Numerical Study of MHD Flow and Heat Transfer of an Unsteady Third Grade Fluid with Viscous Dissipation

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***Abstract*—In ongoing investigation the unsteady magneto-hydrodynamics flow and heat transfer with viscous dissipation of a third grade fluid passed between two long parallel flat porous plates is discussed. With the physical assumptions a coupled system of non-linear partial differential equation is procured as governing MHD flow and heat transfer problem, which is then transformed to a system of non-linear algebraic equations by descretising using fully implicit finite difference scheme and solved numerically by damped-Newton method. Finally the problem is coded using MATLAB programming. The numerical results which characterize the imposing physical parameters are described through various graphs. The foremost finding of the present study is that out two elastic parameters the visco-elastic parameterα has a dominant role over the non-Newtonian parameter γ and has a control over reducing thevelocity flow of third grade fluid. Again more viscous dissipation energy is generated when the parameter values of *E* increases, as a result temperature profile increases.**

***Index Terms*— magneto-hydrodynamics, unsteady flow, viscous-dissipation, third grade fluid, Porousplates.**

I. Introduction

THEstudy of non-Newtonian fluids is intensified due to its numerous application in industry and engineering. To describe various characteristics of non-Newtonian fluid different constitutive fluid models are developed. Second grade fluid model is one of the simplest subclass of these models which is capable of describing normal stress differences. But this model is inadequate to describe shear thinning/thickening phenomenon observed in many non-Newtonian fluids as described by Joseph and Fosdick(1973), Beavers and Joseph(1975). Such characteristics of visco-elastic fluids can be predicted by third grade fluid model. Many researcher like Ariel(2002), Hayat et al.(2002), Hayat et al.(2008), Abbasbandy et al.(2008), Sahoo(2009), Narain and Kara(2010),Sahoo and Poncet(2011), Nayak et al.(2012), Gital et al.(2014), Awais(2015),O Samuel and D Oluwole(2015),O Samuel and Falade(2015), Nayak et al.(2016), Qian and Cai(2018), are involved in studying the non-Newtonian fluid models due to its important technological application.

Recently Wang et al.(2016) have studied flow and heat transfer of third grade fluid within two micro parallel plates in presence of magnetic field and an externally imposed electrical field. The problem is analyzed both by analytic and numerical method. Then finally a comparison is done between these solutions. Okoya(2016) have analyzed the criticality and transition of a third grade fluid through a pipe with Reynolds number viscosity. The steady incompressible flow of an exothermic reactive third grade fluid is solved numerically and discussed for both cases of viscosity and Reynolds number viscosity model. Carapau and Corria (2017) have discussed the numerical simulation of third grade fluid in a tube through a contraction. They used numerical simulation to analyze unsteady flow over a finite set or a tube with contraction is performed. Opangue et al.(2017) have investigated entropy generation of hydro-magnetic couple stressfluid flow through a channel filled with a non—Darcian medium. Then they solve the coupled fourth order non-linear set of differential equation using Adomain decomposition method and differential transform method. Carapau et al.(2017) used a non-dimensional hierarchical approach to solve generalized third grade fluid with shear dependent visco-elastic effect model. They consider the particular case of a flow through a straight and rigid tube with constant circular cross section. Then a real three dimensional flow problem is reduced to exact three dimensionalsystems as an ordinary differential equation and final solution is obtained using the Runge-Kutta method.

In present study, it is attempted to analyze the most generalized case of unsteady magneto-hydrodynamics flow and heat transfer of a third grade fluid passingwithin an infinite porous channel with viscous dissipation. Where the upper plate is fixed and lower plate suddenly gets in motion with a velocity whichvaries with respect to time. A comparison study is made by taking smaller as well as larger values of all emerging physical parameters. Now a day, study of non-Newtonian fluid with magnetic field is used in different branches of technology as exploiting liquid metal, bio-medical engineering, magneto-hydrodynamic power generation and many more. Thus this study is advantageous because of its important technological application and efficacious mathematical features. Also to my knowledge so far in above cited studies the present numerical investigation is not done.

A fully implicit finite difference scheme is used to convert governing coupled non-linear partial differential equation into a system of non-linear algebraic equations and then numerical solution is obtained using quadratic convergent damped Newton method. The advantage of the employed scheme is that, it is valid for small as well as large value of elastic parameters. Further this method does not require repeated derivation of the problem for every change

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in boundary condition.

Rest of the paper is organized successfully. Section 2 is concerned with theproblem formulation. The convergence and stability of the scheme for the MHD flow and heat transfer is discussed in section 3. Section 4 gives a detailed account regarding impact of several physical parameters on fluid flow with graphical representation. Section 5 concludes the paper.

II. Construction of Problem

The stress tensor P for an incompressible homogeneous and thermodynamically compatible third grade fluid by Fosdick and Rajagopal (1980) is

$$P = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_3 (trA_1^2) A_1 \qquad (2.1)$$

$where\ \ \mu = viscosity$

$\alpha_1, \alpha_2 = normal\ stress\ moduli$

$\beta_3 = higher\ order\ viscosity$

$p = pressure$

$I = identitytensor$

$A_1, A_2 = kinematicaltensor,$ defined by Rivlin and Erickson (1995)

Here lower plate is taken along x'-axis and y'-axis is normal to it. The upper plate isspecified by y'=1. Let both plates can extends to infinite in either sides of the x'-axis, where the upper plate is assumed to be fixed and lower plate moves witha time varying velocity F(t). Let V > 0 indicate the suction velocity at the plate. A transverse uniform magnetic field with strength $B_0$is applied at the lower plate.

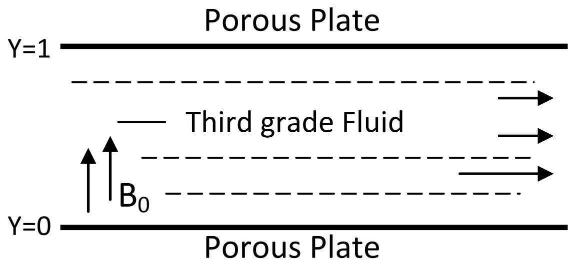

Fig. 0, Fluid flow between porous plates

With the above physical assumptions and the stress components given in eq.(2.1) the governing third grade flow equation becomes

$$\rho\left(\frac{\partial w'}{\partial t'} + V\frac{\partial w'}{\partial y'}\right) = \mu\frac{\partial^2 w'}{\partial y'^2} + \alpha_1\frac{\partial^3 w'}{\partial y'^2 \partial t'} + 6\beta_3 (\frac{\partial w'}{\partial y'})^2 \frac{\partial^2 w'}{\partial y'^2} + \alpha_1 V\frac{\partial^3 w'}{\partial y'^3} - \frac{\sigma B_0^2}{\rho} w' \qquad \mathbf{(2.2)}$$

With subjected initial and boundary conditions are

$$t' = 0:\ w' = 0, \quad \forall y'$$

$$t' > 0:\ w' = F(t)\ fory = 0 \qquad \mathbf{(2.3)}$$

$$w' = 0 \quad fory' = 1$$

Where $F(t) = A\sin(w't')$

$$y = \frac{y'}{\sqrt{\vartheta T}}, t = \frac{t'}{T}, w = \frac{w'}{A}\ ,\ \theta = \frac{\theta' - \theta_1}{\theta_2 - \theta_1}\ ,\ \ \omega = \omega T \mathbf{(2.4)}$$

$$Re = \frac{V\sqrt{T}}{\sqrt{\vartheta}}\ , \alpha = \frac{\alpha_1}{\vartheta T \rho}\ , \gamma = \frac{6\beta_3 A^2}{\rho \vartheta^2 T}\ , m^2 = \frac{\sigma B_0^2 T}{\rho}\ ,$$

$$p_r = \frac{\vartheta \rho\, c_p}{k}\ \ ,\ E = \frac{A^2}{c_p(\theta_2 - \theta_1)}\ .$$

Where Re – local Reynolds number, α - visco-elastic parameter, γ - third grade elastic parameter, m- Hartmann number, Pr – Prandtl number, E –viscous dissipation parameter, ϑ – kinematic coefficient of viscosity. A= constant.

Then the non-dimensionalized field equation of velocity can be obtained as

$$\left(\frac{\partial w}{\partial t} + Re\frac{\partial w}{\partial y}\right) = \frac{\partial^2 w}{\partial y^2} + \alpha\frac{\partial^3 w}{\partial y^2 \partial t} + \gamma(\frac{\partial w}{\partial y})^2 \frac{\partial^2 w}{\partial y^2} + Re\alpha\frac{\partial^3 w}{\partial y^3} - m^2 w \qquad \mathbf{(2.5)}$$

Along with the initial and boundary condition as follows:

$$t = 0: w = 0\ , \qquad \forall y$$

$t > 0: w = \sin\omega t\ ,\ fory = 0$ **(2.6)**$w = 0,\ fory = 1$

The thermal field equation for the thermodynamically compatible third grade fluid with viscous dissipation

$$\rho c_p\left(\frac{\partial \theta'}{\partial t'} + V\frac{\partial \theta'}{\partial y'}\right) = k\frac{\partial^2 \theta'}{\partial y'^2} + \mu\left(\frac{\partial w'}{\partial y'}\right)^2 + \alpha_1\left(\frac{\partial^2 w'}{\partial y' \partial t'}\right)\left(\frac{\partial w'}{\partial y'}\right) + \alpha_1 V\left(\frac{\partial^2 w'}{\partial y'^2}\right)\left(\frac{\partial w'}{\partial y'}\right) + 2\beta_3\left(\frac{\partial w'}{\partial y'}\right)^4 + \sigma B_0^2 w' \qquad \mathbf{(2.7)}$$

With the initial and boundary condition which are subjected to the equation (2.7) are

$$t' = 0:\ \theta' = 0 \quad \forall y'$$

$$t' > 0: \theta' = \theta_2 fory' = 0 \qquad \mathbf{(2.8)}$$

$$\theta' = 0 fory' = 1$$

ρ = densityoffluid, σ = electrical conductivity, μ = viscosity, c_p = specificheat, k = thermalconductivity andθ' = temperature .

Non-dimensionalized field equation of energy with initial and boundary condition is secure as

$$\left(\frac{\partial \theta}{\partial t} + Re\frac{\partial \theta}{\partial y}\right) = \frac{1}{p_r}\frac{\partial^2 \theta}{\partial y^2} + E\left(\frac{\partial w}{\partial y}\right)^2 + \alpha E\left(\frac{\partial^2 w}{\partial y \partial t}\right)\left(\frac{\partial w}{\partial y}\right)$$

$$+\alpha Re E\left(\frac{\partial^2 w}{\partial y^2}\right)\left(\frac{\partial w}{\partial u}\right) + \frac{\gamma E}{3}\left(\frac{\partial w}{\partial y}\right)^4 + E m^2 w \qquad \mathbf{(2.9)}$$

With the initial and boundary conditions:

$$t = 0:\ \theta = 0, \qquad forally$$

$$t > 0: \theta = 1, \qquad fory = 0 \qquad \mathbf{(2.10)}$$

$$\theta = 0, \quad fory = 1$$

The above system of non-linear coupled differential equations is solved under the relevant initial and boundary conditions with the help of implicit finite difference scheme. Then the numerical solution of the system is obtained using above said method described in Conte, De Boor (1980).

III. NUMERICAL SOLUTION METHOD

The equation (2.5), (2.6), (2.9) and (2.10) are solved by finite difference scheme of crank- Nicolson type. Taking a uniform mesh of step h and time step Δt the grid points generated are

$$(y_j\ ,t_j) = (ih\ ,j\Delta t), \qquad i = 0,1,2,\dots,N+1,$$

$$j = 0,1,2,\dots,M-1$$

Following notations and difference approximations to the derivative are taken at different nodes asf_{ij}'s.

Notations:

$$f_{1i}^{j} = w_{i+1}^{j} - w_{i-1}^{j}$$

$$f_{2i}^{j} = w_{i+1}^{j} - 2w_i^j + w_{i-1}^{j}$$

$$f_{3i}^{j} = -w_{i-2}^{j} + 2w_{i-1}^{j} - 2w_{i+1}^{j} + w_{i+2}^{j}$$

$$f_{3i}^{'j} = -3w_{i-1}^{j} + 10w_i^j - 12w_{i+1}^{j} + 6w_{i+2}^{j} - w_{i+3}^{j}$$

$$f_{3i}^{''j} = w_{i-3}^{j} - 6w_{i-2}^{j} + 12w_{i-1}^{j} - 10w_i^j + 3w_{i+1}^{j}$$

The difference approximation to the derivatives at nodes $(ih\ ,j\Delta t),\ i = 1,2,\dots,N+1$ and

$j = 0,1,\dots,M+1$are written as

$$\frac{\partial w}{\partial t} \approx \frac{w_i^{j+1} - w_i^j}{\Delta t}$$

$$\frac{\partial w}{\partial y} \approx \frac{f_{1i}^{j+1} + f_{1i}^{j}}{4h}$$

$$\frac{\partial^2 w}{\partial y^2} \approx \frac{f_{2i}^{j+1} + f_{2i}^{j}}{2h^2} \qquad \textbf{(3.1)}$$

$$\frac{\partial^3 w}{\partial y^3} \approx \frac{f_{3i}^{j+1} + f_{3i}^{j}}{2h^3}$$

$$\frac{\partial^3 w}{\partial y^2 \partial t} \approx \frac{f_{2i}^{j+1} - f_{2i}^{j}}{h^2 \Delta t}$$

At the point (1, jΔt) and (N, $j\Delta t$) the third grade derivative $\frac{\partial^3 w}{\partial y^3}$ is replaced respectively by

$\frac{f_{3i}^{'j+1} + f_{3i}^{'j}}{2h^3}$and $\frac{f_{3i}^{''j+1} + f_{3i}^{''j}}{2h^3}$.

Here the difference scheme is seen to be unconditionally stable and has second order convergence in time and space.

With the implementation of above said notations in (3.1), the governing equation of velocity is discretized as

$$\frac{w_i^{j+1} - w_i^j}{\Delta t} + Re\frac{f_{1i}^{j+1} + f_{1i}^{j}}{4h} = \frac{f_{2i}^{j+1} + f_{2i}^{j}}{2h^2} + \alpha\frac{f_{2i}^{j+1} - f_{2i}^{j}}{h^2\Delta t} + Re\alpha\frac{f_{3i}^{j+1} + f_{3i}^{j}}{2h^3} + \frac{\gamma}{32h^4}(f_{1i}^{j+1} + f_{1i}^{j})^2(f_{2i}^{j+1} + f_{2i}^{j}) - m^2(\frac{w_i^{j+1} + w_i^j}{2}) \qquad \textbf{(3.2)}$$

For i = *2,3,...,N-1, j=0,1,2,...,M.*

The initial and boundary conditions in discretized form as follows:

$$w_i^0 = 0, i = 0,1,\dots,N+1$$

$$w_0^j = \sin \omega j \Delta t \text{and} \qquad \textbf{(3.3)}$$

$$w_{N+1}^j = 0,\ \ j = 1,\dots,M$$

The governing equation of energy in discretized form is written as

$$\frac{\theta_i^{j+1} - \theta_i^j}{\Delta t} + \frac{Re}{4h}\left[\left(\theta_{i+1}^{j+1} - \theta_{i-1}^{j+1}\right) + (\theta_{i+1}^{j} - \theta_{i-1}^{j})\right] = \frac{1}{p_r 2h^2}\left[\left(\theta_{i+1}^{j+1} - 2\theta_i^{j+1} + \theta_{i-1}^{j+1}\right) + \left(\theta_{i+1}^{j} - 2\theta_i^{j} + \theta_{i-1}^{j}\right)\right] + E(\frac{f_{1i}^{j+1} + f_{1i}^{j}}{4\text{h}})^2 + \frac{\gamma E}{3\times 16^2 \times h^4}(f_{1i}^{j+1} + f_{1i}^{j})^4 + \frac{E\alpha}{8h^2\Delta t}\left(f_{1i}^{j+1} + f_{1i}^{j}\right)\left(f_{1i}^{j+1} - f_{1i}^{j}\right) + \frac{\alpha ReE}{8h^3}\left(f_{1i}^{j+1} + f_{1i}^{j}\right)\left(f_{2i}^{j+1} + f_{2i}^{j}\right) + Em^2(\frac{w_i^{j+1} + w_i^j}{2}) \qquad \textbf{(3.4)}$$

With initial and boundary conditions on θ in discretised form

$$\theta_i^0 = 0, \qquad i = 0,1,2,\dots,N+1$$

$$\theta_0^j = 1\ and \qquad \textbf{(3.5)}$$

$$\theta_{N+1}^j = 0,\ j = 1,2,\dots,M.$$

To impose the boundary conditions at infinity, we choose N so that the absolute difference of two solutions obtained by assuming the boundary conditions at∞ to hold at $\big((N+1)h\ ,j\Delta t\big) and\ ((N+2)h\ ,j\Delta t)$ successively, so that the difference between the two consecutive solutions obtained is less than a prescribed error tolerance∈ given in Jain (1984).

A tri-diagonal system of equations is obtained after rearrangement of energy equation. Then it is solved using special form of Gaussian elimination method with the help of initial velocities which are obtained again by solving the tri-diagonal system for velocity formed by making the parameter m, αandγbecome zero in constitutive equation of velocity given in eq.(3.2). Similarly initial choice of temperature is made converting the energy equation into a tri-diagonal system by making the parameter Eand α become zero in eq.(3.4) and then it is solved using the exponentially fitted scheme described in Morton(1996).

Then the system of non-linear equations for velocity (3.2) with its initial and boundary conditions (3.3) is solved by damped-Newton method. For implementation of this method the residuals ($R_i, i = 0,1,2,\dots,N$) and the non-zero elements of the Jacobian matrix$\left(\frac{\partial R_i}{\partial w_j}\right)$,$i = 1,2,\dots,Nandj = 1,2,\dots,M$ of the system are computed. The above method isquadratically convergent and a new approximation x^{m+1}being accepted as $\left(x^m + {}^{h}/_{2^i}\right)$ for some suitable i, only when it satisfies$\left\|f\left(x^m + {}^{h}/_{2^i}\right)\right\|_2 < \|f(x^m)\|_2$,otherwise the failure occur. This ensuresthere must be decrease in residual error after every iteration. Hence it establishes the convergence of the iteration scheme. The MATLAB coding for the above numerical scheme is verified with the existing result given in Conte, De Boor (1980) and the results are found to be correct up to 10^{-5}.

Hence the aforementioned iteration scheme is stable and convergent. Finally procure results are presented through graphs and its physical concepts are described in following section.

IV. RESULTS& DISCUSSION

The current investigation shows the unsteady MHD flow and heat transfer characteristics of third grade fluid placed between two porous plates subjected to a uniform magnetic field. Results are plotted graphically in fig-1 to fig-24 for different physical parameters.

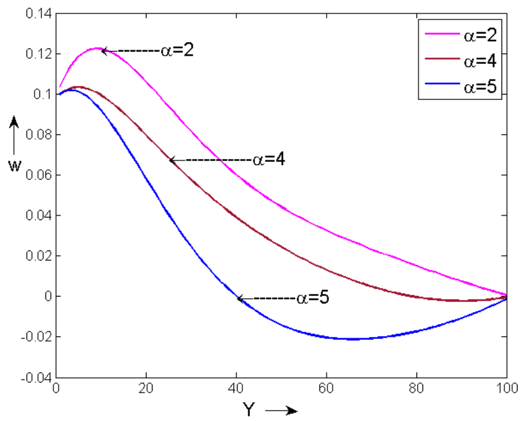

Fig. 1, Effect of α on Velocity field (γ = 3,Re=7, w=1, pr=.3, E=.7, m=7, n=100)

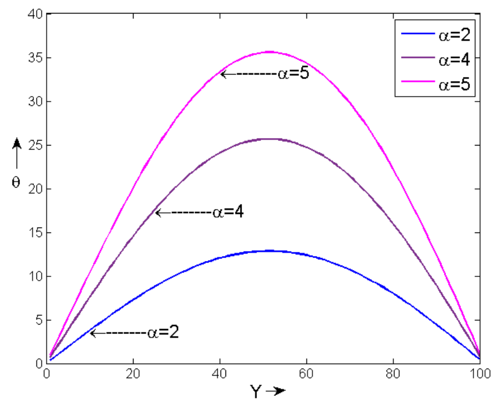

Fig. 2, Effect of α on Temp. field (γ = 3,Re=7, w=1, pr=.3, E=.7, m=7, n=100)

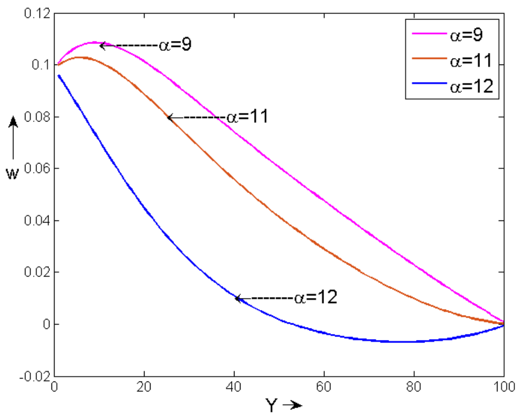

Fig. 3, Effect of α on Velocity field (γ = 5,Re=10, w=1, pr=.3, E=.7, m=5, n=100)

Fig-1, 2, 3, 4 ($\alpha > 1$) it is observed that with increasing the second grade elastic parameter αvalues, elasticity of the fluid increases that reduces fluid velocity profile. A sudden hike of velocity revealed near the upper plate and then velocity decreases towards the lower plate. Whereas a significant increase of temperature profile is observed throughout the flow field and the maximum effect is seen at the center of the plates.

Fig-5, 6($\alpha < 1$) illustrates a similar impact as to previous case for both the fields, which is obtained with the reduction of non-Newton parametric value γ to1 and magnetic parameter value m to 5 than the previous case.

Fig-7, 8 (γ>1) Forlarge values of third grade elastic parameter γ, it is observed that velocity profile increases and temperature profile decreases with increase of the non-Newtonian parameter γ.

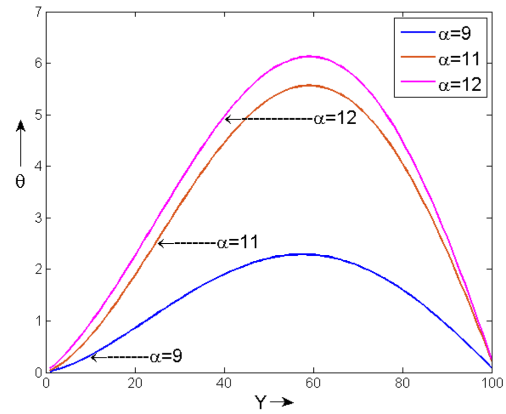

Fig. 4,Effect of α on Temp.field (γ = 5,Re=10, w=1, pr=.3, E=.7, m=5, n=100)

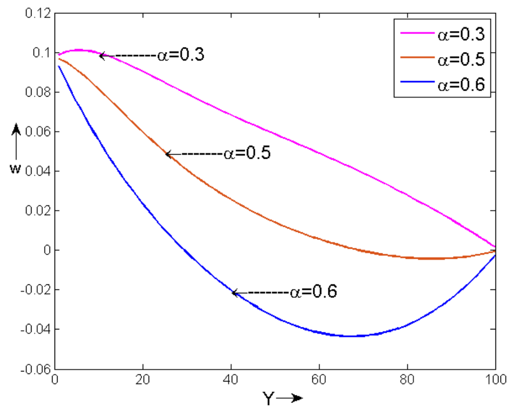

Fig. 5, Effect of αon Velocity field (γ = 1,Re=7, w=1, pr=.3, E=.7, m=5, n=100)

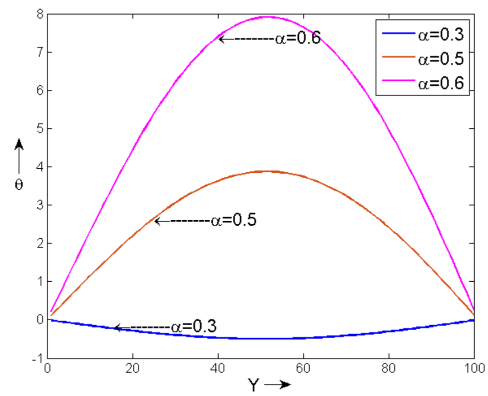

Fig. 6, Effect of α on Temp. field (γ = 1,Re=7, w=1, pr=.3, E=.7, m=5, n=100)

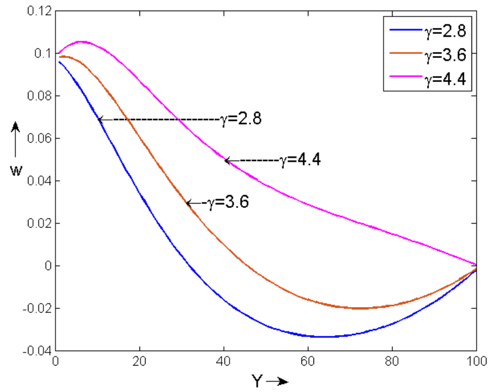

Fig. 7, Effect of γ on Velocity field (α = 3, Re=8, w=1, pr=.3, E=.7, m=7, n=100)

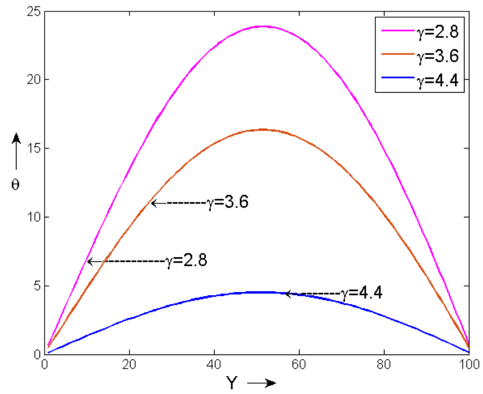

Fig. 8, Effect of γ on Temp (α = 3 ,Re=8, w=1, pr=.3, E=.7, m=7, n=100)

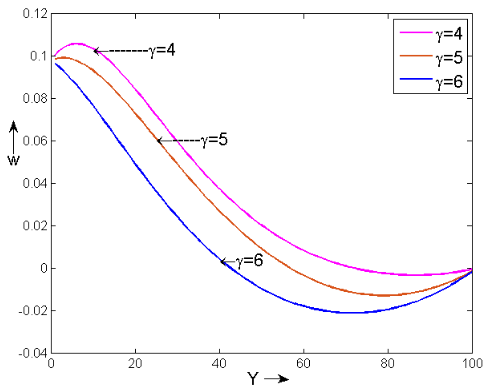

Fig. 9, Effect of γ on Velocity (α = 4, Re=8, w = 1, pr =.3, E =.7, m= 7, n = 100)

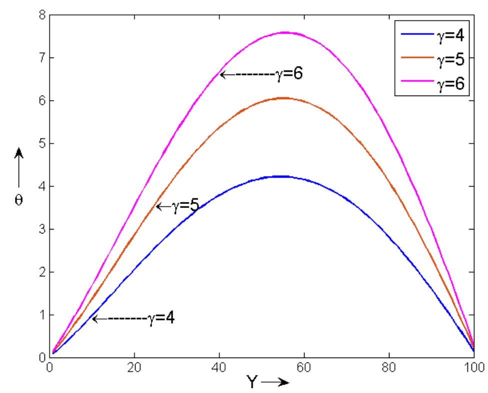

Fig. 10,Effect ofγon Temp. (α = 4, Re = 8, w = 1, pr =.3, E =.7, m= 7, n = 100)

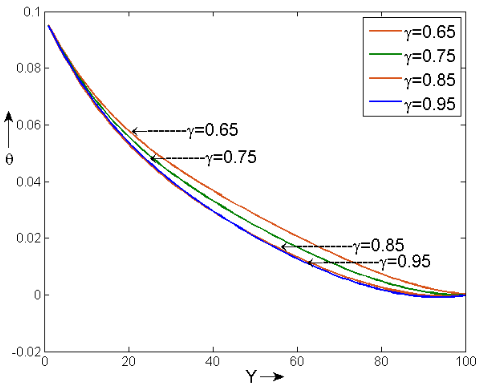

Fig. 11, Effect of γ on Velocity field (α = 3, Re = 8, w = 1, pr =.3, E =.7, m= 7, n = 100)

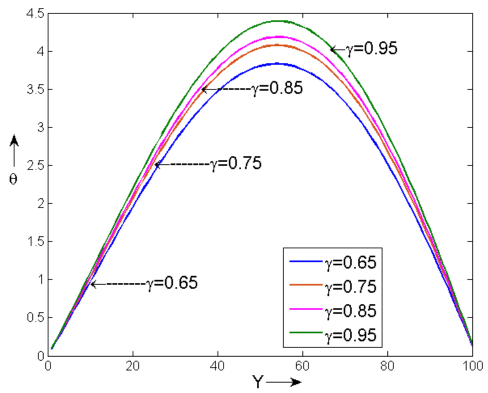

Fig. 12,Effect of γ on Temp. (α = 3, Re = 8, w = 1, pr =.3, E =.7, m= 7, n = 100

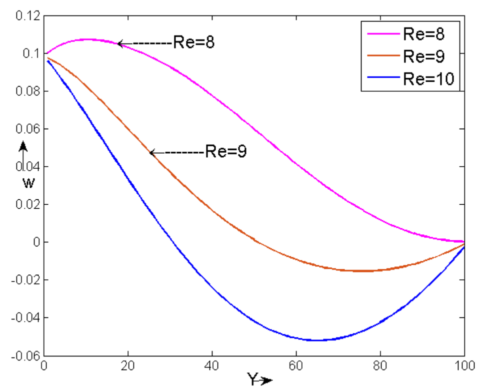

Fig. 13, Effect of Re on Vel. (α = 3, γ = 3, w = 1, pr =.3, E=.7,m=5, n = 100)

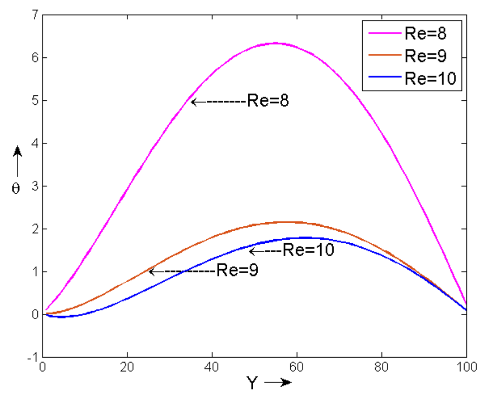

Fig. 14, Effect of Re on Temp. (α = 3, γ = 3, w = 1, pr =.3, E =.7, m=5, n = 100)

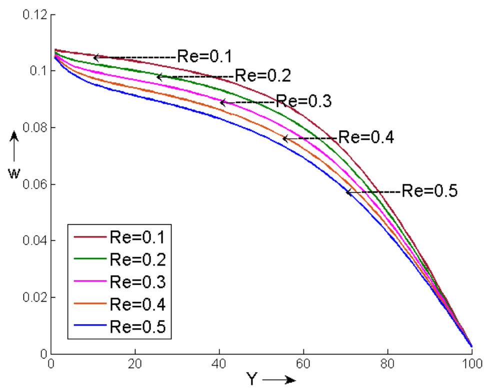

Fig. 15, Effect of Re on velocity field (α = .01, γ = 1, w = 1, pr =.3, E =.7,m=0.5, n = 100)

Fig-9, 10 But when second grade elastic parameter value of α raised to 4 keeping rest parameters fixed, a decreasing behavior of velocity profile is observed with increasing the values ofγ. Also it slow down the velocity as well as temperature profile and conforms that influence of second grade elastic parameter α is more on velocity profile than third grade elastic parameter γ.

Fig-11, 12 (γ<1) Again for small values of third grade elastic parameter γ the momentum boundary layer thickness gradually decreases with increase of γ values. This might occur, because when γ values are small, the influence of second grade elastic parameter is more on flow field and that causes gradual deceleration of velocity profile. The effect is not pronounced near the boundary, but a noticeable effect is seen away from boundary. The temperature field increases with increase of the parameter valuesγ.

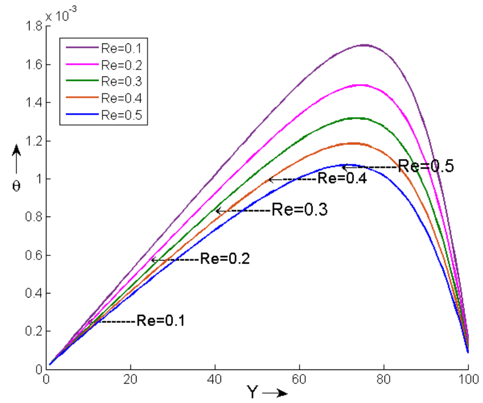

Fig. 16,Effect of *Re* on Temp. (α = .01, γ = 1, w = 1, pr =.3, E =.7,m=.5, n = 100)

Fig-13, 14(R_e >1) Depict the impact of Reynolds number R_e on velocity and temperature field. It shows when values of R_e increases, viscous force of the fluid gradually increases which causes decrease in both velocity as well as temperature profile at all points of the domain of fluid flow.

Fig-15, 16(R_e <1) Similar effect is seen on velocity and temperature profile when Reynolds number values are less than 1 obtained along with a reduction inthe values of elastic parameters α, γand magnetic parameter m. Here the momentum and thermal boundary layer influence near the plates observed to be insignificant.

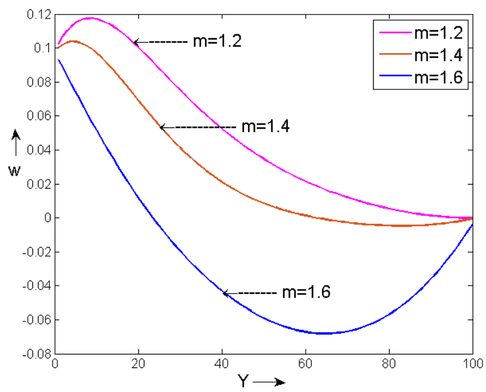

Fig. 17, Effect of m on velocity field (α = 3, γ = 3, Re = 7, w = 1, pr =.3,E=.7,n = 100)

Fig-17, 18($m >$ 1) Show that velocity gradually decreases when magnetic parameter m values increases. Due to increase in magnetic field strength the transverse Lorentz force increases on flow field, which in turns reduces the fluid velocity. But a substantial increase is seen on temperature field

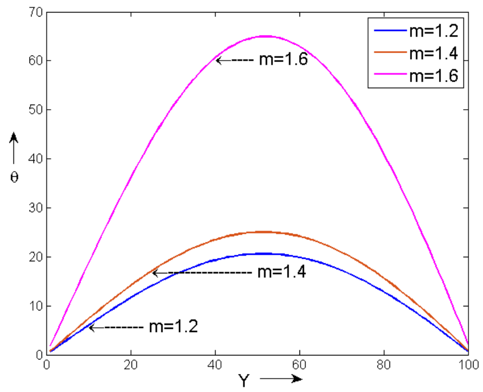

Fig.18, Effect of m on Temp. (α = 3, γ = 3, Re = 7, w = 1, pr =.3,E=.7, n = 100)

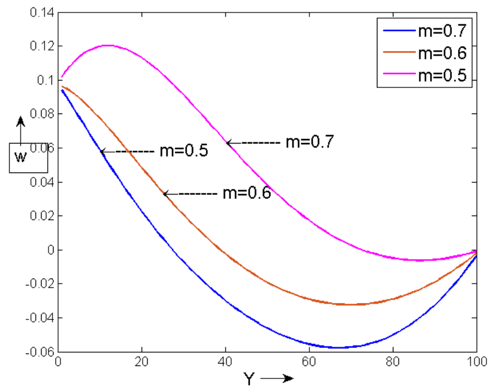

Fig. 19, Effect of m on vel. (α = 3, γ = 3, Re = 7, w = 1,pr=.3, E =.7, n = 100)

Fig-19, 20 (m< 1) Portray the effect of smaller magnitude magnetic field effect on velocity and temperature profile. It is observed here that the effect of Lorentz force become negligible and that provides less amount of resistance to the fluid flow, so a profound increase of velocity profile is seen. But the temperature field observed to be decreasing.

Fig-21, 22 recite the effect of viscous dissipation parameter *E* on temperature profile. It is seen that temperature profile increases with increase of every value of viscous dissipation parameter E.

So when *E* increases viscous dissipation heat generated is more which causes rise in the temperature field.

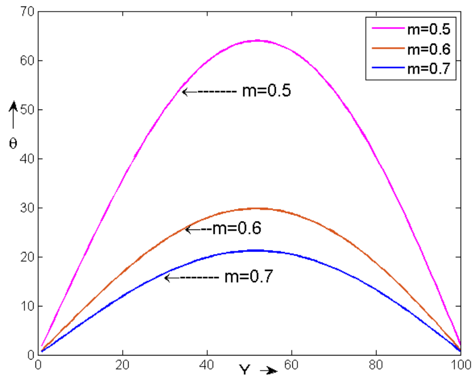

Fig. 20,Effect of m on Temp. (α = 3, γ = 3, Re = 7, w = 1,pr=.3, E =.7, n = 100)

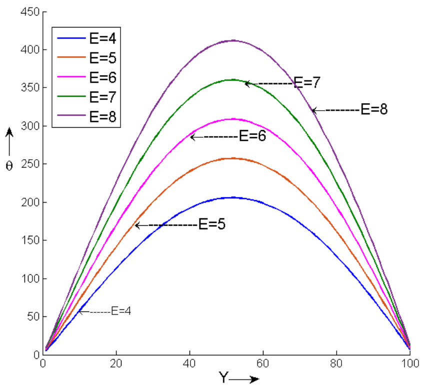

Fig. 21,Effect of E on Temp. (α = 2, γ = 3, Re = 7, w = 1, pr =.3,m = 7, n = 100)

Finally fig-23, 24 Depicts when $p_r < 1$ temperature profile decreases with increase of Prandtl number. Where the effect is noticeable near the boundary and gradually diminishes away from the boundary. For $p_r > 1$ it is observed that when the parameter values increases, the temperature field decreases near the surface of the plate and then gradually increases away from the plate and that causes a cross over temperature profile.

V. Conclusion

This study analyses unsteady magneto-hydrodynamic flow and heat transfer of third grade fluid in a porous channel. The results are pictorially presented through several graphs and a comparison study is made between small and large values of physical parameters on velocity and temperature field. The imperative finding of the present study are as follows

- Velocity profile decreases and temperature profile increases with increase of low as well as high values of second grade elastic parameter α.
- Velocity profile increases and temperature decreases with increase of low as well as high value of third grade elastic parameter γ whenα = 3.But the effect become opposite on both the fields for valuesof γbecome 4 or more and strengthening second grade elastic parameter α to 4.
- Velocity decreases and temperature increases with increase of the magnetic parameter m values but on reduction of parametric values of m, a reversed effect is seen on both the fields.

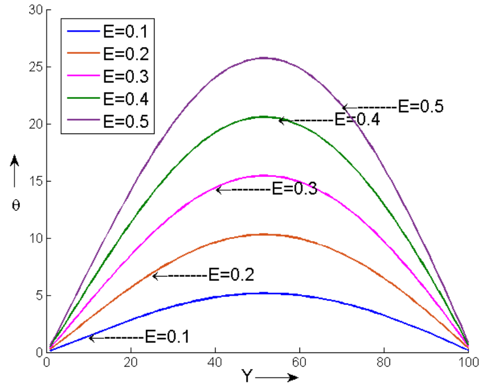

Fig. 22, Effect of E on Temp.

(α = 2, γ = 3, Re = 7, w = 1, pr =.3,m = 7, n = 100)

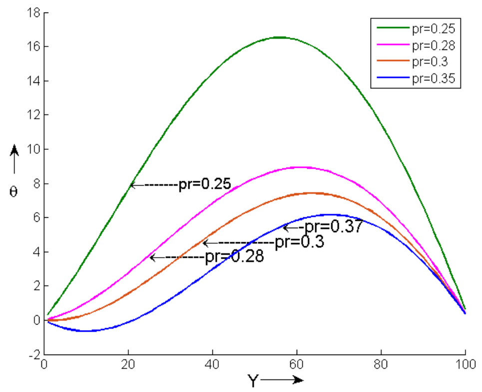

Fig. 23,Effect of Pr on Temp. (α = 3, γ = 3, Re = 9, w = 1, E =.7, m= 7, n = 100)

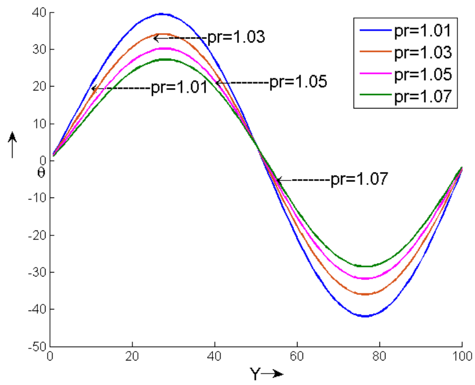

Fig. 24Effect of Pr on Temp. (α = 3, γ = 3, Re = 9, w = 1, E =.7, m= 7, n = 100)

- Moreover when viscous dissipation factor E increases, temperature profile increases whatever the Eckert number E is large or small.

This means large values of visco-elastic parameter α and magnetic parameter m create hindrance on fluid flow and slow down the velocity profile but the temperature field increases substantially.

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