# Distributed Optimal Control for Multi-agent Systems with Impulsive Effects

Lijuan Chen and Jitao Sun*

*Abstract*—**In this paper, we investigate the distributed optimal control for multi-agent systems with impulsive effects. We present sufficient condition to guarantee that the distributed protocol can make desired performance index reach minimal value for the impulsive multi-agent systems and the general case, respectively. An example is also presented to illustrate the efficiency of the obtained results.**

*Index Terms*—**multi-agent system, inverse optimal control, consensus performance, impulse**

## I. Introduction

NOWADAYS, more and more researchers([1]-[5]) pay attention to networks of systems, especially, the multi-agent systems (MASs) due to its wide application in various areas, for example, flocking in biology, camera network in airports, formation control in unmanned aerial vehicles and so on. Optimal control for MASs is recently studied by lots of authors. For example, Cao et al.[6] investigated the optimal consensus of first-order multi-agent systems by using algebraic Riccati equation to get the optimal control input and then proved that the communication topology is a complete directed graph. Dong[7] used a local observer to obtain the global information but only presented a suboptimal design with respect to a performance index. In [8], Liu and Geng investigated the optimal control problem for the second-order multi-agent systems by using the Pontrayagin maximum principle. Liao et al.[9] derived the controller of multi-agent systems with error integral and preview action that can guarantee the achievement of cooperative optimal preview tracking.

However, as was pointed out in [10] and [11], the global optimization problem generally needs the global information which is not easily obtained int the multi-agent networked system. So the most common difficulty about the cooperative optimal control problem is that the obtained protocol is not distributed([7]-[9]). Indeed, we can investigate the cooperative optimal control problem by the inverse optimality theory[12]. The basic idea of the inverse optimal design is to analyze the performance of a stabilizing control by computing its cost and demonstrating optimality with respect to some well defined and meaningful performance index. Recently, a considerable amount of the existing literature is dedicated to finding optimal strategy subject to the inverse optimal control problem for the multi-agent system. In [10] and [11], the authors obtained the optimal distributed consensus protocols by constructing a specified global performance index. In [13], the sufficient and necessary conditions are provided for globally optimal cooperative control problems on directed graphs. Feng et al.[14] designed optimal distributed consensus protocols for general identical linear continuous time cooperative systems which not only minimize some local quadric performances, but also regulate the consensus rate for the multi-agent systems. Chen and Sun[15] investigated the distributed optimal control of a multi-agent consensus problem in an obstacle-laden environment. For the general linear discrete time multi-agent systems on a fixed, directed graph, Zhang et al.[16] designed a novel linear quadratic regulator (LQR)-based optimal distributed cooperative synchronization control.

At the same time, it has been noticed that all the above literatures pay attention to either continuous-time or discrete-time systems. In practice, impulsive effects, which mean sudden jumps of some state variables at some instants, usually exist in the real world, such as frequency-modulated signal processing systems and bursting rhythm systems in pathology. This kind of system dramatically reduces the cost needed, which makes it more efficient and applicable than other systems, thus having received considerable attention. In the recent several decades, systems with impulse have aroused the interest of many authors([17]-[21]), to name just a few. Of particular relevances to this paper are the works, Jiang et al.[19] designed an impulsive control protocol for multi-agent linear dynamic systems on undirected graphs. Xiong et al.[20] presented a criterion to guarantee the consensus for a multi-agent directed network with nonlinear perturbations provided that each row sum of the impulse matrix is equal to zero. Guan et al.[21] studied the problem of guaranteed performance consensus in second-order multi-agent systems. Ma et al.[22] investigated the problem of cooperative synchronization of nonlinear multi-agent systems with time delays and impulsive disturbances. In Han et al.[23], for the multi-consensus problem of the second order multi-agent networks with a directed topology, three rectangular impulsive protocols were proposed to solve the stationary multi-consensus and the dynamic multi-consensus. However, to the best of the authors' knowledge, to this day, with few exception[24], there exist no other literatures which discuss the problem that the distributed consensus protocol can make the specified performance reach minimal value for multi-agent systems with impulse. Different with [24] which pays attention to the hybrid protocol, this paper aims to design the distributed optimal consensus or tracking protocol for both the leaderless and leader-following cases.

The organization of this paper is as follows. In Section II, we show some concepts and results of matrix theory and graph theory. In Section III, we present the consensus or synchronization problem for the impulsive multi-agent system with impulsive effects. Then we investigate

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the optimal cooperative regulator problem and the optimal cooperative tracker problem for the above impulsive system and the general case, respectively. In Section IV, numerical simulation is presented to illustrate the feasibility of our main results. In the last section, we give a brief discussion.

## II. Preliminaries

In this section, we will give several notations and some results in matrix theory and algebraic graph theory which shall be used throughout this paper.

First we provide some notations as follows. Let $Z_+ = \{1, 2, \ldots\}$ and $N_+ = \{1, 2, \ldots, N\}$. $R^n$ denotes the $n-$dimensional Euclidean space, $R^{n\times m}$ is the set of all $n \times m$ real matrices and $I_n \in R^{n\times n}$ is an identity matrix. $1_N = (1, 1, \ldots, 1)^T \in R^N$. A S.P.D. matrix $A$ or $A > 0$ means that $A$ is symmetric and positive definite. Also, a S.P.S-D. matrix $A$ or $A \geq 0$ means that $A$ is symmetric and positive semi-definite. $\lambda_{\min}(A)(\lambda_{\max}(A))$ denotes the s-mallest(largest) eigenvalue of S.P.S-D. matrix $A$. $\lambda_{>0\min}(A)$ means the smallest positive eigenvalue of S.P.S-D. matrix $A$. For $A \in R^{N\times N}$, let $\|A\|$ indicate the norm of $A$ induced by the Euclidean vector norm, i.e., $\|A\| = \sqrt{\lambda_{\max}(A^TA)}$. $A \otimes B$ denotes the Kronecker product of matrix $A$ and $B$. $d(x, S) = \inf_{y\in S} d(x, y)$ is the distance of a point from the manifold $S$ as given by the distance function $d$ of the embedding space. A function $\varphi$ defined in $[0, +\infty)$ is a $\kappa$ class function which means that $\varphi$ is continuous, strictly monotone increasing, $\varphi \geq 0$ and $\varphi(0) = 0$.

Let $G = (N_+, E, A)$ be a graph consisting of a vertex set $N_+ = \{1, 2, \ldots, N\}$, an edge set $E = \{(j, i) : i, j \in N_+\} \subset N_+ \times N_+$ and an adjacency matrix $A = (a_{ij}) \in R^{N\times N}$. An edge $(j, i) \in E$ implies that the agent $i$ can access the information of agent $j$. The set of neighbors of vertex $i$ is denoted by $N_i = \{j \in N_+ : (j, i) \in E, j \neq i\}$. The degree matrix $\Xi \in R^{N\times N}$ is a diagonal matrix with $\Xi_{ii} = \sum_{j\in N_i} a_{ij}$ and the Laplacian matrix $L = \Xi - A$ and $L = (l_{ij}), i, j \in N_+$. Also, a path between any two distinct vertices $i$ and $j$ is meant as a sequence of distinct edges of $G$ of the form $(i, k_1),\ (k_1, k_2), \ldots, (k_l, j)$. The graph is said to contain a directed spanning tree if there exists a vertex, $v_0$, such that every other vertex can be connected to $v_0$ by a directed path starting from $v_0$. Such a special vertex is then called a root node. $L$ has a simple zero eigenvalue if and only if the directed graph contains a spanning tree. In this paper, we assume that the graph has a directed spanning tree. When considering the leader-following case, we assume that the directed spanning tree with at least one non-zero pinning gain connecting to a root node. Define $d_i \geq 0,\ i \in N_+$, by the pinning gain coefficient and $D = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$ is the pinning matrix. And $d_i > 0$ when the $i$th agent is directed connected to the leader, while $d_i = 0$ otherwise.

## III. Inverse optimal control

As we know, the traditional first-order multi-agent system is

$$\dot{x}_i(t) = u_i(t), \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n$ and $u_i(t) = (u_{i1}(t), u_{i2}(t), \ldots, u_{in}(t))^T \in R^n$ are the state and the control input of $i$th agent at time $t$, respectively. Motivated by [25], we construct a hybrid controller $u_i(t) = u_i^{(1)}(t) + u_i^{(2)}(t)$ for (1) as follows.

$$u_i^{(1)}(t) = u_i^{(c)}(t)l_k(t),\ u_i^{(2)}(t) = \sum_{k=1}^{\infty} B_k x_i(t)\delta(t - t_k), \tag{2}$$

where $u_i^{(c)}(t)$ is the continuous control input. $l_k(t) = 1$ as $t \neq t_k$, and otherwise, $l_k(t) = 0$. $\delta(\cdot)$ is the Dirac impulse. Here $0 = t_0 < t_1 < t_2 < \ldots < t_{k-1} < t_k < \ldots$ and $\lim_{k\to\infty} t_k = +\infty$. Under the protocol (2), the system (1) becomes a multi-agent system with impulsive effects as follows.

$$\begin{cases} \dot{x}_i(t) = u_i(t),\ t \neq t_k, \\ \Delta x_i(t_k) = B_k x_i(t_k),\ k \in Z_+. \end{cases} \tag{3}$$

For simplicity, here we still denote $u_i^{(c)}(t)$ by $u_i(t)$. $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-)$ and $B_k \in R^{n\times n}$ is impulse matrix where $k \in Z_+$. Also, $x(t_0) = (x_1(t_0), x_2(t_0), \ldots, x_N(t_0))^T = x_0,\ x_i(t_k^+) = \lim_{t\to t_k^+} x_i(t)$ and $x_i(t_k) = x_i(t_k^-) = \lim_{t\to t_k^-} x_i(t)$.

For the single-integrator multi-agent systems with impulsive effects, an important and interesting problem is whether the distributed control which guarantees the consensus is optimal for some specified performance index or not. In this section, we will discuss this problem by the inverse optimal approach of the impulsive system. Following we will present a useful lemma about the inverse optimal approach of the impulsive system[24].

*Lemma 1:* Consider the impulsive system

$$\begin{cases} \dot{x}(t) = A_1x(t) + A_2u_c(t),\ x(t_0) = x_0,\ t \neq t_k, \\ \Delta x(t_k) = (A_{3k} - I_n)x(t_k) + A_{4k}u_d(t_k),\ k \in Z_+, \end{cases} \tag{4}$$

with quadratic hybrid performance functional

$$\begin{aligned} J(x_0, u_c(\cdot), u_d(\cdot)) = &\int_0^{+\infty}[x^T(t)Q_1x(t) + u_c^T(t)Ru_c(t)]dt \\ &+ \sum_{k\in Z_+}[x^T(t_k)Q_{3k}x(t_k) + u_d^T(t_k)Q_{4k}u_d(t_k)], \end{aligned} \tag{5}$$

where $(u_c(\cdot), u_d(\cdot))$ is a stable control with respect to a target manifold $S$. Here $A_1, A_2, A_{3k}$ and $A_{4k}$ are matrices with compatible dimension. Assume that there exists a positive semi-definite matrix $P^T = P$ such that

$$A_1^TP + PA_1 + Q_1 - PA_2R^{-1}A_2^TP = 0, \tag{6}$$

$$\begin{aligned} &A_{3k}^TPA_{4k}(Q_{4k} + A_{4k}^TPA_{4k})^{-1}A_{4k}^TPA_{3k} \\ &-A_{3k}^TPA_{3k} + P - Q_{3k} = 0. \end{aligned} \tag{7}$$

Then the hybrid feedback control law

$$\begin{aligned} &\phi_c(x(t)) = -R^{-1}A_2^TPx(t), t \neq t_k, \\ &\phi_d(x(t_k)) = -(Q_{4k} + A_{4k}^TPA_{4k})^{-1}A_{4k}^TPA_{3k}x(t_k) \end{aligned} \tag{8}$$

is asymptotically stable with respect to the manifold $S$ which is the null space of $P$. Moreover, (8) can make $J(x_0, u_c(\cdot), u_d(\cdot))$ subject to the system (4) achieve the minimal value $J(x_0, \phi_c(\cdot), \phi_d(\cdot)) = x_0^TPx_0$.

By using of the above lemma, next we will present the distributed optimal control results for both the leaderless and the leader-following cases, respectively. Firstly we will consider the following linear quadratic regular problem

$$\begin{aligned} J(x_0, u) = &\int_0^{+\infty}(x^T(t)Q_1x(t) + u^T(t)Ru(t))dt \\ &+ \sum_{k\in Z_+} x^T(t_k)Q_{3k}x(t_k) \end{aligned} \tag{9}$$

subject to the system (3). For a given control

$$u(t) = -(L \otimes K)x(t), \tag{10}$$

our purpose is to give the sufficient conditions to guarantee that the control law (10) minimizes the cost performance index (9) for some specified S.P.S-D. matrices $Q_1, Q_{3k}$ and $R = R_1 \otimes R_2$ where $R_1$ and $R_2$ are two given S.P.D. matrices.

*Theorem 1:* Consider the system (3) and the performance index (9). Suppose that $P_1 = R_1 L$ is a S.P.S-D. matrix and $P_2$ is a S.P.D. matrix which satisfies that $M_k = P_2 - (I_n + B_k)^T P_2 (I_n + B_k)$ is a S.P.S-D. matrix. Let the feedback gain matrix $K = R_2^{-1} P_2$. Then the control law (10) is optimal with respect to the following performance index

$$\begin{array}{l} J(x_0, u) = \int_0^{+\infty} [x^T(t)(P_1 R_1^{-1} P_1) \otimes (P_2 R_2^{-1} P_2) x(t) \\ +u^T(t)(R_1 \otimes R_2) u(t)] dt + \sum\limits_{k \in Z_+} x^T(t_k)(P_1 \otimes M_k) x(t_k), \end{array} \tag{11}$$

and is asymptotically stable with respect to the null space of $L \otimes I_n$. Moreover, if the graph $G$ has a directed spanning tree, the consensus is reached.

*Proof:* Let $A_1 = 0$, $A_2 = I_N \otimes I_n$, $A_{3k} = I_N \otimes (I_n + B_k)$, $A_{4k} = 0$, $R = R_1 \otimes R_2$ and $Q_{4k} = 0$. Let $P = P_1 \otimes P_2$. Here $P_1$ and $P_2$ are two matrices which satisfy all the conditions of Theorem 1. It is easy to verify that the conditions (6) and (7) hold. From Lemma 1, we obtain that $u = -(L \otimes K)x$ is optimal with respect to the performance index (11), i.e., $Q_1 = (P_1 R_1^{-1} P_1) \otimes (P_2 R_2^{-1} P_2)$ and $Q_{3k} = P_1 \otimes M_k$. Moreover, we obtain that under the distributed control law $u = -(L \otimes K)x$, the time derivative of the Lyapunov function $V(x) = x^T P x$ along the solution of the system (3) is

$$\begin{aligned} \dot{V}(x) &= -2x^T (P_1 \otimes P_2)(L \otimes K)x \\ &= -2x^T (P_1 \otimes P_2)(R_1^{-1} \otimes R_2^{-1})(P_1 \otimes P_2)x \\ &= -2x^T P R^{-1} P x, \ t \neq t_k \end{aligned}$$

and

$$\begin{array}{l} \quad V((I_N \otimes (I_n + B_k))x(t_k)) - V(x(t_k)) \\ = x^T(t_k)[P_1 \otimes ((I_n + B_k)^T P_2 (I_n + B_k) - P_2)] x(t_k) \\ = -x^T(t_k)(P_1 \otimes M_k) x(t_k), k \in Z_+ \end{array}$$

which imply asymptotic stability to the null space of $P$. Because $P_2$ and $R_1$ are nonsingular, it follows that the null space of $P$ equals the null space of $L \otimes I_n$. As a result, $u = -(L \otimes K)x$ is asymptotically stable with respect to the null space of $L \otimes I_n$. Furthermore, if the graph $G$ has a directed spanning tree, then the consensus is also achieved, thus we complete the proof.

■

*Remark 1:* If $B_k = 0$ in the system (3), i.e., the multi-agent systems don't have impulse, then we can choose $M_k = 0$ and the corresponding result is reduced to Theorem 5.2 in [11].

*Remark 2:* The conditions of Theorem 1 can be satisfied. For example, when $B_k = -pI_n (0 < p < 1)$, it naturally follows that $M_k$ is symmetric positive semi-definite for any $k \in Z_+$. Also, for the case that the graph is undirected and connected, we can choose $R_1 = I$. Then the condition that $R_1 L$ is a S.P.S-D. matrix can be satisfied.

Moreover, the optimal control result in Theorem 1 can be easily extended to the general first-order multi-agent systems as follows.

*Theorem 2:* Consider the system

$$\left\{ \begin{array}{l} \dot{x}(t) = (I_N \otimes F)x(t) + (I_N \otimes G)u(t), \ t \neq t_k, \\ \Delta x(t_k) = (I_N \otimes B_k)x(t_k), \ k \in Z_+ \end{array} \right. \tag{12}$$

and the performance index (9). Suppose that $P_1 = cR_1 L$ is a S.P.S-D. matrix where the positive coupling gain $c > \frac{c_1}{c_2}$. Assume that $P_2$ is a S.P.D. matrix which satisfies that $M_k = P_2 - (I_n + B_k)^T P_2 (I_n + B_k)$ and $N = P_2 G R_2^{-1} G^T P_2 - F^T P_2 - P_2 F$ are S.P.S-D. matrices. Let the feedback gain matrix $K = R_2^{-1} G^T P_2$. Then the distributed control law $u = -c(L \otimes K)x$ is optimal with respect to the following performance index

$$\begin{array}{l} J(x_0, u) = \int_0^{+\infty} (x^T(t) Q_1 x(t) + u^T(t)(R_1 \otimes R_2) u(t)) dt \\ \quad + \sum\limits_{k \in Z_+} x^T(t_k)(P_1 \otimes M_k) x(t_k), \end{array} \tag{13}$$

where $c_1 = \lambda_{\max}(R_1 L \otimes (N - K^T R K))$, $c_2 = \lambda_{>0\min}(L^T R_1 L \otimes (K^T R_2 K))$ and $Q_1 = c^2 L^T R_1 L \otimes (K^T R_2 K) - cR_1 L \otimes (F^T P_2 + P_2 F)$.

Lastly, for the leader-following case, we also can give the optimal control analysis. Now we will consider the leader-following case:

$$\left\{ \begin{array}{l} \dot{x}_i(t) = F x_i(t) + G u_i(t), \ t \neq t_k, \\ \Delta x_i(t_k) = B_k (x_i(t_k) - x_l(t_k)), \end{array} \right. \tag{14}$$

and

$$\dot{x}_l(t) = F x_l(t). \tag{15}$$

Here $x_i(t)$, $i \in N_+$ and $x_l(t)$ mean the position of the $i$th agent (follower) and the leader at time $t$, respectively. Let $e_i(t) = x_i(t) - x_l(t)$, $i \in N_+$, we can obtain the error system

$$\left\{ \begin{array}{l} \dot{e}(t) = (I_N \otimes F)e(t) + (I_N \otimes G)u(t), \ t \neq t_k, \\ \Delta e(t_k) = (I_N \otimes B_k)e(t_k), \ k \in Z_+. \end{array} \right. \tag{16}$$

Following we pay attention to the linear quadratic regular problem

$$\begin{array}{l} J(x_0, u) = \int_0^{+\infty} (x^T(t) \bar{Q}_1 x(t) + u^T(t) \bar{R} u(t)) dt \\ \quad + \sum\limits_{k \in Z_+} x^T(t_k) \bar{Q}_{3k} x(t_k) \end{array} \tag{17}$$

subject to the system (16). Here $\bar{Q}_1, \bar{Q}_{3k}$ are S.P.S-D. matrices and $\bar{R} = \bar{R}_1 \otimes \bar{R}_2$ where $\bar{R}_1$ and $\bar{R}_2$ are S.P.D. matrices. It is not difficult for us to obtain the following result.

*Theorem 3:* Consider the system (16) and the performance index (17). Suppose that $\bar{P}_1 = c\bar{R}_1(L+D)$ is a S.P.D. matrix, where the positive coupling gain $c$ satisfies $c > \frac{c_3}{c_4}$. Assume that $\bar{P}_2$ is a S.P.D. matrix which satisfies that are S.P.S-D. matrices. Let the feedback gain matrix $\bar{K} = \bar{R}_2^{-1} G^T \bar{P}_2$. Then the distributed protocol $u = -c((L + D) \otimes \bar{K})e(t)$ is optimal with respect to the following performance index

$$\begin{array}{l} J(x_0, u) = \int_0^{+\infty} (e^T(t) \bar{Q}_1 e(t) + u^T(t)(\bar{R}_1 \otimes \bar{R}_2) u(t)) dt \\ \quad + \sum\limits_{k \in Z_+} e^T(t_k)(\bar{P}_1 \otimes \bar{M}_k) e(t_k). \end{array} \tag{18}$$

Here we have $c_3 = \lambda_{\max}(\bar{R}_1(L + D) \otimes (\bar{N} - \bar{K}^T \bar{R}_2 \bar{K}))$, $c_4 = \lambda_{>0\min}((L + D)^T \bar{R}_1 (L + D) \otimes (\bar{K}^T \bar{R}_2 \bar{K}))$ and $\bar{Q}_1 = c^2 (L+D)^T \bar{R}_1 (L+D) \otimes (\bar{K}^T \bar{R}_2 \bar{K}) - c\bar{R}_1 (L+D) \otimes (F^T \bar{P}_2 + \bar{P}_2 F)$.

*Remark 3:* In Theorem 2-3, we extend the corresponding result in [10] and [11] to the impulsive case.

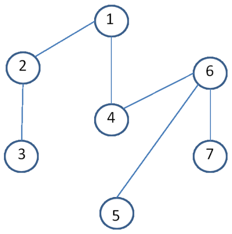


Fig. 1. The communication topology $G_1$

## IV. Example

In this section, we give an example to show the feasibility of our main results. We choose the communication topology as $G_1$ and consider a multi-agent system consisting of seven agents with the dynamics depicted in (3) where $n = 2$. Also, $B_k = -0.1I_2$, $t_k - t_{k-1} = 0.3$, $R_1 = I_7$ and $R_2 = P_2 = I_2$. We have given the comparison between the special performance index (11) and other kinds of performance indexes. For example, in Table 1, under the control law (10), we have presented the performance indexes (11), $J_1 = \int_0^{+\infty}(x^T(t)(P_1R_1^{-1}P_1) \otimes (P_2R_2^{-1}P_2)x(t) + u^T(t)u(t))dt + \sum\limits_{k\in Z_+} x^T(t_k)x(t_k)$ and $J_2 = \int_0^{+\infty}(x^T(t)x(t) + u^T(t)u(t))dt + \sum\limits_{k\in Z_+} x^T(t_k)x(t_k)$, respectively. From Table 1, one can see that the value of the performance index (11) is less than that of the traditional performance index under three different initial cases, i.e., $(a)(0\ 1\ 1\ 1\ \ -1\ 0\ 0\ \ -1\ \ -1\ \ -1\ 2\ 0\ 1\ 1)$, $(b)(0.4\ 0.5\ 1.5\ 1\ \ -0.5\ 0\ 0\ 1\ \ -0.5\ \ -1\ 1\ 0.5\ \ -1\ 0)$ and $(c)(-0.5\ 0.6\ 1\ 0.5\ 0.35\ 0\ 0.5\ 0.55\ -1\ 0.5\ -1\ -0.45\ 0\ -0.65)$.

TABLE I
THE PERFORMANCE INDEX UNDER DIFFERENT INITIAL CONDITIONS

| | $J$ | $J_1$ | $J_2$ |
|---|---|---|---|
| case $(a)$ | 0.0612 | 2.3818 | 19.5677 |
| case $(b)$ | 0.0874 | 0.5164 | 3.7680 |
| case $(c)$ | 0.2296 | 11.2853 | 92.7865 |

## V. Conclusion

In this paper, we have investigated the distributed optimal control for multi-agent system with impulsive effects. We have provided the sufficient conditions to ensure that the consensus protocol can make the specified performance index reach minimal value. In addition, for the general first-order multi-agent system, we have discussed the optimal cooperative regulator and the optimal cooperative tracker, respectively. Lastly, an example has been presented to illustrate our main results.

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