+) - x(t_n^-) = C_n x(t_n)$, that is

$$x(t_n^+) = (I + C_n)x(t_n),$$

then

$$\begin{aligned}V(t_n^+) &= x^T(t_n^+)Px(t_n^+) \\ &= x^T(t_n)(I + C_n)^T P(I + C_n)x(t_n).\end{aligned}$$

Let $S = (I + C_n)^T P(I + C_n)$. Since S is symmetric, then there exist an orthogonal matrix U and a diagonal matrix $\Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_n)$, such that

$$U^T \Lambda U = S, U^T U = I.$$

Therefore,

$$\begin{aligned}V(t_n^+) &= (Ux(t_n))^T \Lambda (Ux(t_n)) \le \tilde{\lambda}(Ux(t_n))^T (Ux(t_n)) \\ &= \tilde{\lambda} x^T(t_n)x(t_n) \le \zeta V(t_n),\end{aligned} \tag{8}$$

where $\tilde{\lambda} = \max\{\lambda_1, \lambda_2, \cdots, \lambda_n\} > 0$, $\zeta = \frac{\tilde{\lambda}}{\lambda_{\min}(P)}$.

It follows from (7) and (8) that

$$V(t) \le \zeta^n e^{-\xi(t-t_0)} V(t_0), t \in (t_n, t_{n+1}]. \tag{9}$$

If $\inf_n \{t_n - t_{n-1}\} \ge \beta$, set $n \le \frac{t-t_0}{\beta}$; If $\sup_n \{t_n - t_{n-1}\} \le \beta$, set $n \ge \frac{t-t_0}{\beta} - 1$; From (9), for arbitrarily $t \ge t_0$, we have

$$V(t) \le \begin{cases} \exp\Big((\frac{\ln \zeta}{\beta} - \xi)(t - t_0) \Big) V(t_0), \\ \qquad \zeta > 1, \inf_n \{t_n - t_{n-1}\} \ge \beta; \\ \frac{1}{\zeta} \exp\Big((\frac{\ln \zeta}{\beta} - \xi)(t - t_0) \Big) V(t_0), \\ \qquad \zeta \in (0,1), \sup_n \{t_n - t_{n-1}\} \le \beta. \end{cases}$$

Notice that $\frac{\ln \zeta}{\beta} - \xi = -\sigma < 0$, then the closed-loop system (4) is globally uniformly exponentially stable under the two cases (i) and (ii).

Next, we consider the case (iii). If $\zeta = 1$, then (9) can be written as

$$V(t) \le e^{-\sigma(t-t_0)} V(t_0), t \ge t_0,$$

that is, then the closed-loop system (4) is globally uniformly exponentially stable. This completes the proof.

Furthermore, we consider the following impulsive T-S fuzzy model with uncertainties:

Plant Rule i:

If $\{z_1(t)$ is $F_{i1}\}$ and $\{z_2(t)$ is $F_{i2}\}$ $\cdots$ and $\{z_p(t)$ is $F_{ip}\}$, then

$$\begin{cases} \dot{x} = A_i x + B_i u + B_i f_i(t,x), i = 1, 2, \cdots, r, t \ne t_n, \\ \Delta x = C_n x, t = t_n. \end{cases} \tag{10}$$

Assume that the functions $f_i (i = 1, 2, \cdots, r)$ in system (10) satisfy

(H_3) $\|f_i(t,x)\| \le \tilde{\rho}(x), \forall t \ge t_0, \forall x \in \mathbb{R}^n$, where $\tilde{\rho}$ is a nonnegative continuous function, such that $\tilde{\rho}(t_0) = 0$.

Let

$$u(t) = \sum_{j=1}^{r} \eta_j K_j x + \tilde{u}, \tag{11}$$

where $\tilde{u}$ is related to the uncertainties, which is chosen in the following form

$$\begin{cases} -\dfrac{B^T P x \tilde{\rho}(x)}{||B^T P x|| + \varepsilon ||x||^2}, & x \neq 0, \\ 0, & x = 0, \end{cases}$$

for a certain $\varepsilon > 0$.

By using the same analysis methods of system (1), the closed-loop system of (10) is given by

$$\begin{cases} \dot{x} = \sum_{i=1}^{r} \eta_i \Big(A_i x + B_i \sum_{j=1}^{r} \eta_j K_j x + B_i \tilde{u} \Big) \\ \quad + \sum_{i=1}^{r} \eta_i B_i f_i(t,x), \\ \Delta x = C_n x, t = t_n, \end{cases} \tag{12}$$

and

$$\begin{aligned} \dot{x} &= \sum_{i=1}^{r} \eta_i \Big(A_i x + B_i \sum_{j=1}^{r} \eta_j K_j x + B_i \tilde{u} \Big) \\ &\quad + \sum_{i=1}^{r} \eta_i B_i f_i(t,x) \\ &= \sum_{i=1}^{r} \eta_i \Big(A_i x + B_i \sum_{j=1}^{r} \eta_j K_j x \Big) + B \tilde{u} \\ &\quad + \sum_{i=1}^{r} \eta_i B_i f_i(t,x) \\ &= \sum_{i=1}^{r} \eta_i^2 G_{ii} x + 2 \sum_{i<j}^{r} \eta_i \eta_j G_{ij} x + B \tilde{u} \\ &\quad + \sum_{i=1}^{r} \eta_i B_i f_i(t,x). \end{aligned}$$

Theorem 2. *Assume that $(H_2) - (H_3)$ hold. If the function $\tilde{\rho}(x)$ satisfies*

$$\begin{aligned} \tilde{\rho}(x) &< \frac{1}{2\varepsilon} \Big(\inf_{i=1,\cdots,r} \lambda_{\min}(Q_i) \sum_{i=1}^{r} \eta_i^2 - 2\mu \\ &\quad - \frac{\ln \zeta}{\beta} \lambda_{\max}(P) \Big) \end{aligned} \tag{13}$$

with $\varepsilon > 0$ and

$$\mu < \frac{1}{2} \inf_{i=1,\cdots,r} \lambda_{\min}(Q_i) \sum_{i=1}^{r} \eta_i^2,$$

where $\beta > 0$ is a constant, $\zeta = \frac{\tilde{\lambda}}{\lambda_{\min}(P)}$, $\tilde{\lambda} = \max\{\lambda_1, \lambda_2, \cdots, \lambda_n\} > 0$, and $\lambda_1, \lambda_2, \cdots, \lambda_n$ be the eigenvalues of $(I + C_n)^T P (I + C_n)$, then

- (i) *If $\zeta > 1$, and $\inf_n \{t_n - t_{n-1}\} \ge \beta > 0$, then the closed-loop system (12) is globally uniformly exponentially stable;*
- (ii) *If $\zeta \in (0,1)$, and $0 < \sup_n \{t_n - t_{n-1}\} \le \beta$, then the closed-loop system (12) is globally uniformly exponentially stable;*
- (iii) *If $\zeta = 1$, then the closed-loop system (12) is globally uniformly exponentially stable.*

Proof: Let $V(t) = x^T P x$. Calculate the derivative of $V(t)$ along the trajectories of the closed-loop system (12), we have

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{r} \eta_i^2 x^T (G_{ii}^T P + P G_{ii}) x \\ &\quad + 2 \sum_{i<j}^{r} \eta_i \eta_j x^T (G_{ij}^T P + P G_{ij}) x \\ &\quad + 2 x^T P B \tilde{u} + 2 x^T P \sum_{i=1}^{r} \eta_i B_i f_i(t,x). \end{aligned}$$

Thus, for $x \neq 0$, we have

$$\begin{aligned} \dot{V}(t) &\le -\sum_{i=1}^{r} \eta_i^2 \lambda_{\min}(Q_i) ||x||^2 + 2\mu ||x||^2 \\ &\quad + 2 ||x^T P B|| \tilde{\rho}(x) - 2 \frac{x^T P B B^T P x \tilde{\rho}(x)}{||B^T P x|| + \varepsilon ||x||^2} \\ &\le -\inf_{i=1,\cdots,r} \lambda_{\min}(Q_i) \sum_{i=1}^{r} \eta_i^2 ||x||^2 + 2\mu ||x||^2 \\ &\quad + 2 \frac{||B^T P x|| \tilde{\rho}(x) \varepsilon ||x||^2}{||B^T P x|| + \varepsilon ||x||^2} \\ &\le -\inf_{i=1,\cdots,r} \lambda_{\min}(Q_i) \sum_{i=1}^{r} \eta_i^2 ||x||^2 + 2\mu ||x||^2 \\ &\quad + 2 \tilde{\rho}(x) \varepsilon ||x||^2 \\ &\le \Big(-\inf_{i=1,\cdots,r} \lambda_{\min}(Q_i) \sum_{i=1}^{r} \eta_i^2 + 2\mu + 2\tilde{\rho}(x) \varepsilon \Big) \\ &\quad \times ||x||^2. \end{aligned}$$

Since $\tilde{\rho}(x)$ satisfies (13), then there exists a sufficient small positive constant $\delta > 0$ such that

$$\dot{V}(t) \le -(\frac{\ln \zeta}{\beta} + \delta) \lambda_{\max}(P) ||x||^2.$$

Similar to the proof in Theorem 1, the closed-loop system (12) is globally uniformly exponentially stable. This completes the proof.

Remark 1. *When $\zeta > 1$, the impulses can potentially destroy the stability, so we require that they not happen too frequently; When $\zeta \in (0,1)$, the continuous flow can potentially destroy the stability, so we require the flows to be persistently interrupted by the impulses with stabilizing effects.*

IV. Numerical Example

Consider the following dynamic system:

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = 1.96 \sin(x_1(t)) - 2x_3(t) - 2x_4(t), \\ \dot{x}_3(t) = 4x_4(t), \\ \dot{x}_4(t) = 7.84 \sin(x_1(t)) - 16x_3(t) - 16x_4(t) \\ \qquad - 4u(t), \end{cases} \tag{14}$$

One can represent exactly the system by the following four-rule fuzzy model:

Rule 1: If z_1 is F_{11} and z_2 is F_{12} then

$$\dot{x}(t) = A_1x(t) + B_1u(t) + B_1f_1(t,x)$$

Rule 2: If z_1 is F_{21} and z_2 is F_{22} then

$$\dot{x}(t) = A_2x(t) + B_2u(t) + B_2f_2(t,x)$$

Rule 3: If z_1 is F_{31} and z_2 is F_{32} then

$$\dot{x}(t) = A_3x(t) + B_3u(t) + B_3f_3(t,x)$$

Rule 4: If z_1 is F_{41} and z_2 is F_{42} then

$$\dot{x}(t) = A_4x(t) + B_4u(t) + B_4f_4(t,x)$$

with

$$z_1 = \begin{cases} \frac{\sin x_1}{x_1}, & x_1 \neq 0 \\ 0, & x_1 = 0 \end{cases} \quad \text{and} \quad z_2 = \sin x_1,$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1.96 & 0 & -2 & -2 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & -16 & -16 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1.96 & 0 & -2 & -2 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & -16 & -16 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1.96 & 0 & -2 & -2 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & -16 & -16 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1.96 & 0 & -2 & -2 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & -16 & -16 \end{bmatrix},$$

$$B_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix},$$

$$f_1(t,x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1.96 \end{bmatrix}, f_2(t,x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1.96 \end{bmatrix},$$

$$f_3(t,x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1.96 \end{bmatrix}, f_4(t,x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1.96 \end{bmatrix}.$$

We have to choose a positive definite matrix Q, take $Q = I$, and to solve the equation in (H_2) to find P. Let $\varepsilon = 0.125$, $\tilde{\rho}(x) \leq 0.1$, using Matlab2014a, we get the following solutions, for K_1, K_2, K_3, K_4 and P:

$$K_1 = \begin{bmatrix} 20.3146 \\ 12.8371 \\ 0.2267 \\ 0.9033 \end{bmatrix}, K_2 = \begin{bmatrix} 20.3146 \\ 12.8371 \\ 0.2267 \\ 0.9033 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 20.3146 \\ 12.8371 \\ 0.2267 \\ 0.9033 \end{bmatrix}, K_4 = \begin{bmatrix} 20.3146 \\ 12.8371 \\ 0.2267 \\ 0.9033 \end{bmatrix},$$

$$P = \begin{bmatrix} 0.0503 & 0.0228 & -0.0035 & -0.0059 \\ 0.0228 & 0.0154 & -0.0011 & -0.0036 \\ -0.0035 & -0.0011 & 0.0027 & 0.0004 \\ -0.0059 & -0.0036 & 0.0004 & 0.0021 \end{bmatrix}.$$

Consider system (14) with impulses

$$\begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \\ \Delta x_3(t) \\ \Delta x_4(t) \end{bmatrix} = C_4 \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}, \tag{15}$$

where C_4 is a 4×4 constant matrix.

By Theorem 2, we get the following results:

If $C_4 = O$, by a direct calculation, $\zeta = 1$, then the closed-loop system is globally uniformly exponentially stable.

If $C_4 = P$, by a direct calculation, $\zeta = 1.1284$ and $\beta = 0.76$. Since $\zeta > 1$, the impulses can potentially destroy the stability, so we require that they not happen too frequently. Let $\inf_n\{t_n - t_{n-1}\} \geq \beta > 0$, the closed-loop system is globally uniformly exponentially stable.

If $C_4 = P - 2I$, by a direct calculation, $\zeta = 0.8794$ and $\beta = 0.21$. Since $\zeta \in (0,1)$, the continuous flow can potentially destroy the stability, so we require the flows to be persistently interrupted by the impulses with stabilizing effects. Let $0 < \sup_n\{t_n - t_{n-1}\} \leq \beta$, the closed-loop system is globally uniformly exponentially stable.

V. Conclusion

The design problems of feedback controller for two types of impulsive T-S fuzzy systems with uncertainties have been studied. Our methods are helpful to improve the existing technologies used in the analysis and control for uncertain impulsive systems. The application of the main results have been done on a dynamic physical model. It is important to notice that the methods used in this paper can be extended to other types of dynamic models; see, for example, [17-20]. Future work will include systems modeling and analysis.

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