Modeling and Analysis of the Impact of Adaptive Defense Strategy on Virus Spreading

Qingyi Zhu, Longqi Li, Chenquan Gan

Abstract—**Nowadays, the dynamical modeling has been an important approach to macroscopically study the propagation behaviors of computer virus on the Internet, and a large number of models have been established in this field. To our knowledge, however, the existing models did not take into account the difference in the ability of computer security defense. As it is known, in reality, due to the different nature and usefulness, not all computers in a system have the same security defense ability. In this paper, a new computer virus propagation model, which addresses the impact of different security defense abilities on computer virus spreading, is proposed and analyzed. In this context, a threshold is given to determine when the computer security defense level is need to be upgraded. Then, three potential equilibria are obtained, and their local and global asymptotic stability are fully studied. Furthermore, the optimal control problem of proposed model is formulated. On this basis, some numerical experiments are made to justify our results.**

Index Terms—**Computer virus, Adaptive defence strategy, Dynamical model, Global stability, Optimal control.**

I. Introduction

WITH the rapid development of information and communication technology, the Internet has become a necessity in daily life. The openness, interactivity and dispersion of the Internet meet the needs of sharing, opening, flexibility and fast, so that work, life and learning have been greatly improved than before. The Internet is widely used for sharing, communication and service to create the ideal space, provide a huge impetus to the progress of human society. However, it is precisely because of the above characteristics of the Internet, it is inevitable to produce a lot of network security issues which seriously affect our lives, and even personal safety[1]. At present, the electronic commerce ages has arrived, network security has become our most concern while enjoying the convenience of the Internet.

In recent years, researchers have achieved great progress in the study of mathematical modelling in virology[2]. Since Kephart and White modeled the computer virus propagation by using epidemiological dynamics firstly[3], [4], the study of the macro spreading behaviors of computer viruses has received more and more global attentions. In recent years, multifarious computer virus propagation models have been proposed by modifying their corresponding biological counterparts, ranging from conventional models [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], to delayed models [14], [15], [16], [17], [18], [19], [20], to impulsive models [21], to stochastic models [22], [23], [24], and optimal control models[25], [26], [27], [28].

As early as 1985, Trusted Computer System Evaluation Criteria (TcsEC)[29] was issued by the United States Department of Defense in order to protect the security of computer. Following this criteria, the security level of computer can be divided into four divisions, gradually improved from low to high, they are D , C, B and A respectively.

D - Minimal protection : It is the lowest level of security, providing minimal security for the system. There is no limit to the access control of the system, and the data can be accessed without landing system. such as DOS.

C - Discretionary protection : It has the function of self-protection and audit in the design, which can be used to audit and restrain the behavior of the subject. Its security strategy is mainly autonomous access control, ensuring that unauthorized users cannot access and security management of personal data can be achieved. C level users must provide proof of identity to be able to achieve normal access control, such as password mechanism.

B - Mandatory protection : It can provide mandatory safety protection. The owner of the information resource does not have the authority to change itself. And the system data is under the supervision of the access control management.

A - Verified protection : The formal security verification method is used to ensure that the system can effectively protect the secret information or other sensitive information stored and processed in the system.

In reality, due to the different nature and usefulness, computers have different levels of security defense in a network autonomous system. However, in the pre-proposed compartment-based computer virus propagation models, they did not take into account the difference in the ability of computer security defense. Inspired by the above criteria, in order to macroscopically study the impact of security defense ability on virus spreading, the security level of computers is divided two divisions: low and high. In this way, a new compartment-based propagation model, in which the traditional S-compartment is divided into S_L-compartment with low security level and S_H-compartment with high security level, is proposed.

As it is known, the computer connected to the Internet is constantly under the threats of viruses. To defend against various computer viruses, a network system needs to fast respond to the complex and dynamical cyber conditions. Therefore, it is of great importance to find a dynamical defense approach that can automatically adjust its parameters to fight against the virus spreading. In the new model, an

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adaptive defense strategy based on security level of computer is introduced, which can adaptively adjust its configurations according to network conditions to suppress the spread of computer virus in the network. In this model, a threshold is given, which determines when the computer security defense level should be automatically upgraded. Besides, in this paper, the existence of equilibria is studied. Both their local and global asymptotic stability are analyzed. The optimal control problem based on this model is also presented. And some numerical experiments are also made to justify the result that computer virus has been effectively controlled after carrying out the adaptive defense strategy. And some measures for containing the propagation of computer virus are recommended.

The rest parts of this paper are organized as following: The new model is established in Section 2. The analysis of equilibria are addressed in Section 3. The local and global stabilities of the equilibria are studied in Section 4 and 5, respectively. Section 6 derives the optimal control solution of proposed model. Numerical experiments are presented in Section 7. Finally, this work is summarized in Section 8.

II. Model formulation

It is assumed that all the computers connected to the network are divided into three compartments : S_L-compartment, S_H-compartment and Infected compartment.

(1) S_L-compartment: the set of susceptible computers in low security level.
(2) S_H-compartment: the set of susceptible computers in high security level.
(3) I-compartment: the set of infected computers.

For the modeling purpose, a series of parameters are introduced and some assumptions are made as follows :

(1) It is assumed that the probability per unit time of external computers through the network connect to the system is b.
(2) Every computer in the system is removed for some reasons with probability per unit time μ, where μ is a positive constant.
(3) It is possible that susceptible computers contact with infected computers in the system, every computer in the S_L-compartment is infected with probability per unit time β_1 and in the S_H-compartment is infected with probability per unit time β_2, where β_1 and β_2 are positive constants and $\beta_1 > \beta_2$.
(4) For some reasons, every infected computer in the system is cured with probability per unit time γ, where γ is a positive constant.
(5) Susceptible computers from S_H-compartment into S_L-compartment with probability per unit time δ, where δ is a positive constant.
(6) I_{max} denotes the threshold which determine when the upgrade of security level is required.
(7) To control the number of infected computers in the system, some measures are taken to upgrade the security level of susceptible computers, the probability per unit time is denoted by a piecewise function $f(I)$. The expression of $f(I)$ as follows :

$$f(I) = \begin{cases} 0 & \text{if } 0 \leqslant I < I_{max} \\ \alpha I, & \text{if } I_{max} \leqslant I \end{cases}$$

Let $S_L(t)$, $S_H(t)$, and $I(t)$ denote, at time t, the average numbers of S_L, S_H and I-compartment computers, respectively. Let $N(t)$ denote the total number of all computers in the system at time t. Unless other stated in the following content, they will be abbreviated as N, S_L, S_H and I respectively. Then, $S_L + S_H + I = N$. The collection of the above parameters and assumptions can be schematically depicted in Fig.1, from which the dynamical model is formulated as the following differential system:

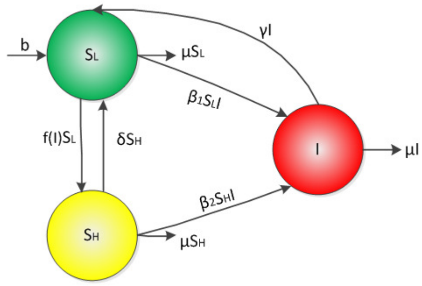

Fig. 1. The transfer diagram of model.

$$\begin{cases} \dot{S_L} = b + \gamma I + \delta S_H - f(I)S_L - \beta_1 S_L I - \mu S_L \\ \dot{S_H} = f(I)S_L - \delta S_H - \beta_2 S_H I - \mu S_H \\ \dot{I} = \beta_1 S_L I + \beta_2 S_H I - \gamma I - \mu I \end{cases} \tag{1}$$

Considering that $S_L + S_H + I = N$. System (1) can be reduced to the following system :

$$\begin{cases} \dot{N} = b - \mu N \\ \dot{S_H} = f(I)(N - S_H - I) - \delta S_H - \beta_2 S_H I - \mu S_H \\ \dot{I} = \beta_1 (N - S_H - I)I + \beta_2 S_H I - \gamma I - \mu I \end{cases} \tag{2}$$

Let $N^* = \frac{b}{\mu}$. Solving the first equations of system (2), it is easy to obtain $\lim\limits_{t\to\infty} N = N^*$. Therefore, system (2) can be reduced to the following limiting system:

$$\begin{cases} \dot{S_2} = f(I)(N^* - S_2 - I) - \delta S_2 - \beta_2 S_2 I - \mu S_2 \\ \dot{I} = \beta_1 (N^* - S_2 - I)I + \beta_2 S_2 I - \gamma I - \mu I \end{cases} \tag{3}$$

The feasible region for system (3) is

$$\Omega = \{(S_H, I) | S_H \geqslant 0, I \geqslant 0, 0 \leqslant S_H + I \leqslant N^*\}$$

which is positively invariant.

III. Equilibria

In this section, the equilibria of system (3) are deduced. To obtain its equilibria, system (3) can be rewritten as follows:

$$\begin{cases} -\delta S_H - \beta_2 S_H I - \mu S_H = 0 \\ \beta_1 (N^* - S_H - I)I + \beta_2 S_H I - \gamma I - \mu I = 0 \end{cases} \tag{4}$$

if $0 \leqslant I < I_{max}$, and

$$\begin{cases} \alpha I(N^* - S_H - I) - \delta S_H - \beta_2 S_H I - \mu S_H = 0 \\ \beta_1 (N^* - S_H - I)I + \beta_2 S_H I - \gamma I - \mu I = 0 \end{cases} \tag{5}$$

if $I_{max} \leqslant I$.

System (4) has a virus-free equilibrium $E_0^* = (0, 0)$. Let $R_0 = \frac{\beta_1 b}{\mu(\mu+\gamma)}$. If $R_0 > 1$, then (4) admits a unique positive solution $E_1^*(S_{H1}^*, I_1^*)$, where

$$S_{H1}^* = 0, \qquad I_1^* = (R_0 - 1)\frac{\mu+\gamma}{\beta_1}.$$

Obviously, E_1^* is a viral equilibrium of (4) if $R_0 > 1$.

From (5) a quadratic equation in I, one can get :

$$AI^2 + BI + C = 0 \tag{6}$$

where

$$\begin{aligned} A &= (\alpha+\beta_1)\beta_2 \\ B &= (\mu+\gamma)(\alpha+\beta_2) + (\mu+\delta)\beta_1 - \beta_2 N^*(\alpha+\beta_1) \\ C &= (\mu+\delta)(\mu+\gamma-\beta_1 N^*) \end{aligned}$$

From (5), we have the following system:

$$\begin{cases} \beta_1(N^* - S_H - I) + \beta_2 S_H = \mu+\gamma \\ (N^* - S_H - I) + S_H + I = N^* \end{cases} \tag{7}$$

That is $(\beta_2 - \beta_1)S_H - \beta_1 I = \mu + \gamma - \beta_1 N^*$. Therefore, $\mu+\gamma-\beta_1 N^* < 0$, that is to say, $C < 0$. $\Delta = B^2 - 4AC > 0$, then (6) admits a unique positive solution $I_2^* = \frac{-B+\sqrt{\Delta}}{2A}$. $E_2^*(S_{H2}^*, I_2^*)$ can be deduced, where

$$S_{H2}^* = \frac{(\mu+\gamma)(R_0-1) - \beta_1 I_2^*}{\beta_1 - \beta_2}, \qquad I_2^* = \frac{-B+\sqrt{\Delta}}{2A}$$

Obviously, E_2^* is a viral equilibrium of (5) if $R_0 > 1 + \frac{\beta_1 I_2^*}{\mu+\gamma}$.

From (4) and (5), we have:

$$\begin{cases} \beta_1(N^* - S_{H1}^* - I_1^*) + \beta_2 S_{H1}^* - \gamma - \mu = 0 \\ \beta_1(N^* - S_{H2}^* - I_2^*) + \beta_2 S_{H2}^* - \gamma - \mu = 0 \end{cases} \tag{8}$$

Thus, $\beta_1(I_2^* - I_1^*) + (\beta_1 - \beta_2)S_{H2}^* = 0$, $I_2^* < I_1^*$ can be deduced.

By summarizing above analysis, we have

Theorem 1: Consider model (3), the following assertions hold.

(1.1) E_0^* always exists;
(1.2) E_1^* exists if and only if $R_0 > 1$ and $I_1^* < I_{max}$;
(1.3) E_2^* exists if and only if $R_0 > 1 + \frac{\beta_1 I_2^*}{\mu+\gamma}$ and $I_{max} \leqslant I_2^*$.

IV. Local stability

To examine the local stability of the equilibria, system (3) can be written as the following :

$$\begin{cases} \dot{S}_H = -\delta S_H - \beta_2 S_H I - \mu S_H \\ \dot{I} = \beta_1(N^* - S_H - I)I + \beta_2 S_H I - \gamma I - \mu I \end{cases} \tag{9}$$

if $0 \leqslant I < I_{max}$, and

$$\begin{cases} \dot{S}_H = \alpha I(N^* - S_H - I) - \delta S_H - \beta_2 S_H I - \mu S_H \\ \dot{I} = \beta_1(N^* - S_H - I)I + \beta_2 S_H I - \gamma I - \mu I \end{cases} \tag{10}$$

if $I_{max} \leqslant I$. For their Jacobian matrices at the equilibria are :

$$J_1 = \begin{pmatrix} -\delta - \beta_2 I - \mu & -\beta_2 S_H \\ -\beta_1 I + \beta_2 I & J_1^{22} \end{pmatrix}$$

and

$$J_2 = \begin{pmatrix} -\alpha I - \delta - \beta_2 I - \mu & J_2^{12} \\ -\beta_1 I + \beta_2 I & J_2^{22} \end{pmatrix}$$

where

$$\begin{aligned} J_1^{22} &= \beta_1 N^* - \beta_1 S_H - 2\beta_1 I + \beta_2 S_H - \gamma - \mu \\ J_2^{12} &= \alpha N^* - \alpha S_H - 2\alpha I - \beta_2 S_H \\ J_2^{22} &= \beta_1 N^* - \beta_1 S_H - 2\beta_1 I + \beta_2 S_H - \gamma - \mu. \end{aligned}$$

(1) The corresponding eigenvalues of system (3) at E_0^* are :

$$\begin{aligned} \lambda_1 &= -\delta - \mu \\ \lambda_2 &= (R_0 - 1)(\mu+\gamma) \end{aligned}$$

By the Lyapunov theorem[30], if only if $R_0 < 1$, the two eigenvalues are both negative, which means E_0^* is locally asymptotically stable.

(2) The corresponding characteristic equation of system (3) at E_1^* is

$$\lambda^2 + a_1\lambda + a_2 = 0,$$

where

$$\begin{aligned} a_1 &= \delta + \mu + (\beta_1 + \beta_2)I_1^* \\ a_2 &= (\delta + \mu + \beta_2 I_1^*)\beta_1 I_1^* \end{aligned}$$

By the Hurwitz criteria[30], if only if $R_0 > 1$ and $I_1^* < I_{max}$, E_1^* is locally asymptotically stable.

(3) The corresponding characteristic equation of system (3) at E_2^* is

$$\lambda^2 + a_3\lambda + a_4 = 0,$$

where

$$\begin{aligned} a_3 &= \delta + \mu + (\alpha + \beta_1 + \beta_2)I_2^* \\ a_4 &= (\delta + \mu + \beta_2 I_2^*)\beta_1 I_2^* + \alpha\beta_2 (I_2^*)^2 + (\beta_1 - \beta_2)(\delta+\mu)S_{H2}^* \end{aligned}$$

By the Hurwitz criteria, if only if $R_0 > 1 + \frac{\beta_1 I_2^*}{\mu+\gamma}$ and $I_{max} \leqslant I_2^*$, E_2^* is locally asymptotically stable.

In summary, we have

Theorem 2: Consider model (3), the following assertions hold.

(2.1) E_0^* is locally asymptotically stable if and only if $R_0 < 1$;
(2.2) E_1^* is locally asymptotically stable if and only if $R_0 > 1$ and $I_1^* < I_{max}$;
(2.3) E_2^* is locally asymptotically stable if and only if $R_0 > 1 + \frac{\beta_1 I_2^*}{\mu+\gamma}$ and $I_{max} \leqslant I_2^*$.

V. Global stability

Theorem 3: Consider model (3), the following assertions hold.

(3.1) E_0^* is globally asymptotically stable if and only if $R_0 < 1$,;
(3.2) E_1^* is globally asymptotically stable if and only if $R_0 > 1$ and $I_1^* < I_{max}$;
(3.3) E_2^* is globally asymptotically stable if and only if $R_0 > 1 + \frac{\beta_1 I_2^*}{\mu+\gamma}$ and $I_{max} \leqslant I_2^*$..

Proof: (3.1) Let

$$\begin{aligned} G(S_H, I) &= -\delta S_H - \beta_2 S_H I - \mu S_H \\ H(S_H, I) &= \beta_1(\tfrac{b}{\mu} - S_H - I)I + \beta_2 S_H I - \gamma I - \mu I \\ B(S_H, I) &= \tfrac{1}{I} \end{aligned} \tag{11}$$

Then, $\frac{\partial(BG)}{\partial S_H} + \frac{\partial(BH)}{\partial I} = -\frac{\delta+\mu}{I} - \beta_1 - \beta_2 < 0$. By Dulac's criteria[30], (3) admits no limit cycle. Consider an arbitrary point, ($\overline{S_H}$, $\overline{I}$), on the boundary of Ω. From (3), $\partial\Omega$ consists of the following three possibilities:

(a) $0 \leqslant \overline{S_H} \leqslant N^*$, $\overline{I} = 0$. Then, $\dot{I}|_{(\overline{S_H},\ \overline{I})} = 0$.
(b) $0 < \overline{I} < N^*$, $\overline{S_H} = 0$. Then, $\dot{S}_H|_{(\overline{S_2},\ \overline{I})} = 0$.
(c) $\overline{S_H} + \overline{I} = N^*$. Then, $\frac{d(S_H+I)}{dt}|_{(\overline{S_H},\ \overline{I})} = -\delta S_H - \mu S_H - (\mu+\gamma)I < 0$.

By the Poincaré-Bendixson theorem[30], if $R_0 < 1$, E_0^* is globally asymptotically stable. A similar analysis can be conducted to obtain the result (3.2).

(3.3) Let

$$
\begin{aligned}
&P(S_H, I) = \alpha I(\tfrac{b}{\mu} - S_H - I) - \delta S_H - \beta_2 S_H I - \mu S_H \\
&Q(S_H, I) = \beta_1(\tfrac{b}{\mu} - S_H - I)I + \beta_2 S_H I - \gamma I - \mu I \\
&D(S_H, I) = \tfrac{1}{I}
\end{aligned} \tag{12}
$$

Then, $\frac{\partial(DP)}{\partial S_H} + \frac{\partial(DQ)}{\partial I} = -\alpha - \frac{\delta+\mu}{I} - \beta_1 - \beta_2 < 0$. By Dulac's criteria, (3) admits no limit cycle. Consider an arbitrary point, ($\overline{S_H}$, $\overline{I}$), on the boundary of Ω. From (3), $\partial\Omega$ consists of the following three possibilities :

(a) $0 \leqslant \overline{S_H} \leqslant N^*$, $\overline{I} = 0$. Then, $\dot{I}|_{(\overline{S_H},\ \overline{I})} = 0$.
(b) $0 < \overline{I} < N^*$, $\overline{S_H} = 0$. Then, $\dot{S}_H|_{(\overline{S_H},\ \overline{I})} = (N^* - I)\alpha I > 0$.
(c) $\overline{S_H} + \overline{I} = N^*$. Then, $\frac{d(S_H+I)}{dt}|_{(\overline{S_H},\ \overline{I})} = -\delta S_H - \mu S_H - (\mu+\gamma)I < 0$.

By the Poincaré-Bendixson theorem, we can obtain the result (3.3). ■

VI. The optimal control model

This section deals with the optimal control problem of system (1). Firstly, by incorporating a Lebesgue square integrable control function $u(t)(0 \leqslant u(t) \leqslant u_{\max})$, the controlled state system can be obtained as:

$$
\begin{cases}
\dot{S} = \lambda - \beta SI + \theta\sigma_1 IP + \sigma_2 I + \gamma P - \mu S, \\
\dot{I} = \beta SI - \sigma_1 IP - (\sigma_2+\mu)I - u(t)I, \\
\dot{P} = (1-\theta)\sigma_1 IP - (\gamma+\mu)P + u(t)I.
\end{cases} \tag{13}
$$

Then the minimized objective functional is defined as follow:

$$
J = \int_0^T I + \frac{1}{2}wu^2 \mathrm{d}t, \tag{14}
$$

where w is the weight index of control costs.

For applying Pontryagin's minimum principle, one can obtain the following corresponding Hamiltonian:

$$
\begin{aligned}
H = I + \frac{1}{2}wu^2 &+ \eta_1(\lambda - \beta SI + \theta\sigma_1 IP + \sigma_2 I + \gamma P - \mu S) \\
&+ \eta_2(\beta SI - \sigma_1 IP - (\sigma_2+\mu)I - u(t)I) \\
&+ \eta_3((1-\theta)\sigma_1 IP - (\gamma+\mu)P + u(t)I).
\end{aligned} \tag{15}
$$

where $\eta_i (i = 1, 2, 3)$ are the adjoint variables. By directly calculation, we can obtain following results.

$$
\begin{aligned}
\dot{\eta_1} =& -\frac{\partial H}{\partial S} \\
=& \beta I(\eta_1 - \eta_2) + \mu\eta_1 \\
\dot{\eta_2} =& -\frac{\partial H}{\partial I}, \\
=& -1 + (\beta S - \theta\sigma_1 P - \sigma_2)\eta_1 - ((1-\theta)\sigma_1 P + u(t))\eta_3 \\
&+ (-\beta S + \sigma_1 P + \sigma_2 + \mu + u(t))\eta_2, \\
\dot{\eta_2} =& -\frac{\partial H}{\partial P} \\
=& -(\theta\sigma_1 I + \gamma)\eta_1 + \sigma_1 I\eta_2 - ((1-\theta)\sigma_1 I - \gamma - \mu)\eta_3.
\end{aligned}
$$

By using the optimality condition, we obtain

$$
\frac{\partial H}{\partial u(t)} = wu(t) + (\eta_3 - \eta_2)I = 0. \tag{16}
$$

Hence, the optimality solution respect to system (5) is

$$
u(t) = \min\{\max\{\frac{(\eta_2 - \eta_3)I}{w}, 0\}, u_{\max}\}. \tag{17}
$$

VII. Numerical examples and discussions

In this section, by choosing the appropriate value for each parameter in the system, some numerical examples are given to justify the global stability of each equilibrium and the efficiency of the optimal control.

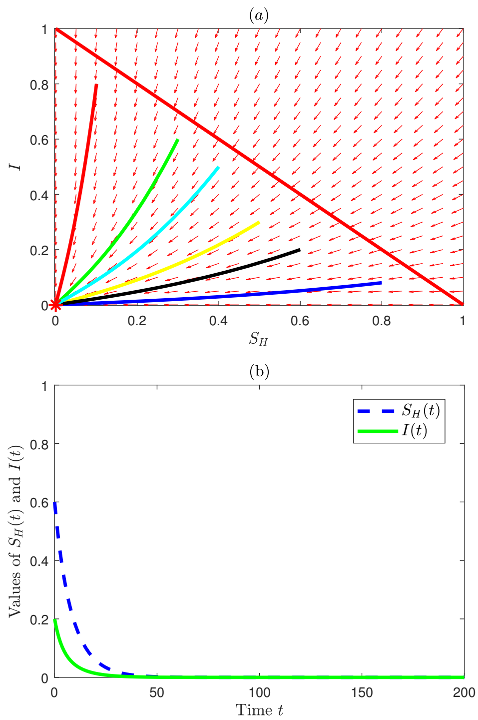

Fig. 2. The virus-free equilibrium E_0^* is globally asymptotically stable for the case b = 0.1, δ = 0.01, β_1 = 0.2, β_2 = 0.1, γ = 0.2, μ = 0.1. In this situation, $R_0 < 1$.

R_0 is the basic reproduction number which determines whether the virus will die out or not. In order to study the effect of parameters on R_0, it can be got:

$$
\frac{\partial R_0}{\partial \beta_1} = \frac{b}{\mu(\mu+\gamma)} > 0,
$$

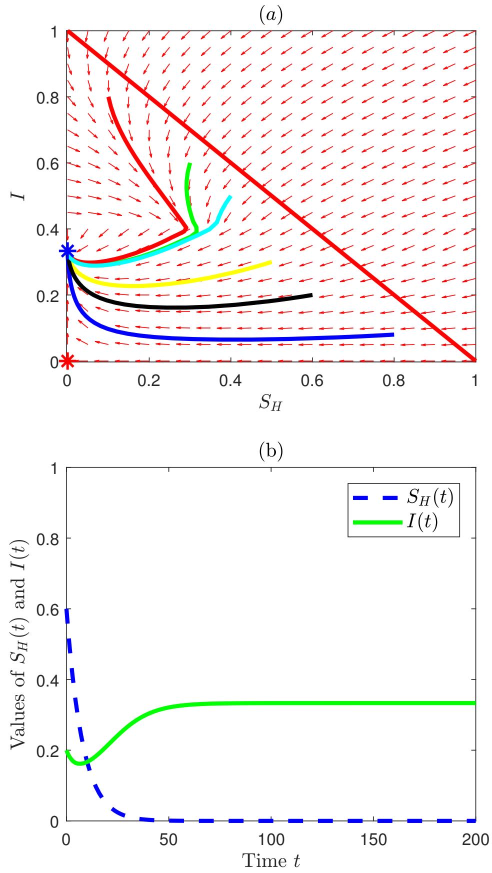

Fig. 3. The viral equilibrium E_1^* is globally asymptotically stable for the case b = 0.1, δ = 0.01, β_1 = 0.3, β_2 = 0.1, γ = 0.1, μ = 0.1, and I_{max} = 0.4. In this situation, $R_0 > 1$, $I_1^* < I_{max}$.

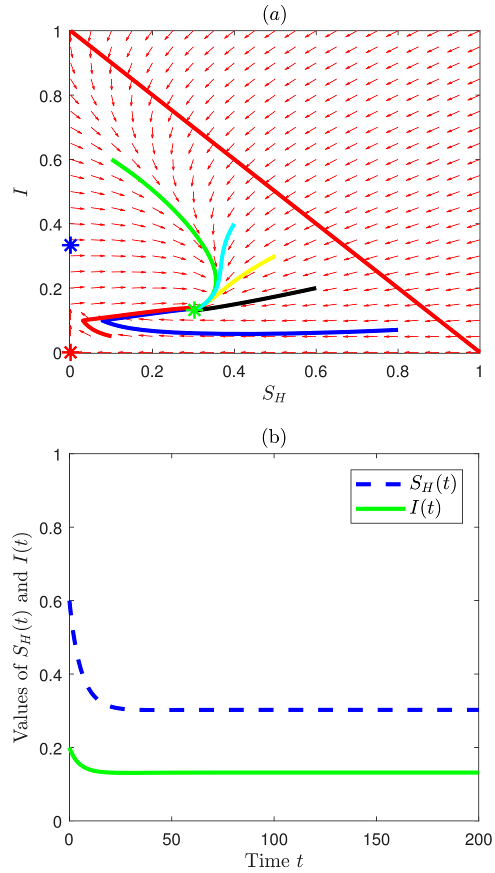

Fig. 4. The viral equilibrium E_2^* is globally asymptotically stable for the case b = 0.1, α = 0.5, δ = 0.01, β_1 = 0.3, β_2 = 0.1, γ = 0.1, μ = 0.1, and I_{max} = 0.1. In this situation, $R_0 > 1$, $I_{max} < I_2^*$.

$$\frac{\partial R_0}{\partial b} = \frac{\beta_1}{\mu(\mu+\gamma)} > 0,$$
$$\frac{\partial R_0}{\partial \mu} = -\frac{\beta_1 b(2\mu+\gamma)}{\mu^2(\mu+\gamma)^2} < 0,$$
$$\frac{\partial R_0}{\partial \gamma} = -\frac{\beta_1 b}{\mu(\mu+\gamma)^2} < 0,$$

Obviously, R_0 is increasing with β_1, b, and is decreasing with μ, γ.

As we can see from the above examples, Fig.2 shows us that the computer virus in the system will die out ultimately, all computers will be in the S_L-compartment. Fig.3 shows us that while the number of computers in the I-compartment is less than the threshold, any initial point in the area above I_{max} will converge to E_1^*. Fig.4 displays that while the number of computers in the I-compartment exceeds the threshold, it will be reduced due to the upgrade of security level. And from Fig.5, one can clearly see that the number of infected computers is significantly reduced after applying the optimal control strategy.

Based on the numerical examples, the following policies are recommended.

1. In the absence of necessity, computers do not connect to the Internet, so that reduce the risk of infection.

2. According to their own situation, configure the security level of firewall while accessing the Internet.

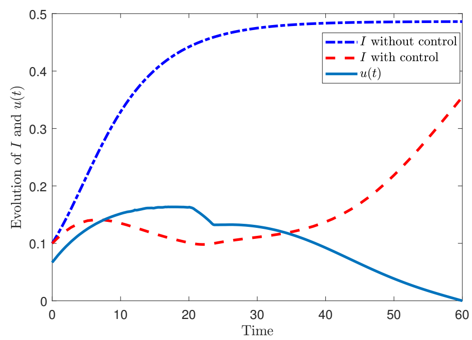

Fig. 5. The optimal control for the case b = 0.1, α = 0.5, δ = 0.01, β_1 = 0.4, β_2 = 0.2, γ = 0.05, μ = 0.1, I_{max} = 0.1, $u_{max} = 0.2$ and $w = 10$.

3. If you installed antivirus software on your computer, it can be set to autorun regularly.

VIII. Conclusions

This paper has studied the impact of adaptive defense rate on virus spreading. Considering that the difference of computer security defense ability, the S-compartment of previous models is divided into two compartments: S_L-compartment with low security level and S_H-compartment with high security level. And a threshold named I_{max} is given which determines when the security defense level of susceptible computers is upgraded. A thorough analysis reveals the global stability of the virus-free equilibrium and the two viral equilibria. This demonstrates that computer virus will eventually die out when the basic reproduction number is less than one, whereas it will persist if the basic reproduction number exceeds one. What is more, while the number of computers in the I-compartment exceeds a threshold value which is given by I_{max}, the susceptible computers in the S_L-compartment become S_H due to the upgrade of the security defense level, and the number of computers in the I-compartment thereby is reduced. Finally, the propagation of computer virus in the system is controlled.

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