Stabilization and L$_2$-gain for a Class of Switched Systems with Nonlinear Disturbance and Time-varying Delay

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Abstract—**This paper focuses on the problem of exponential stabilization and L_2-gain analysis for a class of switched nonlinear systems with time-varying delay. Based on the Lyapunov stability theory, a novel multi-Lyapunov-Krasovskii functional dependent on the size of time delay is constructed. Specifically, the integer terms in the multi-Lyapunov-Krasovskii functional that taking the bound of upper and lower about delay are processed with the help of delay decomposition technique. Moreover, by utilizing the free-weighting matrix and the average dwell time approach, some sufficient conditions in forms of linear matrix inequalities are presented to ensure the exponential stability with weighted L_2-gain performance of the switched nonlinear system. Based on the obtained results, the stabilizing feedback controllers of switched nonlinear systems are designed through special operations of matrices and Schur complement. Finally, two numerical examples and an application example are carried out to demonstrate the effectiveness of the proposed method in this paper.**

Index Terms—**Switched nonlinear systems, Exponential stabilization, L_2-gain, Delay decomposition approach, Average dwell time.**

I. Introduction

IT is well known that hybrid systems are becoming increasingly important in contemporary society both in science and technology. Switched system is one of the special hybrid systems, which is composed of a family of continuous-time or discrete-time subsystems and a switching sequence that orchestrates the switching among the subsystems to ensure stability. In the last decades, switched system has attracted extensive attention from domestic and foreign scientific research due mainly to two aspects. Firstly, the switched system has been extensively applied in power systems, engineering systems and physical systems, such as flight control, artificial intelligence, communication, networked control, power electronic and automatic highway [1-4]. Secondly, some complex systems can be simplified into multiple switching subsystems for easy research and analysis. So far, many important advances and significant achievements for various types of switched systems have been studied [5-9].

Most physical systems are nonlinear dynamical systems with time delay. Time delay and nonlinearity are inevitable in some practical systems, which are often the main cause for instability, oscillation and undesirable system performance of a dynamical system. Hence, nonlinearity naturally makes the analysis and control design of nonlinear systems with time delay more complicated. And as far as the many properties of dynamical systems are concerned, it is worth mentioning that stability analysis is one of the fundamental problems. During the past few decades, many results concerning stability analysis of systems with time delay and nonlinearity problems have been reported[10-14]. With respect to those problem, we just mention here some representative work. [15] investigate the problems of robust stability and robust stabilization of uncertain neutral systems with distributed delays, and new delay-dependent sufficient conditions for robust stability and robust stabilization are formulated in terms of linear matrix inequalities(LMIs). By following this idea, robust guaranteed cost control for a class of uncertain neutral system with time-varying delays is investigated in [16], and delay-dependent and delay-independent criteria are proposed for the stabilization of considered systems, and state feedback control is considered to stabilize the uncertain neutral system and upper bounds on the closed-loop cost function are also given. Based on this approach, in [17], the problem of robust stability for a class of stochastic interval neural networks with discrete and distributed time-varying delays is discussed. Moreover, by using the It$\hat{o}$ differential formula and stochastic stability theory, new delay-range-dependent criteria for stochastic interval neural networks with time-varying delays are also derived in [17]. And the method therein reduces the conservativeness of methods involving a fixed model transformation. [18] focuses on the issue of robustly exponential stability for uncertain neutral systems with mixed time-varying delays and nonlinear perturbations by applying an integral inequality. Based on the above discussion, the theory of time-delay systems can be divided into two classes: delay independent control and delay dependent control. To the best of our knowledge, delay-dependent stabilization condition gives less conservative result than the delay-independent one as it makes full use of information of the system. Specifically, switched systems with time-delay and nonlinearity have a deep engineering background and social applications. However, results concerning switched systems with time delay and nonlinearity problems are relatively infrequent. This is motivated by the need for systematic approach to investigate switched nonlinear systems with delay.

On the other hand, since exogenous disturbance is encountered in various physical and engineering systems and often result in instability and performance degradation. The L_2-gain problem for a variety of dynamical systems

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has received increasing attention [19-22], which is quite practical and energy efficient. With the great development of switched systems, it is noted that some valid results have appeared to studying L_2-gain analysis of nonlinear switched systems with different performance in the past few years. For instance, [23] study stability and L_2-gain for a class of switched systems with time-varying delays, and sufficient conditions with delay-dependent for the exponential stability and weighted L_2-gain are obtained by a common Lyapunov functional (CLF), which may lead to certain degree of conservatism. To address this issue, the problem of exponential stabilization and L_2-gain for a class of uncertain switched nonlinear systems with time-varying delay is studied in [24] by multiple Lyapunov functional (MLF) and the dwell time approach. Recently analogous results are found in [25], finite-time boundedness problem for switched neural networks subject to L_2-gain disturbance is considered. Moreover, some new delay-dependent criteria guaranteeing finite-time boundedness and stabilizability with L_2-gain analysis performance are developed by resorting to the average dwell time approach and Lyapunov-Krasovskii functional technology. In addition, some improved results were obtained in [26], since the switching instants of the controllers lag behind those of the subsystems, [26] consider the stability and L_2-gain analysis problem for a class of switched linear systems under asynchronous switching. However, the results mentioned above, the problem of exponential stabilization and L_2-gain analysis for a class of switched nonlinear systems with time-varying delay has not been well reported and remains important.

This paper is concerned with the problem of exponential stabilization and L_2-gain analysis for a class of switched nonlinear systems with time-varying delay. Compared with the existing results, we mainly consider the following issues in this paper. Firstly, we consider interval time-varying delay. It is natural to look for an alternative view to derive a less conservative condition for exponential stabilization of nonlinear switched systems with interval time-varying delay. Therefore, a novel multi-Lyapunov-Krasovskii functional dependent on the size of time delay is constructed based on the Lyapunov stability theory. Secondly, motivated by delay-dependent and Jensens Inequality technique, we obtain less conservative sufficient conditions for exponential stability and a guaranteed weighted L_2-gain disturbance attenuation performance of the switched nonlinear systems with time-varying delay under the average dwell time approach. Specifically, we design feedback controller for switched nonlinear systems with time-varying delay by matrix deformation technique and Schur compensation.

The remainder of this paper is organized as follows. In Sections 2, the problem description and preliminaries and some necessary lemmas are presented. Section 3 is devoted to derive the results on exponential stabilization L_2-gain analysis for switching signals by the average dwell time approach and delay-dependent multi-Lyapunov-Krasovskii functional, and feedback controller was designed of nonlinear switched nonlinear systems with interval time-varying delay, which is the main result of this paper. In Section 4, some examples are carried out to illustrate the effectiveness of the proposed approach. The paper is concluded in Section 5.

II. Problem formulation and preliminaries

In this section, a class of switched nonlinear systems with interval-time-varying delay is considered, which is represented as follows:

$$\begin{aligned}\dot{x}(t) &= A_{1\sigma(t)} + A_{2\sigma(t)}x(t-h(t)) + C_{\sigma(t)}u(t)\\ &\quad + \omega(t) + f(t, x(t-h(t)))\\ x(t) &= \varphi(t), t \in [-h_M, 0],\\ z(t) &= D_{\sigma(t)}x(t) + F_{\sigma(t)}x(t).\end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $z(t) \in R^n$ is the measured output, $\omega(t) \in L_2[0,\infty)$ is an exogenous disturbance, the switching signal $\sigma(t) : [0,\infty] \to L = \{1,2,\ldots,l\}$ is a piecewise continuous (from the right) function, where l is the number of subsystems. Specifically, denote, $\sigma(t) : \{(t_0,\sigma(t)),\cdots,(t_k,\sigma(t)),\cdots,k=0,1,2,\cdots\}$, where t_0 is the initial switching instant and t_k denotes the kth switching instant. For any $i \in M, A_{1i}, A_{2i}, C_i, D_i, F_i$ are constant matrices. $\varphi(s) \in R^n$ is the initial condition, $h(t)$ denotes the time-varying delay satisfying $0 \le h_m \le h(t) \le h_M, \dot{h}(t) \le h < 1$. $f(t, x(t-h(t)))$ is an nonlinear perturbation function, which satisfies

$$\|f_i(t, x(t-h(t)))\| \le |V_i x(t-h(t))\|, \tag{2}$$

where V_i are known real constant matrices.

For system (1), we consider the state feedback given by

$$u(t) = K_{\sigma(t)}x(t). \tag{3}$$

Combining(1) with (3), the closed-loop system of ith subsystem with the ith controller can be expressed as

$$\begin{aligned}\dot{x}(t) &= (A_{1i} + C_iK_i)x(t) + A_{2i}x(t-h(t))\\ &\quad + \omega(t) + f(t, x(t-h(t))).\end{aligned} \tag{4}$$

For convenience of discussion, we denote $\bar{A}_{1i} = A_{1i} + C_iK_i$. Then, we obtain

$$\dot{x}(t) = \bar{A}_{1i} + A_{2i}x(t-h(t)) + \omega(t) + f(t, x(t-h(t))). \tag{5}$$

For the sake of facilitating the description and proofing the main results, we now introduce the following definitions and lemmas.

Definition 1.([22]) *The equilibrium $x^* = 0$ of system (1) with $\omega(t) = 0$ is said to be exponentially stable under switching signal $\sigma(t)$, if there exist constants $k \ge 1, \lambda > 0$ such that every solution $x(t)$ of system(1) satisfies that*

$$\|x(t)\| \le k \sup_{-H_M \le \theta \le 0} \|x(t_0+\theta)\| e^{-\lambda(t-t_0)}, \forall t \ge t_0.$$

Definition 2.([25]) *For any $T_2 > T_1 \ge 0$, $N_\sigma(T_1,T_2)$ is the switching number of $\sigma(t)$ on an interval (T_1,T_2). If*

$$N_\sigma(T_1,T_2) \le N_0 + (T_2-T_1)/\tau_\alpha, \tag{6}$$

holds for given $N_0 \ge 0, \tau_\alpha \ge 0$, then the constant τ_α is called the average dwell time and N_0 is the chatter bound. Without loss of generality, we choose $N_0 = 0$ in this paper.

Definition 3.([26]) *For $\alpha > 0$ and $\gamma > 0$, the switched system (1) is said to have weighted L_2-gain γ, if under zero initial condition $\varphi(t) = 0, t \in [-h_M, 0]$, it holds that*

$$\int_0^\infty e^{-\alpha s} Z^T(s)Z(s)ds \le \gamma^2 \int_0^\infty \omega^T(s)\omega(s)ds \tag{7}$$

Lemma 1.([13]) *For any symmetric and positive definite constant matrix $G \in R^{l\times l}$ and scalar $0 \leq r(t) \leq r$, if there exists a vector function $\chi : [0, r] \to R^l$ such that integrals $\int_0^{r(t)} \chi^T G\chi(s)ds$ and $\int_0^{r(t)} \chi(s)ds$ are well defined, then the following inequality holds*

$$(\int_0^{r(t)} \chi(s)ds)\chi^T G(\int_0^{r(t)} \chi(s)ds) \leq r\int_0^{r(t)} \chi^T G\chi(s)ds.$$

Lemma 2.([27]) *Given constant matrices $\Omega_1, \Omega_2, \Omega_3$, where $\Omega_1 = \Omega_1^T$ and $\Omega_2 = \Omega_2^T > 0$, then $\Omega_1 + \Omega_3^T\Omega_2^{-1}\Omega_3 < 0$ if and only if*

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0.$$

III. Main results

In the section, the sufficient conditions in forms of linear matrix inequalities is presented to ensure the exponential stability with weighted L_2-gain performance for the switched nonlinear systems (1).

A. Stability analysis

In the following, applying the average dwell time approach, we give sufficient conditions for the exponential stabilization of system (1) with $\omega(t) = 0$.

Theorem 1. *For given positive constants α, ε and $\mu \geq 1$, if there exist symmetric and positive definite matrices $P_i, Q_{1i}, Q_{2i}, Q_{3i}, R_{1i}, R_{2i}$ such that the following matrix inequalities hold for all $i, j \in M, i \neq j$,*

$$\begin{aligned} &P_i \leq \mu P_j, Q_{1i} \leq \mu Q_{1j}, Q_{2i} \leq \mu Q_{2j}, \\ &Q_{3i} \leq \mu Q_{3j}, R_{1i} \leq \mu R_{1j}, R_{2i} \leq \mu R_{2j}, \end{aligned} \tag{8}$$

$$\Xi_i = \begin{pmatrix} \phi_{11}^i & \phi_{12}^i & 0 & 0 & P_i & 0 & 0 \\ * & \phi_{22}^i & 0 & 0 & 0 & 0 & 0 \\ * & * & \phi_i^{33} & 0 & 0 & 0 & 0 \\ * & * & * & \phi_i^{44} & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 & 0 \\ * & * & * & * & * & \phi_{66}^i & 0 \\ * & * & * & * & * & * & \phi_i^{77} \end{pmatrix} < 0 \tag{9}$$

where

$$\begin{aligned} \phi_{11}^i &= \bar{A}_{1i}^T P_i + P_i\bar{A}_{1i} + Q_{1i} + Q_{2i} + Q_{3i} + {h_m}^2 R_{1i} \\ &\quad + {h_M}^2 R_{2i} + \alpha P_i, \\ \phi_{12}^i &= P_i A_{2i}, \quad \phi_{22}^i = \varepsilon V_i^T V_i - (1-d)e^{-\alpha h_M} Q_{1i}, \\ \phi_{33}^i &= -e^{-\alpha h_m} Q_{2i}, \phi_{44}^i = -e^{-\alpha h_M} Q_{3i}, \\ \phi_{66}^i &= -e^{-\alpha h_m} R_{1i}, \quad \phi_{77}^i = -e^{-\alpha h_M} R_{2i}, \end{aligned}$$

then system (1) is exponentially stabilizable under the feedback control (3) for any switching signal with the average dwell time satisfying

$$\tau_a > \tau_a^* = \frac{\ln\mu}{\alpha} \tag{10}$$

$$\begin{aligned} \tilde{\varphi}_{11i} &= \hat{A}_{1i}^T P_i + P_i\hat{A}_{1i} + Q_{1i} + Q_{2i} + Q_{3i} + \delta^2 R_i \\ &\quad + \alpha P_i + \Gamma^T\Gamma, \\ \tilde{\varphi}_{22i} &= -(1-d)e^{-\alpha h_M} Q_{1i} + \Lambda^T\Lambda, \\ \varphi_{33i} &= -e^{-\alpha h_m} Q_{2i}, \quad \varphi_{44i} = -e^{-\alpha h_M} Q_{3i}. \end{aligned}$$

Then system (7) is exponentially stabilizable under the feedback control (6) for any switching signal with the average dwell time satisfying

$$\tau_a > \tau_a^* = \frac{\ln\mu}{\alpha}. \tag{11}$$

Proof: We choose the multi-Lyapunov-Krasovskii functional candidate as follows:

$$V(t) = V_{\sigma(t)}(t) = \sum_{s=1}^{6} V_{j\sigma(t)}(t), \tag{12}$$

where

$$\begin{aligned} V_{1\sigma(t)}(t) &= x^T(t)P_{\sigma(t)}x(t), \\ V_{2\sigma(t)}(t) &= \int_{t-h(t)}^t e^{\alpha(s-t)}x^T(s)Q_{1\sigma(t)}x(s)ds, \\ V_{3\sigma(t)}(t) &= \int_{t-h_m}^t e^{\alpha(s-t)}x^T(s)Q_{2\sigma(t)}x(s)ds, \\ V_{4\sigma(t)}(t) &= \int_{t-h_M}^t e^{\alpha(s-t)}x^T(s)Q_{3\sigma(t)}x(s)ds, \\ V_{5\sigma(t)}(t) &= h_m\int_{-h_m}^0\int_{t+\theta}^t e^{\alpha(s-t)}x^T(s)R_{1i}x(s)dsd\theta, \\ V_{6\sigma(t)}(t) &= h_M\int_{-h_M}^0\int_{t+\theta}^t e^{\alpha(s-t)}x^T(s)R_{2i}x(s)dsd\theta. \end{aligned}$$

We can get the time derivative of $V_\sigma(t)$ as follows.

$$\begin{aligned} \dot{V}_{1i} &= 2x^T(t)P_i\dot{x}(t), \\ \dot{V}_{2i} &= -(1-\dot{h}(t))e^{-\alpha h(t)}x^T(t-h(t))Q_{1i}x(t-h(t)) \\ &\quad + x^T(t)Q_{1i}x(t) - \alpha\textstyle\int_{t-h(t)}^t e^{\alpha(s-t)}x^T(s)Q_{1i}x(s)ds \\ &\leq -(1-d)e^{-\alpha h_M}x^T(t-h(t))Q_{1i}x(t-h(t)) \\ &\quad + x^T(t)Q_{1i}x(t) - \alpha\textstyle\int_{t-h(t)}^t e^{\alpha(s-t)}x^T(s)Q_{1i}x(s)ds, \\ \dot{V}_{3i} &= -e^{-\alpha h_m}x^T(t-h_m)Q_{2i}x(t-h_m) + x^T(t)Q_{2i}x(t) \\ &\quad - \alpha\textstyle\int_{t-h_m}^t e^{\alpha(s-t)}x^T(s)Q_{2i}x(s)ds, \\ \dot{V}_{4i} &= -e^{-\alpha h_M}x^T(t-h_M)Q_{3i}x(t-h_M) + x^T(t)Q_{3i}x(t) \\ &\quad - \alpha\textstyle\int_{t-h_M}^t e^{\alpha(s-t)}x^T(s)Q_{3i}x(s)ds, \\ \dot{V}_{5i} &= -\alpha h_m\textstyle\int_{-h_m}^0\int_{t+\theta}^t e^{\alpha(s-t)}x^T(s)R_{1i}x(s)dsd\theta \\ &\quad - h_m\textstyle\int_{-h_m}^0 e^{\alpha\theta}x^T(t+\theta)R_{1i}x(t+\theta)d\theta \\ &\quad + h_m^2 x^T(t)R_{1i}x(t) \\ &\leq -\alpha h_m\textstyle\int_{-h_m}^0\int_{t+\theta}^t e^{\alpha(s-t)}x^T(s)R_{1i}x(s)dsd\theta \\ &\quad - h_m\textstyle\int_{t-h_m}^t e^{-\alpha h_m}x^T(s)R_{1i}x(s)ds \\ &\quad + h_m^2 x^T(t)R_{1i}x(t), \\ \dot{V}_{6i} &= -\alpha h_M\textstyle\int_{-h_M}^0\int_{t+\theta}^t e^{\alpha(s-t)}x^T(s)R_{2i}x(s)dsd\theta \\ &\quad - h_M\textstyle\int_{-h_M}^0 e^{-\alpha h_M}x^T(t+\theta)R_{2i}x(t+\theta)d\theta \\ &\quad + h_M^2 x^T(t)R_{2i}x(t) \\ &\leq -\alpha h_M\textstyle\int_{-h_M}^0\int_{t+\theta}^t e^{\alpha(s-t)}x^T(s)R_{2i}x(s)dsd\theta \\ &\quad - h_M\textstyle\int_{t-h_M}^t e^{-\alpha h_M}x^T(s)R_{2i}x(s)ds \\ &\quad + h_M^2 x^T(t)R_{2i}x(t). \end{aligned} \tag{13}$$

The inequality (2) can be written as

$$\begin{aligned} &\varepsilon x^T(t-h(t))V_i^T V_i x(t-h(t)) \\ &\quad - \varepsilon f^T(t, x(t-h(t)))f(t, x(t-h(t))) \geq 0. \end{aligned} \tag{14}$$

where ε is known positive constants.

By Lemma 2, it is clear that

$$\begin{aligned}&-h_m\int_{t-h_m}^{t}e^{-\alpha h_m}x^T(s)R_{1i}x(s)ds\\&\le -e^{-\alpha h_m}\left[\int_{t-h_m}^{t}x(s)ds\right]^T R_{1i}\left[\int_{t-h_m}^{t}x(s)ds\right],\\&-h_M\int_{t-h_M}^{t}e^{-\alpha h_M}x^T(s)R_{2i}x(s)ds\\&\le -e^{-\alpha h_M}\left[\int_{t-h_M}^{t}x(s)ds\right]^T R_{2i}\left[\int_{t-h_M}^{t}x(s)ds\right],\end{aligned} \tag{15}$$

We can obtain the following inequalities by combining (14) with (15).

$$\begin{aligned}\dot V(t) &+ \alpha V(t)\\&\le x^T(t)[\bar A_{1i}^T P_i + P_i\bar A_{1i} + Q_{1i} + Q_{2i}\\&+ Q_{3i} + h_m^2 R_{1i} + h_M^2 R_{2i} + \alpha P_i]x(t)\\&+ x^T(t-h(t))[\varepsilon V_i^T V_i - (1-d)\\&\times e^{-\alpha h_M}Q_{1i}]x(t-h(t))\\&- e^{-\alpha h_M}x^T(t-h_M)Q_{3i}x(t-h_M)\\&- f^T(t,x(t-h(t)))f(t,x(t-h(t)))\\&+ x^T(t)P_i f(t,x(t-h(t)))\\&+ f^T(t,x(t-h(t)))P_i x(t)\\&+ x^T(t-h(t))A_{2i}^T P_i x(t)\\&- e^{-\alpha h_m}x^T(t-h_m)Q_{2i}x(t-h_m)\\&- e^{-\alpha h_m}\left[\int_{t-h_m}^{t}x(s)ds\right]^T R_{1i}\left[\int_{t-h_m}^{t}x(s)ds\right]\\&- e^{-\alpha h_M}\left[\int_{t-h_M}^{t}x(s)ds\right]^T R_{2i}\left[\int_{t-h_M}^{t}x(s)ds\right],\end{aligned} \tag{16}$$

Therefore,

$$\dot V_i(t) + \alpha V_i(t) \le \zeta_i^T \Xi_i \zeta_i, \tag{17}$$

where Ξ_i is given by (9), and

$$\begin{aligned}\zeta_i = [&x^T(t)\quad x^T(t-h(t))\quad x^T(t-h_m)\quad x^T(t-h_M)\\&f^T(t,x(t-h(t)))\quad (\int_{t-h_m}^{t}x(s)ds)^T\quad (\int_{t-h_M}^{t}x(s)ds)^T]^T.\end{aligned} \tag{18}$$

So we get

$$\dot V_i(t) + \alpha V_i(t) < 0.$$

Then,

$$(e^{\alpha t}V_i(t))' = \alpha e^{\alpha t}V_i(t) + e^{\alpha t}\dot V_i(t) < 0. \tag{19}$$

When $t \in [t_k, t_{k+1})$, integrating both sides of (19) from t_k to t, we get

$$V_{\sigma(t)}(t) \le V_{\sigma(t_k)}(t_k)e^{-\alpha(t-t_k)}, \quad t_k \le t < t_{k+1}. \tag{20}$$

Using (8), at the switching time t_k, we have

$$V_i(t_k) \le \mu V_j({t_k}^-), \quad \forall i,j \in N, k = 1,2,\ldots. \tag{21}$$

Therefore, recalling the relation $L = N_\sigma(t_0,t) \le (t-t_0)/\tau_\alpha$, it follows that

$$\begin{aligned}V_{\sigma(t)}(t) &\le \mu V_{\sigma({t_k}^-)}({t_k}^-)e^{-\alpha(t-t_k)} \le \cdots\\&\le \mu^L V_{\sigma(t_0)}(t_0)e^{-\alpha(t-t_0)}\\&\le e^{-(t-t_0)(\alpha-\ln\mu/\tau_a)}V_{\sigma(t_0)}(t_0)\end{aligned} \tag{22}$$

Using (12), we get

$$V_i(t) \ge a\|x(t)\|^2, \quad V_{\sigma(t_0)}(t_0) \le b\sup_{-h_M\le\theta\le 0}\|x(t_0+\theta)\|^2,$$

where

$$\begin{aligned}a &= \min_{i\in N}\lambda_{\min}(P_i),\\b &= \max_{i\in N}\lambda_{\max}(P_i) + h_M\max_{i\in N}\lambda_{\max}(Q_{1i})\\&\quad + h_m\max_{i\in N}\lambda_{\max}(Q_{2i}) + h_M\max_{i\in N}\lambda_{\max}(Q_{3i})\\&\quad + \tfrac{h_m^3}{2}\max_{i\in N}\lambda_{\max}(R_{1i}) + \tfrac{h_M^3}{2}\max_{i\in N}\lambda_{\max}(R_{2i}).\end{aligned}$$

Hence,

$$\|x(t)\| \le \sqrt{\frac{b}{a}}\sup_{-h_M\le\theta\le 0}\|x(t_0+\theta)\|\,e^{-\frac{1}{2}\left(\alpha-\frac{ln\mu}{\tau_a}\right)(t-t_0)}. \tag{23}$$

By Definition 1, we know that system (1) is exponentially stable.

Remark 1. *Based on the Lyapunov stability theory, a novel multi-Lyapunov-Krasovskii functional dependent on the size of time delay is constructed. It is noticed that the Lyapunov-Krasovskii functional is delay-dependent. Furthermore, the important information of h_m and h_M are taken into a full consideration, which derive a less conservative condition of exponential stabilization for nonlinear switched systems with $\omega(t) = 0$. Moreover, the exponential stabilization criterion is necessary for the L_2-gain analysis of system (1).*

B. L_2-gain analysis

In the section, we present the sufficient conditions of the exponential stabilization with L_2-gain property for the system (1).

Theorem 2. *For given constants $\alpha > 0, \gamma > 0, \varepsilon > 0$ and $\mu \ge 1$, if there exist symmetric and positive definite matrices $P_i, Q_{1i}, Q_{2i}, Q_{3i}, R_{1i}, R_{2i}$, such that the following matrix inequalities hold for all $i, j \in M$,*

$$\begin{aligned}&P_i \le \mu P_j, Q_{1i} \le \mu Q_{1j}, Q_{2i} \le Q_{2j},\\&Q_{3i} \le \mu Q_{3j}, R_{1i} \le \mu R_{1j}, R_{2i} \le \mu R_{2j},\end{aligned} \tag{24}$$

$$\Xi_i = \begin{pmatrix}\tilde\phi_{11}^i & \tilde\phi_{12}^i & 0 & 0 & P_i & \tilde\phi_{16}^i & 0 & 0\\ * & \tilde\phi_{22}^i & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & \tilde\phi_{33}^i & 0 & 0 & 0 & 0 & 0\\ * & * & * & \tilde\phi_{44}^i & 0 & 0 & 0 & 0\\ * & * & * & * & -\varepsilon I & 0 & 0 & 0\\ * & * & * & * & * & \tilde\phi_{66}^i & 0 & 0\\ * & * & * & * & * & * & \tilde\phi_{77}^i & 0\\ * & * & * & * & * & * & * & \tilde\phi_{88}^i\end{pmatrix} < 0 \tag{25}$$

where

$$\begin{aligned}\phi_{11}^i &= \bar A_{1i}^T P_i + P_i\bar A_{1i} + Q_{1i} + Q_{2i} + Q_{3i}\\&\quad + {h_m}^2 R_{1i} + {h_M}^2 R_{2i} + \alpha P_i + D_i^T D_i\\\phi_{12}^i &= P_i A_{2i}, \quad \phi_{16}^i = P_i + D_i^T F_i,\\\phi_{22}^i &= \varepsilon V_i^T V_i - (1-d)e^{-\alpha h_M}Q_{1i}, \phi_{33}^i = -e^{-\alpha h_m}Q_{2i},\\\phi_{44}^i &= -e^{-\alpha h_M}Q_{3i}, \quad \phi_{66}^i = F_i^T F_i - \gamma^2 I,\\\phi_{77}^i &= -e^{-\alpha h_m}R_{1i}, \quad \phi_{88}^i = -e^{-\alpha h_M}R_{2i},\end{aligned}$$

then the system (1)is exponentially stabilizable and has weighted L_2-gain γ under the feedback control(3) for any switching signal with the average dwell time defined (10).

Proof: By Theorem 1, the exponential stabilizable of system (1)with $\omega(t) = 0$ is ensured. To show the weighted L_2-gain, we choose Lyapunov-Krasovskii functional as (12). We have

$$\begin{aligned}
\dot{V}_i(t) &+ \alpha V_i(t) + Z^T(t)Z(t) - \gamma^2\omega^T(t)\omega(t)\\
&\leq x^T(t)[\bar{A}_{1i}^T P_i + P_i\bar{A}_{1i} + Q_{1i} + Q_{2i} + Q_{3i}\\
&+ h_m^2 R_{1i} + h_M^2 R_{2i} + \alpha P_i + D_i^T D_i]x(t)\\
&- e^{-\alpha h_m} x^T(t-h_m)Q_{2i}x(t-h_m)\\
&+ x^T(t-h(t))[\varepsilon V_i^T V_i - (1-d)e^{-\alpha h_M}\\
&\times x(t-h(t)) - e^{-\alpha h_M} x^T(t-h_M)Q_{3i}x(t-h_M)\\
&+ x^T(t)(P_i + D_i^T F_i)\omega(t)\\
&+ x^T(t)P_i A_{2i} x(t-h(t))\\
&+ \omega^T(t)(F_i^T D_i + P_i)x(t)\\
&+ \omega^T(t)(F_i^T F_i - \gamma^2 I)\omega(t)Q_{1i}]\\
&- f^T(t, x(t-h(t)))f(t, x(t-h(t)))\\
&+ x^T(t)P_i f(t, x(t-h(t)))\\
&+ f^T(t, x(t-h(t)))P_i x(t)\\
&+ x^T(t-h(t))A_{2i}^T P_i x(t)\\
&- e^{-\alpha h_m}\left[\int_{t-h_m}^t x(s)ds\right]^T R_{1i}\left[\int_{t-h_m}^t x(s)ds\right]\\
&- e^{-\alpha h_M}\left[\int_{t-h_M}^t x(s)ds\right]^T R_{2i}\left[\int_{t-h_M}^t x(s)ds\right],
\end{aligned} \tag{26}$$

Considering equations (39),we have

$$\dot{V}_i(t) + \alpha V_i(t) + Z^T(t)Z(t) - \gamma^2\omega^T(t)\omega(t) \leq \eta^T\bar{\Xi}_i\eta(t), \tag{27}$$

where

$$\begin{aligned}
\eta(t) = [&x^T(t) \quad x^T(t-h(t)) \quad x^T(t-h_m) \quad x^T(t-h_M)\\
&f^T(t, x(t-h(t))) \quad \omega^T(t)\\
&(\int_{t-h_m}^t x(s)ds)^T \quad (\int_{t-h_M}^t x(s)ds)^T]^T.
\end{aligned} \tag{28}$$

It is easy to get

$$\dot{V}_i(t) + \alpha V_i(t) + Z^T(t)Z(t) - \gamma^2\omega^T(t)\omega \leq 0. \tag{29}$$

When $t \in [t_t, t_{k+1})$,integrating both sides of (29)from t_k to t we have

$$V(t) \leq e^{-\alpha(t-t_k)}V(t_k) - \int_{t_k}^t e^{-\alpha(t-s)}\Delta(s)ds, \tag{30}$$

where $\Delta(s) = Z^T(t)Z(t) - \gamma^2\omega^T\omega(t)$.

Combining (21) and (30), we can get

$$\begin{aligned}
V_i(t) &\leq \mu e^{-\alpha(t-t_k)}V(t_k^-) - \int_{t_k}^t e^{-\alpha(t-s)}\Delta(s)ds\\
&\leq \mu^k V(t_0)e^{-\alpha t} - \mu^k\int_0^{t_1} e^{-\alpha(t-s)}\Delta(s)ds\\
&\quad - \mu^{k-1}\int_{t_1}^{t_2} e^{-\alpha(t-s)}\Delta(s)ds - \cdots\\
&\quad - \int_{t_1}^{t_2} e^{-\alpha(t-s)}\Delta(s)ds\\
&\leq e^{-\alpha t + N_\sigma(0,t)\ln u}V(t_0)\\
&\quad - \int_0^t e^{-\alpha(t-s) + N_\sigma(s,t)\ln u}\Delta(s)ds.
\end{aligned} \tag{31}$$

Under zero initial condition, (31) gives

$$0 \leq -\int_0^t e^{-\alpha(t-s) + N_\sigma(s,t)\ln u}\Delta(s)ds. \tag{32}$$

Using $e^{-N_\sigma(0,t)\ln\mu}$ to pre-multiply and post-multiply the left term of (32), we have

$$\begin{aligned}
&\int_0^t e^{-\alpha(t-s) - N_\sigma(0,s)\ln u}Z^T(s)Z(s)ds\\
&\quad \leq \int_0^t e^{-\alpha(t-s) - N_\sigma(0,s)\ln u}\gamma^2\omega^T(s)\omega(s)ds.
\end{aligned} \tag{33}$$

We consider that $N_\sigma(0,s) \leq \frac{s}{\tau_a}$ and $\tau_a > \tau_a^* = \frac{\ln u}{\alpha}$. So we obtain that $N_\sigma(0,s)\ln u \leq \alpha s$ holds for any $s > 0$. It follows form (33) that

$$\begin{aligned}
&\int_0^t e^{-\alpha(t-s) - \alpha s}Z^T(s)Z(s)ds\\
&\quad \leq \int_0^t e^{-\alpha(t-s)}\gamma^2\omega^T(s)\omega(s)ds.
\end{aligned} \tag{34}$$

Then

$$\int_0^\infty e^{-\alpha s}Z^T(s)Z(s)ds \leq \int_0^\infty \gamma^2\omega^T(s)\omega(s)ds.$$

The proof is completed.

Remark 2. *If time-varying delay satisfies $\dot{h}(t) = 0$, we can obtain exponential criteria of switched nonlinear systems with constant delays.*

Consider the following switched nonlinear systems with constant delays.

$$\begin{aligned}
\dot{x}(t) = A_{1\sigma(t)} &+ A_{2\sigma(t)}x(t-h) + C_{\sigma(t)}u(t)\\
&+ \omega(t) + f(t, x(t-h)).
\end{aligned} \tag{35}$$

For system (35), we consider the state feedback given by (3).

The following result presents a sufficient condition of the exponential stabilization with weighted L_2-gain for the switched system (35).

Corollary 1. *For given constants $\alpha > 0, \gamma > 0, \varepsilon > 0$ and $\mu \geq 1$, if there exist symmetric and positive definite matrices P_i, Q_i, R_i, such that the following matrix inequalities hold for all $i, j \in M$,*

$$P_i \leq \mu P_j, Q_i \leq \mu Q_j, R_i \leq \mu R_j, \tag{36}$$

$$\Pi_i = \begin{pmatrix} \varpi_{11}^i & \varpi_{12}^i & P_i & \varpi_{14}^i & 0 \\ * & \varpi_{22}^i & 0 & 0 & 0 \\ * & * & -\varepsilon I & 0 & 0 \\ * & * & * & \varpi_{44}^i & 0 \\ * & * & * & * & \varpi_{55}^i \end{pmatrix} < 0, \quad (37)$$

where

$$\varpi_{11}^i = \bar{A}_{1i}^T P_i + P_i \bar{A}_{1i} + Q_i + h^2 R_i + \alpha P_i + D_i^T D_i,$$
$$\phi_{14}^i = P_i + D_i^T F_i, \varpi_{12}^i = P_i A_{2i}, \phi_{22}^i = -e^{-\alpha h} Q_i,$$
$$\varpi_{44}^i = F_i^T F_i - \gamma_I^2, \quad \varpi_{55}^i = -e^{-\alpha h} R_i,$$

then the system (35) is exponentially stabilizable and has weighted L_2-gain γ under the feedback control (3) for any switching signal with the average dwell time defined (10).

Proof: In order to show the weighted L_2-gain, we choose Lyapunov-Krasovskii functional as follows:

$$\begin{aligned} V_{\sigma(t)}(t) =& x^T(t) P_{\sigma(t)} x(t) + \int_{t-h}^{t} e^{\alpha(s-t)} x^T(s) Q_{\sigma(t)} x(s) ds \\ &+ h \int_{h}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} x^T(s) R_{\sigma(t)} x(s) ds d\theta. \end{aligned} \quad (38)$$

We have

$$\begin{aligned} \dot{V}_i(t) &+ \alpha V_i(t) + Z^T(t)Z(t) - \gamma^2 \omega^T(t)\omega(t) \\ &\leq x^T(t)[\bar{A}_{1i}^T P_i + P_i \bar{A}_{1i} + Q_i + h^2 R_i + \alpha P_i \\ &+ D_i^T D_i] x(t) + x^T(P_i + D_i^T F_i)\omega(t) \\ &+ x^T(t) P_i A_{2i} x(t-h(t)) \\ &+ \omega^T(t)(F_i^T D_i + P_i) x(t) \\ &+ \omega^T(t)(F_i^T F_i - \gamma^2 I)\omega(t) \\ &+ x^T(t-h)[\varepsilon V_i^T V_i - e^{-\alpha h} Q_i] x(t-h) \\ &- f^T(t, x(t-h(t))) f(t, x(t-h(t))) \\ &+ f^T(t, x(t-h(t))) P_i x(t) \\ &+ x^T(t-h(t)) A_{2i}^T P_i x(t) \\ &+ x^T(t) P_i f(t, x(t-h(t))) \\ &- e^{-\alpha h} \left[\int_{t-h}^{t} x(s) ds\right]^T R_i \left[\int_{t-h}^{t} x(s) ds\right]. \end{aligned} \quad (39)$$

So

$$\begin{aligned} &\dot{V}_i(t) + \alpha V_i(t) + Z^T(t)Z(t) - \gamma^2 \omega^T(t)\omega(t) \\ &\leq \psi^T(t) \Pi_i \psi(t) \end{aligned} \quad (40)$$

where

$$\begin{aligned} \psi(t) = [x^T(t) \quad & x^T(t-h) \quad f^T(t, x(t-h)) \\ & \omega^T(t) \quad (\int_{t-h}^{t} x(s) ds)^T]^T. \end{aligned} \quad (41)$$

So we can get

$$\dot{V}_i(t) + \alpha V_i(t) + Z^T(t)Z(t) - \gamma^2 \omega^T(t)\omega(t) \leq 0$$

Therefore,

$$\begin{aligned} V_i(t) &\leq \mu e^{-\alpha(t-t_k)} V(t_k^-) - \int_{t_k}^{t} e^{-\alpha(t-s)} \Delta(s) ds \\ &\leq \mu^k V(t_0) e^{-\alpha t} - \mu^k \int_{0}^{t_1} e^{-\alpha(t-s)} \Delta(s) ds \\ &\quad - \mu^{k-1} \int_{t_1}^{t_2} e^{-\alpha(t-s)} \Delta(s) ds - \cdots \\ &\quad - \int_{t_1}^{t_2} e^{-\alpha(t-s)} \Delta(s) ds \\ &\leq e^{-\alpha t + N_{\sigma(0,t)} \ln u} V(t_0) \\ &\quad - \int_{0}^{t} e^{-\alpha(t-s) + N_\sigma(s,t) \ln u} \Delta(s) ds. \end{aligned} \quad (42)$$

Using the same method in the proof of Theorem 2, we can get that switched system (35) is exponential stabilization with weighted L_2-gain.

C. Controller design

In the following, the design method of the controllers for system (1) is shown.

Theorem 3. *For given constants $\alpha, \varepsilon, \gamma$ and $\mu \geq 1$, if there exist symmetric and positive definite matrices $X_i, T_{1i}, T_{2i}, T_{3i}, O_{1i}, O_{2i}$, any matrices Y_i satisfying the following matrix inequalities for $i, j \in M$,*

$$\begin{aligned} &X_j \leq \mu X_i, T_{1j} \leq \mu T_{1i}, T_{2j} \leq \mu T_{2i}, \\ &T_{3j} \leq \mu T_{3i}, O_{1j} \leq \mu O_{1i}, O_{2j} \leq \mu O_{2i}, \quad \forall i, j \in M, \end{aligned} \quad (43)$$

$$\begin{bmatrix} \Omega_{1i} & \Omega_{2i} \\ * & \Omega_{3i} \end{bmatrix} < 0, \quad (44)$$

where

$$\Omega_{1i} = \begin{bmatrix} \hat{\phi}_{11}^i & \hat{\phi}_{12}^i & 0 & 0 & I & \hat{\phi}_{16}^i & 0 & 0 \\ * & \hat{\phi}_{22}^i & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \hat{\phi}_{33}^i & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\phi}_{44}^i & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 & 0 & 0 \\ * & * & * & * & * & \tilde{\phi}_{66}^i & 0 & 0 \\ * & * & * & * & * & * & \hat{\phi}_{77}^i & 0 \\ * & * & * & * & * & * & * & \hat{\phi}_{88}^i \end{bmatrix},$$

$$\Omega_{2i} = \begin{bmatrix} X_i D_i^T & 0 & X_i h_m & X_i h_M & X_i & X_i & X_i \\ 0 & X_i V_i^T & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Omega_{3i} = diag\{-I, -\varepsilon^{-1} I, -O_{1i}, -O_{2i}, -T_{1i}, -T_{2i}, -T_{3i}\},$$
$$\hat{\phi}_{11}^i = A_{1i} X_i + C_i Y_i + (A_{1i} X_i + C_i Y_i)^T + \alpha X_i,$$
$$\hat{\phi}_{33}^i = e^{-\alpha h_m}(T_{2i} - 2X_i), \hat{\phi}_{12}^i = A_{2i} X_i,$$
$$\hat{\phi}_{22}^i = (1-d) e^{-\alpha h_M}(T_{1i} - 2X_i), \quad \hat{\phi}_{44}^i = e^{-\alpha h_M}(T_{3i} - 2X_i),$$
$$\hat{\phi}_{16}^i = I + X_i D_i^T F_i \quad \hat{\phi}_{77}^i = e^{-\alpha h_m}(O_{1i} - 2X_i),$$
$$\hat{\phi}_{88}^i = e^{-\alpha h_M}(O_{2i} - 2X_i),$$

then system (1) is exponentially stabilizable and has weighted L_2-gain γ under the feedback control (3) for any switching *signal with the average dwell time satisfying (10). Moreover, the controller gains are constructed by*

$$K_i = Y_i X_i^{-1}, i \in M \tag{45}$$

Proof: From $T_{pi} > 0(p = 1, 2, 3), O_{qi} > 0(q = 1, 2)$, we can get

$$\begin{aligned}(T_{pi} - X_i)^T {T_{pi}}^{-1}(T_{pi} - X_i) \geq 0,\\ (O_{qi} - X_i)^T {O_{qi}}^{-1}(O_{qi} - X_i) \geq 0.\end{aligned}$$

Then the following inequality can be obtained:

$$\begin{aligned}T_{pi} - 2X_i \geq -X_i {T_{pi}}^{-1} X_i \quad (p = 1, 2, 3),\\ O_{qi} - 2X_i \geq -X_i {O_{qi}}^{-1} X_i \quad (q = 1, 2).\end{aligned} \tag{46}$$

Substituting (45) into (43) and using $diag\{X_i^{-1}, X_i^{-1}, X_i^{-1}, X_i^{-1}, I, I, X_i^{-1}, X_i^{-1}, I, I, I, I, I, I, I\}$ to pre- and post- multiply the left term of (43), and the following inequality is obtained:

$$\begin{bmatrix} \bar{\Omega}_{1i} & \bar{\Omega}_{2i} \\ * & \Omega_{3i} \end{bmatrix} < 0, \tag{47}$$

where

$$\bar{\Omega}_{1i} = \begin{bmatrix} \bar{\phi}_{11}^i & \bar{\phi}_{12}^i & 0 & 0 & I & \bar{\phi}_{16}^i & 0 & 0 \\ * & \bar{\phi}_{22}^i & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \bar{\phi}_{33}^i & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \bar{\phi}_{44}^i & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 & 0 & 0 \\ * & * & * & * & * & \hat{\phi}_{66}^i & 0 & 0 \\ * & * & * & * & * & * & \bar{\phi}_{77}^i & 0 \\ * & * & * & * & * & * & * & \bar{\phi}_{88}^i \end{bmatrix},$$

$$\bar{\Omega}_{2i} = \begin{bmatrix} D_i^T & 0 & h_m & h_M & I & I & I \\ 0 & V_i^T & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned}\bar{\phi}_{11}^i &= X_i^{-1} A_{1i} + X_i^{-1} C_i Y_i X_i^{-1} + (X_i^{-1} A_{1i} \\ &\quad + X_i^{-1} C_i Y_i X_i^{-1})^T + \alpha X_i^{-1},\\ \bar{\phi}_{12}^i &= X_i^{-1} A_{2i}, \quad \bar{\phi}_{22}^i = -(1-d) e^{-\alpha h_M} T_{1i}^{-1},\\ \bar{\phi}_{33}^i &= -e^{\alpha h_m} T_{2i}^{-1}, \bar{\phi}_{44}^i = -e^{-\alpha h_M} T_{3i}^{-1},\\ \bar{\phi}_{16}^i &= X_i^{-1} + D_i^T F_i, \bar{\phi}_{77}^i = -e^{-\alpha h_m} O_{1i}^{-1},\\ \bar{\phi}_{88}^i &= -e^{-\alpha h_M} O_{2i}^{-1},\end{aligned}$$

Then setting

$$\begin{aligned}Y_i = K_i X_i, \quad {X_i}^{-1} = P_i,\\ {T_{pi}}^{-1} = Q_{pi}, \quad {O_{qi}}^{-1} = R_{qi}.\end{aligned} \tag{48}$$

and using Lemma 1 in (47), it can be concluded that (25) holds. This means that (44) implies (25). From (45), the controller gains are given by (48).

The following result presents the design method of the controllers for system with constant delays.

Corollary 2. *For given constants $\alpha > 0, \varepsilon > 0, \gamma > 0$ and $\mu \geq 1$, if there exist symmetric and positive definite matrices X_i, T_i, L_i, and any matrices Y_i satisfying the following matrix inequalities for $i, j \in M$,*

$$X_j \leq \mu X_i, T_j \leq \mu T_i, O_j \leq \mu O_i, \quad \forall i, j \in M, \tag{49}$$

$$\begin{pmatrix} \vartheta_{11}^i & \vartheta_{12}^i & X_i & \vartheta_{14}^i & 0 & X_i D^T & 0 & h X_i & X_i \\ * & \vartheta_{22}^i & 0 & 0 & 0 & 0 & X_i V_i^T & 0 & 0 \\ * & * & -\varepsilon I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \vartheta_{44}^i & 0 & 0 & 0 & 0 & \\ * & * & * & * & \vartheta_{55}^i & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & 0 & -\varepsilon^{-1} I & 0 & 0 \\ * & * & * & * & * & 0 & 0 & -L_i & 0 \\ * & * & * & * & * & 0 & 0 & 0 & -T_i \end{pmatrix} < 0, \tag{50}$$

where

$$\begin{aligned}\vartheta_{11}^i &= A_{1i} X_i + C_i Y_i + (A_{1i} X_i + C_i Y_i)^T + \alpha X_i,\\ \vartheta_{12}^i &= A_{2i} X_i, \quad \vartheta_{22}^i = e^{-\alpha h}(T_i - 2X_i),\\ \vartheta_{14}^i &= I + X_i D_i^T F_i, \quad \vartheta_{44}^i = F_i^T F_i - \gamma^2 I,\\ \vartheta_{55}^i &= e^{-\alpha h}(O_i - 2X_i),\end{aligned}$$

then system (35) is exponentially stabilizable and has weighted L_2-gain γ under the feedback control (3) for any switching signal with the average dwell time satisfying (10). Moreover, the controller gains are constructed by

$$K_i = Y_i X_i^{-1}, i \in M \tag{51}$$

Proof: From $T_i > 0, O_i > 0$, we can get

$$\begin{aligned}(T_i - X_i)^T {T_i}^{-1}(T_i - X_i) \geq 0,\\ (O_i - X_i)^T {O_i}^{-1}(O_i - X_i) \geq 0.\end{aligned}$$

Then

$$\begin{aligned}T_i - 2X_i &\geq -X_i {T_i}^{-1} X_i,\\ O_i - 2X_i &\geq -X_i {O_i}^{-1} X_i.\end{aligned} \tag{52}$$

Substituting (52) into (50) and using $diag\{X_i^{-1}, X_i^{-1}, I, I, X_i^{-1}, I, I, I, I\}$to pre- and post- multiply the left term of (38), and the following inequality is obtained:

$$\begin{pmatrix} \tilde{\vartheta}_{11}^i & \tilde{\vartheta}_{12}^i & X_i^{-1} & \tilde{\vartheta}_{14}^i & 0 & D_i^T & 0 & h & I \\ * & \tilde{\vartheta}_{22}^i & 0 & 0 & 0 & 0 & V_i^T & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \tilde{\vartheta}_{44}^i & 0 & 0 & 0 & 0 & \\ * & * & * & * & \tilde{\vartheta}_{55}^i & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & 0 & -\varepsilon^{-1} I & 0 & 0 \\ * & * & * & * & * & 0 & 0 & -O_i & 0 \\ * & * & * & * & * & 0 & 0 & 0 & -T_i \end{pmatrix} < 0, \tag{53}$$

where

$$\begin{aligned}\tilde{\vartheta}_{11}^i &= X_i^{-1} A_{1i} + X_i^{-1} C_i Y_i X_i^{-1} \\ &\quad + (X_i^{-1} A_{1i} + X_i^{-1} C_i Y_i X_i^{-1})^T + \alpha {X_i}^{-1},\\ \tilde{\vartheta}_{12}^i &= X_i^{-1} A_{2i}, \quad \tilde{\vartheta}_{22}^i = -e^{-\alpha h} T_i^{-1},\\ \tilde{\vartheta}_{14}^i &= X_i^{-1} + D_i^T F_i, \tilde{\vartheta}_{44}^i = F_i^T F_i - \gamma^2 I,\\ \tilde{\vartheta}_{55}^i &= -e^{-\alpha h} O_i,\end{aligned}$$

Then setting

$$\begin{aligned}Y_i = K_i X_i, \quad {X_i}^{-1} = P_i,\\ {T_i}^{-1} = Q_i, \quad {O_i}^{-1} = R_i.\end{aligned} \tag{54}$$

Using the same method in the proof of Theorem 3. the controller gains are given by (54).

Remark 3. *In terms of switched systems, a common Lyapunov function for all subsystems is often employed to characterize stability in some literature. For instance, in [23], stability and L_2-gain for a class of switched linear system with time varying delays have been studied based on the common Lyapunov Krasovskii function for all subsystems. To the best of our knowledge, a common Lyapunov function may not exist or be too conservative and it is reasonable that each subsystem has its own Lyapunov function. Thus, multiple storage functions to describe stability for switched systems are developed naturally. In this paper, we investigate the problem of exponential stabilization and L_2-gain analysis for a class of switched nonlinear systems with time-varying delay by multi-Lyapunov-Krasovskii functional. Compared with [23], the conservativeness of our results is lower.*

Remark 4. *In [11], a feedback stabilization problem for switched linear systems with time-delay in detection of switching signal is formulated. We understand that time delay is constant delay and nonlinear disturbance is not involve in [11]. Specifically, [11] can be seen as a special case of this paper. Therefore, this paper has a greater advantage when dealing with complex systems in practice.*

IV. Numerical examples

In this section, two examples are presented to confirm the effectiveness of the proposed approach.

Example 1. Consider system (1) composed of two subsystems with the following parameters:

$$A_{11}=\begin{bmatrix}-0.8 & 0.2\\ 0 & -0.8\end{bmatrix},\ A_{21}=\begin{bmatrix}-0.8 & 0\\ 0.2 & -0.6\end{bmatrix},$$
$$A_{12}=\begin{bmatrix}-0.9 & 0\\ 0.1 & -0.8\end{bmatrix},\ A_{22}=\begin{bmatrix}-0.8 & 0.1\\ 0 & -0.7\end{bmatrix},$$
$$D_1=\begin{bmatrix}-0.9 & 0.2\\ 0 & -0.8\end{bmatrix},\ D_2=\begin{bmatrix}-0.8 & 0.1\\ 0.2 & -0.9\end{bmatrix},$$
$$F_1=\begin{bmatrix}-0.6 & 0\\ 0.2 & -0.7\end{bmatrix},\ F_2=\begin{bmatrix}-0.7 & 0.1\\ 0.2 & -0.1\end{bmatrix},$$
$$V_1=\begin{bmatrix}-0.8 & 0\\ 0.2 & -0.6\end{bmatrix},\ V_2=\begin{bmatrix}-0.7 & 0.3\\ 0.1 & -0.9\end{bmatrix},$$
$$B_1=\begin{bmatrix}0.2\\ 0.3\end{bmatrix},\qquad B_2=\begin{bmatrix}0.2\\ 0.4\end{bmatrix}.$$

Choose $\alpha = 0.4, \mu = 1.6, h_m = 0.2, h_M = 0.9, \varepsilon = 0.6, \gamma = 0.3, \omega_1(t) = 0.3^{t-1}, \omega_2(t) = 0.2^t, h(t) = 0.01sin(t)$, $\omega(t) = \begin{bmatrix}\omega_1(t), \omega_2(t)\end{bmatrix}^T$,

$$f(t, x(t-h(t))) = \begin{bmatrix}0.01cos(x_1(t-h(t)))\\ 0.02sin(x_2(t))\end{bmatrix},$$

Then the average dwell time is

$$\tau_a > \tau_a^* = \frac{\ln \mu}{\alpha} = 1.1750.$$

By solving (43) and (44), we can get

$$X_1=\begin{bmatrix}0.2002 & 0.0976\\ 0.0976 & 0.2302\end{bmatrix}, X_2=\begin{bmatrix}0.1840 & 0.0148\\ 0.0148 & 0.1746\end{bmatrix},$$
$$T_{11}=\begin{bmatrix}1.5883 & 0.7756\\ 0.7756 & 1.5841\end{bmatrix}, T_{12}=\begin{bmatrix}1.6834 & 0.6260\\ 0.6260 & 1.7008\end{bmatrix},$$
$$T_{21}=\begin{bmatrix}0.3162 & 0.1064\\ 0.1064 & 0.2822\end{bmatrix}, T_{22}=\begin{bmatrix}0.3359 & 0.1465\\ 0.1465 & 0.5197\end{bmatrix},$$
$$T_{31}=\begin{bmatrix}0.7078 & 0.8384\\ 0.8384 & 0.6949\end{bmatrix}, T_{32}=\begin{bmatrix}1.2852 & 0.4559\\ 0.4559 & 1.3509\end{bmatrix},$$
$$L_{11}=\begin{bmatrix}1.8520 & 0.1304\\ 0.1304 & 0.1536\end{bmatrix}, L_{12}=\begin{bmatrix}0.2041 & 0.1780\\ 0.1780 & 0.3763\end{bmatrix},$$
$$L_{21}=\begin{bmatrix}1.1103 & 0.5333\\ 0.5333 & 1.0969\end{bmatrix}, L_{22}=\begin{bmatrix}1.1193 & 0.5550\\ 0.5550 & 1.1919\end{bmatrix},$$
$$Y_1=\begin{bmatrix}0.1338 & 0.2071\end{bmatrix},\quad Y_2=\begin{bmatrix}0.6072 & 0.6299\end{bmatrix}.$$

Then the controller gains constructed by (51) are

$$K_1=\begin{bmatrix}0.2896 & 0.7769\end{bmatrix},\ K_2=\begin{bmatrix}3.0305 & 3.3508\end{bmatrix}.$$

According to Theorem 3, we can get that the system (1) is exponentially stabilizable for any switching signal under the feedback control and has weighted L_2-gain γ. Switching signal and state response diagrams are shown in Figs.1 and 2 with the initial state is $x(0) = (0, 1)^T$, respectively. The effectiveness of the results obtained by Fig 2

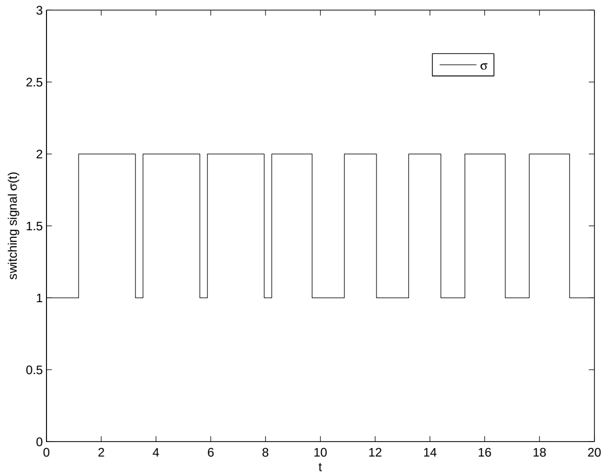

Fig. 1: The switching law.

Example 2. Consider system (35) composed of two subsystems with the following parameters:

$$A_{11}=\begin{bmatrix}-0.7 & 0.1\\ 0 & -1\end{bmatrix},\ A_{21}=\begin{bmatrix}-0.6 & 0\\ 0.2 & -0.9\end{bmatrix},$$
$$A_{12}=\begin{bmatrix}-1 & 0\\ 0.1 & -0.7\end{bmatrix},\ A_{22}=\begin{bmatrix}-1.2 & 0.1\\ 0 & -0.8\end{bmatrix},$$
$$D_1=\begin{bmatrix}-0.7 & 0.2\\ 0 & -0.6\end{bmatrix}, D_2=\begin{bmatrix}-0.8 & 0.1\\ 0.2 & -0.7\end{bmatrix},$$
$$F_1=\begin{bmatrix}-0.9 & 0\\ 0.2 & -0.9\end{bmatrix},\ F_2=\begin{bmatrix}-0.7 & 0.1\\ 0.2 & -0.6\end{bmatrix},$$
$$V_1=\begin{bmatrix}-0.6 & 0\\ 0.2 & -0.8\end{bmatrix},\ V_2=\begin{bmatrix}-0.8 & 0.2\\ 0.1 & -0.8\end{bmatrix},$$
$$B_1=\begin{bmatrix}0.1\\ 0.4\end{bmatrix}, B_2=\begin{bmatrix}0.3\\ 0.5\end{bmatrix}.$$

Choose $\alpha = 0.5, \mu = 1.6, h = 0.4, \varepsilon = 0.4, \gamma = 0.5$, then the average dwell time is

$$\tau_a > \tau_a^* = \frac{\ln \mu}{\alpha} = 0.9400.$$

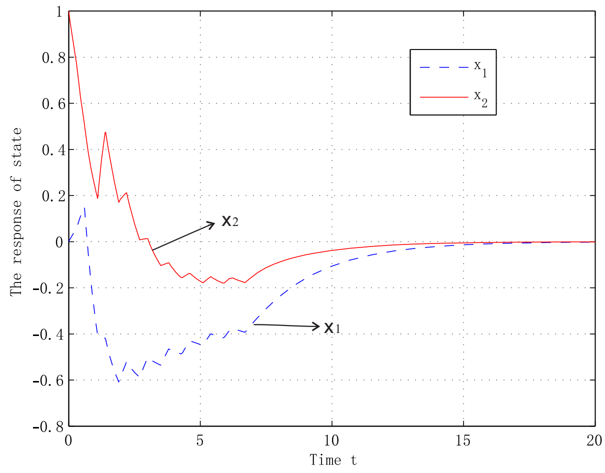

Fig. 2: State response of the closed-loop system.

By solving (43) and (44), we can get

$$X_1 = \begin{bmatrix} 0.1862 & 0.0163 \\ 0.0163 & 0.0421 \end{bmatrix}, X_2 = \begin{bmatrix} 0.1451 & 0.3641 \\ 0.3641 & 0.2330 \end{bmatrix},$$
$$T_1 = \begin{bmatrix} 0.4116 & 0.0150 \\ 0.0150 & 0.6258 \end{bmatrix}, T_2 = \begin{bmatrix} 0.7188 & 0.3459 \\ 0.3459 & 0.7857 \end{bmatrix},$$
$$L_1 = \begin{bmatrix} 0.4368 & 0.0075 \\ 0.0075 & 0.6286 \end{bmatrix}, L_2 = \begin{bmatrix} 0.7188 & 0.3459 \\ 0.3459 & 0.7857 \end{bmatrix},$$
$$Y_1 = \begin{bmatrix} 0.5869 & 0.0576 \end{bmatrix}, Y_2 = \begin{bmatrix} 0.3676 & 0.6315 \end{bmatrix}.$$

Then the controller gains constructed by (51) are

$$K_1 = \begin{bmatrix} 3.1386 & 0.1530 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 1.4609 & 0.4274 \end{bmatrix}.$$

According to corollary 2, we can get that the system (35) is exponentially stabilizable for any switching signal under the feedback control.

Example 3. Water pollution is a huge challenge facing today's society. The design of water pollution control system is of great significance for sustainable development. In the following, an example of applying this system to water pollution control systems will be demonstrated.

In order to facilitate the simulation in a reach of a polluted river, we record $m(t)$ and $p(t)$ as the concentrations per unit volume of biochemical oxygen demand and dissolved oxygen, respectively. Specifically, let m^* and p^* indicate the desired steady values of $m(t)$ and $p(t)$ in a reach of a polluted river, respectively. Moreover, we take m^* and p^* as corresponding to some measure of water quality standards. The following definitions are given:

$$x_1(t) = m(t) - m^*,\ x_2(t) = p(t) - p^*,\ x(t) = [x_1^T(t)\ x_2^T(t)]^T$$

As a result, the dynamic equation for $x(t)$ can be expressed as:

$$\dot{x}(t) = A_1 x(t) + A_2 x(t - h(t)) + B u(t) + \omega(t), \qquad (55)$$

where

$$A_1 = \begin{bmatrix} -l_1 - \varepsilon_1 - \varepsilon_2 & 0 \\ -l_3 & -l_2 - \varepsilon_1 - \varepsilon_2 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} \varepsilon_2 & 0 \\ 0 & \varepsilon_2 \end{bmatrix},\ B = \begin{bmatrix} \varepsilon_1 \\ 1 \end{bmatrix},$$

$l_i (i = 1,2,3), \varepsilon_1$ and ε_2 are known constants, and $\omega(t)$ is the external disturbance of dynamic system. Moreover, $u(t) = [u_1^T(t)\ u_2^T(t)]^T$ is the control variable of river pollution system. The physical meaning of the parameters mentioned above is easy to find in [27].

This paper assumes that system actuators have good performance or failure, and according to the actual situation, we know that at least one actuator can ensure the normal operation of the river pollution system. In addition, for simulation of our purposes, we do not consider the nonlinear perturbation term, and the nonlinear perturbation term is not also considered in [1]. As a consequence, the river pollution system (55) can be modeled as a switched system consisting of two subsystems:

$$\dot{x}(t) = \begin{cases} A_{11}(t)x(t) + A_{21}(t)x(t - h(t)) + B_1 u(t) + \omega(t), \\ \qquad no\ failures\ occur \\ A_{12}(t)x(t) + A_{22}(t)x(t - h(t)) + B_2 u(t) + \omega(t), \\ \qquad failures\ occur \end{cases} \qquad (56)$$

In order to get the simulation results, we choose $l_1 = 1.3, l_2 = 0.8, l_3 = 1.2, \varepsilon_1 = 0.6, \varepsilon_2 = 0.5$, and get that

$$A_1 = \begin{bmatrix} -2.4 & 0 \\ -1.2 & -1.9 \end{bmatrix}, A_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, B = \begin{bmatrix} 0.6 \\ 1 \end{bmatrix}.$$

Let $h(t) = 0.02sin(t), \omega(t) = [\omega_1(t)\ \ \omega_2(t)]^T, \omega_1(t) = 0.2^{t-0.05}, \omega_1(t) = 0.1^t$. Therefore, we can obtain a set of switching sequences to stabilize the system (55) by the parameters in Theorem 1. By (43) and (44), we have

$$X_1 = \begin{bmatrix} 0.2734 & 0.0523 \\ 0.0523 & 0.2587 \end{bmatrix},$$
$$X_2 = \begin{bmatrix} 0.3104 & 0.0182 \\ 0.0182 & 0.3241 \end{bmatrix},$$
$$T_{11} = \begin{bmatrix} 1.2713 & 0.2132 \\ 0.2132 & 1.8151 \end{bmatrix},$$
$$T_{12} = \begin{bmatrix} 2.3127 & 0.2781 \\ 0.2781 & 2.0128 \end{bmatrix},$$
$$T_{21} = \begin{bmatrix} 2.5615 & 0.0237 \\ 0.0237 & 1.0342 \end{bmatrix},$$
$$T_{22} = \begin{bmatrix} 1.0129 & 0.2108 \\ 0.2108 & 0.8654 \end{bmatrix},$$
$$T_{31} = \begin{bmatrix} 2.1264 & 0.8525 \\ 0.8525 & 2.0213 \end{bmatrix},$$
$$T_{32} = \begin{bmatrix} 1.8353 & 0.3289 \\ 0.3289 & 0.9713 \end{bmatrix},$$
$$L_1 = \begin{bmatrix} 1.4328 & 0.2197 \\ 0.2197 & 1.2785 \end{bmatrix},$$
$$L_2 = \begin{bmatrix} 1.6782 & 0.5234 \\ 0.5234 & 1.2713 \end{bmatrix},$$
$$Y_1 = \begin{bmatrix} 0.7741 & 0.9326 \end{bmatrix},$$
$$Y_2 = \begin{bmatrix} 0.3276 & 0.0356 \end{bmatrix}.$$

Then the controller gains constructed by (27) are

$$K_1 = \begin{bmatrix} 2.2279 & 3.1545 \end{bmatrix}, K_2 = \begin{bmatrix} 1.0524 & 0.0507 \end{bmatrix}.$$

Fig.3 and 4 describe state response of the subsystem 1 and 2 with the initial condition $x(0) = (-1,1)^T$ for the system (55), respectively. Through the designed switching signal and our approach, we can get that the system (55) with the initial condition $x(0) = (0,1)^T$ is exponential stability with weighted L_2-gain performance of the switched

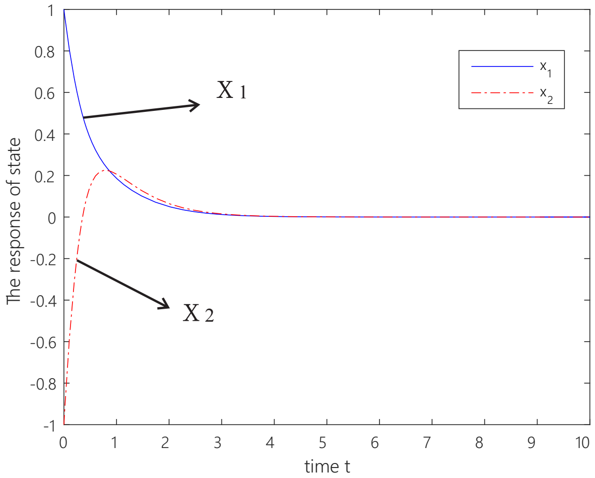

Fig. 3: State response of the subsystem 1

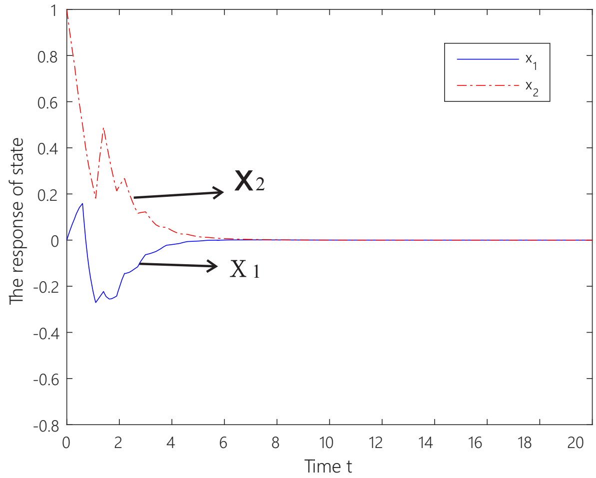

Fig. 5: State response of the system (55)

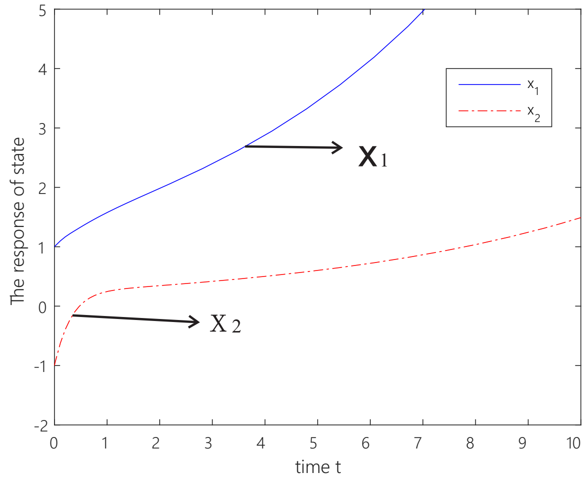

Fig. 4: State response of the subsystem 2

nonlinear system form Fig.5. As a consequence, this verifies the effectiveness of our results in the control of river pollution process.

V. Conclusions

In this paper, we have investigated the problem of exponential stabilization and L_2-gain analysis for a class of switched nonlinear systems with time-varying delay. Firstly, interval time-varying delay is considered and a novel multi-Lyapunov-Krasovskii functional dependent on the size of time delay is also constructed. Based on the matrix inequality technique and the average dwell time approach. The exponential stabilization criteria and weighted L_2-gain disturbance attenuation performance for nonlinear switched systems with interval time-varying delay are obtained. Then, the proposed approach is extended to design state feedback controller for switched nonlinear systems by special operations of matrices and Schur complement. Finally, three numerical examples illustrates the effectiveness of the theoretical results.

Through the research of this paper, it is important to derive a less conservative condition for exponential stabilization and L_2-gain disturbance attenuation performance for nonlinear switched systems with interval time-varying delay. Hence, we take the bound of upper and lower about delay are processed with the help of delay decomposition technique. Compared with the existing results, our results are more practical and less conservative. In order to better study the issue of switched systems with time delay[28,29], our future work will focus on extending the proposed method to delay-dependent robust dissipative problem for a class of nonlinear switched system with mixed delays, and expand theoretical to other fields[30,31].

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