## Speeding up Multi-lane Traffic Cellular Automata Simulation

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Abstract—Understanding driving behaviors on the roads is a complicated research topic. To describe accurate speed, flow and density of a multi-lane traffic flow, an adequate traffic flow model is needed. Unfortunately, the model with complicated behaviors always takes lots of computing time. In this study, traffic cellular automata models are considered and two speed-up strategies are proposed. The first strategy employs an analytical solution instead of simulation. However, the solution, which is based on the mean field theory does not agree with the result of simulation. The second strategy considers 3-lane traffic to be equivalent to 2-lane traffic under heavy traffic situation. The computing time for various simulation scenarios is also presented so as to analyze the computational efficiency. According to the results, 2-lane simulation could be a good estimation of 3-lane simulation under the same simulation parameters. In addition, the computational efficiency can be improved up to 42.7% in the best case.

*Index Terms*—traffic flow simulation, cellular automata, multi-lane traffic, speeding up simulation

#### I. INTRODUCTION

In the real world, traffic flow involves complex phenomena, such as acceleration, deceleration. dawdling lane-changing and multiple driving behaviors. Therefore, various models are developed to understand single-lane traffic, multi-lane traffic, lane-changing behavior and network traffic situations. The traffic flow theory provides a description for the fundamental traffic flow characteristics and analytical techniques so as to predict, control, manage and plan traffic flow systems [1]. The anisotropic property and consistency of model are two necessary conditions for traffic flow modeling and simulation. Generally, microscopic models have good agreement with these two aspects. The traffic cellular automata (TCA) model [2] is one of the microscopic models. In TCA, a road is represented as a string of cells, which are either empty or occupied by exactly one vehicle. Movement takes place by propagating along the string of cells.

Traffic problems in daily life involve many aspects, such as geometric design, different driving behaviors, different types of vehicles, weather conditions, lane usage, network topology and so on. There are diverse research topics related to traffic flow. Generally, multi-lane traffic is quite common in the real world. Simulating multi-lane traffic involves

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acceleration, deceleration, lane-changing, and passing of vehicles, which takes lots of computing time especially under high vehicle density. However, to improve the performance of traffic control, it is necessary to describe and predict the multi-lane traffic flow.

In this study, two speed-up strategies are proposed and examined. Firstly, an analytical solution of TCA is presented. If the simulation result of TCA can be replaced by the analytical solution, the computing time will be saved. Next, an equivalent concept is employed because lane-changing behavior occurs in dilute traffic. When traffic is congested, vehicles are unable to change lanes and thus multi-lane traffic may behave as single-lane traffic. The simulation of single-lane traffic is much faster than that of multi-lane traffic. The paper is organized as follows. In section 2, an introduction of multi-lane traffic CA models is briefly reviewed. Section 3 presents the speed-up strategy of the analytical solution. Then, an equivalent strategy is presented in Sec. 4. Finally, section 5 concludes with a short summary and discussion of our findings.

#### II. MULTI-LANE TRAFFIC CELLULAR AUTOMATA MODELS

In CA, a road is represented as a string of same-sized cells. The size of the cells is chosen to be equal to the speed of the vehicle that moves forward one cell during one time step. The vehicle's speed is assumed to be a limited number of discrete values ranging from zero to  $v_{max}$ , which is the maximum speed of vehicle. The process of the NaSch model [2] can be split up into four steps:

- (Step 1) Acceleration. If time step is less than the total simulation time, let each vehicle with speed lower than its maximum speed  $v_{max}$  accelerate to a higher speed, i.e.  $v = \min(v_{max}, v+1)$ .
- (Step 2) Deceleration. If the speed is greater than the distance gap d to the preceding vehicle, the vehicle will decelerate:  $v = \min(v, d)$ .
- (Step 3) Dawdling. With the given slow-down probability p, the speed of a vehicle decreases spontaneously:  $v = \max(v-1, 0)$ .
- (Step 4) Propagation. Let each vehicle move forward *v* cells and let time step increase one. Then, repeat the procedure: acceleration, deceleration, dawdling and propagation.

Based on the NaSch model, Takayasu and Takayasu (the TT model or the T2 model) [3] suggested a slow-to-start rule firstly. The TT model describes that a standing vehicle (i.e., a vehicle with the instantaneous speed v = 0) with exactly one empty cell in front accelerates with probability  $q_t = 1-p_t$ , while all other vehicles accelerate deterministically. The other steps of the update rule (Step 2 to Step 4) of the NaSch

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model are kept unchanged. The TT model reduces to the NaSch model in the limit  $p_t = 0$ . Another slow-to-start model is the BJH model [4]. An extra step is introduced to implement the slow-to-start rule. According to the BJH model, a standing vehicle will move on only with probability  $1-p_s$ . Step 1, 3 and 4 of the BJH model are the same as those of the NaSch model. An extra step (Step 1.5) is introduced and Step 2 is modified as follows:

(Step 1.5) Slow-to-start. If flag = 1, then let v = 0 with probability  $p_s$ .

(Step 2') Blockage.  $v = \min(v, d)$  and let flag = 1 if v = 0; else flag = 0.

Here, flag is a label for distinguishing vehicles. If flag = 1, the vehicles should obey the slow-to-start rule. If flag = 0, the vehicles can accelerate immediately. Obviously, when  $p_s = 0$ , the above rules reduce to those of the NaSch model. The slow-to-start rule of the TT model is a spatial rule. In contrast, the BJH slow-to-start rule requires memory, i.e., it is a temporal rule depending on the number of trials and not depending on the free space available in front of the vehicle. Velocity dependent randomization (VDR model) is a generalized BJH model [5], which considers a larger slow-down probability while the velocity is zero in the last time step. In addition, Maerivoet [6, 7] proposed a time-oriented delay model, which considers a safety threshold. If the gap between two vehicles is smaller than the threshold, then the following vehicle cannot accelerate. The average speed obtained by the model would be smaller than that of the other models under the same simulation scenario.

Fukui and Ishibashi [8] proposed a high speed CA model, which is the so-called FI model. The FI model considers that a driver would not dawdle unless he is driving at the maximum speed. Thus, the Step 3 of the NaSch model is modified as follows:

(Step 3') Dawdling. Let each vehicle with speed equal to its maximum speed  $v_{max}$  have a probability p to slow down to  $v_{max}$ -1. Otherwise, vehicles will keep the same speed and will accelerate in the next time step.

In multi-lane traffic, overtaking maneuver uses an extended neighborhood behind and ahead of the vehicle on both lanes. Technically, one can say that there must be a gap of size  $d^-+1+d^+$ . The label + (-) belongs to the gap on the target lane in front of (behind) the vehicle that wants to change lanes. In the following, we characterize the security criterion (no accident condition) by the boundaries  $[-d^-, d^+]$  of the required gap on the target lane relative to the current position of the vehicle considered for changing lanes. If one vehicle has enough gaps to change lanes, it still has a probability  $p_{cl}$  to keep in the same lane so as to avoid the ping-pong phenomenon [9-13].

#### III. SPEED-UP BY AN ANALYTICAL SOLUTION

Generally, the analytical solution of the cellular automata model is derived by the mean field theory. The basic idea of the mean field theory is based on the conservation of probability. The definition of notations is listed in Table I.

| TABLE I                 |  |  |  |  |
|-------------------------|--|--|--|--|
| DEFINITION OF NOTATIONS |  |  |  |  |
| Notation                | Definition                                     |  |  |  |
| m                       | The number of cells between two cars.          |  |  |  |
| $P_m(t)$                | The probability that there are <i>m</i> cells  |  |  |  |
|                         | between two cars and the speed of the          |  |  |  |
|                         | following car is 1 at time <i>t</i> .          |  |  |  |
| $B_m(t)$                | The probability that there are <i>m</i> cells  |  |  |  |
|                         | between two cars and the speed of the          |  |  |  |
|                         | following car is 0 at time <i>t</i> .          |  |  |  |
| р                       | Dawdling probability.                          |  |  |  |
| q                       | <i>q</i> =1- <i>p</i> °                        |  |  |  |
| $g_m(t)$                | The probability of the preceding car that      |  |  |  |
|                         | moves forward $m$ cells at time $t, m$ is 0 or |  |  |  |
|                         | 1.   |  |  |  |

In the four steps of the NaSch model, the stochastic deceleration is in the third step. Let  $v_{max} = 1$  and derive the analytical solution by the mean field theory [14, 15]. In this case, the speed is 0 or 1. Since the four steps are recurring, we can rearrange the order to make the formulation easier. Let the deceleration be the first step and dawdling be the next. The third step moves forward and the last step is acceleration. Thus, the speed of all cars will be 1 after the four steps. According to the conservation of probability, we will have the following systematic equations:

$$P_0(t+1) = g_0(t) \{ P_0(t) + q P_1(t) \}, \tag{1}$$

$$P_1(t+1) = g_1(t)P_0(t) + \{qg_1(t) + pg_0(t)\}P_1(t) + qg_0(t)P_2(t), \quad (2)$$

$$P_{n}(t+1) = pg_{1}(t)P_{n-1}(t) + \{qg_{1}(t) + pg_{0}(t)\}P_{n}(t) + qg_{0}(t)P_{n+1}(t), \quad (3)$$

where Eq.(1) is the probability that the speed of the following car is 1 at time t+1 when there is no gap between the following car and the preceding car. Equation (2) is the probability that the speed of the following car is 1 at time t+1 when the gap between the following car and the preceding car is 1. Equation (3) is the general form of Eqs.(1) and (2). Since  $g_m(t)$  denotes the probability of the preceding car that moves forward *m* cells at time *t*, we can derive the conservative equations as follows:

$$g_0(t) = P_0(t) + p \sum_{n \ge 1} P_n(t) = p + q P_0(t), \qquad (4)$$

$$g_1(t) = q \sum_{n \ge 1} P_n(t) = q \{1 - P_0(t)\}, \qquad (5)$$

$$\sum_{n>0} P_n(t) = 1, \qquad (6)$$

$$\sum_{n\geq 0} (n+1)P_n(t) = \frac{1}{k}$$
(7)

After solving the simultaneous equations (1) to (7), the flow equation is derived.

$$f(k,p) = uk = g_1 k = \frac{1 - \sqrt{1 - 4qk(1-k)}}{2}, \qquad (8)$$

where k is traffic density, u is average speed and f(k, p) is traffic volume.

According to our study [16], a three-step TCA is proposed to simplify the moving procedure of TCA. The three steps are given as follows:

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(Step 1) Adjusting. If time step is less than the total simulation time, let each vehicle with speed  $v < v_{max}$  have a probability 1-p to accelerate to a higher speed. One vehicle with its maximum speed  $v_{max}$  has a probability p to decrease its speed to  $v_{max}-1$ . That is,

If (probability < p), 
$$\begin{cases} \text{if } (v = v_{max}), & v = v_{max} - 1, \\ \text{otherwise,} & v = v, \end{cases}$$
  
otherwise,  $v = \max(v+1, v_{max}).$ 

- (Step 2) Deceleration. If the speed is greater than the distance gap *d* to the preceding vehicle, the vehicle will decelerate:  $v = \min(v, d)$ .
- (Step 3) Propagation. Let each vehicle move forward *v* cells and let time step increase one. Then, repeat the procedure: adjusting, deceleration and propagation.

Figure 1 illustrates the single-lane flow-density relation of the NaSch and the 3-step models. Obviously, the critical density ( $k_c$ ) of the 3-step model is larger than that of the NaSch model. The critical density is defined as the density when the maximum flow occurs.  $k_c$  of the NaSch model is 0.17 and  $k_c$  of the 3-step model is 0.20. The flow-density relation of the NaSch and 3-step models is very close. In single-lane traffic, the flow of the 3-step model is larger than that of the NaSch model, especially when density belongs to the interval [0.16, 0.75].



Fig. 1. Flow-density relation of the NaSch and 3-step models for single-lane traffic.

Also, we derive the analytical solution of the 3-step TCA by the mean field theory. Let  $v_{max} = 1$ ,  $B_I(t)$  denotes the probability that there is 1 cell between two cars and the speed of the following car is 0 at time *t*. By conservation of probability, we have

$$P_0(t+1) = qg_0(t)\{P_1(t) + B_1(t)\},$$
(9)

$$P_1(t+1) = qg_1(t)\{P_1(t) + B_1(t)\} + qg_0(t)\{P_2(t)\} + B_2(t), \quad (10)$$

$$P_n(t+1) = qg_1(t)\{P_n(t) + B_n(t)\} + qg_0(t)\{P_{n+1}(t) + B_{n+1}(t)\}, n \ge 2, (11)$$

where Eq.(9) is the probability that the speed of the following car is 1 and there is no gap between the preceding car and the following car at time t+1. Equation (10) shows the probability that the speed of the following car is 1 and there is no gap between the preceding car and the following car at time t+1. The general form is given in Eq. (11). From the viewpoint of  $B_n(t)$ , we can derive Eqs. (12) to (15).

$$B_0(t+1) = g_0(t) \{ P_0(t) + B_0(t) \},$$
(12)

$$B_{1}(t+1) = g_{1}(t) \{P_{0}(t) + B_{0}(t)\} + pg_{0}(t) \{P_{1}(t)\} + B_{1}(t), \quad (13)$$

$$B_{2}(t+1) = pg_{1}(t)\{P_{1}(t) + B_{1}(t)\} + pg_{0}(t)\{P_{2}(t) + B_{2}(t)\}, \qquad (14)$$

$$B_n(t+1) = pg_1(t)\{P_n(t) + B_n(t)\} + pg_0(t)\{P_{n+1}(t) + B_{n+1}(t)\}, \ n \ge 3.$$
(15)

Then, the flow equation  $g_0(t)$  can be obtained by the relationship between  $P_n(t)$  and  $B_n(t)$ .

$$g_0(t) = P_0(t) + B_0(t) + p \sum_{n \ge 1} [P_n(t) + B_n(t)], \qquad (16)$$

$$g_{1}(t) = q \sum_{n \ge 1} [P_{n}(t) + B_{n}(t)]$$
(17)

The summation of  $P_n(t)$  and  $B_n(t)$  is 1. That is,

$$\sum_{n>0} \left[ P_n(t) + B_n(t) \right] = 1 \cdot \tag{18}$$

Next, convert the probability to density, we have

$$\sum_{n\geq 0} (n+1) [P_n(t) + B_n(t)] = \frac{1}{k}$$
 (19)

After solving the simultaneous equations (11) to (19), the speed-flow equation of the 3-step TCA model is derived.

$$u = g_1 = \frac{1 - \sqrt{1 - 4qk(1 - k)}}{2k},$$
(20)

$$f(k, p) = uk = g_1 k = \frac{1 - \sqrt{1 - 4qk(1 - k)}}{2}.$$
 (21)

In Eqs. (8) and (21), the analytical solution of the NaSch model is the same as that of the 3-step TCA. However, the simulation results of the two models are different. The reason is that the mean field theory is derived under steady-state conditions and the variables are independent to the moving procedure. That is, the derivation of the analytical solution is procedure-independent. If the relationship between steps must be considered, the conditional mean field theory should be employed to derive the analytical solution of TCA. However, the simultaneous equations of the conditional mean field theory can only be solved numerically, which cannot save computing time. In addition, the derivation of the mean field theory generates simultaneous equations and the number of the equations is equal to  $(v_{max} + 1)^n$ , where *n* is the number of neighboring cells. Therefore, speeding up the simulation of traffic cellular automata by adopting the analytical solution is not a good alternative.

#### IV. SPEED-UP BY AN EQUIVALENT STRATEGY

The simulation results and the computational efficiency are compared in this section. The settings of simulation are given as follows. Typically, the length of a cell is suggested to be 7.5m. If time step is 1 second and  $v_{max}$  is 5, the corresponding speed will be 135 kilometers per hour (kph). In Taiwan, the upper speed limit of National Freeway No. 1 is 100 kph and that of National Freeway No. 2 is 110 kph. Therefore, the length of a cell is considered to be 7 m, and the maximum speed  $v_{max}$  is 5 (i.e., 126 kph). With dawdling probability, the corresponding speed will be close to the real situation in Taiwan. The CA results are obtained from simulation on a chain of 1,000 sites, which is 7 km. A periodic boundary condition is assumed so that both the total number of vehicles and density are conserved at each simulated point. For each initial configuration of simulation, results are obtained by averaging over 10,000 time steps after the first 10,000 steps, so that the long-time limit is approached. This criterion was found to be sufficient to guarantee a steady-state being reached.

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In this study, four combinations of parameters are simulated by 1-lane, 2-lane and 3-lane TCA models. Two maximal speeds ( $v_{max}$ ), which are 4 and 5, and two dawdling probabilities (*p*), which are 0.2 and 0.9, are considered. The slow-down probability is *p*=0.25 and the lane-changing probability ( $p_{cl}$ ) is 0.5.

Figures 2 to 5 illustrate the speed-density curves of 1-lane, 2-lane and 3-lane models for four simulation scenarios, respectively. From the figures, the speed of 1-lane model is obviously smaller than that of 2-lane and 3-lane models in four cases. When p=0.2, the speed of 2-lane model is in good agreement with that of 3-lane model in both cases of  $v_{max}=4$  and 5. When p=0.9, the speed of 3-lane model is larger than that of 2-lane model slightly. The absolute error (AE) and absolute percentage error (APE) are employed to compare the difference between simulation scenarios. Equations (22) and (23) provide the definitions.

$$AE_{me} = \left| u_m - u_e \right|, \tag{22}$$

$$APE_{me} = \frac{|u_m - u_e|}{u_m} \times 100\% , \qquad (23)$$

where  $u_m$  is the speed of multi-lane models, m=2 or 3.  $u_e$  is the speed of 1-lane or 2-lane model, e=1 or 2. If e=1, m=2 or 3. If e=2, m=3. That is,  $AE_{12}$  represents the difference in speed between 1-lane and 2-lane models, whereas  $APE_{12}$  represents the percentage difference. Table II shows the same observation as Figs. 2 to 5.



Fig. 2 Speed-density curve of  $v_{max} = 4$  and p = 0.2.



Fig. 3 Speed-density curve of  $v_{max} = 4$  and p = 0.9.



Fig. 4 Speed-density curve of  $v_{max} = 5$  and p=0.2.



Fig. 5 Speed-density curve of  $v_{max} = 5$  and p = 0.9.

According to Table II, the speed of 1-lane model only has good agreement with that of 2-lane and 3-lane models in the case of  $v_{max}=4$ , p=0.2 with a density of 14.29 veh/km. Although  $AE_{m1}$  randomly fluctuates with density,  $APE_{m1}$ increases with density, where m=2 or 3. The reason is that when density increases, speed decreases, and a little difference between the models will cause a large difference in percentage. 1-lane model is not a good approximation of 2-lane and 3-lane models. It is evident from Table II that  $AE_{32}$ is much smaller than  $AE_{21}$  and  $AE_{31}$ . Also,  $APE_{32}$  is much smaller than  $APE_{21}$  and  $APE_{31}$ . When  $v_{max}=5$ , the largest  $APE_{32}$  is 3.33% and  $AE_{32}$  is only 2.30 kph. When  $v_{max}=4$ , the largest  $APE_{32}$  is 5.24% and  $AE_{32}$  is only 3.52 kph. Therefore, 2-lane model is an acceptable approximation of 3-lane model. Next, the computing time of 1-lane, 2-lane and 3-lane models is discussed.

The computing time of 1-lane, 2-lane and 3-lane models for all scenarios is illustrated in Figs. 6 to 8. From the figures, the computing time increases with density because when density increases, the number of simulated cars increases. If the number of lanes is given, the computing time will almost increase linearly with density. In the case of the largest density, the computing time of 1-lane, 2-lane and 3-lane models is about 50 seconds, 170 seconds and 250 seconds, respectively. When the number of lanes increases from 1 to 3, the computing time increases from 1 to 5 times. Although the computing time of 1-lane model is much less than that of the other two models, 1-lane model is

unfortunately a poor approximation of 2-lane and 3-lane models. Therefore, approximating 3-lane model by 2-lane model is acceptable. Table III and IV provide the difference and the percentage difference in computing time between 2-lane and 3-lane models. By using 2-lane model to approximate 3-lane model, 114 seconds can be saved in the best case (congested traffic), which is about 42.7% of the total computing time. Even in the worst case (dilute traffic), 21.82% of speed-up is achieved.

TABLE II ABSOLUTE ERROR AND ABSOLUTE PERCENTAGE ERROR FOR ALL SCENARIOS

| 4 0.0   |  |   |  |  |   |  |
|---|--|---|--|--|---|--|
| $v_{max} = 4, p = 0.2$  |  |   |  |  |   |  |
| density   | absolute difference APE (%)  |   |  |  |   |  |
| (veh/km)  |  | (km/hr)   |  | -  |   |  |
|   | $AE_{21}$  | $AE_{31}$   | $AE_{32}$  | $APE_{21}$   | $APE_{31}$  | $APE_{32}$   |
| 14.29   | 0.02   | 0.20  | 0.21   | 0.02   | 0.21  | 0.23   |
| 28.57   | 2.07   | 5.59  | 3.52   | 3.19   | 8.59  | 5.24   |
| 42.86   | 4.87   | 5.99  | 1.12   | 11.96  | 14.70   | 2.45   |
| 57.14   | 5.02   | 5.51  | 0.49   | 18.00  | 19.75   | 1.48   |
| 71.43   | 4.50   | 4.78  | 0.28   | 22.58  | 23.98   | 1.14   |
| 85.71   | 4.00   | 4.16  | 0.16   | 28.00  | 29.14   | 0.89   |
| 100.00  | 3.61   | 3.72  | 0.12   | 35.98  | 37.13   | 0.84   |
| 114.29  | 3.60   | 3.70  | 0.10   | 55.68  | 57.20   | 0.98   |
| 128.57  | 4.09   | 4.10  | 0.02   | 128.88   | 129.46  | 0.26   |
|   |  | $v_m$   | ax = 4, p  | = 0.9  |   |  |
| density   | absol  | ute diffe   | rence  |  | APE (%)   |  |
| (veh/km)  |  | (km/hr)   |  |  |   |  |
|   | $AE_{21}$  | $AE_{31}$   | $AE_{32}$  | $APE_{21}$   | $APE_{31}$  | $APE_{32}$   |
| 14.29   | 6.35   | 8.95  | 2.60   | 35.33  | 49.76   | 10.6%  |
| 28.57   | 3.37   | 4.30  | 0.93   | 40.58  | 51.77   | 7.96   |
| 42.86   | 2.27   | 3.11  | 0.84   | 46.70  | 63.94   | 11.75  |
| 57.14   | 1.85   | 2.47  | 0.62   | 59.08  | 78.85   | 12.43  |
| 71.43   | 1.57   | 2.07  | 0.50   | 74.91  | 98.69   | 13.59  |
| 85.71   | 1.44   | 1.75  | 0.31   | 101.01   | 122.43  | 10.66  |
| 100.00  | 1.19   | 1.68  | 0.54   | 124.64   | 184.07  | 26.46  |
| 114.29  | 1.16   | 1.45  | 0.29   | 207.43   | 260.28  | 17.19  |
| 128.57  | 1.05   | 1.21  | 0.16   | 389.75   | 451.26  | 12.56  |
|   |  | Vm  | ax = 5, p  | = 0.2  |   |  |
| density   | abso   | lute diffe  | erence   |  | APE (%)   |  |
| (veh/km)  |  | (km/hr)   |  |  |   |  |
|   | $AE_{21}$  | $AE_{31}$   | $AE_{32}$  | $APE_{21}$   | $APE_{31}$  | $APE_{32}$   |
| 14.29   | 2.79   | 0.06  | 2.73   | 2.33   | 0.05  | 2.34   |
| 28.57   | 3.40   | 5.70  | 2.30   | 5.17   | 8.67  | 3.33   |
| 42.86   | 5.28   | 6.31  | 1.03   | 12.88  | 15.40   | 2.23   |
| 57.14   | 5.44   | 5.83  | 0.39   | 19.54  | 20.92   | 1.16   |
| 71.43   | 4.71   | 4.93  | 0.21   | 23.71  | 24.79   | 0.87   |
| 85.71   | 4.00   | 4.23  | 0.23   | 27.98  | 29.59   | 1.26   |
| 100.00  | 3.71   | 3.77  | 0.06   | 37.06  | 37.68   | 0.45   |
| 114.29  |  |   |  | 57.00  | 57.00   | 0.45   |
|   | 3.65   | 3.66  | 0.02   | 56.31  | 56.57   | 0.43   |
| 128.57  | 3.65   | 3.66<br>4.17  | 0.02   | 56.31<br>128.44  | 56.57<br>131.67   | 0.17   |
| 128.57  | 3.65<br>4.07   | 3.66<br>4.17  | 0.02<br>0.10<br>qr = 5, p  | 56.31<br>128.44<br>= 0.9   | 56.57<br>131.67   | 0.17 1.42  |
| density   | 3.65<br>4.07   | 3.66<br>4.17<br>$v_m$<br>lute diffe   | 0.02<br>0.10<br>ax = 5, p<br>erence  | 56.31<br>128.44<br>= 0.9   | 56.57<br>131.67   | 0.43<br>0.17<br>1.42   |
| 128.57<br>density<br>(veh/km)   | 3.65<br>4.07<br>absol  | $3.66$ $4.17$ $v_m$ lute diffe (km/hr)  | 0.02 $0.10$ $ax = 5, p$ erence   | 56.31<br>128.44<br>= 0.9   | 56.57<br>131.67<br>APE (%)  | 0.43<br>0.17<br>1.42   |
| 128.57<br>density<br>(veh/km)   | 3.65<br>4.07<br>absol  | $3.66$ $4.17$ $v_m$ lute diffe (km/hr) $AE_{31}$  | $0.02$ $0.10$ $ax = 5, p$ erence $AE_{32}$   | 56.31<br>128.44<br>= 0.9   | APE (%)   | 0.43<br>0.17<br>1.42   |
| 128.57<br>density<br>(veh/km)   | $ \begin{array}{c} 3.65 \\ 4.07 \\ \hline absol \\ AE_{2l} \\ 7.03 \\ \end{array} $  | $3.66$ $4.17$ $v_m$ lute difference (km/hr) $AE_{31}$ $8.20$  | 0.02<br>0.10<br>ax = 5, p<br>perence<br>$AE_{32}$<br>1.17  | $56.31$ $128.44$ $= 0.9$ $APE_{21}$ $38.92$  | APE (%)   | $     \begin{array}{r}       0.43 \\       0.17 \\       1.42 \\       \hline       APE_{32} \\       4.68 \\     \end{array} $  |
| 128.57<br>density<br>(veh/km)<br>14.29<br>28.57   | $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | $ \begin{array}{c c} 3.66 \\ 4.17 \\ v_m \\ \hline v_m \\ $ | $\begin{array}{c} 0.02 \\ 0.10 \\ a_x = 5, p \\ erence \\ \hline AE_{32} \\ 1.17 \\ 1.15 \\ \end{array}$   | $56.31$ $128.44$ $= 0.9$ $APE_{21}$ $38.92$ $43.81$  | 37.00           56.57           131.67           APE (%)           APE <sub>31</sub> 45.42           57.73  | $ \begin{array}{r} 0.17 \\ 0.17 \\ 1.42 \\ \end{array} $ $ \begin{array}{r} APE_{32} \\ 4.68 \\ 9.68 \\ \end{array} $  |
| 128.57<br>density<br>(veh/km)<br>14.29<br>28.57<br>42.86  | $ \begin{array}{c} 3.65 \\ 4.07 \\ \hline \\ AE_{21} \\ 7.03 \\ 3.62 \\ 2.36 \\ \end{array} $  | $ \begin{array}{c c} 3.66 \\ 4.17 \\ \hline v_m \\ \text{lute diffe} \\ (km/hr) \\ AE_{31} \\ 8.20 \\ 4.77 \\ 3.31 \\ \end{array} $   | $\begin{array}{c} 0.02 \\ 0.10 \\ ax = 5, p \\ perence \\ \hline AE_{32} \\ 1.17 \\ 1.15 \\ 0.94 \\ \end{array}$   | $56.31$ $128.44$ $= 0.9$ $APE_{21}$ $38.92$ $43.81$ $47.82$  | 57.00           56.57           131.67           APE (%)           APE <sub>31</sub> 45.42           57.73           66.90  |  |
| 128.57<br>density<br>(veh/km)<br>14.29<br>28.57<br>42.86<br>57 14                                       | 3.65<br>4.07<br>abso<br><u>AE<sub>21</sub></u><br>7.03<br>3.62<br>2.36<br>2.07   | $ \begin{array}{c} 3.66 \\ 4.17 \\ \hline v_m \\ \text{lute diffe} \\ (km/hr) \\ AE_{31} \\ 8.20 \\ 4.77 \\ 3.31 \\ 2.81 \\ \end{array} $   | $\begin{array}{c} 0.02 \\ 0.10 \\ ax = 5, p \\ erence \\ \hline AE_{32} \\ 1.17 \\ 1.15 \\ 0.94 \\ 0.75 \\ \end{array}$                                    | $56.31$ $128.44$ $= 0.9$ $APE_{21}$ $38.92$ $43.81$ $47.82$ $65.43$                                    | 37.00           56.57           131.67           APE (%)           APE <sub>31</sub> 45.42           57.73           66.90           89.10  |  |
| 128.57<br>density<br>(veh/km)<br>14.29<br>28.57<br>42.86<br>57.14<br>71.43                              | $\begin{array}{c c} 3.65 \\ \hline 4.07 \\ \hline \\ absol \\ \hline \\ \hline \\ AE_{21} \\ \hline \\ 7.03 \\ \hline \\ 3.62 \\ \hline \\ 2.36 \\ \hline \\ 2.07 \\ \hline \\ 1.62 \end{array}$   | $\begin{array}{c} 3.66 \\ 4.17 \\ v_m \\ \text{lute diffe} \\ (\text{km/hr}) \\ \hline AE_{31} \\ 8.20 \\ 4.77 \\ 3.31 \\ 2.81 \\ 2.41 \\ \end{array}$  | $\begin{array}{c} 0.02 \\ 0.02 \\ 0.10 \\ ax = 5, p \\ erence \\ \hline AE_{32} \\ 1.17 \\ 1.15 \\ 0.94 \\ 0.75 \\ 0.79 \\ \end{array}$                    | $56.31$ $128.44$ $= 0.9$ $APE_{21}$ $38.92$ $43.81$ $47.82$ $65.43$ $77.32$                            | 37.00           56.57           131.67           APE (%)           APE <sub>31</sub> 45.42           57.73           66.90           89.10           114.85   | $ \begin{array}{r}             0.17 \\             1.42 \\             \hline             APE_{32} \\             4.68 \\             9.68 \\             12.91 \\             14.31 \\             21.16 \\         \end{array} $ |
| 128.57<br>density<br>(veh/km)<br>14.29<br>28.57<br>42.86<br>57.14<br>71.43<br>85 71                     | $\begin{array}{c c} 3.65 \\ \hline 4.07 \\ \hline \\ absol \\ \hline \\ AE_{21} \\ \hline \\ 7.03 \\ \hline \\ 3.62 \\ \hline \\ 2.36 \\ \hline \\ 2.07 \\ \hline \\ 1.62 \\ \hline \\ 1.68 \end{array}$                                     | $\begin{array}{c} 3.66 \\ 4.17 \\ v_m \\ \text{lute diffe} \\ (\text{km/hr)} \\ AE_{31} \\ 8.20 \\ 4.77 \\ 3.31 \\ 2.81 \\ 2.41 \\ 2.29 \end{array}$  | $\begin{array}{c} 0.02 \\ 0.10 \\ a_x = 5, p \\ p \\ erence \\ \hline AE_{32} \\ 1.17 \\ 1.15 \\ 0.94 \\ 0.75 \\ 0.79 \\ 0.61 \\ \end{array}$              | $56.31$ $128.44$ $= 0.9$ $APE_{21}$ $38.92$ $43.81$ $47.82$ $65.43$ $77.32$ $119.65$                   | APE (%)           APE (%)           APE 31           45.42           57.73           66.90           89.10           114.85           162.98  | $\begin{array}{r} 0.43\\ 0.17\\ 1.42\\ \hline \\ 4.68\\ 9.68\\ 12.91\\ 14.31\\ 21.16\\ 19.73\\ \end{array}$  |
| 128.57<br>density<br>(veh/km)<br>14.29<br>28.57<br>42.86<br>57.14<br>71.43<br>85.71<br>100.00           | $\begin{array}{c c} 3.65 \\ \hline 4.07 \\ \hline \\ absol \\ \hline \\ AE_{21} \\ \hline \\ 7.03 \\ \hline \\ 3.62 \\ \hline \\ 2.36 \\ \hline \\ 2.07 \\ \hline \\ 1.62 \\ \hline \\ 1.68 \\ \hline \\ 1.50 \end{array}$                   | $\begin{array}{c} 3.66 \\ 4.17 \\ \hline v_m \\ \text{lute diffe} \\ (\text{km/hr)} \\ \hline AE_{31} \\ 8.20 \\ 4.77 \\ 3.31 \\ 2.81 \\ 2.41 \\ 2.29 \\ 2.01 \\ \end{array}$   | $\begin{array}{c} 0.02 \\ 0.10 \\ a_{x} = 5, p \\ p \\ rence \\ \hline AE_{32} \\ 1.17 \\ 1.15 \\ 0.94 \\ 0.75 \\ 0.79 \\ 0.61 \\ 0.52 \\ \end{array}$     | $56.31$ $128.44$ $= 0.9$ $APE_{21}$ $38.92$ $43.81$ $47.82$ $65.43$ $77.32$ $119.65$ $162.86$          | 37.00           56.57           131.67           APE (%)           APE_31           45.42           57.73           66.90           89.10           114.85           162.98           219.00  | $\begin{array}{r} 0.43\\ 0.17\\ 1.42\\ \hline \\ 4.68\\ 9.68\\ 12.91\\ 14.31\\ 21.16\\ 19.73\\ 21.36\\ \end{array}$  |
| 128.57<br>density<br>(veh/km)<br>14.29<br>28.57<br>42.86<br>57.14<br>71.43<br>85.71<br>100.00<br>114.29 | $\begin{array}{c c} 3.65 \\ \hline 4.07 \\ \hline \\ absol \\ \hline \\ AE_{21} \\ \hline \\ 7.03 \\ \hline \\ 3.62 \\ \hline \\ 2.36 \\ \hline \\ 2.07 \\ \hline \\ 1.62 \\ \hline \\ 1.68 \\ \hline \\ 1.50 \\ \hline \\ 1.43 \end{array}$ | $\begin{array}{c} 3.66 \\ 4.17 \\ v_m \\ \text{lute diffe} \\ (\text{km/hr}) \\ \hline AE_{3l} \\ 8.20 \\ 4.77 \\ 3.31 \\ 2.81 \\ 2.41 \\ 2.29 \\ 2.01 \\ 1.69 \\ \end{array}$  | $\begin{array}{c} 0.02 \\ 0.10 \\ a_{x} = 5, p \\ prence \\ \hline AE_{32} \\ 1.17 \\ 1.15 \\ 0.94 \\ 0.75 \\ 0.79 \\ 0.61 \\ 0.52 \\ 0.26 \\ \end{array}$ | $56.31$ $128.44$ $= 0.9$ $APE_{21}$ $38.92$ $43.81$ $47.82$ $65.43$ $77.32$ $119.65$ $162.86$ $257.76$ | $\begin{array}{r} 57.00\\ \hline 56.57\\ \hline 131.67\\ \hline APE (\%)\\ \hline APE_{31}\\ \hline 45.42\\ \hline 57.73\\ \hline 66.90\\ \hline 89.10\\ \hline 114.85\\ \hline 162.98\\ \hline 219.00\\ \hline 305.12\\ \end{array}$ | $\begin{array}{r} 0.43\\ 0.17\\ 1.42\\ \hline \\ 4.68\\ 9.68\\ 12.91\\ 14.31\\ 21.16\\ 19.73\\ 21.36\\ 13.24\\ \end{array}$  |



Fig. 6 Computing time of 1-lane model.



Fig. 7 Computing time of 2-lane model.



Fig. 8 Computing time of 3-lane model.

| DIFFERENCE IN COMPUTING TIME BETWEEN 2-LANE AND 3-LANE MODELS |  |                            |                            |                    |  |
|---|--|----------------------------|----------------------------|--------------------|--|
| density   | difference in computing time (seconds) |                            |                            |                    |  |
| (veh/km)  | v <sub>max</sub> =4, p=0.2             | v <sub>max</sub> =4, p=0.9 | v <sub>max</sub> =5, p=0.2 | $v_{max}=5, p=0.9$ |  |
| 14.29   | 16                                     | 12                         | 13                         | 13                 |  |
| 28.57   | 29                                     | 24                         | 24                         | 26                 |  |
| 42.86   | 39                                     | 35                         | 33                         | 37                 |  |
| 57.14   | 51                                     | 46                         | 47                         | 47                 |  |
| 71.43   | 62                                     | 59                         | 55                         | 58                 |  |
| 85.71   | 76                                     | 68                         | 63                         | 67                 |  |
| 100.00  | 88                                     | 79                         | 75                         | 77                 |  |
| 114.29  | 99                                     | 88                         | 83                         | 89                 |  |
| 128.57  | 114                                    | 101                        | 94                         | 92                 |  |

TABLE III

| PERCENTAGE DIFFERENCE IN COMPUTING TIME BETWEEN 2-LANE AND |   |                            |                    |                    |  |
|--|---|----------------------------|--------------------|--------------------|--|
| density  | percentage difference in computing time (%) |                            |                    |                    |  |
| (veh/km)   | v <sub>max</sub> =4, p=0.2                  | v <sub>max</sub> =4, p=0.9 | $v_{max}=5, p=0.2$ | $v_{max}=5, p=0.9$ |  |
| 14.29  | 29.09                                       | 21.82                      | 23.21              | 23.21              |  |
| 28.57  | 34.94                                       | 29.27                      | 28.92              | 31.33              |  |
| 42.86  | 36.11                                       | 32.41                      | 30.28              | 33.64              |  |
| 57.14  | 37.50                                       | 34.07                      | 34.56              | 34.56              |  |
| 71.43  | 38.75                                       | 36.42                      | 34.59              | 35.80              |  |
| 85.71  | 40.86                                       | 36.56                      | 34.81              | 36.22              |  |
| 100.00   | 41.12                                       | 37.44                      | 36.23              | 36.67              |  |

# TARI F IV

#### V. CONCLUSIONS AND PERSPECTIVES

37.13

37.97

36.24

36.86

38.03

35.25

114.29

128.57

41.60

42.70

Traffic forecasting is an important part of management modernization of transportation systems. The accurate forecasting results for traffic parameters can help traffic control center in transportation systems to reduce traffic congestion and to improve the mobility of transportation. In addition, rapid and accurate evaluation of control strategies is another important part of relieving traffic congestion. Generally, macroscopic traffic flow models are advantage in rapid computing. However, modeling control strategies in macroscopic models is difficult. On the contrary, modeling control strategies in microscopic traffic flow models is much easier, but simulation of microscopic models is time consuming. In this study, two strategies are proposed to speed up TCA simulation. The speed-up strategy of the analytical solution fails to be achieved because the mean field theory is procedure-independent. If the conditional mean field theory is used to solve the TCA model, explicit equations cannot be derived and a numerical simulation is still needed. Therefore, speeding up TCA simulation by adopting the analytical solution is not possible. The second strategy aims to speed up simulation by adopting an equivalent concept. According to our findings, 1-lane model takes the least amount of computing time, but it is a poor approximation of 2-lane and 3-lane models. On the other hand, 2-lane model is a good approximation of 3-lane model and 42.7% of the computing time is saved in the best case. Thus, we can speed up TCA simulation by employing the equivalent strategy.

In the real world, different driving behaviors can be observed on the road. If we simulate different driving behaviors, the logical judgement of TCA model will be more complicated and the computing time will increase. In this study, the behavior of simulated cars is controlled by the same parameters. That is, only one driving behavior is considered. In dilute traffic condition, drivers might behave as they wish. Yet, in congested traffic condition, drivers are restricted to one lane, which means that all drivers behave the same way. Consequently, the equivalent concept might be employed to deal with the simulation of multi-behavior traffic flow. The simulation and comparison are left for further studies.

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