A Novel Generalized Simplified Neutrosophic Number Einstein Aggregation Operator

Baolin Li, Juanru Wang*, Lihua Yang, and Xuetao Li

Abstract—**In order to handle the indeterminate information and inconsistent information in some complex decision-making environment, some novel aggregation operators are firstly proposed based on Einstein operational laws and generalized aggregation operators under simplified neutrosophic environment, where the truth-membership degree, indeterminacy-membership degree and falsity-membership degree of each element are singleton subsets in $[0,1]$. Firstly, to avoid some impractical operations in certain cases, some new operational laws of simplified neutrosophic numbers (SNNs) based on Einstein operations are defined. Then, Einstein operations and generalized weighted average operator are combined in order to make use the advantages of them, and a novel generalized simplified neutrosophic number Einstein weighted aggregation (GSNNEWA) operator is proposed, and some desirable properties of the new operator are also analyzed. Furthermore, some special cases of the proposed operator are also discussed. Finally, to solve multi-criteria decision-making (MCDM) problems, an illustrative example on the basis of the GSNNEWA operator proposed and similarity measures based on Hamming distance is shown to verify the effectiveness and practicality of the proposed method.**

Index Terms—**multi-criteria decision-making, Einstein, generalized aggregation operator, simplified neutrosophic sets**

I. INTRODUCTION

Although the theories of fuzzy set have been generalized, it cannot handle all kinds of uncertainties in real life. Therefore, as a generalization of the fuzzy set [1], intuitionistic fuzzy set [2], and hesitant fuzzy set [7], neutrosophic set (NS) [9] has attracted wide attentions owing to it can deal with not only the incomplete information but also the indeterminate information and inconsistent information. True-membership, indeterminacy-membership and false-membership in neutrosophic set are completely independent, whereas the uncertainty is dependent on the true-membership and false-membership in intuitionistic fuzzy set.

To date, some research achievements about NS have been made. One hand, different kinds of NS is developed with specific description in order to apply NS in the real application. A single-valued neutrosophic set (SVNS) and some properties were proposed [10] and analyzed. Ye [11] defined the concept of simplified neutrosophic sets (SNSs), and introduced a MCDM method using a simplified neutrosophic weighted arithmetic average operator and a simplified neutrosophic weighted geometric average operator. Wang [12] introduced the definition of interval neutrosophic set (INS) and discussed operators of INS. Peng [13] defined multi-valued NSs, and applied the power aggregation operators to solve MCGDM problems. Liu [14] proposed the concept of interval neutrosophic hesitant fuzzy set, and presented the operations and developed generalized hybrid weighted aggregation operators. On the other hand, the correlation coefficient, similarity measure and entropy of NS are developed. Ye [15] introduced the correlation coefficient and weighted correlation coefficient of SVNS. Majumdar [16]defined similarity measures between two SVNSs and investigated entropy of SVNSs. Ye[17] proposed the cross-entropy of SVNSs. Ye [18]defined distances between INSs, and proposed the similarity measures between INSs based on the relationship between similarity measures and distances. Wang [28] proposed some new distance measures for dual hesitant fuzzy sets, and study the properties of the measures.

The information aggregation operators are very important to process the fuzzy decision-making problems in different fields. Yager [19] and Xu [20] defined weighted arithmetic average operator and weighted geometric average operator. Zhao [21]developed generalized aggregation operators for IFS. Wang and Liu [22] proposed intuitionistic fuzzy geometric aggregation operators based on Einstein operations. Liu [23] proposed some Hamacher aggregation operators for the interval-valued intuitionistic fuzzy numbers. Furthermore, some information aggregation operators for NS are also discussed. Liu [23] proposed a multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean. Zhang [25] defined the operations for INSs, and developed two interval neutrosophic number aggregation operators. Yang [26]

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proposed a multi-criteria decision-making method using power aggregation operators for single-valued neutrosophic sets.

However, the operations of SNNs provided by Ye [11] may be irrational in some situations, in order to avoid this shortcoming, the neutrosophic number aggregation operators based on the Einstein t-norm and t-conorm operational rules in the context of simplified neutrosophic environment are proposed in this paper. Meanwhile, the generalized aggregation operator has become the focus of research because it can be reduced to arithmetic, geometric and harmonic aggregation operators. However, until to now, there is no research about generalized neutrosophic number aggregation operators based on Einstein operations under simplified neutrosophic environment.

Therefore, the aim of the paper is to combine Einstein operations and generalized aggregation operators to SNNS, and then propose the novel GSNNEWA aggregation operator, Where the simplified neutrosophic number Einstein weighted average (SNNEWA) operator, the simplified neutrosophic number Einstein weighted geometric average (SNNEWGA) operator, and the simplified neutrosophic number Einstein weighted harmonic average (SNNEWHA) operator are special cases of the GSNNEWA operator proposed.

The remainder of this paper is organized as follows. In section Ⅱ, some novel operational laws of SNNS based on Einstein operations are defined. In section Ⅲ, generalized simplified neutrosophic number Einstein weighted aggregation (GSNNEWA) operator is proposed, and some desirable properties are analyzed. Moreover, some special cases of the proposed operator are also discussed in section Ⅲ. The new method for multi-criteria decision making based on the proposed operator is presented and an illustrative example is shown in section Ⅳ. Finally, the main conclusions of this paper are summarized in Section Ⅴ.

II. NOVEL OPERATIONS BASED ON EINSTEIN

A NS A in X is characterized by $T_A(x)$, $I_A(x)$ and $F_A(x)$, which are singleton subintervals /subsets in the real standard $[0,1]$, that is

$T_A(x): X \to [0,1]$, $I_A(x): X \to [0,1]$, and $F_A(x): X \to [0,1]$.

Then, a simplification of A is denoted by [11]:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\},$$

For convenience, we will use the SNS where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are single values in the real standard $[0,1]$ rather than subintervals /subsets in the real standard $[0,1]$. Therefore, each SNS can be represented by three real numbers in $[0,1]$ .we can adapt $A = < T_A(x), I_A(x), F_A(x) >$ to represent an element in SNS. In particular, if X has only one element, and call it a simplified neutrosophic number (SNN). The set of all SNNs is represented as SNNS.

Let $A = (T_1, I_1, F_1)$ and $B = (T_2, I_2, F_2)$ be two SNSs, then operational relations are defined as follows [11]:

(1) $A \oplus B = < T_1 + T_2 - T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2 >$

(2) $A \otimes B = < T_1T_2, I_1I_2, F_1F_2 >$

(3) $\lambda A = < 1-(1-T_1)^\lambda, 1-(1-I_1)^\lambda, 1-(1-F_1)^\lambda > \quad \lambda > 0$

(4) $A^\lambda = < T_1^\lambda, I_1^\lambda, F_1^\lambda > \quad \lambda > 0$

However, in some situations, there are some limitations in definition above. Theoretically, the sum of any element and the maximum element is equal to the maximum value, and the multiple of any number and the minimum number is equal to the any one. But this may be incorrect in some cases. For instance, Let $A = < 0.6, 0.5, 0.5 >$ and $B = < 1,0,0 >$ be two SNSs. Obviously, $B = (1,0,0)$ is the larger value of these SNSs. However, according to operations above, $A \oplus B = < 1, 0.5, 0.5 > \neq B$, similarly, $A = < 0.6, 0.5, 0.5 >$ is the smaller value of these SNSs. However, according to operations above, $A \otimes B = < 0.6, 0, 0 > \neq A$ .Thus, the operations defined above are irrational. Therefore, some novel operations are proposed in this paper.

The t-norm and t-conorm play an important role in the building process of operation rules and aggregation operators. A strict Archimedean t-norm and a strict Archimedean t-conorm can be represented as $T(x,y) = k^{-1}(k(x)+k(y))$ and $S(x,y) = l^{-1}(l(x)+l(y))$, where $l(t) = k(1-t)$. Einstein t-norm and t-conorm are as follows

$$T(x,y) = \frac{xy}{1+(1-x)(1-y)}, \quad S(x,y) = \frac{x+y}{1+xy}$$

Here,

$$k(t) = \log\frac{2-t}{t}, \quad k^{-1}(t) = \log\frac{2}{e^t+1}, \quad l(t) = \log\frac{2-(1-t)}{1-t}, \quad l^{-1}(t) = 1-\frac{2}{e^t+1}.$$

Then, some novel operational rules of SNNs based on Einstein operations are defined.

Let $A = (T_1, I_1, F_1)$ and $B = (T_2, I_2, F_2)$ be two SNSs, then the new operational relations based on Einstein operations are defined as follows:

(1) $A \oplus B = < \dfrac{T_1+T_2}{1+T_1T_2}, \dfrac{I_1I_2}{1+(1-I_1)(1-I_2)}, \dfrac{F_1F_2}{1+(1-F_1)(1-F_2)} >$

(2) $A \otimes B = < \dfrac{T_1T_2}{1+(1-T_1)(1-T_2)}, \dfrac{I_1+I_2}{1+I_1I_2}, \dfrac{F_1+F_2}{1+F_1F_2} >$

(3) $\lambda A = < \dfrac{(1+T_1)^\lambda-(1-T_1)^\lambda}{(1+T_1)^\lambda+(1-T_1)^\lambda}, \dfrac{2(I_1)^\lambda}{(2-I_1)^\lambda+(I_1)^\lambda}, \dfrac{2(F_1)^\lambda}{(2-F_1)^\lambda+(F_1)^\lambda} > \quad \lambda > 0$

(4) $A^\lambda = < \dfrac{2(T_1)^\lambda}{(2-T_1)^\lambda+(T_1)^\lambda}, \dfrac{(1+I_1)^\lambda-(1-I_1)^\lambda}{(1+I_1)^\lambda+(1-I_1)^\lambda}, \dfrac{(1+F_1)^\lambda-(1-F_1)^\lambda}{(1+F_1)^\lambda+(1-F_1)^\lambda} > \quad \lambda > 0$

Take into account the aforementioned example again. When applying new operational laws, $A \oplus B = < 1,0,0 > = B$, $A \otimes B = < 0.6, 0.5, 0.5 > = A$,and these results coincide with the theory. Therefore, the new operational relations above based on Einstein operations can avoid irrational result in certain cases. The novel operations proposed in this paper is better than the operations in literature [11]. Subsequently, the novel operation laws proposed will be used to establish new aggregation operators.

III. GENERALIZED SIMPLIFIED NEUTROSOPHIC NUMBER EINSTEIN WEIGHTED AGGREGATION OPERATORS

In this section, some generalized simplified neutrosophic number Einstein aggregation operators are developed, and some properties are discussed.

A. GSNNEWA operator

Considering advantages of generalized weighted average (GWA) operator [27], we extend it to simplified neutrosophic environment.

Let $A_j=\langle T_j,I_j,F_j\rangle(j=1,2,\cdots,n)$ be a collection of SNNs, and $w=(w_1,w_2,\cdots,w_n)$ be the weight vector of $A_j(j=1,2,\cdots,n)$, satisfying $w_j\ge 0$ and $\sum_{j=1}^{n}w_j=1$. The generalized simplified neutrosophic number Einstein weighted average (GSNNEWA) operator of dimension n is the mapping GSNNEWA: $SNN^n\to SNN$, then the GSNNEWA operator is defined as

$$\mathrm{GSNNEWA}(A_1,A_2,\cdots,A_n)=(\sum_{j=1}^{n}w_jA_j^{\lambda})^{\frac{1}{\lambda}}$$

For a SNNs $A_j(j=1,2,\cdots,n)$, we have the following result.

$$\mathrm{GSNNEWA}(A_1,A_2,\cdots,A_n)$$
$$=\left\langle \frac{2(\prod_{j=1}^{n}u_j^{w_j}-\prod_{j=1}^{n}v_j^{w_j})^{\frac{1}{\lambda}}}{(\prod_{j=1}^{n}u_j^{w_j}+3\prod_{j=1}^{n}v_j^{w_j})^{\frac{1}{\lambda}}+(\prod_{j=1}^{n}u_j^{w_j}-\prod_{j=1}^{n}v_j^{w_j})^{\frac{1}{\lambda}}}, \frac{(\prod_{j=1}^{n}t_j^{w_j}+3\prod_{j=1}^{n}x_j^{w_j})^{\frac{1}{\lambda}}-(\prod_{j=1}^{n}t_j^{w_j}-\prod_{j=1}^{n}x_j^{w_j})^{\frac{1}{\lambda}}}{(\prod_{j=1}^{n}t_j^{w_j}+3\prod_{j=1}^{n}x_j^{w_j})^{\frac{1}{\lambda}}+(\prod_{j=1}^{n}t_j^{w_j}-\prod_{j=1}^{n}x_j^{w_j})^{\frac{1}{\lambda}}}, \frac{(\prod_{j=1}^{n}y_j^{w_j}+3\prod_{j=1}^{n}z_j^{w_j})^{\frac{1}{\lambda}}-(\prod_{j=1}^{n}y_j^{w_j}-\prod_{j=1}^{n}z_j^{w_j})^{\frac{1}{\lambda}}}{(\prod_{j=1}^{n}y_j^{w_j}+3\prod_{j=1}^{n}z_j^{w_j})^{\frac{1}{\lambda}}+(\prod_{j=1}^{n}y_j^{w_j}-\prod_{j=1}^{n}z_j^{w_j})^{\frac{1}{\lambda}}}\right\rangle \quad (1)$$

Where $u_j=(2-T_j)^{\lambda}+3(T_j)^{\lambda}, v_j=(2-T_j)^{\lambda}-(T_j)^{\lambda}$,

$t_j=(1+I_j)^{\lambda}+3(1-I_j)^{\lambda}, x_j=(1+I_j)^{\lambda}-(1-I_j)^{\lambda}$,

$y_j=(1+F_j)^{\lambda}+3(1-F_j)^{\lambda}, z_j=(1+F_j)^{\lambda}-(1-F_j)^{\lambda}$,

Then the aggregated result using the GSNNEWA operator is still a SNN.

Proof. The proof can be done by using the mathematical induction.

1) When n=2, then

$$A_1^{\lambda}=\left\langle \frac{2(T_1)^{\lambda}}{(2-T_1)^{\lambda}+(T_1)^{\lambda}}, \frac{(1+I_1)^{\lambda}-(1-I_1)^{\lambda}}{(1+I_1)^{\lambda}+(1-I_1)^{\lambda}}, \frac{(1+F_1)^{\lambda}-(1-F_1)^{\lambda}}{(1+F_1)^{\lambda}+(1-F_1)^{\lambda}}\right\rangle$$

$$w_1A_1^{\lambda}=\left\langle \frac{(1+\frac{2(T_1)^{\lambda}}{(2-T_1)^{\lambda}+(T_1)^{\lambda}})^{w_1}-(1-\frac{2(T_1)^{\lambda}}{(2-T_1)^{\lambda}+(T_1)^{\lambda}})^{w_1}}{(1+\frac{2(T_1)^{\lambda}}{(2-T_1)^{\lambda}+(T_1)^{\lambda}})^{w_1}+(1-\frac{2(T_1)^{\lambda}}{(2-T_1)^{\lambda}+(T_1)^{\lambda}})^{w_1}}, \frac{2(\frac{(1+I_1)^{\lambda}-(1-I_1)^{\lambda}}{(1+I_1)^{\lambda}+(1-I_1)^{\lambda}})^{w_1}}{(2-\frac{(1+I_1)^{\lambda}-(1-I_1)^{\lambda}}{(1+I_1)^{\lambda}+(1-I_1)^{\lambda}})^{w_1}+(\frac{(1+I_1)^{\lambda}-(1-I_1)^{\lambda}}{(1+I_1)^{\lambda}+(1-I_1)^{\lambda}})^{w_1}}, \frac{2(\frac{(1+F_1)^{\lambda}-(1-F_1)^{\lambda}}{(1+F_1)^{\lambda}+(1-F_1)^{\lambda}})^{w_1}}{(2-\frac{(1+F_1)^{\lambda}-(1-F_1)^{\lambda}}{(1+F_1)^{\lambda}+(1-F_1)^{\lambda}})^{w_1}+(\frac{(1+F_1)^{\lambda}-(1-F_1)^{\lambda}}{(1+F_1)^{\lambda}+(1-F_1)^{\lambda}})^{w_1}}\right\rangle$$

$$=\left\langle \frac{((2-T_1)^{\lambda}+3(T_1)^{\lambda})^{w_1}-((2-T_1)^{\lambda}-(T_1)^{\lambda})^{w_1}}{((2-T_1)^{\lambda}+3(T_1)^{\lambda})^{w_1}+((2-T_1)^{\lambda}-(T_1)^{\lambda})^{w_1}}, \frac{2((1+I_1)^{\lambda}-(1-I_1)^{\lambda})^{w_1}}{((1+I_1)^{\lambda}+3(1-I_1)^{\lambda})^{w_1}+((1+I_1)^{\lambda}-(1-I_1)^{\lambda})^{w_1}}, \frac{2((1+F_1)^{\lambda}-(1-F_1)^{\lambda})^{w_1}}{((1+F_1)^{\lambda}+3(1-F_1)^{\lambda})^{w_1}+((1+F_1)^{\lambda}-(1-F_1)^{\lambda})^{w_1}}\right\rangle$$

Thus,

$$w_1A_1^{\lambda}\oplus w_2A_2^{\lambda}$$
$$=\left\langle \frac{\frac{(u_1^{w_1}-v_1^{w_1})}{(u_1^{w_1}+v_1^{w_1})}+\frac{(u_2^{w_2}-v_2^{w_2})}{(u_2^{w_2}+v_2^{w_2})}}{1+\frac{(u_1^{w_1}-v_1^{w_1})}{(u_1^{w_1}+v_1^{w_1})}\cdot\frac{(u_2^{w_2}-v_2^{w_2})}{(u_2^{w_2}+v_2^{w_2})}}, \frac{\frac{2x_1^{w_1}}{t_1^{w_1}+x_1^{w_1}}\cdot\frac{2x_2^{w_2}}{t_2^{w_2}+x_2^{w_2}}}{1+(1-\frac{2x_1^{w_1}}{t_1^{w_1}+x_1^{w_1}})(1-\frac{2x_2^{w_2}}{t_2^{w_2}+x_2^{w_2}})}, \frac{\frac{2z_1^{w_1}}{y_1^{w_1}+z_1^{w_1}}\cdot\frac{2z_2^{w_2}}{y_2^{w_2}+z_2^{w_2}}}{1+(1-\frac{2z_1^{w_1}}{y_1^{w_1}+z_1^{w_1}})(1-\frac{2z_2^{w_2}}{y_2^{w_2}+z_2^{w_2}})}\right\rangle \quad (2)$$

$$=\left\langle \frac{u_1^{w_1}u_2^{w_2}-v_1^{w_1}v_2^{w_2}}{u_1^{w_1}u_2^{w_2}+v_1^{w_1}v_2^{w_2}}, \frac{2x_1^{w_1}x_2^{w_2}}{t_1^{w_1}t_2^{w_2}+x_1^{w_1}x_2^{w_2}}, \frac{2z_1^{w_1}z_2^{w_2}}{y_1^{w_1}y_2^{w_2}+z_1^{w_1}z_2^{w_2}}\right\rangle$$

2) When n=k, we get

$$w_1A_1^{\lambda}\oplus w_2A_2^{\lambda}\oplus\cdots\oplus w_kA_k^{\lambda}$$
$$=\left\langle \frac{\prod_{j=1}^{k}u_j^{w_j}-\prod_{j=1}^{k}v_j^{w_j}}{\prod_{j=1}^{k}u_j^{w_j}+\prod_{j=1}^{k}v_j^{w_j}}, \frac{2\prod_{j=1}^{k}x_j^{w_j}}{\prod_{j=1}^{k}t_j^{w_j}+\prod_{j=1}^{k}x_j^{w_j}}, \frac{2\prod_{j=1}^{k}z_j^{w_j}}{\prod_{j=1}^{k}y_j^{w_j}+\prod_{j=1}^{k}z_j^{w_j}}\right\rangle \quad (3)$$

3) When n=k+1, we can get

$$\sum_{j=1}^{k} w_j A_j^{\lambda} \oplus w_{k+1} A_{k+1}^{\lambda}$$

$$= \left\langle \frac{\dfrac{\prod_{j=1}^{k} u_j^{w_j} - \prod_{j=1}^{k} v_j^{w_j}}{\prod_{j=1}^{k} u_j^{w_j} + \prod_{j=1}^{k} v_j^{w_j}} + \dfrac{(u_{k+1}^{w_{k+1}} - v_{k+1}^{w_{k+1}})}{(u_{k+1}^{w_{k+1}} + v_{k+1}^{w_{k+1}})}}{1 + \dfrac{\prod_{j=1}^{k} u_j^{w_j} - \prod_{j=1}^{k} v_j^{w_j}}{\prod_{j=1}^{k} u_j^{w_j} + \prod_{j=1}^{k} v_j^{w_j}} \cdot \dfrac{(u_{k+1}^{w_{k+1}} - v_{k+1}^{w_{k+1}})}{(u_{k+1}^{w_{k+1}} + v_{k+1}^{w_{k+1}})}}, \right.$$

$$\frac{\dfrac{2\prod_{j=1}^{k} x_j^{w_j}}{\prod_{j=1}^{k} t_j^{w_j} + \prod_{j=1}^{k} x_j^{w_j}} \cdot \dfrac{2x_{k+1}^{w_{k+1}}}{t_{k+1}^{w_{k+1}} + x_{k+1}^{w_{k+1}}}}{1 + (1 - \dfrac{2\prod_{j=1}^{k} x_j^{w_j}}{\prod_{j=1}^{k} t_j^{w_j} + \prod_{j=1}^{k} x_j^{w_j}})(1 - \dfrac{2x_{k+1}^{w_{k+1}}}{t_{k+1}^{w_{k+1}} + x_{k+1}^{w_{k+1}}})}$$

$$\left. \frac{\dfrac{2\prod_{j=1}^{k} z_j^{w_j}}{\prod_{j=1}^{k} y_j^{w_j} + \prod_{j=1}^{k} z_j^{w_j}} \cdot \dfrac{2z_{k+1}^{w_{k+1}}}{y_{k+1}^{w_{k+1}} + z_{k+1}^{w_{k+1}}}}{1 + (1 - \dfrac{2\prod_{j=1}^{k} z_j^{w_j}}{\prod_{j=1}^{k} y_j^{w_j} + \prod_{j=1}^{k} z_j^{w_j}})(1 - \dfrac{2z_{k+1}^{w_{k+1}}}{y_{k+1}^{w_{k+1}} + z_{k+1}^{w_{k+1}}})} \right\rangle$$

$$= \left\langle \frac{\prod_{j=1}^{k+1} u_j^{w_j} - \prod_{j=1}^{k+1} v_j^{w_j}}{\prod_{j=1}^{k+1} u_j^{w_j} + \prod_{j=1}^{k+1} v_j^{w_j}}, \frac{2\prod_{j=1}^{k+1} x_j^{w_j}}{\prod_{j=1}^{k+1} t_j^{w_j} + \prod_{j=1}^{k+1} x_j^{w_j}}, \frac{2\prod_{j=1}^{k+1} z_j^{w_j}}{\prod_{j=1}^{k+1} y_j^{w_j} + \prod_{j=1}^{k+1} z_j^{w_j}} \right\rangle \quad (4)$$

Therefore, considering these results, we can get Equation (1) holds for any n.

4) Finally, we can get

$$\mathrm{GSNNEWA}(A_1, A_2, \cdots, A_n)$$

$$= \left\langle \frac{2(\dfrac{\prod_{j=1}^{k} u_j^{w_j} - \prod_{j=1}^{k} v_j^{w_j}}{\prod_{j=1}^{k} u_j^{w_j} + \prod_{j=1}^{k} v_j^{w_j}})^{\frac{1}{\lambda}}}{(2 - \dfrac{\prod_{j=1}^{k} u_j^{w_j} - \prod_{j=1}^{k} v_j^{w_j}}{\prod_{j=1}^{k} u_j^{w_j} + \prod_{j=1}^{k} v_j^{w_j}})^{\frac{1}{\lambda}} + (\dfrac{\prod_{j=1}^{k} u_j^{w_j} - \prod_{j=1}^{k} v_j^{w_j}}{\prod_{j=1}^{k} u_j^{w_j} + \prod_{j=1}^{k} v_j^{w_j}})^{\frac{1}{\lambda}}}, \frac{(1 + \dfrac{2\prod_{j=1}^{k} x_j^{w_j}}{\prod_{j=1}^{k} t_j^{w_j} + \prod_{j=1}^{k} x_j^{w_j}})^{\frac{1}{\lambda}} - (1 - \dfrac{2\prod_{j=1}^{k} x_j^{w_j}}{\prod_{j=1}^{k} t_j^{w_j} + \prod_{j=1}^{k} x_j^{w_j}})^{\frac{1}{\lambda}}}{(1 + \dfrac{2\prod_{j=1}^{k} x_j^{w_j}}{\prod_{j=1}^{k} t_j^{w_j} + \prod_{j=1}^{k} x_j^{w_j}})^{\frac{1}{\lambda}} + (1 - \dfrac{2\prod_{j=1}^{k} x_j^{w_j}}{\prod_{j=1}^{k} t_j^{w_j} + \prod_{j=1}^{k} x_j^{w_j}})^{\frac{1}{\lambda}}}, \right.$$

$$\left. \frac{(1 + \dfrac{2\prod_{j=1}^{k} z_j^{w_j}}{\prod_{j=1}^{k} y_j^{w_j} + \prod_{j=1}^{k} z_j^{w_j}})^{\frac{1}{\lambda}} - (1 - \dfrac{2\prod_{j=1}^{k} z_j^{w_j}}{\prod_{j=1}^{k} y_j^{w_j} + \prod_{j=1}^{k} z_j^{w_j}})^{\frac{1}{\lambda}}}{(1 + \dfrac{2\prod_{j=1}^{k} z_j^{w_j}}{\prod_{j=1}^{k} y_j^{w_j} + \prod_{j=1}^{k} z_j^{w_j}})^{\frac{1}{\lambda}} + (1 - \dfrac{2\prod_{j=1}^{k} z_j^{w_j}}{\prod_{j=1}^{k} y_j^{w_j} + \prod_{j=1}^{k} z_j^{w_j}})^{\frac{1}{\lambda}}} \right\rangle$$

$$= \left\langle \frac{2(\prod_{j=1}^{n} u_j^{w_j} - \prod_{j=1}^{n} v_j^{w_j})^{\frac{1}{\lambda}}}{(\prod_{j=1}^{n} u_j^{w_j} + 3\prod_{j=1}^{n} v_j^{w_j})^{\frac{1}{\lambda}} + (\prod_{j=1}^{n} u_j^{w_j} - \prod_{j=1}^{n} v_j^{w_j})^{\frac{1}{\lambda}}}, \frac{(\prod_{j=1}^{n} t_j^{w_j} + 3\prod_{j=1}^{n} x_j^{w_j})^{\frac{1}{\lambda}} - (\prod_{j=1}^{n} t_j^{w_j} - \prod_{j=1}^{n} x_j^{w_j})^{\frac{1}{\lambda}}}{(\prod_{j=1}^{n} t_j^{w_j} + 3\prod_{j=1}^{n} x_j^{w_j})^{\frac{1}{\lambda}} + (\prod_{j=1}^{n} t_j^{w_j} - \prod_{j=1}^{n} x_j^{w_j})^{\frac{1}{\lambda}}}, \right.$$

$$\left. \frac{(\prod_{j=1}^{n} y_j^{w_j} + 3\prod_{j=1}^{n} z_j^{w_j})^{\frac{1}{\lambda}} - (\prod_{j=1}^{n} y_j^{w_j} - \prod_{j=1}^{n} z_j^{w_j})^{\frac{1}{\lambda}}}{(\prod_{j=1}^{n} y_j^{w_j} + 3\prod_{j=1}^{n} z_j^{w_j})^{\frac{1}{\lambda}} + (\prod_{j=1}^{n} y_j^{w_j} - \prod_{j=1}^{n} z_j^{w_j})^{\frac{1}{\lambda}}} \right\rangle$$

The proof is completed, and the novel GSNNEWA operator is calculated.

B. Properties of GSNNEWA operator

We can easily prove that the GSNNEWA operator has the following properties.

1) Idempotency: Let $A_j = \langle T_{A_j}, I_{A_j}, F_{A_j} \rangle (j = 1,2,\cdots,n)$ be a collection of SNNs, and $A = \langle T_A, I_A, F_A \rangle$ be a SNN. If $A_j = A$ $(j = 1,2,\cdots,n)$, then $GSNNEWA(A_1, A_2, \cdots, A_n) = A$.

2) Commutativity: Let $A_j = \langle T_{A_j}, I_{A_j}, F_{A_j} \rangle (j = 1,2,\cdots,n)$ be a collection of SVNNs, if $A_j^{*} (j = 1,2,\cdots,n)$ is any permutation of A_j $(j = 1,2,\cdots,n)$, then

$$GSNNEWA(A_1, A_2, \cdots, A_n) = GSNNEWA(A_1^{*}, A_2^{*}, \cdots, A_n^{*}).$$

3) Boundedness: Let $A_j^{-} = \langle T_{A_j^{-}}, I_{A_j^{-}}, F_{A_j^{-}} \rangle (j = 1,2,\cdots,n)$, $A_j = \langle T_{A_j}, I_{A_j}, F_{A_j} \rangle (j = 1,2,\cdots,n)$, and $A_j^{+} = \langle T_{A_j^{+}}, I_{A_j^{+}}, F_{A_j^{+}} \rangle (j = 1,2,\cdots,n)$ be three collections of SNNs. If for all j, $T_{A_j^{-}} \le T_{A_j} \le T_{A_j^{+}}$, $I_{A_j^{+}} \le I_{A_j} \le I_{A_j^{-}}$, and $F_{A_j^{+}} \le F_{A_j} \le F_{A_j^{-}}$, then

$$GSNNEWA(A_1^{-}, A_2^{-}, \cdots, A_n^{-}) \le GSNNEWA(A_1, A_2, \cdots, A_n) \le GSNNEWA(A_1^{+}, A_2^{+}, \cdots, A_n^{+})$$

4) Monotonicity:

Let $A_j = \langle T_{A_j}, I_{A_j}, F_{A_j} \rangle$ and $A_j' = \langle T_{A_j'}, I_{A_j'}, F_{A_j'} \rangle$ be two collections of SNNs, if $A_j \le A_j'$ for all $(j = 1,2,\cdots,n)$, then

$$GSNNEWA(A_1, A_2, \cdots, A_n) \le GSNNEWA(A_1', A_2', \cdots, A_n')$$

C. Special cases

Some special operators based on the different parameter λ of GSNNEWA operator will be discussed in the following.

1) If $\lambda = 1$, then the GSNNEWA operator will be reduced to the simplified neutrosophic number Einstein weighted average (SNNEWA) operator. Thus,

$u_j = 2 + 2T_j, v_j = 2 - 2T_j, t_j = 4 - 2I_j, x_j = 2I_j, y_j = 4 - 2F_j, z_j = 2F_j$,

SNNEWA operator is defined as follows:

$$\mathrm{SNNEWA}(A_1, A_2, \cdots, A_n) = \left\langle \frac{\prod_{j=1}^{n} (1+T_j)^{w_j} - \prod_{j=1}^{n} (1-T_j)^{w_j}}{\prod_{j=1}^{n} (1+T_j)^{w_j} + \prod_{j=1}^{n} (1-T_j)^{w_j}}, \frac{2\prod_{j=1}^{n} I_j^{w_j}}{\prod_{j=1}^{n} (2-I_j)^{w_j} + \prod_{j=1}^{n} I_j^{w_j}}, \frac{2\prod_{j=1}^{n} F_j^{w_j}}{\prod_{j=1}^{n} (2-F_j)^{w_j} + \prod_{j=1}^{n} F_j^{w_j}} \right\rangle$$

2) If $\lambda = -1$, then the GSNNEWA operator will be reduced to the simplified neutrosophic number Einstein weighted harmonic average(SNNEWHA) operator which is defined as follows:

$$u_j = \frac{6 - 2T_j}{(2 - T_j)T_j}, v_j = \frac{2T_j - 2}{(2 - T_j)T_j}, t_j = \frac{4 + 2I_j}{(1 + I_j)(1 - I_j)}, x_j = \frac{-2I_j}{(1 + I_j)(1 - I_j)},$$

$$y_j = \frac{4 + 2F_j}{(1 + F_j)(1 - F_j)}, z_j = \frac{-2F_j}{(1 + F_j)(1 - F_j)},$$

$$\mathrm{SNNEWHA}(A_1, A_2, \cdots, A_n) = \left\langle \frac{\prod_{j=1}^{n} (3-T_j)^{w_j} + 3\prod_{j=1}^{n} (T_j - 1)^{w_j}}{\prod_{j=1}^{n} (3-T_j)^{w_j} + \prod_{j=1}^{n} (T_j - 1)^{w_j}}, \frac{-2\prod_{j=1}^{n} (-I_j)^{w_j}}{\prod_{j=1}^{n} (2+I_j)^{w_j} + \prod_{j=1}^{n} (-I_j)^{w_j}}, \frac{-2\prod_{j=1}^{n} (-F_j)^{w_j}}{\prod_{j=1}^{n} (2+F_j)^{w_j} + \prod_{j=1}^{n} (-F_j)^{w_j}} \right\rangle$$

3) if $\lambda \to 0$, then the GSNNEWA operator will be reduced to the simplified neutrosophic number Einstein weighted geometric average(SNNEWGA) operator which is defined as follows:

$$\mathrm{SNNEWGA}(A_1, A_2, \cdots, A_n) = \left\langle \frac{2\prod_{j=1}^{n} T_j^{w_j}}{\prod_{j=1}^{n} (2-T_j)^{w_j} + \prod_{j=1}^{n} T_j^{w_j}}, \frac{\prod_{j=1}^{n} (1+I_j)^{w_j} - \prod_{j=1}^{n} (1-I_j)^{w_j}}{\prod_{j=1}^{n} (1+I_j)^{w_j} + \prod_{j=1}^{n} (1-I_j)^{w_j}}, \frac{\prod_{j=1}^{n} (1+F_j)^{w_j} - \prod_{j=1}^{n} (1-F_j)^{w_j}}{\prod_{j=1}^{n} (1+F_j)^{w_j} + \prod_{j=1}^{n} (1-F_j)^{w_j}} \right\rangle$$

From the above discussion, we can easily know that GSNNEWA operator is more generalized.

IV. AN ILLUSTRATIVE EXAMPLE

In this section, we will use the GSNNEWA operator to deal with the multi-criteria decision-making problems under the simplified neutrosophic environment, where the alternative values are in the form of SNNs and the criteria weights are in the form of crisp values.

A. Hamming distance measure

Let $x=(T,I,F)$ be a SNN, and the cosine similarity measure $S(x)$ between SNN x and the ideal alternative $(1,0,0)$ can be defined as follows [15]:

$$S(x)=\frac{T}{\sqrt{T^2+I^2+F^2}}$$

However, the cosine similarity measure for SVNNs might be unacceptable. For example, let $a_1=<0.3,0,0>$ and $a_2=<0.7,0,0>$ be two alternatives, and the ideal alternative is $a^*=<1,0,0>$. According to the calculated formula of similarity measure under simplified neutrosophic environment, $S_1(a_1,a^*)=S_2(a_2,a^*)=1$ can be obtained. However, it cannot indicate which alternative is better. In fact, it is clear that a_2 is superior to a_1. Therefore, we introduce Hamming distance to rank alternatives instead of cosine similarity measure.

Let $x=(T_1,I_1,F_1)$ and $y=(T_2,I_2,F_2)$ be any two SVNNs, then the Hamming distance between x and y can be defined as follows:

$$d(x,y)=|T_1-T_2|+|I_1-I_2|+|F_1-F_2|$$

We know that the Hamming distance can solve the problem in the literature [15]. Let us consider the same example above, according to the calculated formula of Hamming distance, $D_1(a_1,a^*)=0.7, D_2(a_2,a^*)=0.3$ can be obtained, which indicate a_1 is inferior to a_2.

B. An example

Next, we will consider the same decision-making problem adapted from Ye [11].

The example is about an investment company with four possible alternatives based on three criteria. A_1, A_2, A_3 and A_4 are four alternatives, where A_1 is a car company, A_2 is a food company, A_3 is a computer company, A_3 is an arms company. C_1, C_2 and C_3 are corresponding criteria, where C_1 represents the risk analysis, C_2 represents the growth analysis, C_3 represents the environmental impact analysis. The weight vector of the criteria is $W=(0.35,0.25,0.4)$.

The simplified netrosophic decision matrix is shown in the following forms:

$$D=\begin{bmatrix}\langle 0.4,0.2,0.3\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0.2,0.2,0.5\rangle\\ \langle 0.6,0.1,0.2\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.5,0.2,0.2\rangle\\ \langle 0.3,0.2,0.3\rangle & \langle 0.5,0.2,0.3\rangle & \langle 0.5,0.3,0.2\rangle\\ \langle 0.7,0.0,0.1\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.4,0.3,0.2\rangle\end{bmatrix}$$

To get the best alternative, the following steps are involved:

Step1: Utilize the GSNNEWA operator to calculate the comprehensive evaluation value of each alternative.

For simplicity, suppose $\lambda\to 0$, then the GSNNEWA operator will be reduced to the simplified neutrosophic number Einstein weighted geometric average (SNNEWGA) operator.

$$\mathrm{SNNEWGA}(A_1,A_2,\cdots,A_n)=\left\langle \frac{2\prod_{j=1}^{n}T_j^{w_j}}{\prod_{j=1}^{n}(2-T_j)^{w_j}+\prod_{j=1}^{n}T_j^{w_j}}, \frac{\prod_{j=1}^{n}(1+I_j)^{w_j}-\prod_{j=1}^{n}(1-I_j)^{w_j}}{\prod_{j=1}^{n}(1+I_j)^{w_j}+\prod_{j=1}^{n}(1-I_j)^{w_j}}, \frac{\prod_{j=1}^{n}(1+F_j)^{w_j}-\prod_{j=1}^{n}(1-F_j)^{w_j}}{\prod_{j=1}^{n}(1+F_j)^{w_j}+\prod_{j=1}^{n}(1-F_j)^{w_j}}\right\rangle$$

Thus, the comprehensive evaluation value of each alternative can be obtained by the above formula. Therefore, we can get

$\alpha_1=\langle 0.3062,0.2,0.3846\rangle$; $\alpha_2=\langle 0.5586,0.1404,0.2\rangle$;

$\alpha_3=\langle 0.4212,0.2406,0.2606\rangle$; $\alpha_4=\langle 0.5446,0.1478,0.1654\rangle$

Step2: Calculate Hamming distance measure $S(\alpha_i,\alpha^*)(i=1,2,3,4)$ to evaluate the similarity of each alternative and the ideal alternative. The ideal alternative is defined as $\alpha^*=\langle 1,0,0\rangle$, as we all known, the smaller the distance value is, the bigger the similarity is, and the better the alternative is.

$D_1(\alpha_1,\alpha^*)=1.2784, D_1(\alpha_2,\alpha^*)=0.7818,$

$D_1(\alpha_3,\alpha^*)=1.08, D_1(\alpha_4,\alpha^*)=0.7686,$

Step3: Give the ranking order of all alternatives based on the obtained distance values. We can get the ranking order of four alternatives. That is $A_4\succ A_2\succ A_3\succ A_1$

Step4: Get the best alternative and the worst alternative. The best alternative is A_4, and the worst alternative is A_1.

V. Conclusion

In the real world, the indeterminate information and inconsistent information existing commonly cannot be deal with by FSs [29, 30] and IFSs. The simplified neutrosophic set is an extension of traditional fuzzy set, which is more suitable for real scientific and engineering application.

In a word, firstly, we proposed some novel operational relations based on Einstein operations which can avoid the disadvantages of some irrational operations, and we extend generalized aggregation operators to accommodate the simplified neutrosophic set. Secondly, we combine Einstein operations and generalized aggregation operators under simplified neutrosophic environment in order to make use of their advantages. Thirdly, the novel GSNNEWA operator is firstly proposed, some desirable properties and some special cases of the proposed operator are also analyzed. Finally, an illustrative example on the basis of the GSNNEWA operator utilizing Hamming distance measure was presented in order to demonstrate the effectiveness and application of the proposed method.

In this paper, there are the same ranking results when we compare the proposed method with the method presented in the literature [11]. However, the GSNNEWA operator proposed in this paper can avoid some irrational operations, which is more general and more flexible, which is the extension of aggregation operator proposed by Ye [11]. Meanwhile, we use the Hamming distance instead of the cosine similarity measure in [15], the Hamming distance measure can avoid some unacceptable situations. Therefore, the example shows that the novel aggregation operator proposed in the paper is more effectiveness and practicality.

In the future, we will study some new aggregation operators under simplified neutrosophic environments, and give the application of the proposed operator to the other fields.

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