Pricing and Green Level Decisions with a Risk Averse Retailer in an Uncertain Green Supply Chain

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Abstract—**This paper considers the greening level and retail price decisions with a risk averse retailer in a green supply chain under uncertain demand environment. The production cost, market demand and investment coefficient are all considered as uncertain variables. Two different kinds of uncertain models including one expected value model and one chance constrained model with risk sensitive retailer are developed, and their optimal solutions are also obtained based on uncertainty theory. Finally, a numerical example is given to compare the optimal solutions in two models. It shows that the wholesale price, green level and retail price are becoming higher with the increasing of the confidence level, and the manufacturer makes more profit when the retailer is more risk sensitive.**

Index Terms—**supply chain, green level, risk sensitive, uncertain environment**

I. Introduction

With the introduction of a low-carbon economy and green GDP, environmental consciousness has received increasingly attention in present times. Supply chain members focus on their operations impacted on environment, and many models have been designed to cope with these problems. Recently, more and more scholars and market administrators have applied the green principles and techniques to develop and solve the green supply chain management (GSCM) problems.

In recent years, a number of research issues have been addressed such as the impact of government behavior on green supply chain members, the pricing and coordination strategy in green supply chain. By using the asymmetrical Nash barging game and backward induction approaches, Sheu [1] studied the problem of negotiations between producers and reverse logistics suppliers for cooperative agreements under government intervention. Sheu and Chen [2] analyzed the impact of governmental intervention via green legislation and financial instruments on competing green supply chains for green-product production by using a three stage game theoretic model. By using evolutionary dynamics, Barari et al. [3] developed a synergetic alliance between the environmental and commercial benefits by establishing coordination between the producer and the retailer to adjudicate their strategies to trigger green practices with the focus on maximizing economic profits of the entities of the supply chain. Sheu [4] presented a multi-objective optimization programming approach to address the issue of nuclear power generation in green supply chain management. Swami and Shah [5] examined the optimal prices charged, optimal reduction in size and optimal shelf-space allocation in green supply chain management. Similar issues were studied by Swami and Shah [6], they proposed a two part tariff contract to coordinate the green channel in which there were both price and non-price variables. Ghosh and Shah [7] proposed different decision making structures including cooperative and individual in a green supply chain, and analyzed the impact of greening costs and consumer sensitivity to greening on the supply chain players. Mirzapour Al-e-hashem [8] developed a two stage stochastic programming model with the assumption of demand fluctuation to deal with aggregate production distribution planning in a green supply chain. Tomasin et al. [9] investigated the elements that could generate an increase in industrial green product sales in Brazil. They found that in order to increase sales, profits and losses of distributors must be considered, and the best distributors of green products had their own sales team. In two different of green supply chain structures: a vertically integrated structure and a decentralized setting, Xie et al. [10] considered the selection of cleaner products in a green supply chain for risk reduction. Xie et al. [11] also investigated the selection of cleaner products with the consideration of the tradeoff between risk and the return of players. Zhang and Liu [12] considered four models for the three-level green supply chain composed of a supplier, a manufacturer and a retailer. They found that the revenue sharing mechanism, the Shapley value method coordination mechanism and the asymmetric Nash coordination mechanism could encourage positive response of the participating members to the cooperation strategy. Zhang et al. [13] also considered the cooperative game and non-cooperative game of green supply chain in hybrid production mode. They showed that cooperative game based on Rubinstein bargaining coordination model could ensure a 33.3% increase in profits of cooperating members' investment from that in non-cooperative game. Yang et al. [14] investigated the influence of low-carbon policies on channel coordination in a two stage supply chain and considered four different models: the basic model, the carbon emission model, the

Manuscript received November 22, 2017; revised May 04, 2018. This work was supported by the Humanity and Social Science Youth Foundation of the Ministry of Education of China (NO.17YJC630116)

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carbon emission trading model and the carbon tax model. Recently, Li et al. [15] examined the optimal pricing policies in a competitive dual-channel supply chain. Sang [16] proposed three different fuzzy decentralized decision models with Manufacturer-Stackelberg game, Retailer-Stackelberg game and Vertical-Nash game in a green supply chain. Zhu and He [17] investigated the green product design problems in green supply chains under competition. Liu and Yi [18] studied the pricing policies of green supply chain considering targeted advertising and product green degree in the Big Data environment. Song and Gao [19] studied the revenue sharing contract in a green supply chain, and showed that this contract could effectively improve the greening level of the products and the total profit of the supply chain.

All studies mentioned above discussed the pricing and green level decisions in a crisp environment, such as a linear market demand and known production cost. However, in real world, especially for some new green products, the relevant precise dates are difficult to obtain due to lack of historical data. In this situation, the market base and production cost can usually be predicted by some experts. We adopt uncertainty theory instead of probability theory to solve issues with such variables. For now, uncertain theory has been successfully applied in supply chain management. Ding [20-21] studied the newsboy problems based on uncertain theory. Sang [22-23] studied the pricing and service decisions in an uncertain supply chain, in which the manufacturer and the retailer provided the services, respectively. Huang and Ke [24] investigated the pricing decision problem with two manufacturers and one common retailer in an uncertain supply chain. Chen et al. [25] studied the pricing and effort decisions for a supply chain with uncertain information. In addition, Wang [26] investigated the pricing competition problem between two retailers and one manufacturer in an uncertain environment, in which the manufacturer was supposed to be a leader and dominated the supply chain. Hong [27] studied the pricing and selling effort decisions in an uncertain supply chain, in which the manufacturer provided the selling effort and dominated the supply chain.

To the best of our knowledge, there is no study that deals with the pricing and green level decisions of supply chain based on uncertain theory. Therefore, in this paper, we discuss the pricing and green level decisions with a manufacturer and a retailer, in which the market base, price elasticity, green level elasticity, manufacturing cost and investment elasticity are all uncertain variables. We mainly discuss the conditions where the manufacturer is the Stackelberg leader, and the retailer is the follower and assumed to be risk averse.

The rest of the paper is organized as follows. Section II presents the uncertain theory related to this paper. Section III describes the problem in our models. Two uncertain models between the manufacturer and the retailer are developed in Section IV. A numerical example to illustrate the results of the proposed models is provided in Section V. Finally, Section VI draws the conclusion and indicates the way to future research.

II. Preliminaries

Definition 1. [27] Let L be a σ-algebra on a nonempty set Γ and M be a set function from L to $[0,1]$. Then M is called an uncertain measure if it satisfies the following four axioms

Axiom 1. (Normality axiom) $M\{\Gamma\}=1$.

Axiom 2. (Duality axiom) $M\{\Lambda\}+M\{\Lambda^c\}=1$, for any event Λ.

Axiom 3. (Subadditivity axiom) For every countable sequence of events $\{\Lambda_i\}$, $i=1,2,\ldots$, we have

$$M\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\}\le\sum_{i=1}^{\infty}M\{\Lambda_i\}$$

Axiom 4. (Product axiom) Let $\{\Gamma_k, L_k, M_k\}$ be an uncertainty space, $k=1,2,\ldots$, the product uncertain measure M is an uncertain measure satisfying

$$M\left\{\prod_{i=1}^{\infty}A_k\right\}=\bigcap_{k=1}^{\infty}M_k\{A_k\}$$

where A_k are arbitrarily chosen events from L_k, $k=1,2,\ldots$, respectively.

Definition 2. [27] An uncertain variable is a measurable function ξ from an uncertainty space $\{\Gamma_k, L_k, M_k\}$ to the set of a real number, i.e., for any Borel set B of real numbers, the set

$$\{\xi\in B\}=\{\gamma\in\Gamma \mid \xi(\gamma)\in B\}$$

is an event.

Definition 3. [27 The uncertain variables $\xi_1,\xi_2,\ldots,\xi_n$ are called independent if

$$M\left\{\prod_{i=1}^{n}\{\xi_i\in B_i\}\right\}=\bigcap_{i=1}^{n}M\{\xi_i\in B_i\}$$

for any Borel sets $B_1,B_2,\ldots,B_n$.

Definition 4. [27] Let ξ be an uncertain variable, and its uncertainty distribution Φ is defined by

$$\Phi(x)=M\{\xi\le x\}$$

for any real number x.

Definition 5. [27] An uncertain variable $\xi=L(a,b)$ is called a linear uncertain variable if it has the following uncertainty distribution

$$\Phi(x)=\begin{cases}0, & x\le a\\ (x-a)/(b-a), & a\le x\le b\\ 1, & x\ge b\end{cases}$$

where a and b are real numbers with $a<b$.

Definition 6. [27] An uncertain variable $\xi=Z(a,b,c)$ is called a zigzag uncertain variable if it has the following uncertainty distribution

$$\Phi(x)=\begin{cases}0, & x<a\\ (x-a)/2(b-a), & a\le x\le b\\ (x+c-2b)/2(c-b), & b<x\le c\\ 1, & x\ge c\end{cases}$$

where a, b and c are real numbers with $a < b < c$.

Lemma 1. [28] Let ξ be an uncertain variable with uncertainty distribution Φ. If the expected value of ξ exists, then

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha)\mathrm{d}\alpha$$

where Φ^{-1} is the inverse function of Φ.

Example 1. Let $\xi = L(a,b)$ be a linear uncertain variable. Then, its inverse uncertainty distribution is

$$\Phi^{-1}(\alpha) = a + (b-a)\alpha,\ \alpha \in [0,1]$$

The expected value can be obtained

$$E[\xi] = \int_0^1 \left(a + (b-a)\alpha\right)\mathrm{d}\alpha = \frac{a+b}{2}$$

Example 2. Let $\xi = Z(a,b,c)$ be a zigzag uncertain variable. Then, its inverse uncertainty distribution is

$$\Phi^{-1}(\alpha) = \begin{cases} a + 2(b-a)\alpha, & 0 \le \alpha \le 0.5 \\ 2b - c + 2(c-b)\alpha, & 0.5 < \alpha \le 1 \end{cases}$$

The expected value can be obtained

$$\begin{aligned} E[\xi] &= \int_0^{0.5} \left(a + 2(b-a)\alpha\right)\mathrm{d}\alpha \\ &\quad + \int_{0.5}^1 \left(2b - c + 2(c-b)\alpha\right)\mathrm{d}\alpha \\ &= \frac{a+2b+c}{4} \end{aligned}$$

Lemma 2. [28] Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. A function $f(x_1, x_2, \ldots, x_n)$ is strictly increasing with respected to $x_1, x_2, \ldots, x_m$ and strictly decreasing with respected to $x_{m+1}, x_{m+2}, \ldots, x_n$. Then the expected value of $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ is

$$E[\xi] = \int_0^1 f\left(\Phi_1^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha)\right)\mathrm{d}\alpha$$

III. Problem Descriptions

Consider a two stage green supply chain consisting of one manufacturer and one retailer. The manufacturer sells his green product to the retailer, and then the retailer retails it to the customer. We assume the manufacturer produces only one green product and the retailer sells only single green product.

The uncertain market demand faced by the manufacturer and the retailer is assumed as a linear function of the retail price p and the green level θ, which is given by

$$q = \tilde{d} - \tilde{\beta} p + \tilde{\gamma}\theta \tag{1}$$

where $\tilde{d}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are all uncertain variables, $\tilde{d}$ denotes the market base, $\tilde{\beta}$ denotes the price elastic coefficient of the demand to retail price, and $\tilde{\gamma}$ denotes the greening elastic coefficient of the demand to green level.

Further, we assume that the marginal cost of the retailer is not affected by the green level. The cost of achieving green level requires fixed investment, which is a quadratic function of green level θ. It is given by $\frac{1}{2}\tilde{\lambda}\theta^2$, where $\tilde{\lambda}$ denotes the investment coefficient, which is an uncertain variable. The notations which will be used in this paper are as follows:

$\tilde{c}$: unite production cost of the green product, an uncertain variable;

w : unite wholesale price of the green product, a decision variable;

p : unite retail price of the green product, a decision variable;

θ : unite green level of the green product, a decision variable.

In order to obtain the closed-form solutions, we give some assumptions as follows.

Assumptions 1. The manufacturer is the Stackelberg leader, and the retailer is the follower.

Assumptions 2. The uncertain variables $\tilde{c}$, $\tilde{d}$, $\tilde{\beta}$, $\tilde{\gamma}$ and $\tilde{\lambda}$ are assumed nonnegative and mutually independent.

Assumptions 3. We assume that the production cost of the green product cannot exceed the wholesale price, and the market demand is positive.

$$M\{w - \tilde{c} \le 0\} = 0, \text{ and } M\{\tilde{d} - \tilde{\beta} p + \tilde{\gamma}\theta \le 0\} = 0.$$

Assumptions 4. The retailer is assumed to be risk averse.

Thus, we can get the manufacturer's and retailer's profits as follows

$$\Pi_M = (w - \tilde{c})(\tilde{d} - \tilde{\beta} p + \tilde{\gamma}\theta) - \frac{1}{2}\tilde{\lambda}\theta^2 \tag{2}$$

$$\Pi_R = (p - w)(\tilde{d} - \tilde{\beta} p + \tilde{\gamma}\theta) \tag{3}$$

IV. Models Analysis

In this section, we consider two kinds of models to distinguish the risk sensitivity of the retailer: (a) expected value (EV) model: both the retailer and the manufacturer are supposed to be risk neutral; (b) chance constrained (CC) model: the manufacturer is supposed to be risk neutral, while the retailer is supposed to be risk averse.

A. Expected value model

In the EV (expected value) model, the manufacturer is the leader and the retailer is a follower. In this model, the wholesale price and green level are released by the manufacturer first, and then retail price is decided by the manufacturer. The manufacturer and the retailer maximize their expected profit, respectively. Then the model can be given as follows

$$\begin{cases} \max\limits_{w,\theta} E[\Pi_M] = E\left[(w - \tilde{c})(\tilde{d} - \tilde{\beta} p^* + \tilde{\gamma}\theta) - \frac{1}{2}\tilde{\lambda}\theta^2\right] \\ M\{w - \tilde{c} \le 0\} = 0 \\ \text{where } p^* \text{ solves the following problem} \\ \max\limits_{p} E[\Pi_R] = E\left[(p - w)(\tilde{d} - \tilde{\beta} p + \tilde{\gamma}\theta)\right] \\ p > w \\ M\{\tilde{d} - \tilde{\beta} p + \tilde{\gamma}\theta \le 0\} = 0 \end{cases} \tag{4}$$

In order to attain the optimal solutions, we should convert the uncertain profits of the manufacturer and the retailer into crisp forms first.

For concise, we define

$$E\left[\tilde{a}^{1-\alpha}\tilde{b}^{\alpha}\right]=\int_0^1\Phi_{\tilde{a}}^{-1}(1-\alpha)\Phi_{\tilde{b}}^{-1}(\alpha)\,\mathrm{d}\alpha,$$

$$E\left[\tilde{a}^{1-\alpha}\tilde{b}^{1-\alpha}\right]=\int_0^1\Phi_{\tilde{a}}^{-1}(1-\alpha)\Phi_{\tilde{b}}^{-1}(1-\alpha)\,\mathrm{d}\alpha.$$

where $\Phi_{\tilde{a}}^{-1}$ and $\Phi_{\tilde{b}}^{-1}$ are the reverse uncertainty distribution of the uncertain variables $\tilde{a}$ and $\tilde{b}$, respectively.

Theorem 1. The expected profits of the manufacturer and the retailer can be transformed as follows

$$E[\Pi_M]=-E[\tilde{d}]w-E[\tilde{\beta}]wp+E[\tilde{\gamma}]w\theta-E\left[\tilde{c}^{1-\alpha}\tilde{d}^{\alpha}\right] +E\left[\tilde{c}^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]p-E\left[\tilde{c}^{1-\alpha}\tilde{\gamma}^{\alpha}\right]\theta-\frac{1}{2}E[\tilde{\lambda}]\theta^2 \quad (5)$$

$$E[\Pi_R]=-E[\tilde{\beta}]p^2+\left(E[\tilde{d}]+E[\tilde{\gamma}]\theta+E[\tilde{\beta}]w\right)p -E[\tilde{d}]w-E[\tilde{\gamma}]w\theta \quad (6)$$

Proof: Let $\tilde{c}$, $\tilde{d}$, $\tilde{\beta}$, $\tilde{\gamma}$ and $\tilde{\lambda}$ be positive uncertain variables with uncertainty distributions $\Phi_{\tilde{c}}$, $\Phi_{\tilde{d}}$, $\Phi_{\tilde{\beta}}$, $\Phi_{\tilde{\gamma}}$ and $\Phi_{\tilde{\lambda}}$, respectively. From Eq. (2), we can find that $E[\tilde{\Pi}_M]$ is monotone decreasing with $\tilde{c}$, $\tilde{\beta}$ and $\tilde{\lambda}$, and monotone increasing with $\tilde{d}$ and $\tilde{\gamma}$. Then referring to Lemma 1 and 2, we have

$$\begin{aligned}E[\Pi_M]&=E\left[(w-\tilde{c})(\tilde{d}-\tilde{\beta}p+\tilde{\gamma}\theta)-\frac{1}{2}\tilde{\lambda}\theta^2\right]\\&=\int_0^1\Big[\left(w-\Phi_{\tilde{c}}(1-\alpha)\right)\left(\Phi_{\tilde{d}}(\alpha)-\Phi_{\tilde{\beta}}(1-\alpha)p+\Phi_{\tilde{\gamma}}(\alpha)\theta\right)\\&\qquad-\frac{1}{2}\Phi_{\tilde{\lambda}}(1-\alpha)\theta^2\Big]\mathrm{d}\alpha\\&=-E[\tilde{d}]w-E[\tilde{\beta}]wp+E[\tilde{\gamma}]w\theta-E\left[\tilde{c}^{1-\alpha}\tilde{d}^{\alpha}\right]\\&\quad+E\left[\tilde{c}^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]p-E\left[\tilde{c}^{1-\alpha}\tilde{\gamma}^{\alpha}\right]\theta-\frac{1}{2}E[\tilde{\lambda}]\theta^2\end{aligned}$$

In the same way, we can derive $E[\Pi_R]$ showed as

$$\begin{aligned}E[\Pi_R]&=E\left[(p-w)(\tilde{d}-\tilde{\beta}p+\tilde{\gamma}\theta)\right]\\&=\int_0^1\left[(p-w)\left(\Phi_{\tilde{d}}(\alpha)-\Phi_{\tilde{\beta}}(1-\alpha)p+\Phi_{\tilde{\gamma}}(\alpha)\theta\right)\right]\mathrm{d}\alpha\\&=E[\tilde{d}]p-E[\tilde{\beta}]p^2+E[\tilde{\gamma}]p\theta-E[\tilde{d}]w\\&\quad+E[\tilde{\beta}]wp-E[\tilde{\gamma}]w\theta\\&=-E[\tilde{\beta}]p^2+\left(E[\tilde{d}]+E[\tilde{\gamma}]\theta+E[\tilde{\beta}]w\right)p\\&\quad-E[\tilde{d}]w-E[\tilde{\gamma}]w\theta\end{aligned}$$

The proof of Theorem1 is completed.

We first obtain the optimal decisions of the retailer.

Theorem 2. In the EV model, if $E[\tilde{\beta}]w-E[\tilde{\gamma}]\theta<E[\tilde{d}]$ and $\mathcal{M}\left\{\tilde{d}-\tilde{\beta}p+\tilde{\gamma}\theta\le 0\right\}=0$ hold, the optimal reaction functions $p^*(w,\theta)$ of the retailer can be given by considering the wholesale price w and green level θ made earlier by the manufacturer

$$p^*(w,\theta)=\frac{E[\tilde{\beta}]w+E[\tilde{\gamma}]\theta+E[\tilde{d}]}{2E[\tilde{\beta}]} \quad (7)$$

Proof. Referring to Eq. (6), the first and second order derivatives of $E[\Pi_R]$ to p can be shown as

$$\frac{\mathrm{d}E[\Pi_R]}{\mathrm{d}p}=-2E[\tilde{\beta}]p+E[\tilde{d}]+E[\tilde{\gamma}]\theta+E[\tilde{\beta}]w \quad (8)$$

$$\frac{\mathrm{d}^2E[\Pi_R]}{\mathrm{d}p^2}=-2E[\tilde{\beta}] \quad (9)$$

Note that the second order derivative of $E[\Pi_R]$ is negative definite, since $\tilde{\beta}$ is a nonnegative uncertain variable. Consequently, $E[\Pi_R]$ is strictly concave in p.

Setting Eq. (8) to zero, the first order condition can be given as follows

$$\frac{\mathrm{d}E[\Pi_R]}{\mathrm{d}p}=-2E[\tilde{\beta}]p+E[\tilde{d}]+E[\tilde{\gamma}]\theta+E[\tilde{\beta}]w=0 \quad (10)$$

Solving Eq. (10), we obtain Eq. (7).

Since $p^*>w$, thus we have $E[\tilde{\beta}]w-E[\tilde{\gamma}]\theta<E[\tilde{d}]$.

The proof of Theorem 2 is completed.

After knowing the retailer's reaction function, the manufacturer would use it to maximize his expected profit by choosing the wholesale price and green level.

Theorem 3. In the EV model, if $4E[\tilde{\beta}]E[\tilde{\lambda}]>\left(E[\tilde{\gamma}]\right)^2$, $p^*>w$, $\mathcal{M}\{w^*-\tilde{c}\le 0\}=0$ and $\mathcal{M}\left\{\tilde{d}-\tilde{\beta}p^*+\tilde{\gamma}\theta^*\le 0\right\}=0$ hold, the optimal solutions of the manufacturer and the retailer are

$$w^*=\frac{2E[\tilde{\lambda}]A_1-E[\tilde{\gamma}]A_2}{4E[\tilde{\beta}]E[\tilde{\lambda}]-\left(E[\tilde{\gamma}]\right)^2} \quad (11)$$

$$\theta^*=\frac{E[\tilde{\gamma}]A_1-2E[\tilde{\beta}]A_2}{4E[\tilde{\beta}]E[\tilde{\lambda}]-\left(E[\tilde{\gamma}]\right)^2} \quad (12)$$

$$p^*=\frac{E[\tilde{\beta}]w^*+E[\tilde{\gamma}]\theta^*+E[\tilde{d}]}{2E[\tilde{\beta}]} \quad (13)$$

where $A_1=E[\tilde{d}]+E\left[\tilde{c}^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]$,

$$A_2=\frac{2E\left[\tilde{c}^{1-\alpha}\tilde{\gamma}^{1-\alpha}\right]E[\tilde{\beta}]-E\left[\tilde{c}^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]E[\tilde{\gamma}]}{E[\tilde{\beta}]}.$$

Proof. Substituting $p^*(w,\theta)$ in Eq. (7) into Eq. (5), we can have the expected profit of the manufacturer $E[\Pi_M]$ as follows

$$E[\Pi_M]=-\frac{1}{2}E[\tilde{\beta}]w^2-\frac{1}{2}E[\tilde{\lambda}]\theta^2+\frac{1}{2}E[\tilde{\gamma}]w\theta+\frac{1}{2}A_1w -\frac{1}{2}A_2\theta+\frac{E\left[\tilde{c}^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]E[\tilde{d}]}{2E[\tilde{\beta}]}-E\left[\tilde{c}^{1-\alpha}\tilde{d}^{\alpha}\right] \quad (14)$$

Referring to Eq. (14), we can get the first order derivatives of $E[\Pi_M]$ to w and θ as follows

$$\frac{\partial E[\Pi_M]}{\partial w}=-E[\tilde{\beta}]w+\frac{1}{2}E[\tilde{\gamma}]\theta+\frac{1}{2}A_1 \quad (15)$$

$$\frac{\partial E[\Pi_M]}{\partial \theta}=-E[\tilde{\lambda}]\theta+\frac{1}{2}E[\tilde{\gamma}]w-\frac{1}{2}A_2 \quad (16)$$

Then, the second order derivatives of $E[\Pi_M]$ to w and

θ can be shown as

$$\frac{\partial^2 E[\Pi_M]}{\partial w^2}=-E\left[\tilde{\beta}\right],\ \frac{\partial^2 E[\Pi_M]}{\partial w\partial\theta}=\frac{1}{2}E[\tilde{\gamma}],$$

$$\frac{\partial^2 E[\Pi_M]}{\partial\theta^2}=-E\left[\tilde{\lambda}\right],\ \frac{\partial^2 E[\Pi_M]}{\partial\theta\partial w}=\frac{1}{2}E[\tilde{\gamma}].$$

Thus, the Hessian matrix can be obtained

$$H=\begin{vmatrix}\frac{\partial^2 E[\Pi_M]}{\partial w^2} & \frac{\partial^2 E[\Pi_M]}{\partial w\partial\theta}\\ \frac{\partial^2 E[\Pi_M]}{\partial\theta\partial w} & \frac{\partial^2 E[\Pi_M]}{\partial\theta^2}\end{vmatrix}=\begin{vmatrix}-E\left[\tilde{\beta}\right] & \frac{1}{2}E[\tilde{\gamma}]\\ \frac{1}{2}E[\tilde{\gamma}] & -E\left[\tilde{\lambda}\right]\end{vmatrix}$$

Note that the Hessian matrix is negative definite, since $\tilde{\beta}$ and $\tilde{\lambda}$ are nonnegative uncertain variables, and $4E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]-\left(E[\tilde{\gamma}]\right)^2>0$. Consequently, $E[\Pi_M]$ is strictly jointly concave in w and θ.

Setting Eqs. (15) and (16) to zero, the first order conditions can be shown as

$$\frac{\partial E[\Pi_M]}{\partial w}=-E\left[\tilde{\beta}\right]w+\frac{1}{2}E[\tilde{\gamma}]\theta+\frac{1}{2}A_1=0 \tag{17}$$

$$\frac{\partial E[\Pi_M]}{\partial\theta}=-E\left[\tilde{\lambda}\right]\theta+\frac{1}{2}E[\tilde{\gamma}]w-\frac{1}{2}A_2=0 \tag{18}$$

Solving Eqs. (17) and (18), we obtain Eqs. (11) and (12).

Substituting w^* and θ^* into Eq. (7), we obtain Eq. (13). The proof of Theorem 3 is completed.

B. Chance constrained model

In the CC (chance constrained) model, the conditions are as the same as the former, and the retailer is supposed to be risk averse and maximize the objective function under a certain confidence level. Then the CC model can be given as follows

$$\begin{cases}\max\limits_{w,\theta} E[\Pi_M]=E\left[(w-\tilde{c})\left(\tilde{d}-\tilde{\beta}p^*+\tilde{\gamma}\theta\right)-\frac{1}{2}\tilde{\lambda}\theta^2\right]\\ M\{w-\tilde{c}\le 0\}=0\\ \text{where } p^* \text{ solves the following problem}\\ \begin{cases}\max\bar{\Pi}_R\\ M\left\{(p-w)\left(\tilde{d}-\tilde{\beta}p+\tilde{\gamma}\theta\right)\ge\bar{\Pi}_R\right\}\ge\alpha\\ p>w\\ M\left\{\tilde{d}-\tilde{\beta}p+\tilde{\gamma}\theta\le 0\right\}=0\end{cases}\end{cases} \tag{19}$$

In the CC model, the retailer is supposed to be risk averse, which means that the confidence level $\alpha>0.5$. To solve the above model, we should transform the uncertain model into an equivalent model first. The equivalent model is as follows

$$\begin{cases}\max\limits_{w,\theta} E[\Pi_M]=E\left[(w-\tilde{c})\left(\tilde{d}-\tilde{\beta}p^{**}+\tilde{\gamma}\theta\right)-\frac{1}{2}\tilde{\lambda}\theta^2\right]\\ M\{w-\tilde{c}\le 0\}=0\\ \text{where } p^{**} \text{ solves the following problem}\\ \begin{cases}\max(p-w)\left(\Phi_{\tilde{d}}^{-1}(1-\alpha)-\Phi_{\tilde{\beta}}^{-1}(\alpha)p+\Phi_{\tilde{\gamma}}^{-1}(1-\alpha)\theta\right)\\ p>w\\ \Phi_{\tilde{d}}^{-1}(1-\alpha)-\Phi_{\tilde{\beta}}^{-1}(\alpha)p+\Phi_{\tilde{\gamma}}^{-1}(1-\alpha)\theta>0\end{cases}\end{cases} \tag{20}$$

We first obtain the optimal decisions of the retailer.

Theorem 4. In the CC model, for given $\alpha\in(0.5,1]$, if $\Phi_{\tilde{\beta}}^{-1}(\alpha)w-\Phi_{\tilde{\gamma}}^{-1}(1-\alpha)\theta<\Phi_{\tilde{d}}^{-1}(1-\alpha)$ holds, the optimal reaction functions $p^{**}(w,\theta)$ of the retailer can be given by considering the wholesale price w and green level θ made earlier by the manufacturer

$$p^*(w,\theta)=\frac{\Phi_{\tilde{\beta}}^{-1}(\alpha)w+\Phi_{\tilde{\gamma}}^{-1}(1-\alpha)\theta+\Phi_{\tilde{d}}^{-1}(1-\alpha)}{2\Phi_{\tilde{\beta}}^{-1}(\alpha)} \tag{21}$$

Proof. The profit of the retailer is

$$\begin{aligned}\Pi_R&=(p-w)\left(\Phi_{\tilde{d}}^{-1}(1-\alpha)-\Phi_{\tilde{\beta}}^{-1}(\alpha)p+\Phi_{\tilde{\gamma}}^{-1}(1-\alpha)\theta\right)\\&=-\Phi_{\tilde{\beta}}^{-1}(\alpha)p^2+\left(\Phi_{\tilde{\beta}}^{-1}(\alpha)w+\Phi_{\tilde{\gamma}}^{-1}(1-\alpha)\theta+\Phi_{\tilde{d}}^{-1}(1-\alpha)\right)p\\&\quad-\Phi_{\tilde{d}}^{-1}(1-\alpha)w-\Phi_{\tilde{\gamma}}^{-1}(1-\alpha)w\theta\end{aligned} \tag{22}$$

Referring to Eq. (22), the first and second order derivatives of Π_R to p can be shown as

$$\frac{\mathrm{d}\Pi_R}{\mathrm{d}p}=-2\Phi_{\tilde{\beta}}^{-1}(\alpha)p+\Phi_{\tilde{\beta}}^{-1}(\alpha)w+\Phi_{\tilde{\gamma}}^{-1}(1-\alpha)\theta+\Phi_{\tilde{d}}^{-1}(1-\alpha) \tag{23}$$

$$\frac{\mathrm{d}^2\Pi_R}{\mathrm{d}p^2}=-2\Phi_{\tilde{\beta}}^{-1}(\alpha) \tag{24}$$

Note that the second order derivative of Π_R is negative definite, since $\tilde{\beta}$ is a nonnegative uncertain variable. Consequently, Π_R is strictly concave in p.

Setting Eq. (23) to zero, the first order condition can be given as follows

$$\frac{\mathrm{d}\Pi_R}{\mathrm{d}p}=-2\Phi_{\tilde{\beta}}^{-1}(\alpha)p+\Phi_{\tilde{\beta}}^{-1}(\alpha)w+\Phi_{\tilde{\gamma}}^{-1}(1-\alpha)\theta+\Phi_{\tilde{d}}^{-1}(1-\alpha)=0 \tag{25}$$

Solving Eq. (25), we obtain Eq. (21).

Since $p^{**}>w$ and $\Phi_{\tilde{d}}^{-1}(1-\alpha)-\Phi_{\tilde{\beta}}^{-1}(\alpha)p^{**}+\Phi_{\tilde{\gamma}}^{-1}(1-\alpha)\theta>0$, thus we have $\Phi_{\tilde{\beta}}^{-1}(\alpha)w-\Phi_{\tilde{\gamma}}^{-1}(1-\alpha)\theta<\Phi_{\tilde{d}}^{-1}(1-\alpha)$.

The proof of Theorem 4 is completed.

After knowing the retailer's reaction function, the manufacturer would use it to maximize his expected profit by choosing the wholesale price and green level.

Theorem 5. In the CC model, for given $\alpha\in(0.5,1]$, if $E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]>B_1^2$, $\Phi_{\tilde{d}}^{-1}(1-\alpha)-\Phi_{\tilde{\beta}}^{-1}(\alpha)p^{**}+\Phi_{\tilde{\gamma}}^{-1}(1-\alpha)\theta^{**}>0$ and $M\{w^*-\tilde{c}\le 0\}=0$ hold, the optimal solutions of the manufacturer and the retailer are

$$w^{**}=\frac{E\left[\tilde{\lambda}\right]B_2-B_1B_3}{E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]-B_1^2} \tag{26}$$

$$\theta^{**}=\frac{B_1B_2-E\left[\tilde{\beta}\right]B_3}{E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]-B_1^2} \tag{27}$$

$$p^{**}=\frac{\Phi_{\tilde{\beta}}^{-1}(\alpha)w^{**}+\Phi_{\tilde{\gamma}}^{-1}(1-\alpha)\theta^{**}+\Phi_{\tilde{d}}^{-1}(1-\alpha)}{2\Phi_{\tilde{\beta}}^{-1}(\alpha)} \tag{28}$$

where $B_1=E[\tilde{\gamma}]-\dfrac{E\left[\tilde{\beta}\right]\Phi_{\tilde{\gamma}}^{-1}(1-\alpha)}{2\Phi_{\tilde{\beta}}^{-1}(\alpha)}$,

$$B_2=E\left[\tilde{d}\right]+\frac{1}{2}E\left[\tilde{c}^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]-\frac{E\left[\tilde{\beta}\right]\Phi_{\tilde{d}}^{-1}(1-\alpha)}{2\Phi_{\tilde{\beta}}^{-1}(\alpha)},$$

$$B_3=E\left[\tilde{c}^{1-\alpha}\tilde{\gamma}^{1-\alpha}\right]-\frac{E\left[\tilde{c}^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]\Phi_{\tilde{\gamma}}^{-1}(1-\alpha)}{2\Phi_{\tilde{\beta}}^{-1}(\alpha)}.$$

Proof. Substituting $p^{**}(w,\theta)$ in Eq. (21) into Eq. (5), we can have the expected profit of the manufacturer $E[\Pi_M]$ as follows

$$E[\Pi_M]=-\frac{1}{2}E\left[\tilde{\beta}\right]w^2-\frac{1}{2}E\left[\tilde{\lambda}\right]\theta^2+B_1w\theta+B_2w-B_3\theta+\frac{E\left[\tilde{c}^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]\Phi_{\tilde{d}}^{-1}(1-\alpha)}{2\Phi_{\tilde{\beta}}^{-1}(\alpha)}-E\left[\tilde{c}^{1-\alpha}\tilde{d}^{\alpha}\right] \quad (29)$$

Referring to Eq. (29), we can get the first order derivatives of $E[\Pi_M]$ to w and θ as follows

$$\frac{\partial E[\Pi_M]}{\partial w}=-E\left[\tilde{\beta}\right]w+B_1\theta+B_2 \quad (30)$$

$$\frac{\partial E[\Pi_M]}{\partial \theta}=-E\left[\tilde{\lambda}\right]\theta+B_1w-B_3 \quad (31)$$

Then, the second order derivatives of $E[\Pi_M]$ to w and θ can be shown as

$$\frac{\partial^2 E[\Pi_M]}{\partial w^2}=-E\left[\tilde{\beta}\right],\ \frac{\partial^2 E[\Pi_M]}{\partial w\partial\theta}=B_1,$$

$$\frac{\partial^2 E[\Pi_M]}{\partial \theta^2}=-E\left[\tilde{\lambda}\right],\ \frac{\partial^2 E[\Pi_M]}{\partial\theta\partial w}=B_1.$$

Thus, the Hessian matrix can be obtained

$$H=\begin{vmatrix}\frac{\partial^2 E[\Pi_M]}{\partial w^2} & \frac{\partial^2 E[\Pi_M]}{\partial w\partial\theta}\\ \frac{\partial^2 E[\Pi_M]}{\partial\theta\partial w} & \frac{\partial^2 E[\Pi_M]}{\partial\theta^2}\end{vmatrix}=\begin{vmatrix}-E\left[\tilde{\beta}\right] & B_1\\ B_1 & -E\left[\tilde{\lambda}\right]\end{vmatrix}$$

Note that the Hessian matrix is negative definite, since $\tilde{\beta}$ and $\tilde{\lambda}$ are nonnegative uncertain variables, and $E\left[\tilde{\beta}\right]E\left[\tilde{\lambda}\right]>B_1^2$. Consequently, $E[\Pi_M]$ is strictly jointly concave in w and θ.

Setting Eqs. (15) and (16) to zero, the first order conditions can be shown as

$$\frac{\partial E[\Pi_M]}{\partial w}=-E\left[\tilde{\beta}\right]w+B_1\theta+B_2=0 \quad (32)$$

$$\frac{\partial E[\Pi_M]}{\partial \theta}=-E\left[\tilde{\lambda}\right]\theta+B_1w-B_3=0 \quad (33)$$

Solving Eqs. (32) and (33), we obtain Eqs. (26) and (27).

Substituting w^{**} and θ^{**} into Eq. (21), we obtain Eq. (28). The proof of Theorem 5 is completed.

V. Numerical Example

Owing to the complicated forms of the solutions, we conduct a numerical example to compare the optimal solutions under two models, and analyze the effects of the risk sensitively of the retailer on equilibrium solutions. Due to lack of the historical date, the market base $\tilde{d}$, the price elasticity $\tilde{\beta}$, the green level elasticity $\tilde{\gamma}$, the investment elasticity $\tilde{\lambda}$, and the cost of the production $\tilde{c}$ are predicted by the experiences of experts showed in Table I.

TABLE I
THE UNCERTAIN VARIABLES

Parameter	Linguistic description	Distribution
Market base $\tilde{d}$	About 300	$Z=(250,300,350)$
Price elasticity $\tilde{\beta}$	About 5	$Z=(4,5,6)$
Greening elasticity $\tilde{\gamma}$	About 4	$Z=(3,4,5)$
Investment elasticity $\tilde{\lambda}$	Between 4 and 6	$L=(4,6)$
Production cost $\tilde{c}$	Between 5 and 7	$L=(5,7)$

From Table I, we obtain

$$E\left[\tilde{d}\right]=\frac{250+2\times300+350}{4}=300,$$

$$E\left[\tilde{\beta}\right]=\frac{4+2\times5+6}{4}=5,\ E\left[\tilde{\gamma}\right]=\frac{3+2\times4+5}{4}=4,$$

$$E\left[\tilde{\lambda}\right]=\frac{4+6}{2}=5,\ E\left[\tilde{c}\right]=\frac{5+7}{2}=6,$$

$$E\left[\tilde{c}^{1-\alpha}\tilde{d}^{\alpha}\right]=\int_0^1\Phi_{\tilde{c}}^{-1}(1-\alpha)\Phi_{\tilde{d}}^{-1}(\alpha)\,\mathrm{d}\alpha=\frac{5450}{3},$$

$$E\left[\tilde{c}^{1-\alpha}\tilde{\beta}^{1-\alpha}\right]=\int_0^1\Phi_{\tilde{c}}^{-1}(1-\alpha)\Phi_{\tilde{\beta}}^{-1}(1-\alpha)\,\mathrm{d}\alpha=\frac{91}{3},$$

$$E\left[\tilde{c}^{1-\beta}\tilde{\gamma}^{\alpha}\right]=\int_0^1\Phi_{\tilde{c}}^{-1}(1-\alpha)\Phi_{\tilde{\gamma}}^{-1}(\alpha)\,\mathrm{d}\alpha=\frac{73}{3}.$$

Based on the above analysis, we present the optimal solutions of the manufacturer and the retailer in the EV model and the CC model in Tables II and III.

TABLE II
OPTIMAL PRICING AND GREEN LEVEL DECISIONS WITH DIFFERENT α

	α	w	θ	p
EV	—	31.037	10.175	49.588
CC	0.55	40.030	14.172	54.355
	0.60	41.314	15.567	54.529
	0.65	43.817	17.010	54.733
	0.70	45.741	18.502	54.964
	0.75	47.690	20.041	55.222

TABLE III
OPTIMAL PROFITS WITH DIFFERENT α

	α	Π_M	Π_R	Π_{SC}
EV	—	2060.000	1720.807	3780.807
CC	0.55	3388.321	1046.529	4434.850
	0.60	3820.699	827.559	4648.258
	0.65	4288.279	631.508	4919.787
	0.70	4793.097	459.282	5252.379
	0.75	5337.315	312.019	5649.334

Based on the results showed in Tables II and III, we find:

1) The wholesale price and the green level of the green product are becoming higher when α increases in the CC model, which means the manufacturer can decide higher wholesale price when the retailer is more sensitive to the risk. The retail price is becoming

higher along with the increasing of α in the CC model.

2) Both the optimal profits of the manufacturer and the supply chain system are becoming higher along with the increasing of α in the CC model, while the optimal profit of the retailer is becoming lower along with the increasing of α in the CC model
3) The optimal profit of the manufacturer is lower in the EV model than that in the CC model, which means that the manufacturer prefers that the retailer is more risk sensitive.
4) The optimal profit of the manufacturer is larger than that of the retailer this is because the manufacturer has the leadership

VI. Conclusion

This paper considered pricing and green level decisions problem with uncertain demand in a green supply chain with risk-averse retailer. The production cost and the demand were considered as uncertain variables rather than stochastic variables. Our studies mainly focused on the impacts of the risk sensitivity of the retailer on the performance of the green supply chain actors with the dominant manufacturer.

The limitation of the paper is that we only focus on one manufacturer and one retailer in a two stage green supply chain, therefore, the pricing and green level decisions with multiple competitive manufacturers or retailers are the important directions for the future research.

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