Finite-Time Control Design for Nonholonomic Mobile Robots Subject to Spatial Constraint

Yanling Shang, Jiacai Huang, Hongsheng Li and Xiulan Wen

Abstract—**This paper studies the problem of finite-time stabilizing control design for nonholonomic mobile robots subject to spatial constraint. A nonlinear mapping is first introduced to transform the constrained system into a new unconstrained one. Then, by employing the adding a power integrator technique and switching control strategy, a state feedback controller is successfully constructed to guarantee that the states of closed-loop system are regulated to zero in a finite time without violation of the constraint.**

Index Terms—**nonholonomic mobile robots, spatial constraint, adding a power integrator, finite-time stabilization.**

I. Introduction

THE nonholonomic systems, that is Lagrange systems with linear nonintegrable constraints, have attracted a great deal of attention during the past decades because they can model many practical systems, such as mobile robots, car-like vehicle, under-actuated satellites and so on [1-4]. Nonholonomic have good flexibility, since they could realize autonomous movement in the case of nobody involving. However, from a famous theorem due to Brockett [1], it is well known that no smooth (or even continuous) time-invariant static state feedback exists for the stabilization of nonholonomic (mobile robots) systems. There are currently several effective control methodologies that overcome the topological obstruction. The idea of using time-varying smooth controllers was first proposed in [6], in order to stabilize a mobile robot. For driftless systems in chained form, several novel approaches have been proposed for the design of periodic, smooth, or continuous stabilizing controllers [7, 8]. Most of the time-varying control scheme suffer from a slow convergence rate and oscillation. However, it has been observed that a discontinuous feedback control scheme usually results in a fast convergence rate. An elegant approach to constructing discontinuous feedback controller was developed in [9]. The drawback is that there is a restriction on the initial conditions of the controlled system. This limitation has been overcome by a switching state or output control scheme [10]. Subsequently, [11-17] further developed the discontinuous feedback control strategy based on different control targets, respectively.

In practical applications, the closed-loop system is desired to possess the property that trajectories converge to the equilibrium in finite time rather than merely asymptotically since system with finite-time convergence may retain not only faster convergence, but also better robustness and disturbance rejection properties [18]. Motivated by this, the finite-time control of nonlinear systems has attained significant amount of interests and efforts over the last years [19-21]. Particularly, by using state feedback, the authors in [22] first addressed the finite-time stabilization of nonholonomic systems with weak drifts, and then the adaptive finite-time stabilization problems were considered for nonholonomic systems with linear parameterization in [23] and nonlinear parameterization [24], respectively. By relaxed the restriction on system growth, [25] studied the finite-time control for a class of nonholonomic systems with low-order nonlinearities. An output feedback controller was developed in [26] to finite-time stabilize a class of nonholonomic systems in feedforward-like form. However, the effect of the constraints is omitted in the above-mentioned results.

As a matter that the constraints which can represent not only physical limitations but also performance requirements are common in practical systems. Violation of the constraints may cause performance degradation or system damage. In recent years, driven by practical needs and theoretical challenges, the control design for constrained nonlinear systems has become an important research topic [27-29]. However, less attention has been paid to the space-constrained nonholonomic mobile robots.

Motivated by the above discussion, this paper focus on solving the finite-time stabilization problem for nonholonomic mobile robots subject to spatial constraint. The contributions can be highlighted as follows. (i) The finite-time stabilization problem of nonholonomic systems with spatial constraint is studied for the first time. (ii) A nonlinear mapping is introduced, under which the constrained interval is mapped to the whole real number field. (iii) Based on a switching strategy to eliminate the phenomenon of uncontrollability of $u_0 = 0$, and by skillfully using the adding a power integrator technique, a systematic state feedback control design procedure is proposed to render the states of closed-loop system to zero in a finite time while the constraint is not violated in a domain.

The rest of this paper is organized as follows. In Section II, the problem formulation and preliminaries are given. Section III presents the input-state-scaling transformation the backstepping design procedure, the switching control strategy and the main result. Finally, concluding remarks are proposed in Section VI.

II. Problem Statement and Preliminaries

Consider a tricycle-type mobile robot shown in Fig. 1. The kinematic equations of this robot are represented by

$$\begin{aligned} \dot{x}_c &= v\cos\theta \\ \dot{y}_c &= v\sin\theta \\ \dot{\theta} &= \omega \end{aligned} \tag{1}$$

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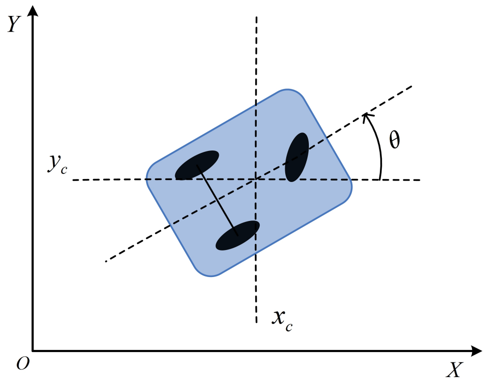

Fig. 1. The planar graph of a mobile robot.

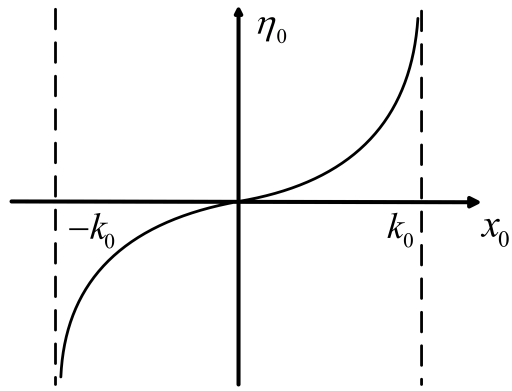

Fig. 2. Schematic illustration of the nonlinear mapping $\mathscr{H}_0$.

where (x_c, y_c) denotes the position of the center of mass of the robot, θ is the heading angle of the robot, v is the forward velocity while ω is the angular velocity of the robot.

Introducing the following change of coordinates

$$\begin{aligned} &x_0 = x_c, \quad x_1 = y_c, \quad x_2 = \tan\theta, \\ &u_0 = v\cos\theta, \quad u_1 = w\sec^2\theta, \end{aligned} \tag{2}$$

system (1) is transformed into the chained form as

$$\begin{aligned} \dot{x}_0 &= u_0 \\ \dot{x}_1 &= u_0 x_2 \\ \dot{x}_2 &= u_1 \end{aligned} \tag{3}$$

Note that the state (x_0, x_1) can be see as the displacement from the parking position. As we all know, when the robots initial position is far away from the parking position, it usually can move directly to the parking position. The robots body angle can be aligned without difficulties and no more maneuvers are needed. However, when the robots initial position is close to the parking position, it might not be feasible to get to the parking position while aligning the robots body angle at the same time. Therefore it is very necessary to develop finite-time control techniques for state-constrained nonholonomic systems for giving this difficulty a straightforward solution.

Due to physical limitations, in this paper we assume that the states x_0 and x_1 are constrained in the compact sets

$$\Omega_{x_i} = \{-k_i < x_i < k_i\}, \quad i = 0, 1 \tag{4}$$

where k_i's are positive constants.

The objective of this paper is to present a state feedback control design strategy which stabilizes the system (3) in a finite time with the constraint being not violated.

Remark 1. Although great progress on constrained control design has been made, for the constrained nonholonomic system (3), how to construct a finite-time stabilizer is still very difficult problem. The crucial obstacle is that the time-varying coefficient u_0 makes the x-subsystem uncontrollable in the case of $u_0 = 0$, and thus the existing constrained control methods mainly based on barrier Lyapunov function are highly difficult to the control problem of the system (3) or even inapplicable. Thereby, how to overcome this obstacle and design a finite-time stabilizer for the output-constrained system (3) is main work of this paper.

The following definitions and lemmas will serve as the basis of the coming control design and performance analysis.

Definition 1. Consider the nonlinear system

$$\dot{x} = f(t, x) \ \ with\ f(t, 0) = 0, \quad x \in R^n \tag{5}$$

where $f : R^+ \times U_0 \to R^n$ is continuous with respect to x on an open neighborhood U_0 of the origin $x = 0$. The equilibrium $x = 0$ of the system is (locally) uniformly finite-time stable if it is uniformly Lyapunov stable and finite-time convergent in a neighborhood $U \subseteq U_0$ of the origin. By "finite-time convergence," we mean: If, for any initial condition $x(t_0) \in U$ at any given initial time $t_0 \geq 0$, there is a settling time $T > 0$, such that every $x(t, t_0, x(t_0))$ of system (5) is defined with $x(t, t_0, x(t_0)) \in U/\{0\}$ for $t \in [t_0, T)$ and satisfies $\lim_{t\to T} x(t, t_0, x(t_0)) = 0$ and $x(t, t_0, x(t_0)) = 0$ for any $t \geq T$. If $U = U_0 = R^n$, the origin is a globally uniformly finite-time stable equilibrium.

Lemma 1[25]**.** Consider the nonlinear system described in (5). Suppose there is a C^1 function $V(t, x)$ defined on $\hat{U} \subseteq U_0 \times R$, where $\hat{U}$ is a neighborhood of the origin, class K functions π_1 and π_2, real numbers $c > 0$ and $0 < \alpha < 1$, for $t \in [t_0, T)$ and $x \in \hat{U}$ such that (i) $\pi_1(|x|) \leq V(t, x) \leq \pi_2(|x|)$, $\forall t \geq t_0$, $\forall x \in \hat{U}$; (ii)$\dot{V}(t, x) + cV^\alpha(t, x) \leq 0$, $\forall t \geq t_0$, $\forall x \in \hat{U}$. Then, the origin of (5) is uniformly finite-time stable with $T \leq \frac{V^{1-\alpha}(t_0, x(t_0))}{c(1-\alpha)}$ for initial condition $x(t_0)$ in some open neighborhood $\hat{U}$ of the origin at initial time t_0. If $\hat{U} = U_0 = R^n$ and π_1 and π_2 are class K_∞ functions, the origin of system (5) is globally uniformly finite-time stable.

Lemma 2[30]**.** For $x \in R$, $y \in R$, $p \geq 1$ and $c > 0$ are constants, the following inequalities hold: (i) $|x + y|^p \leq 2^{p-1}|x^p + y^p|$, (ii) $(|x| + |y|)^{1/p} \leq |x|^{1/p} + |y|^{1/p} \leq 2^{(p-1)/p}(|x| + |y|)^{1/p}$, (iii) $||x| - |y||^p \leq ||x|^p - |y|^p|$, (iv) $|x|^p + |y|^p \leq (|x| + |y|)^p$, (v) $|[x]^{1/p} - [y]^{1/p}| \leq 2^{1-1/p}|x - y|^{1/p}$, (vi) $|[x]^p - [y]^p| \leq c|x - y||\,|x - y|^{p-1} + |y|^{p-1}|$.

Lemma 3[31]**.** For any positive real numbers c, d and any real-valued function $\pi(x, y) > 0$, $|x|^c|y|^d \leq \frac{c}{c+d}\pi(x, y)|x|^{c+d} + \frac{d}{c+d}\pi^{-c/d}(x, y)|y|^{c+d}$.

Lemma 4[30]**.** $f(s) = sgn(s)|s|^a$ is continuously differentiable, and $\dot{f}(s) = a|s|^{a-1}\dot{s}$, where $a \geq 1$, $x \in R$. Moreover, if $s = x(t)$, $t \geq 0$, then $\frac{df(x(t))}{dt} = a|x|^{a-1}\dot{x}(t)$.

III. Nonlinear Mapping

To prevent the states x_0 and x_1 from violating the constraints, we define a nonlinear mapping that will be used to develop the control design and the main results.

Define a one-to-one nonlinear mapping $\mathscr{H} : (x_0, x) \to (\eta_0, \eta)$ as follows:

$$\begin{aligned}\eta_0 &= \mathscr{H}_0(x_0) = ln\Big(\frac{k_0 + x_0}{k_0 - x_0}\Big)\\ \eta_1 &= \mathscr{H}_1(x_1) = ln\Big(\frac{k_1 + x_1}{k_1 - x_1}\Big)\\ \eta_2 &= \mathscr{H}_2(x_2) = x_2\end{aligned} \tag{6}$$

where $\mathscr{H}_0$ is shown in Fig. 2. It is clear that function $\mathscr{H}_0$ is a continuous elementary function. From (6), we get

$$x_0 = \mathscr{H}_0^{-1} = k_0\Big(1 - \frac{2}{e^{\eta_0} + 1}\Big) \tag{7}$$

then the derivative of x_0 is given by

$$\dot{x}_0 = \frac{2k_0 e^{\eta_0}}{(e^{\eta_0} + 1)^2}\dot{\eta}_0 \tag{8}$$

Substituting (8) into the first equation of (3), we have

$$\dot{\eta}_0 = \frac{1}{2k_0}(e^{\eta_0} + e^{-\eta_0} + 2)u_0 \tag{9}$$

Similarly, we can obtain

$$\dot{\eta}_1 = \frac{1}{2k_1}(e^{\eta_1} + e^{-\eta_1} + 2)x_2 \tag{10}$$

By noting that $\dot{\eta}_2 = \dot{x}_2$, we can rewrite the system (3) as

$$\begin{aligned}\dot{\eta}_0 &= d_0(\eta_0)u_0\\ \dot{\eta}_1 &= d_1(\eta_1)u_0\eta_2\\ \dot{\eta}_2 &= u_1\end{aligned} \tag{11}$$

where

$$\begin{aligned}d_0(\eta_0) &= \frac{1}{2k_0}(e^{\eta_0} + e^{-\eta_0} + 2)\\ d_1(\eta_1) &= \frac{1}{2k_1}(e^{\eta_1} + e^{-\eta_1} + 2)\end{aligned} \tag{12}$$

IV. Finite-Time Control Design

In this section, we focus on designing finite-time controller for system (11) provided that $\eta_0(0) \neq 0$, while the case where the initial condition $\eta_0(0) = 0$ will be treated in Section 5. The inherently triangular structure of system (11) suggests that we should design the control inputs u_0 and u_1 in two separate stages.

A. Design u_0 for η_0-subsystem

For η_0-subsystem, we take the following control law

$$u_0 = -\frac{1}{d_0}(\lambda_0 + \bar{\varphi}_0)[\eta_0]^{1+\omega} \tag{13}$$

where k_0 is a positive design parameter.

Consequently, the following lemma can be established.

Lemma 5. Under the control law (13), the η_0-subsystem is globally uniformly finite-time stable within any given settling time T.

Proof. Choosing the Lyapunov function $V_0 = \eta_0^2/2$, a simple computation gives

$$-(\lambda_0 + 2\bar{\varphi}_0)|\eta_0|^{2+\omega} \le \dot{V}_0 \le -\lambda_0|\eta_0|^{2+\omega} \le 0 \tag{14}$$

which implies $|\eta_0(t)| \le |\eta_0(0)|$. Setting $\underline{K} = max_{|\eta_0(t)|\le|\eta_0(0)|}\{\lambda_0 + 2\bar{\varphi}_0\}$, from (11), it follows that

$$-\underline{K}|(\eta_0|^{2+\omega} \le \dot{V}_0 \le -\lambda_0|\eta_0|^{2+\omega} \tag{15}$$

Furthermore

$$-2\underline{K}V_0^{\frac{2+\omega}{2}} \le \dot{V}_0 \le -\lambda_0 V_0^{\frac{2+\omega}{2}} \tag{16}$$

Thus by Lemma 1, η_0 tends to 0 within a settling time denoted by T_0. Moreover,

$$\frac{V_0(0)^{\frac{-\omega}{2}}}{-\omega\underline{K}} \le T_0 \le \frac{2V_0(0)^{\frac{-\omega}{2}}}{-\omega\lambda_0} \tag{17}$$

Hence by taking design parameter λ_0 as $\lambda_0 \ge 2V_0(0)^{\frac{-\omega}{2}}/(-T\omega)$, the lemma follows.

Remark 2. From (17), it can be see that if we take $T_* < V_0(0)^{(-\omega)/2}/(-2\underline{K}\omega)$, then we can obtain $|\eta_0(0)| \ge |\lambda_0(t)| \ge 2^{1/\omega}|\eta_0(0)|$ during $[0, T_*]$ without changing the sign of $\eta_0(t)$ provided that $x_0(0) \neq 0$ [25,26]. Furthermore, from (13), we know that u_0 is bounded and its sign remains unchanged during $[0, T_*]$.

B. Input-state-scaling transformation

Since it has already proven that η_0 can be globally regulated to zero as $t \to T_0$. Next, we only need to stabilize the η-subsystem

$$\begin{aligned}\dot{\eta}_1 &= d_1(\eta_1)u_0\eta_2\\ \dot{\eta}_2 &= u_1\end{aligned} \tag{18}$$

within the given settling time T_*. To facilitate the design of controller u_1, the following discontinuous input-state-scaling transformation is utilized for system (18).

$$z_i = \frac{\eta_i}{u_0^{n-i}}, \quad i = 1, 2 \tag{19}$$

under which, the η-subsystem is transformed into

$$\begin{aligned}\dot{z}_1 &= \tilde{d}_1(z_1)u_0z_2 + \tilde{f}_1(t, \eta_0, z_1, u_0)\\ \dot{z}_2 &= u_1\end{aligned} \tag{20}$$

where

$$\begin{aligned}\tilde{d}_1(z_1) &= d_1(\eta_1)\\ \tilde{f}_1(t, \eta_0, \bar{z}_i, u_0) &= \frac{\bar{f}_1}{u_0} - \frac{z_i}{u_0}\frac{\partial u_0}{\partial \eta_0}(d_0u_0 + \bar{f}_0)\end{aligned} \tag{21}$$

The following lemmas gives the estimations of nonlinear function $\tilde{d}_1$ and $\bar{f}_i$.

Lemma 6. For $t \in [0, T_*]$, there are nonnegative $\eta_0(0)$-dependent smooth functions c_{11}, c_{12} and $\tilde{\varphi}_i$ such that

$$\begin{aligned}&0 < c_{11}(z_1) \le \tilde{d}_1(z_1) \le c_{12}(z_1)\\ &|\tilde{f}_1(t, \eta_0, \bar{z}_2, u_0)| \le \tilde{\varphi}_2(\bar{z}_2)\sum_{j=1}^{2}|z_j|^{\frac{r_i+\tau}{r_j}}\end{aligned} \tag{22}$$

where $\tau \in (-1/n, 0)$ and $r_i = 1 + (i-1)\tau > 0$, $i = 1, 2, 3$ are constants.

Proof. Noting that the boundedness of η_0 and u_0, the estimations can be easily obtained from (12), (20) and the transformation (19). The detailed proof is omitted here.

C. Recursive design for u_1

In this subsection, we shall construct a continuous state feedback controller u_1 which is addressed in a step-by-step manner. for For the consistency of the following inductive steps, we denote $\tilde{d}_2 = 1$.

Step 1. Let $\xi_1 = [z_1]$ and choose a Lyapunov function candidate

$$V_1 = W_1 = \int_{z_1^*}^{z_1} [[s] - z_1^*]^{2-r_2} ds \tag{23}$$

with $z_1^* = 0$. From (20) and (22), it follows that

$$\dot{V}_1 = \tilde{d}_1[\xi_1]^{2-r_2} z_2 + [\xi_1]^{2-r_2} \tilde{f}_1 \leq \tilde{d}_1[\xi_1]^{2-r_2} z_2 + \xi_1^2 \tilde{\varphi}_1 \tag{24}$$

Obviously, the C^0 virtual controller

$$z_2^* = -\frac{1}{c_{11}}(M + n - 1 + \bar{\varphi}_1)[\xi_1]^{r_2} := -\beta_1^{r_2}[\xi_1]^{r_2} \tag{25}$$

where $M > 0$ is a constant to be determined later, and $\beta_1 > 0$ is smooth function, results in

$$\dot{V}_1 \leq -(M + n - 1)\xi_1^2 + \tilde{d}_1[\xi_1]^{2-r_2}(z_2 - z_2^*) \tag{26}$$

Step 2. Let $\xi_2 = [z_2]^{\frac{1}{r_2}} - [z_2^*]^{\frac{1}{r_2}}$ and consider the Lyapunov function candidate

$$V_2 = V_1 + W_2 = V_1 \int_{z_2^*}^{z_2} \left[[s]^{\frac{1}{r_2}} - [z_2^*]^{\frac{1}{r_2}} \right]^{2-r_3} ds \tag{27}$$

From Lemma 4, it is clear that

$$\begin{aligned} \dot{V}_2 \leq& -(M+1)\xi_1^2 + \tilde{d}_1[\xi_1]^{2-r_2}(z_2 - z_2^*) \\ &+ [\xi_2]^{2-r_3} u_1 + \frac{\partial W_2}{\partial z_1}\dot{z}_1 + \frac{\partial W_2}{\partial \eta_0}\dot{\eta}_0 \end{aligned} \tag{28}$$

Now we estimate each term on the right-hand side of (28). To begin with,

Note that $0 > \tau > -\frac{1}{n}$ and $r_2 = r_1 + \tau$, we have $0 < r_k < 1$.It follows from Lemma 2 that

$$|z_2 - z_2^*| \leq 2^{1-r_2} \left| [z_2]^{\frac{1}{r_2}} - [z_2^*]^{\frac{1}{r_2}} \right|^{r_2} = 2^{1-r_2}|\xi_2|^{r_2} \tag{29}$$

By (29) and Lemma 3, it can be obtained that

$$\tilde{d}_1|\xi_1|^{2-r_1}|z_1 - z_1^*| \leq \frac{1}{2}\xi_1^2 + \xi_2^2 l_{21} \tag{30}$$

where $l_{21} \geq 0$ is a smooth function.

Second, by using Lemmas 2 and 3, after tedious calculations, there holds

$$\frac{\partial W_2}{\partial z_1}\dot{z}_1 + \frac{\partial W_2}{\partial \eta_0}\dot{\eta}_0 \leq \frac{1}{2}z_1^2 + l_{21}\xi_2^2 \tag{31}$$

where $l_{21} > 0$ is a constant.

Substituting (30)and (31) into (28) yields

$$\dot{V}_2 \leq -z_1^2 + [\xi_2]^{2-r_3} u_1 + \xi_2(l_{21} + l_{22} + l_{23}) \tag{32}$$

Now, it easy to see that the smooth actual controller

$$\begin{aligned} u_1 &= -(M + l_{21} + l_{22} + l_{23})\xi_2^{r_3} \\ &:= -\beta_2^{r_3}[\xi_2]^{r_2} \end{aligned} \tag{33}$$

renders

$$\dot{V}_2 \leq -M(z_1^2 + \xi_2^2) \tag{34}$$

Lemma 7. If control laws (13) and (33) are applied with an appropriate choice of the design parameters, then the system (20) is finite-time regulated at the origin for $\eta_0(0) \neq 0$.

Proof. Since we have already proven that η_0 can be globally finite-time regulated to zero above, we just need to show that $\eta(t)$ converges to zero within settling time $T_1 \leq T_*$. From Step 2, we easily see that V_2 is positive definite and radially unbounded. Then, by Lemma 4.3 in [32], we know that there exist K_∞ functions π_1 and π_2 such that

$$\pi_1(|\xi(t)|) \leq V_2(\xi(t)) \leq \pi_2(|\xi(t)|) \tag{35}$$

On the other hand, by using Lemma 2, it is easy to see that

$$\begin{aligned} W_2 &= \int_{z_2^*}^{z_2} \left[[s]^{\frac{1}{r_2}} - [z_2^*]^{\frac{1}{r_2}} \right]^{2-r_3} ds \\ &\leq \left| [z_2]^{\frac{1}{r_2}} - [z_2^*]^{\frac{1}{r_2}} \right|^{2-r_3} |z_k - z_k^*| \\ &\leq 2^{1-r_2}|\xi_2|^{2-\tau} \end{aligned} \tag{36}$$

So we have the following estimation

$$V_2 = \sum_{k=1}^{n} W_k \leq 2\sum_{k=1}^{2} |\xi_k|^{2-\tau} \tag{37}$$

Letting $\alpha = 2/(2-\tau)$, with (37) and (34) in mind, by Lemma 2, we can obtain that

$$\dot{V}_2 \leq -\frac{1}{2}MV_2^\alpha \tag{38}$$

By Lemma 1, system (20) under control law (33) is finite time stable with settling time

$$T_1 \leq \frac{2V_n^{(1-\alpha)}(0)}{M(1-\alpha)} \tag{39}$$

Hence, by choosing a large enough M as

$$M \geq \frac{2V_n^{(1-\alpha)}(0)}{T_*(1-\alpha)} \tag{40}$$

the settling time $T_1 < T_*$ is guaranteed. This together with transformation (19) implies that the η-subsystem is finite-time regulated at the origin. Thus, the proof is completed.

V. Switching Control Design and Main Result

In the preceding section, we have given controller design for $\eta_0(0) \neq 0$. Now, we discuss how to select the control laws u_0 and u_1 when $\eta_0(0) = 0$. In the absence of disturbances, the most commonly used control strategy is using constant control $u_0 = u_0^* \neq 0$ in time interval $[0, t_s)$. However, for system (11) with non-Lipschitz nonlinearities, the choice of constant feedbacks may lead to the solution of the η_0-subsystem blow up before the given switching time t_s. In order to prevent this finite escape phenomenon from happening, we give the switching control strategy for control input u_0 by the use of state measurement of the η_0-subsystem in (11) instead of frequently-used time measurement.

When $\eta_0(0) = 0$, we choose u_0 as follow:

$$u_0 = u_0^*, \;\; u_0^* > 0$$

At $\eta_0(0) = 0$, we know that $\dot{\eta}_0(0) = d_0(0)u_0(0) = d_0(0)u_0^* > 0$. Thus for a small positive constant δ, there exists a small neighborhood Ω of $\eta_0(0) = 0$ such that $|d_0\eta_0| \leq \delta$. Suppose that η_0^* satisfies $|\eta_0^*| = \delta$. In Ω, η_0 is increasing until $|\eta_0| > \delta$.

Now, we define the switching control law u_0 as

$$u_0 = u_0^*, \;\; u_0^* > 0, \;\; |\eta_0| \leq |\eta_0^*| < \delta \tag{41}$$

During the time period satisfying $|\eta_0| \leq |\eta_0^*|$, using u_0 defined in (41) and new $u_1 = u_1^*(\eta_0, z)$ obtained by the similar control design method as (33), it is concluded that the η-state of (11) cannot blow up for $|\eta_0| \leq |\eta_0^*|$. At this time, $\eta_0(t_s)$ is not zero ($|\eta_0(t_s)| > |\eta_0^*|$), then, we switch to the control inputs u_0 and u_1 into (13) and (33), respectively. Thus, the following results are obtained.

Lemma 8. If the proposed control design procedure together with the above switching control strategy is applied to system (11), then, for any initial conditions in the state space $(\eta_0, \eta)^T \in R^{n+1}$, uncertain system (9) is regulated at origin within a finite settling time.

Proof. According to the above analysis, it suffices to prove the statement in the case where $\eta_0(0) \neq 0$. Therefore, following the same line of the proof of Lemma 7, one can completed the proof.

With the help of Lemma 8, we are ready to state the main results of this paper.

Theorem 1. If the proposed control design procedure together with the above switching control strategy is applied to system (3), then, for any initial conditions $(x_0(0), x(0)) \in \Theta = \{(x_0, x)^T \in R^{n+1}| - k_i < x_i < k_i, i = 0, 1\}$, the following properties hold.

(i) The states x_0 and x_1 stay in the compact sets $\Omega_{x_i} = \{-k_i < x_i < k_i\}, i = 0, 1$.

(ii) All the states of closed-loop system are regulated to zero within a finite settling time.

Proof. From Lemma 8, we can easily see that the states $\eta_i(t)$, $i = 0, 1, 2$ are bounded, and moreover there exists a finite settling time T such that $\lim_{t\to T} \eta_i(t) = 0$. The bounded states $\eta_i(t)$, $i = 0, 1$ together with the nonlinear mapping (6) lead to

$$|x_0(t)| = k_0\Big|1 - \frac{2}{e^{\eta_0(t)} + 1}\Big| < k_0 \tag{42}$$

and

$$|x_1(t)| = k_1\Big|1 - \frac{2}{e^{\eta_1(t)} + 1}\Big| < k_1 \tag{43}$$

that is, the states x_i will remain in the sets Ω_{x_i}, $i = 0, 1$ and never violate the constraints. Furthermore, $\lim_{t\to T} \eta_i(t) = 0$, $i = 0, 1, \cdots, n$ and (6) imply that implies that $\lim_{t\to T} x_2(t) = 0$ and

$$\lim_{t\to T} x_0 = \lim_{t\to T} k_0\Big(1 - \frac{2}{e^{\eta_0(t)} + 1}\Big) = 0 \tag{44}$$

$$\lim_{t\to T} x_1 = \lim_{t\to T} k_1\Big(1 - \frac{2}{e^{\eta_1(t)} + 1}\Big) = 0 \tag{45}$$

Thus, the proof is completed.

VI. Conclusion

This paper has studied the problem of finite-time stabilization by state feedback for nonholonomic mobile robots subject to spatial constraint. Based on the nonlinear mapping, and by skillfully using the method of adding a power integrator, a constructive design procedure for state feedback control is given. Together with a novel switching control strategy, the designed controller can guarantee that the closed-loop system states are finite-time regulated to zero while the constraint is not violated in a domain. In this direction, there are still remaining problems to be investigated. For example, an interesting research problem is how to design a finite-time output feedback stabilizing controller for such constrained systems studied in the paper.

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