Fractality and Multifractality Analysis of News Sentiments Time Series

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Abstract—Using artificial intelligence algorithms, providers of news analytics calculate the sentiment score of almost every economic and financial news in real time. The sentiment score of negative, neutral, positive news are assigned to be -1, 0, 1, respectively. We constructed time series of news sentiments as follows: a nine-month period of 2015 was divided into non-overlapping consecutive intervals of equal length, and then we calculated the sum of sentiment scores of all news within each time interval. In this paper, we examine long-range dependance and self-similarity of time series of sentiments of economic and financial news using the Fluctuation Analysis (FA), the Detrended Fluctuation Analysis of order 1 (DFA), Rescaled Range Analysis (R/S), Average Wavelet Coefficient Method (AWC) and Fourier Transform Method (FTM). Moreover, in this paper, we use Multifractal Detrended Fluctuation Analysis (MDFA) to examine multifractality of time series of news sentiments. In addition, we analyzed three time series of news sentiments for companies traded on the London Stock Exchange (LSE), the New York Stock Exchange (NYSE) and The Stock Exchange of Hong Kong (SEHK). Empirical results obtained by this method show that time series of news sentiments exhibit self-similarity (as well as a long memory property). The Hurst exponent (as well as the long-range correlation exponent) is greater than 0.55 over four orders of magnitude in time ranging from several minutes to dozen of days. Moreover, we show that both NYSE and LSE time series of news sentiments exhibit multifractality. On the other hand, we demonstrate that the SEHK time series have monofractal behavior.

Index Terms—long-range correlation, time series, detrended fluctuation analysis, Hurst exponent.

I. INTRODUCTION

A multifractal system could be considered as a generalization of a fractal system. Multifractal is an extension of a fractal in which a single Hurst exponent is not sufficient to characterize its behavior, and therefore we need a continuous spectrum of Hurst exponents. Many papers defined a multifractal structure as a superposition of homogeneous monofractal structures.

Multifractal analysis was initially introduced for the study of turbulence and was applied for the measurement of the turbulent flow velocity in the study of energy dissipation. Over the last thirty years, multifractal as well as fractal analysis have been applied extensively for analysis of medical signals [1], [2], [3], fluid flow [4], time series [5], [6], [7], [8], traffic signals [9], images [10], earthquake signals [11], [12], and in many other areas. The survey of multifractal spectrum computing methods can be found in [13].

Long range dependance and self-similarity of financial time series have been the focus of attention for many researchers in the last several years [14]–[25]. The recent years have seen interest in multifractality analysis of financial time series [19], [20], [22]–[30]. However, there are very few works devoted to the self-similarity or multifractality analysis of the statistical properties of the news flow. Following papers [31], [32], [33] and [34], in this work, we analyze time series of news sentiments.

News analytics is an unconventional approach to the analysis of news flow based on artificial intelligence techniques. Introduction to the news analytics tools and techniques can be found in [35] and [36]. For each piece of news, news analytic providers find its sentiment score using artificial intelligence algorithms [35]. The sentiment score of a neutral news is assumed to be 0. Every positive piece of news has sentiment score 1. If news is negative then its sentiment score is assigned to be -1. Then we constructed time series of news sentiments as follows. We use the final data sample and divide the whole period into non-overlapping consecutive intervals of equal length δ = 1 minute and calculated the sum of all news sentiments within each time interval. Note that the sentiment extraction from news is a difficult task and in many real applications it is necessary to use sophisticated techniques [37] and big data approaches [38].

This work uses five estimators

- Fluctuation Analysis (FA),
- Detrended Fluctuation Analysis of orders 1 (DFA),
- Rescaled Range Analysis (R/S),
- Average Wavelet Coefficient Method (AWC),
- Fourier transform method
to detect long-range auto-correlation and self-similarity of time series of news sentiments. In addition to the work [31], in this paper, we use Multifractal Detrended Fluctuation Analysis (MDFA) to examine multifractality of time series of news sentiments.

Our analysis is based on a 189 trading days period from news analytic data, taken from January 1, 2015 to September 22, 2015. In addition to analyzing time series of intensity for all available news from all stock exchanges, the following research question was of our interest: is the multifractal structure of news sentiments similar for different stock exchanges? We examine time series of news sentiments related to different time zones. We consider news affecting companies trading on the European, American and Asian stock exchanges separately. More precisely, we analyze companies trading on the London Stock Exchange (LSE), the New York Stock Exchange (NYSE) and The Stock Exchange of Hong Kong (SEHK). As the paper [39] we excluded from our analysis periods of low news intensity, e.g., weekends and holidays as well as night hours at local time.

In different time zones, hours of high and low news activity do not coincide, although overlap partially. It may be explained by the partial intersection of the lists of trading companies and agents that generate news. We assume that the fractal structure of time series of news sentiments is
more complex for bigger stock exchanges and for larger
differentiation of the agents generated news.

The results of this work show that the long-range power-
law correlation takes place in time series of news sentiments.
All methods showed the presence of a long-range correlation
in the time series of news sentiments (the Hurst exponent is
greater than 0.5). Moreover, this paper shows that time series
of news sentiments exhibit multifractality.

II. METHODS FOR DETECTING AUTO-CORRELATION

A. Auto-correlation types

Let $X = (x_t)_{t=1}^n$ be a time series with large $n$ and let $s \in \mathbb{N}$, $s \ll n$. Let $C(s)$ denote the (auto) correlation between
$X_1 = (x_t)_{t=1}^{n-s}$ and $X_2 = (x_{t+s})_{t=1}^{n-s}$. The correlation can be
of several types:

1) $x_t$ are uncorrelated; it is obvious that if $X_1$ and $X_2$
are uncorrelated then $C(s)$ must be zero, $C(s) = 0$;
2) short-range correlations of $(x_t)_{t=1}^n$ results in exponential
decaying of $C(s)$, i.e. $C(s) \sim \exp(-s/\eta)$, where $s_0$
is a parameter of decay;
3) long-range correlation of the $(x_t)_{t=1}^n$, $C(s)$ should
follow a power-law dependence: $C(s) \sim s^{-\gamma}$, $0 < \gamma < 1$.

We use the following methods for estimation of the
correlation.

B. Fluctuation analysis (FA)

FA algorithm consists of two steps:

1) Integration. We calculate the cumulative deviate series as
follows

$$y_k = \sum_{t=1}^k (x_t - \bar{x}), \quad k = 1, 2, \ldots, n, \quad (1)$$

where $\bar{x} = \sum_{t=1}^n x_t$.

2) The FA fluctuation function is defined by

$$F(s) = F_{FA}(s) = \left(1 - \frac{s}{n-s} \sum_{k=1}^{n-s} (y_{k+s} - y_k)\right)^{1/2}. \quad (2)$$

Note that (2) is actually a special case of the structure
function in turbulence [40].

C. DFA method

DFA was proposed in the papers [41], [42]. This method
is used for studying the indirect scaling of the long-range
dependence. DFA method was effectively applied for analysis
of economical and finance time series in papers [14]–[18].

While DFA has some drawbacks [43], the work [44] remarks
that DMA and DFA stay the choices of choice for evaluating
the long-range correlation of time series. DFA algorithm
includes five stages:

1) Integration. We calculate $y_k$, $k = 1, 2, \ldots, n$, using
(1).

2) Cutting. We divide the $(y_k)_{k=1}^n$ into $n_s = [n/s]$ non-
overlapping intervals with each length $s$, starting with $y_1$.

3) Fitting. For each interval $l = 1, \ldots, n_s$ we construct a
fitting linear function $P_l$ (trend) by means of least-
square fit of the data $(y_k)_{k=(l-1)s+1}^{ls}$. Denote $y_k^* = P_l(k)$.

4) Detrending. The detrended time series are obtained by
$\epsilon_k = y_k - y_k^*$.

5) We calculate variance of residuals $\epsilon_i$:

$$F_i^2(s) = \frac{1}{s} \sum_{l=1}^s \epsilon_k^{2(l-1)s+i}(s), \quad l = 1, \ldots, n_s.$$

6) DFA fluctuation function is defined by

$$F(s) = F_{DFA}(s) = \left(\frac{1}{n_s} \sum_{l=1}^{n_s} F_i^2(s)\right)^{1/2}.$$

Derivation of DFA can be found in the work [45].

D. The auto-correlation parameter

The fluctuation functions $F(s)$ deduced in DFA let us
examine the s-dependence of $F$. In the case of long-range
power-law correlation of $x_t$, the $F$ must follow a power-law
$F(s) \sim s^\alpha$

for large enough $s$. The fluctuation parameter $\alpha$ is connected
with the value of correlation exponent $\gamma$ in the following
way (see [46]) $\alpha = 1 - \gamma/2$, $0 < \gamma < 1$. The correlation
parameter $\alpha$ reflects the auto-correlation properties of time
series as follows:

1) $\alpha = 1/2$, the time series uncorrelated (white noise) or
short-range correlated;
2) $\alpha < 1/2$, it is anti-correlated;
3) $\alpha > 1/2$, it is long-range power-law correlated;
4) $\alpha = 1$, pink noise ($1/f$ noise).

E. Rescaled Range Analysis (R/S)

The R/S algorithm is one of the most widely used methods
for scale exponent estimation. R/S algorithm estimates the
value of the Hurst exponent based on an empirical data set
for the long-range dependent process that generated the data set.
R/S-analysis uses a heuristic approach developed in [47],
[48], [49], [50], [51].

R/S algorithm consists of the following steps:

1) Cutting. We divide the $(x_i)_{i=1}^n$ into $n_s = [n/s]$ non-
overlapping intervals $X(i)^{(1)}(s) = \{x_{(i-1)s+1}, \ldots, x_{is}\}$,
$i = 1, \ldots, n_s$, each of length $s$, starting with $x_1$.

2) Accumulation. We calculate the accumulated series for
each window $X(i)^{(1)}(s)$, $i = 1, \ldots, n_s$, as follows

$$y_j^{(1)}(s) = \sum_{t=(i-1)s+1}^{j} (x_t - \bar{x}(s)), \quad (3)$$

where $\bar{x}(s) = 1/s \sum_{t=(i-1)s+1}^{is} x_t$ is the mean of ith
window of size $s$, and $j = (i-1)s + 1, \ldots, is$.

3) Range calculation. We compute the range of deviation
within each window for the accumulated series as follows
$R(i)^{(1)}(s) = \max_{(i-1)s+1 \leq j \leq is} \{y_j^{(1)}(s)\} - \min_{(i-1)s+1 \leq j \leq is} \{y_j^{(1)}(s)\}$.

4) Standard deviation calculation. For each window $i = 1, \ldots, n_s$ we find the standard deviation:

$$S^{(1)}(s) = \sqrt{\frac{1}{s} \sum_{t=(i-1)s+1}^{is} (x_t - \bar{x}(s))^2}.$$

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5) Computation of R/S statistics:
\[ (R(s)/S(s)) := \frac{1}{n_s} \sum_{i=1}^{n_s} \frac{R(i)}{S(i)} \quad s = s_1, \ldots, s_L. \]

6) Estimation of Hurst exponent. We solve the following least mean square problem
\[ \sum_{j=1}^{L} (\log ((R(s_j)/S(s_j))) - H \log s_j - b)^2 \rightarrow \min_{H,b} \]

R/S algorithm computes the R/S-statistic for different \( s \) and plots the resulting estimates versus the \( s \) on loglog scale. Then the Hurst parameter can be estimated via the slope of the resulting log-log plot. As it pointed out in [52], classical R/S-analysis is not suitable for small samples, but can be highly effective for quite large samples and it often provides a rather accurate picture of the presence or absence of long-range dependence in a given empirical data sets. Moreover, R/S-analysis demonstrates relative robustness under heavy tail distributions with infinite variance in the marginal distribution of the data [47]-[50].

On the other hand, R/S-analysis is quite sensitive relative to the presence of explicit short-range dependence structures and its bias. Therefore, these shortcomings of R/S analysis lead to the fact that many researchers do not consider this algorithm as a rigorous statistical method.

III. WAVELET-BASED METHODS

A. The Hurst exponent

The self-similarity parameter \( 0 < H < 1 \) of self-affine processes is also called the Hurst (or roughness) exponent [53]. The Hurst exponent is commonly used for measuring the duration of long-range dependence of a stochastic process. There are three possibilities:

- If \( H = 0.5 \) then \( C(s) = 0 \) which means that past and future increments are uncorrelated (Brownian motion);
- In case \( H > 0.5 \) we have \( C(s) > 0 \) and the increments are positively correlated (the process \( \{X(t)\} \) is called persistent);
- If \( H < 0.5 \) then \( C(s) < 0 \) and increments are negatively correlated (the process \( \{X(t)\} \) is called anti-persistent or anti-correlated).

B. The Average Wavelet Coefficient Method

The Average Wavelet Coefficient Method (AWC) was proposed in papers [54] and [55]. The method is used for measuring the temporal self-affine correlations of a time series by estimating its Hurst exponent. The AWC method is based on the wavelet transform (good review of the wavelet transform can be found in books [56] and [57]).

The strategy for the data-analysis by the AWC method consists of three main steps:

1) Wavelet transformation of the data \( X(t) \) into the wavelet domain, \( \hat{W}[X](a,b) \), where \( a, b \) are scale and location parameters, respectively [56], [57].
2) Then for a given scale \( a \) we can find a representative wavelet amplitude for that particular scale, and to study its scaling. To do so we calculate the averaged wavelet coefficient \( \hat{W}[X](a) \) according to the equation
\[ \hat{W}[X](a) = \langle |\hat{W}[X](a,b)| \rangle, \]
where \( \langle \cdot \rangle \) denotes the standard arithmetic mean value operator with respect to the \( b \).
3) For a self-affine process \( X(t) \) with exponent \( H \), the spectrum \( W[h](a) \) should scale as \( a^{H+0.5} \) [55]. Therefore, to find \( H + 0.5 \) we should plot \( W[h](a) \) against scale \( a \) in a log-log plot. As it is pointed out in [55], a scaling regime consisting of a straight line in this plot implies a self-affine behavior of the data.

C. Fourier transform method

Let us recall that the Fourier transform of the function \( f \) and inverse transform are defined by
\[ \hat{f}(\xi) = \sum_t e^{-i\xi t} f(t), \quad (4) \]
\[ f(t) = \sum_{\xi} e^{i\xi t} \hat{f}(\xi). \quad (5) \]

Fourier inversion theorem states that \( f \) can be reconstructed from \( \hat{f} \).

If \( \rho_x(s) \) is autocorrelation function of \( x(t) \), \( \rho_x(s) = \sum_t x(t)x(t+s) \), then the Fourier transform of \( \rho_x(s) \) is
\[ \hat{\rho}(\xi) = |\hat{x}(\xi)|^2. \quad (6) \]

Therefore, to find the value of \( \rho_x(s) \) one should make the following steps:

1) Find \( \hat{x}(\xi) \), the Fourier transform of \( x(t) \), using (4).
2) Find \( \hat{\rho}(\xi) \), the Fourier transform of \( \rho_x(s) \), using (6).
3) Compute \( \hat{\rho}(s) \) as the inverse transform of \( \hat{\rho}(\xi) \) using (5).

Fourier transform is a classical way of correlation exponent estimation \( \gamma \), but it is often not appropriate due to noisy nature, non-stationarity and imperfect measurement of data \( x_t \).

\( \hat{\rho}(\xi) \) is a power spectrum \( P(\omega) \) of the Fourier transform, \( \omega = 0 : Fs/n : Fs/2 \) is a frequency increment (angular frequency), where \( Fs \) is sampling frequency, and \( n \) - number of samples.

IV. MULTIFRACTAL DETRENDED FLUCTUATION ANALYSIS (MDFA)

The multifractal generalization of DFA [30] can be briefly sketched as follows.

It is well-known that the self-similarity of fractal structures can be described by the global exponent Hölder, or the local Hurst exponent \( \alpha \). If the fractal is monofractal, then it can be described by only one Hölder exponent, and in the case of a multifractal, different parts of the structure are characterized by different values of \( \alpha \), which means the existence of a spectrum \( f(\alpha) \).

MDFA algorithm includes seven stages:

1) Integration. We calculate \( y_k = \sum_{i=1}^{k} (x_i - \bar{x}), \quad k = 1, 2, \ldots, n, \) where \( \bar{x} = \sum_{i=1}^{n} x_i. \)
2) Cutting. We divide the \( (y_k)_{k=1}^{n} \) into \( n_s = [n/s] \) non-crossing intervals, each of length \( s \), starting with \( y_1 \).
3) Fitting. For each interval \( l = 1, \ldots, n_s \) we construct a fitting linear function \( P_l \) (trend) by means of least-square fit of the data \((y_k)_{k=(l-1)s+1}^{ls}\) Denote \( y_k^* = P_l(k) \).

4) Detrending. The detrended time series are obtained by \( \epsilon_k(s) = y_k - y_k^* \).

5) We calculate variance of residuals \( \epsilon_l \):

\[
F_l^2(s) = \frac{1}{s} \sum_{i=1}^{n_s} \epsilon_i^2(l-1)s+1, \quad l = 1, \ldots, n_s.
\]

6) MDFA fluctuation function is defined by

\[
F_q(s) = \left( \frac{1}{n_s} \sum_{l=1}^{n_s} F_l^q(s) \right)^{1/q},
\]

where \( q \) can take any real value except for \( q = 0 \). When \( q = 0 \), we have

\[
F_0(s) = \exp \left( \frac{1}{n_s} \sum_{l=1}^{n_s} \ln[F_l^2(s)] \right) \approx s^{h_0},
\]

according to L'Hospital's rule.

7) Changing the size of the segment \( s \), we can find a power relation between the function \( F_q(s) \) and the size \( s \):

\[
F_q(s) \sim s^{h(q)}.
\]

Then we calculate the multifractal scale factor \( \tau(q) \) which can be considered as a characteristic of multifractality:

\[
\tau(q) = qh(q) - D_f
\]

where \( D_f \) denotes the fractal dimension of the geometric support of the multifractal measure \([30], h(q) \) is generalized Hurst exponent or Holder exponent, which coincides with Hurst exponent for \( q = 2 \). \( h(q) \). When analyzing time series, we have \( D_f = 1 \). If \( \tau(q) \) is a nonlinear function of \( q \) then the signal is multifractal.

Another characteristic of multifractality for time series may be the singularity spectrum \( f(\alpha) \), which is connected with \( \alpha(q) \) through the Legendre transformation as follows [58]:

\[
\begin{align*}
\alpha(q) &= d\tau(q)/dq \\
\tau(q) &= \alpha(q) - \tau(q)
\end{align*}
\]

where \( \alpha(q) \) is Holder singular index which is also called singular strength and \( f(\alpha) \) is a multifractal spectrum which determines dimensions of subsets of the time series that are characterized by \( \alpha(q) \).

The curve \( \alpha \sim f(\alpha) \) is a single-humped function for a multifractal and is a single point for a mono-fractal. The shape and length of the curve \( f(\alpha) \) give us an important information on characteristics of the distribution of the data set. First, we can determine the width of the spectrum \( \Delta \alpha = \alpha_{max} - \alpha_{min} \). The parameter \( \Delta \alpha \) describes the inhomogeneity of the probability distribution measured by the total fractal structure and determines the degree of multifractality. The function \( f(\alpha) \) has the maximum value \( f_{max} \) at some point \( \alpha_0, f_{max} = f(\alpha_0) \), which corresponds to the peak of the multifractal spectrum [12]. Larger values of \( \Delta \alpha \) and \( f_{max} \) correspond to a stronger degree of multifractality.

A. Time series of news sentiments

Providers of news analytics obtain and aggregate data from different sources (including news agencies and business reports) and social media. Our data cover the period from January 1, 2015 to September 22, 2015 (i.e. 189 trading days). We consider all the news released during this period. Initially we performed data selection and cleaning process as described in [32] or [34].

For each piece of news, news analytic providers find its sentiment score using artificial intelligence algorithms. The sentiment score of neutral news is assumed to be 0. Every positive piece of news has sentiment score 1. If news is negative then its sentiment score is assigned to be -1.

Then we constructed time series of news sentiments as follows. We divided 189-day period \( \Delta \) into \( n \) non-crossing consecutive segments \( \Delta_1, \ldots, \Delta_n \) of the same longevity \( \delta \) minutes, \( \Delta = \Delta_1 \cup \ldots \cup \Delta_n \). We found \( x_t \), the sum of all sentiment scores of all economical and finance news reported in the world during each interval \( \Delta_t, t = 1, 2, \ldots, n \). The sequence \( x_1, x_2, \ldots, x_n \) is the time series of news sentiments with the \( \delta \) minutes window. The overall sentiment of news for the 189-day period is 2011463. Table I shows the summary statistics. We used the time window \( \delta = 1 \).

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>SUMMARY STATISTICS OF TIME SERIES, ( \delta = 1 ) MINUTES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All news</td>
</tr>
<tr>
<td>( n )</td>
<td>( 2.7 \cdot 10^7 )</td>
</tr>
<tr>
<td>Sum sentiment</td>
<td>279750</td>
</tr>
<tr>
<td>Mean</td>
<td>1.03</td>
</tr>
<tr>
<td>Minimum</td>
<td>-45</td>
</tr>
<tr>
<td>Maximum</td>
<td>237</td>
</tr>
<tr>
<td>St. dev.</td>
<td>4.03</td>
</tr>
<tr>
<td>Median</td>
<td>0</td>
</tr>
<tr>
<td>Skewness</td>
<td>11.54</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>282.18</td>
</tr>
</tbody>
</table>

Fig. 1 plots an example of time series of news sentiments with 1-min window. Time series are quite volatile and demonstrate a non-stationary behavior. The amount of positive news is much greater than the amount of negative news during the period for all stock exchanges.
B. Self-Similarity Analysis

In this subsection we demonstrate the auto-correlation and self-similarity analysis with the use of FA, DFA, R/S analysis, AWC and Fourier transform method.

First we use FA and DFA (of order 1) methods to evaluate the correlation and Hurst exponent. Then, we obtained values of $F(s_i)$ for different interval lengths $s_i \in [10^1, 10^{1.5}]$ using FA and DFA. The task of selecting a needed length for scaling range has been examined in the papers [59], [60], [61]. The influence of scaling range on the efficiency of some detrending techniques has been investigated in the same works. We used two ranges $[10^1, 10^{1.5}]$ (for 1-min data) and $[10^1, 10^2]$ (for 5-min data) in our analysis. This values belong to the scaling range suggested by these papers.

Figures 2, 3, 4 and 5 plot the results of FA and DFA application to all news, NYSE news, LSE news and SEHK time-series of news sentiments respectively (with $\delta = 1$). Results of DFA show that the estimates of the Hurst exponent are equal to 0.69 (all news), 0.64 (NYSE), 0.55 (LSE), 0.69 (SEHK) which indicates the presence of long-range correlation in the corresponding time series. Linear regressions are highly significant, since the determination coefficients $R^2$ are close to 1. It must be pointed out that Detrended Fluctuation Analysis let us discover the crossover effect shown in the study [46].

AWC method gives similar results with DFA method in identical estimates of the Hurst exponent on the whole scale of $s$ both for all time series and NYSE news, LSE news and SEHK time-series of news sentiments (Table II). The differences between estimates of the Hurst exponent obtained by means of different methods are less than 0.1. It should be noted that regression errors are very small and the value of $R^2$ is close to 1 for FA, DFA and AWC methods. We noted that AWC method also allowed to detect the so called crossover effect (Fig. 7). It should be mentioned that the crossover effect matches with 1 day period (or 24 hours). Estimates of the Hurst exponent, for small (about one day) as well as large scales (more than a day) using DFA and AWC methods yield similar results, the difference being less than 0.1, on the whole scale of all news, NYSE news, LSE news and SEHK time-series of news sentiments.

Fig. 6 demonstrates the log-log plot of dependance $R/S$ statistics of $s$ for all news time series with $\delta = 1$ obtained by R/S analysis. In this case, though the value of the Hurst exponent is less than $H = 0.63$, it still demonstrates that there are positive long-range correlations in time series. Linear regressions are also highly significant. In contrast to the DFA, R/S analysis as well as FA failed to reveal the presence of the crossover effect.

The Fourier transform analysis of the all news time series of news sentiments (with $\delta = 1$) are presented in Fig. 8. The figure shows the raw power spectrum, $P(w)$ vs. angular frequency $w$ for the news sentiments data. We can see that the raw power spectrum is too noisy, which does not allow us to accurately estimate the Hurst exponent and certainly prevents us from detecting the effect of the crossover. We can apply the log-binning technique [62] to reduce its noise. The result of the log-binning smoothing is shown in Fig. 6. The dashed lines of figures correspond to the slope of $(-2H - 1)$ with the value of $H = 0.61$ for all news time series with time window $\delta = 1$. It should be noted that while the regression is significant, it is considerably worse than regressions obtained by FA, DFA, AWC and R/S analysis. Moreover, it is not possible to identify the effect of the crossover even after the smoothing of power spectrum.

Table II presents the estimates of the Hurst exponent obtained by FA, DFA, R/S, AWC and Fourier transform methods for our time series.
C. Multifractality Analysis

In this subsection we examine the multifractality of time series of news sentiments using the MDFA algorithm. Figure 10 shows the dependence of log-values of fluctuation function on the length of non-crossing intervals for different values of $q := -5 : 0.5 : 5$. One can see that slopes are different at distinct points $q$, which is evidence of multifractality. For periods longer than one day, the alignment of points and slopes can be observed.

In addition, Fig. 11 shows the multifractal spectrum of news sentiments time series. It should be noted that $f(\alpha)$ is in a wide range ($\alpha = 0.82$), and curves are concentrated over a sufficiently large range, which is the indication of multifractality. NYSE and LSE time series exhibit multifractality as well, however their degree of multifractality is less than for the time series generated by all news. At the same time $f(\alpha)$ for SEHK time series is in a narrow range ($\alpha = 0.38$) and the curve is concentrated on a small range, which indicates mono-fractality.

Table II

<table>
<thead>
<tr>
<th>Method</th>
<th>All</th>
<th>NYSE</th>
<th>LSE</th>
<th>SEHK</th>
</tr>
</thead>
<tbody>
<tr>
<td>FA</td>
<td>0.711</td>
<td>0.627</td>
<td>0.543</td>
<td>0.710</td>
</tr>
<tr>
<td>DFA</td>
<td>0.690</td>
<td>0.638</td>
<td>0.554</td>
<td>0.693</td>
</tr>
<tr>
<td>R/S analysis</td>
<td>0.631</td>
<td>0.618</td>
<td>0.521</td>
<td>0.710</td>
</tr>
<tr>
<td>AWC</td>
<td>0.683</td>
<td>0.722</td>
<td>0.648</td>
<td>0.687</td>
</tr>
<tr>
<td>Fourier transform</td>
<td>0.609</td>
<td>0.643</td>
<td>0.588</td>
<td>0.611</td>
</tr>
</tbody>
</table>

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Fig. 11. Multifractal spectrum of $f(\alpha) - \alpha$

VI. CONCLUSION

In this paper we use the FA, DFA, R/S analysis, AWC method and Fourier transform technique to examine the presence of long-range correlation of financial and economic time series of news sentiments. The results of this paper show that the long-range power-law correlation take place in time series of news sentiments. The paper shows that the behavior of long range dependence for time series of sentiment intensity most similar to the news flow intensity [32]. The results show that the self-similarity property is a stable characteristic of the sentiment of news information flow which serves the financial industry and stock markets. All methods showed the presence of a long-range correlation in the time series of news sentiments (the Hurst exponent is greater than 0.5) both for time series of all news sentiments and for three time series of news sentiments for three major stock exchanges (NYSE, LSE, SEHK). Results obtained by the DFA and AWC for sufficiently large scale have revealed the effect of crossover (corresponding to one day). Shortcomings of the classic FA method, R/S analysis and Fourier technique do not allow us to determine the effect of crossover.

Moreover, we show that the time series of news sentiments exhibit multifractality. This statement can also be attributed to the time series of news sentiments related to individual stock exchanges (NYSE and LSE). On the other hand, we demonstrate that the SEHK time series have monofractal behavior. We can conclude that the fractal structures of time series of sentiments are more complex for large stock exchanges with a wide differentiation of operating agents. At the same time, the presence of multifractality in the time series of news sentiments can be explained by the fact that news is formed in the process of information interaction within a complex system with a large number of subsystems and participants. The elements of the system are more than 34000 largest companies located around the world, financial market participants, investors, traders, analytical and news agencies, media, etc. In our opinion, these are the main reasons why the time series of news sentiments are multifractal and have long-range memory. This paper also revealed a long-range power dependence in high-frequency data for relatively limited time periods (up to 1 week). In our opinion, at such relatively short intervals of time, the correlation between news events is caused more by the delay in the reaction of different agents to events (the time sequence of interactions between agents) than the temporal correlation between news events.

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