The Cost Sharing Contract of Greening Level and Pricing Policies in a Green Supply Chain

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Abstract—**This paper addresses the problem of the impacts of the cost sharing contract on the optimal decisions in a two-level green supply chain consisting of a manufacturer and a retailer. The market demand depends on the greening level and the retail price of the green product. Three different kinds of game modes, including MS (Manufacturer-Stackelberg) game, RS (Retailer-Stackelberg) game and VN (Vertical-Nash) game are developed, and their optimal solutions are also derived and compared. Finally, the solutions of proposed models are analyzed via a numerical example. The results show that the VN game is a preferred policy for the customers. Moreover, the greening level, the wholesale price, the retail price, and the profit of the manufacturer increase as the cost sharing proportion increases, while the profit of the supply chain system increases first and then decreases in the three games.**

Index Terms—**green supply chain, cost sharing contract, game model, greening level**

I. Introduction

With the expansion of the global supply chain, resource consumption and environmental pollution problems have aroused people's attention. People are more likely to purchase green products than traditional ones, though the latter is cheaper than the former. Therefore, how to make the optimal greening level and pricing decisions is a challenge for the manufacturer and the retailer.

In recent years, many scholars and researchers have shown interest in greening level and pricing decisions in a green supply chain. For example, Ghosh and Shah [1] studied the game models of green supply chain and showed how the greening level, the prices and the profits were influenced by channel structures. Liu and Yi [2] studied the pricing policies of green supply chain considering targeted advertising and product green degree in the Big Data environment. Chen et al. [3] studied the pricing policies and green strategies with vertical and horizontal competition in green supply chains. Zhu and He [4] investigated the green product development problems with different supply chain structures including coordinated supply chain, vertical competing supply chain, and horizontal competing supply chains. Hafezalkotob [5] developed the price-energy-saving competition and cooperation models for two green supply chains with government financial intervention. Taleizadeh and Heydarian [6] studied a joint pricing and refund optimization problem under cooperative and non-cooperative strategies in a green supply chain. Xing et al. [7] suggested that the green manufacturer always adopted the integrated channel strategy to pursue channel efficiency. Ma et al. [8] examined the pricing policies for substitutable products with two competitive manufacturers and one retailer in a competitive green supply chain. Taleizadeh et al. [9] studied the pricing strategy of the green product, in which the market demand was stochastic and depended on the final price of product and carbon reduction rate. In addition, Sang [10] examined the pricing and retail service decisions in an uncertain supply chain, in which the costs and market demand were uncertain variables. Sang [11] also studied the pricing and service decisions in a supply chain with fairness reference.

Some researchers also studied the coordination issues of green supply chain. Zhang and Liu [12] investigated the coordination mechanism in a three-level green supply chain under one cooperative game and three non-cooperative games. Zhang et al. [13] also studied the pricing and coordination strategy of green supply chain under hybrid production mode where the green product and the non-green products co-existed with and substituted each other. Basiri and Heydari [14] analyzed the green channel coordination issues with a new substitutable green product and a traditional one. Swami and Shah [15] studied the channel coordination issue by using a two-part tariff contract and showed that this contract could coordinate green supply chain. Zhang et al. [16] also used a two-part tariff contract to coordinate a supply chain with green innovation in a dynamic setting. Li et al. [17] discussed the pricing and greening strategies in a competitive dual-channel green supply chain and proposed a two-part tariff contract to coordinate the supply chain actors. Song and Gao [18] established a green supply chain game model with two kinds of revenue-sharing contracts. Raj et al. [19] proposed five different contract types, namely, wholesale price, linear two part tariff, greening cost sharing, revenue sharing, and revenue and greening-cost sharing contracts in a green supply chain. Recently, Yang and Xiao [20] developed three game models of a green supply chain with governmental interventions under fuzzy environment. Sang [21] studied the green policies with three different decentralized decision models of green supply chain, in which the production cost and market demand were fuzzy.

A cost sharing contract plays an important role in coordinating the distribution of benefits among the supply chain members in a supply chain. Some researchers use the

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cost sharing contract to coordinate the supply chain. For example, Ghosh and Shah [22] explored two cost sharing contracts in a green supply chain, one in which the retailer offered a cost sharing contract and the other in which the retailer and the manufacturer bargained on the cost sharing contract. Yang et al. [23] investigated the role of a retailer in a manufacturer's capacity investment strategies by two capacity cost sharing contracts, namely, the full capacity cost sharing contract and the partial capacity cost sharing contract. Bai et al. [24] proposed a revenue and promotional cost sharing contract to coordinate the sustainable supply chain systems with deteriorating items. Xu et al. [25] used the cost sharing contract to study the supply chain coordination problem under cap-and-trade regulation. Xie et al. [26] studied the pricing and servicing policies with the cost sharing contract in a dual-channel closed-loop supply chain.

By analyzing the literature above, it clearly shows that the exiting research on greening level and pricing decisions with the cost sharing contract is rare. Therefore, we discuss the two-level green supply chain game models with a cost sharing contract, which can tell both the manufacturer and the retailer how to make their optimal decisions when they pursue different power structures. We mainly discuss the conditions where the manufacturer and the retailer pursue three non-cooperative games: pursuing the Manufacturer-Stackelberg game, playing Retailer-Stackelberg game and acting in Vertical-Nash game.

The rest of this paper is organized as follows. In Section II, we describe the problem and some necessary assumptions related to the paper. In Section III, we present three non-cooperative game models with the cost sharing contract in a green supply chain. In Section IV, we provide a numerical example to illustrate the results of the proposed models. Finally, In Section V, we conclude our work and offer directions for further research

II. Problem Description and Assumptions

We consider a two-level green supply chain model consisting of a manufacturer with a retailer. The manufacturer produces green production to protect the environment. The wholesale price and greening level of the green production are determined by the manufacturer, then the manufacturer sells it to the retailer, while the retailer sets the retail price and sells it to the green sensitive consumers. Both the manufacturer and the retailer benefit out of the green sensitive market demand. We make the following assumptions, and the parameters and meanings are listed in Table I.

Assumption 1. The demand function is given as

$$q=\alpha-\beta p+\gamma\theta \text{, where } \alpha,\beta,\gamma>0,\quad p=w+m \tag{1}$$

where α represents the market potential, β represents the sensitivity of demand to price changes, p represents the retail price, which equal to the wholesale price w plus the profit margin m, γ represents the demand expansion effectiveness coefficient of the greening level, and θ represents the greening level of the green product.

Assumption 2. To improve the greening level of the product, the manufacturer needs to invest funds for new product research and development. The cost of achieving green innovation is assumed to be a quadratic function of the level of greening level θ. It is given by $\frac{1}{2}\eta\theta^2$, where η is the investment coefficient.

Assumption 3. In order to encourage the manufacturer to improve its greening level of the product, Φ is the proportion of investments that the retailer agrees to make, with the manufacturer contributing $1-\Phi$ to the green innovation investment in the cost sharing contract.

Assumption 4. In order to ensure that the manufacturer and the retailer obtain positive profits, we assume $0<\Phi<1-\frac{\gamma^2}{\beta\eta}$.

TABLE I
PARAMETERS AND ITS MEANING

Parameter	Meaning
α	The market potential
β	Customer sensitive to price
γ	Customer sensitive to greening level
p	The retail price
θ	The greening level
η	The investment coefficient
q	The market demand
w	The wholesale price
m	The profit margin
c	The cost of the producing green product
Φ	The cost sharing proportion of investments
Π_M	The manufacturer's profit
Π_R	The retailer's profit

Based on the above assumptions, the profit functions of the manufacturer and the retailer are given as

$$\Pi_M=(w-c)(\alpha-\beta p+\gamma\theta)-\frac{1}{2}(1-\Phi)\eta\theta^2 \tag{2}$$

$$\Pi_R=(p-w)(\alpha-\beta p+\gamma\theta)-\frac{1}{2}\Phi\eta\theta^2 \tag{3}$$

III. Models Analysis

In this section, we analyze the manufacturer and the retailer how to set their optimal policies when they pursue different power structures in the cost sharing contract. We mainly discuss the conditions where they pursue three non-cooperative games: the manufacturer leads the supply chain, the retailer leads the supply chain, and they have the same power.

A. MS game model

In the MS (Manufacturer-Stackelberg) game model, the manufacturer and the retailer present a typical Stackelberg game and the manufacturer is the leader. That is, firstly, the manufacturer sets the greening level and the wholesale price with the consideration of the reaction function of the retailer. Then, the retailer sets the retail price with the given wholesale price and the greening level so as to maximize his profit. Thus, the MS game model can be given as follows

$$\max_{\theta,w}\Pi_M = (w-c)(\alpha-\beta p+\gamma\theta)-\frac{1}{2}(1-\Phi)\eta\theta^2$$
$$\text{s.t.}\begin{cases} p=\arg\max\Pi_R \\ \max_{p}\Pi_R=(p-w)(\alpha-\beta p+\gamma\theta)-\frac{1}{2}\Phi\eta\theta^2\end{cases} \quad (4)$$

Theorem 1. In the MS game model, the optimal solutions of the manufacturer and the retailer are as follows

$$\theta^*=\frac{\gamma(\alpha-\beta c)}{4(1-\Phi)\beta\eta-\gamma^2} \quad (5)$$
$$w^*=\frac{2(1-\Phi)\eta(\alpha-\beta c)}{4(1-\Phi)\beta\eta-\gamma^2}+c \quad (6)$$
$$p^*=\frac{3(1-\Phi)\eta(\alpha-\beta c)}{4(1-\Phi)\beta\eta-\gamma^2}+c \quad (7)$$

Proof. First we solve the profit function of the retailer as follows

$$\max_{p}\Pi_R=(p-w)(\alpha-\beta p+\gamma\theta)-\frac{1}{2}\Phi\eta\theta^2 \quad (8)$$

The first order condition is

$$\frac{\mathrm{d}\Pi_R}{\mathrm{d}p}=-2\beta p+\alpha+\beta w+\gamma\theta$$

Then the second order condition is

$$\frac{\mathrm{d}^2\Pi_R}{\mathrm{d}p^2}=-2\beta$$

Note that the second order condition of Π_R is negative definite, since $\beta>0$. Consequently, Π_R is strictly concave in p.

Hence, let the first order condition be zero, we can get the optimal response function of the retailer as

$$p^*(\theta,w)=\frac{\alpha+\beta w+\gamma\theta}{2\beta} \quad (9)$$

Next we solve the profit function of the manufacturer

$$\max_{\theta,w}\Pi_M=(w-c)(\alpha-\beta p+\gamma\theta)-\frac{1}{2}(1-\Phi)\eta\theta^2 \quad (10)$$

Substituting $p^*(\theta,w)$ into (10), we get

$$\max_{\theta,w}\Pi_M=\frac{1}{2}(w-c)(\alpha-\beta w+\gamma\theta)-\frac{1}{2}(1-\Phi)\eta\theta^2 \quad (11)$$

The first order conditions are

$$\frac{\partial\Pi_M}{\partial\theta}=-(1-\Phi)\eta\theta+\frac{1}{2}\gamma w-\frac{1}{2}\gamma c$$
$$\frac{\partial\Pi_M}{\partial w}=-\beta w+\frac{1}{2}\gamma\theta+\frac{1}{2}\alpha+\frac{1}{2}\beta c$$

Therefore, the Hessian matrix of Π_M is

$$\mathrm{H}=\begin{bmatrix}\frac{\partial^2\Pi_M}{\partial\theta^2} & \frac{\partial^2\Pi_M}{\partial\theta\partial w}\\ \frac{\partial^2\Pi_M}{\partial w\partial\theta} & \frac{\partial^2\Pi_M}{\partial w^2}\end{bmatrix}=\begin{bmatrix}-(1-\Phi)\eta & \frac{1}{2}\gamma\\ \frac{1}{2}\gamma & -\beta\end{bmatrix}$$

Note that the Hessian matrix of Π_M is negative definite, since $\beta>0$, $\gamma>0$, $\eta>0$ and $0<\Phi<1-\frac{\gamma^2}{\beta\eta}$. Consequently, Π_M is strictly jointly concave in θ and w. Hence, let the first order conditions be zero, we get the optimal solutions θ^* and w^* of the manufacturer as follows

$$\theta^*=\frac{\gamma(\alpha-\beta c)}{4(1-\Phi)\beta\eta-\gamma^2}$$
$$w^*=\frac{2(1-\Phi)\eta(\alpha-\beta c)}{4(1-\Phi)\beta\eta-\gamma^2}+c$$

Substituting θ^* and w^* into (9), we get

$$p^*=\frac{3(1-\Phi)\eta(\alpha-\beta c)}{4(1-\Phi)\beta\eta-\gamma^2}+c$$

The proof of Theorem 1 is completed.

By combining (5), (6) and (7) with (2) and (3), we derive the optimal profits of the manufacturer and the retailer in the MS game model as follows

$$\Pi_M^*=\frac{(1-\Phi)\eta(\alpha-\beta c)^2}{2\left[4(1-\Phi)\beta\eta-\gamma^2\right]} \quad (12)$$
$$\Pi_R^*=\frac{\left[2(1-\Phi)^2\beta\eta-\Phi\gamma^2\right]\eta(\alpha-\beta c)^2}{2\left[4(1-\Phi)\beta\eta-\gamma^2\right]^2} \quad (13)$$

Proposition 1. In the MS game model

1) The greening level, the wholesale price and the retail price increase with Φ.
2) The profit of the manufacturer increases with Φ.
3) If $0<\Phi<\frac{\gamma^2}{8\beta\eta}$, then the profit of the retailer increases with Φ, and if $\frac{\gamma^2}{8\beta\eta}<\Phi<1-\frac{\gamma^2}{\beta\eta}$, then the profit of the retailer decreases with Φ

Proof. The first derivatives of the optimal greening level, wholesale price, retail price, manufacturer's profit and retailer's profit are as follows

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}\Phi}=\frac{4\beta\gamma\eta(\alpha-\beta c)}{\left[4(1-\Phi)\beta\eta-\gamma^2\right]^2}>0$$
$$\frac{\mathrm{d}w^*}{\mathrm{d}\Phi}=\frac{2\gamma^2\eta(\alpha-\beta c)}{\left[4(1-\Phi)\beta\eta-\gamma^2\right]^2}>0$$
$$\frac{\mathrm{d}p^*}{\mathrm{d}\Phi}=\frac{3\gamma^2\eta(\alpha-\beta c)}{\left[4(1-\Phi)\beta\eta-\gamma^2\right]^2}>0$$
$$\frac{\mathrm{d}\Pi_M^*}{\mathrm{d}\Phi}=\frac{\gamma^2\eta(\alpha-\beta c)^2}{2\left[4(1-\Phi)\beta\eta-\gamma^2\right]^2}>0$$
$$\frac{\mathrm{d}\Pi_R^*}{\mathrm{d}\Phi}=\frac{(\gamma^2-8\Phi\beta\eta)\gamma^2\eta(\alpha-\beta c)^2}{2\left[4(1-\Phi)\beta\eta-\gamma^2\right]^3}$$

When $0<\Phi<\frac{\gamma^2}{8\beta\eta}$, then $\frac{\mathrm{d}\Pi_R^*}{\mathrm{d}\Phi}>0$, and when $\frac{\gamma^2}{8\beta\eta}<\Phi<1-\frac{\gamma^2}{\beta\eta}$, then $\frac{\mathrm{d}\Pi_R^*}{\mathrm{d}\Phi}<0$.

The proof of Proposition 1 is completed.

Remark 1. If Φ=0, then the cost sharing contract is not considered, and the optimal policies of the manufacturer and the retailer under the MS game are as follows

$$\theta^*=\frac{\gamma(\alpha-\beta c)}{4\beta\eta-\gamma^2}$$

$$w^* = \frac{2\eta(\alpha-\beta c)}{4\beta\eta-\gamma^2}+c$$

$$p^* = \frac{3\eta(\alpha-\beta c)}{4\beta\eta-\gamma^2}+c$$

$$\Pi_M{}^* = \frac{\eta(\alpha-\beta c)^2}{2(4\beta\eta-\gamma^2)}$$

$$\Pi_R{}^* = \frac{\beta\eta^2(\alpha-\beta c)^2}{(4\beta\eta-\gamma^2)^2}$$

B. RS game model

In the RS (Retailer-Stackelberg) game model, the retailer and the manufacturer present a typical Stackelberg game and the retailer is the leader. That is, firstly, the retailer sets the profit margin using the response functions of the manufacturer. Then the manufacturer sets the greening level and the wholesale price with the given profit margin. Thus, the RS game model can be given as follows

$$\max_m \Pi_R = m\left[\alpha-\beta(w+m)+\gamma\theta\right]-\frac{1}{2}\Phi\eta\theta^2$$
$$\text{s.t.}\begin{cases}\theta, w = \arg\max \Pi_M \\ \max\limits_{\theta,w}\Pi_M = (w-c)\left[\alpha-\beta(w+m)+\gamma\theta\right]-\frac{1}{2}(1-\Phi)\eta\theta^2\end{cases} \quad (14)$$

Theorem 2. In the RS game model, the optimal solutions of the manufacturer and the retailer are as follows

$$\theta^{**} = \frac{\gamma(\alpha-\beta c)}{2\left[2(1-\Phi)\beta\eta-\gamma^2\right]} \quad (15)$$

$$w^{**} = \frac{(1-\Phi)\eta(\alpha-\beta c)}{2\left[2(1-\Phi)\beta\eta-\gamma^2\right]}+c \quad (16)$$

$$p^{**} = \frac{\left[3(1-\Phi)\beta\eta-\gamma^2\right](\alpha-\beta c)}{2\beta\left[2(1-\Phi)\beta\eta-\gamma^2\right]}+c \quad (17)$$

Proof. First we solve the profit function of the manufacturer as follows

$$\max_{\theta,w}\Pi_M = (w-c)\left[\alpha-\beta(w+m)+\gamma\theta\right]-\frac{1}{2}(1-\Phi)\eta\theta^2 \quad (18)$$

The first order conditions are

$$\frac{\partial\Pi_M}{\partial\theta} = -(1-\Phi)\eta\theta+\gamma w-\gamma c$$

$$\frac{\partial\Pi_M}{\partial w} = -2\beta w+\gamma\theta+\alpha-\beta m+\beta c$$

Therefore, the Hessian matrix of Π_M is

$$\mathrm{H} = \begin{bmatrix}\frac{\partial^2\Pi_M}{\partial\theta^2} & \frac{\partial^2\Pi_M}{\partial\theta\partial w} \\ \frac{\partial^2\Pi_M}{\partial w\partial\theta} & \frac{\partial^2\Pi_M}{\partial w^2}\end{bmatrix} = \begin{bmatrix}-(1-\Phi)\eta & \gamma \\ \gamma & -2\beta\end{bmatrix}$$

Note that the Hessian matrix of Π_M is negative definite, since $\beta>0$, $\gamma>0$, $\eta>0$ and $0<\Phi<1-\frac{\gamma^2}{\beta\eta}$. Consequently, Π_M is strictly jointly concave in θ and w.

Hence, let the first order conditions be zero, we get the optimal response functions of the manufacturer as follows

$$\theta^{**}(m) = \frac{\gamma(\alpha-\beta m-\beta c)}{2(1-\Phi)\beta\eta-\gamma^2} \quad (19)$$

$$w^{**}(m) = \frac{(1-\Phi)\eta(\alpha-\beta m-\beta c)}{2(1-\Phi)\beta\eta-\gamma^2}+c \quad (20)$$

Next we solve the profit function of the retailer

$$\max_m \Pi_R = m\left[\alpha-\beta(w+m)+\gamma\theta\right]-\frac{1}{2}\Phi\eta\theta^2 \quad (21)$$

Substituting $\theta^{**}(m)$ and $w^{**}(m)$ into (21), we get

$$\max_m \Pi_R = \frac{(1-\Phi)\beta\eta m(\alpha-\beta m-\beta c)}{2(1-\Phi)\beta\eta-\gamma^2}-\frac{1}{2}\Phi\eta\theta^2 \quad (22)$$

The first order condition is

$$\frac{\mathrm{d}\Pi_R}{\mathrm{d}m} = \frac{(1-\Phi)\beta\eta(-2\beta m+\alpha-\beta c)}{2(1-\Phi)\beta\eta-\gamma^2}$$

Then the second order condition is

$$\frac{\mathrm{d}^2\Pi_R}{\mathrm{d}m^2} = -\frac{2(1-\Phi)\beta^2\eta}{2(1-\Phi)\beta\eta-\gamma^2}$$

Note that the second order condition of Π_R is negative definite, since $\beta>0$, $\eta>0$ and $0<\Phi<1-\frac{\gamma^2}{\beta\eta}$ $\beta>0$. Consequently, Π_R is strictly concave in m.

Hence, let the first order condition be zero, we can get the optimal profit margin of the retailer as

$$m^{**} = \frac{\alpha-\beta c}{2\beta} \quad (23)$$

Substituting m^{**} into (19) and (20), we get

$$\theta^{**} = \frac{\gamma(\alpha-\beta c)}{2\left[2(1-\Phi)\beta\eta-\gamma^2\right]}$$

$$w^{**} = \frac{(1-\Phi)\eta(\alpha-\beta c)}{2\left[2(1-\Phi)\beta\eta-\gamma^2\right]}+c$$

Then, the optimal retail price can be obtained as

$$p^{**} = w^{**}+m^{**} = \frac{\left[3(1-\Phi)\beta\eta-\gamma^2\right](\alpha-\beta c)}{2\beta\left[2(1-\Phi)\beta\eta-\gamma^2\right]}+c$$

The proof of Theorem 2 is completed.

By combining (15), (16) and (17) with (2) and (3), we derive the optimal profits of the manufacturer and the retailer in the RS game model as follows

$$\Pi_M{}^{**} = \frac{(1-\Phi)\eta(\alpha-\beta c)^2}{8\left[2(1-\Phi)\beta\eta-\gamma^2\right]} \quad (23)$$

$$\Pi_R{}^{**} = \frac{\left[4(1-\Phi)^2\beta\eta-(2-\Phi)\gamma^2\right]\eta(\alpha-\beta c)^2}{8\left[2(1-\Phi)\beta\eta-\gamma^2\right]^2} \quad (24)$$

Proposition 2. In the RS game model

1) The greening level, the wholesale price and the retail price increase with Φ.
2) The profit of the manufacturer increases with Φ.
3) If $0<\Phi<\frac{1}{3}-\frac{\gamma^2}{6\beta\eta}$, then the profit of the retailer increases with Φ, and if $\frac{1}{3}-\frac{\gamma^2}{6\beta\eta}<\Phi<1-\frac{\gamma^2}{\beta\eta}$, then

the profit of the retailer decreases with Φ

Proof. The first derivatives of the optimal greening level, wholesale price, retail price, manufacturer's profit and retailer's profit are as follows

$$\frac{\mathrm{d}\,\theta^{**}}{\mathrm{d}\,\Phi}=\frac{\beta\gamma\eta(\alpha-\beta c)}{\left[2(1-\Phi)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\mathrm{d}\,w^{**}}{\mathrm{d}\,\Phi}=\frac{\gamma^2\eta(\alpha-\beta c)}{2\left[2(1-\Phi)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\mathrm{d}\,p^{**}}{\mathrm{d}\,\Phi}=\frac{\gamma^2\eta(\alpha-\beta c)}{2\left[2(1-\Phi)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\mathrm{d}\,\Pi_M{}^{**}}{\mathrm{d}\,\Phi}=\frac{\gamma^2\eta(\alpha-\beta c)^2}{8\left[2(1-\Phi)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\mathrm{d}\,\Pi_R{}^{**}}{\mathrm{d}\,\Phi}=\frac{\left[2(1-3\Phi)\beta\eta-\gamma^2\right]\gamma^2\eta(\alpha-\beta c)^2}{8\left[2(1-\Phi)\beta\eta-\gamma^2\right]^3}$$

When $0<\Phi<\frac{1}{3}-\frac{\gamma^2}{6\beta\eta}$, then $\frac{\mathrm{d}\,\Pi_R{}^{**}}{\mathrm{d}\,\Phi}>0$, and when $\frac{1}{3}-\frac{\gamma^2}{6\beta\eta}<\Phi<1-\frac{\gamma^2}{\beta\eta}$, then $\frac{\mathrm{d}\,\Pi_R{}^{**}}{\mathrm{d}\,\Phi}<0$.

The proof of Proposition 2 is completed.

Remark 2. If Φ=0, then the cost sharing contract is not considered, and the optimal policies of the manufacturer and the retailer under the RS game are as follows

$$\theta^{**}=\frac{\gamma(\alpha-\beta c)}{2(2\beta\eta-\gamma^2)}$$

$$w^{**}=\frac{\eta(\alpha-\beta c)}{2(2\beta\eta-\gamma^2)}+c$$

$$p^{**}=\frac{(3\beta\eta-\gamma^2)(\alpha-\beta c)}{2\beta(2\beta\eta-\gamma^2)}+c$$

$$\Pi_M{}^{**}=\frac{\eta(\alpha-\beta c)^2}{8(2\beta\eta-\gamma^2)}$$

$$\Pi_R{}^{**}=\frac{\eta(\alpha-\beta c)^2}{4(2\beta\eta-\gamma^2)}$$

C. VN game model

In the VN (Vertical-Nash) game model, the manufacturer and the retailer have the same bargaining power. That is, the manufacturer determines the greening level θ and the wholesale price w, and the retailer makes the retail price simultaneously and independently, so as to maximize their profits. Thus, the VN game model can be given as follows

$$\begin{cases}\max\limits_{\theta,w}\Pi_M=(w-c)(\alpha-\beta p+\gamma\theta)-\frac{1}{2}(1-\Phi)\eta\theta^2\\ \max\limits_{p}\Pi_R=(p-w)(\alpha-\beta p+\gamma\theta)-\frac{1}{2}\Phi\eta\theta^2\end{cases}\tag{25}$$

Theorem 3. In the VN game model, the optimal solutions of the manufacturer and the retailer are as follows

$$\theta^{***}=\frac{\gamma(\alpha-\beta c)}{3(1-\Phi)\beta\eta-\gamma^2}\tag{26}$$

$$w^{***}=\frac{(1-\Phi)\eta(\alpha-\beta c)}{3(1-\Phi)\beta\eta-\gamma^2}+c\tag{27}$$

$$p^{***}=\frac{2(1-\Phi)\eta(\alpha-\beta c)}{3(1-\Phi)\beta\eta-\gamma^2}+c\tag{28}$$

Proof. First we solve the profit function of the manufacturer as follows

$$\max_{\theta,w}\Pi_M=(w-c)(\alpha-\beta p+\gamma\theta)-\frac{1}{2}(1-\Phi)\eta\theta^2\tag{29}$$

The first order conditions are

$$\frac{\partial\Pi_M}{\partial\theta}=-(1-\Phi)\eta\theta+\gamma w-\gamma c$$

$$\frac{\partial\Pi_M}{\partial w}=-w+\gamma\theta+\alpha-\beta p+\beta c$$

Therefore, the Hessian matrix of Π_M is

$$\mathrm{H}=\begin{bmatrix}\frac{\partial^2\Pi_M}{\partial\theta^2} & \frac{\partial^2\Pi_M}{\partial\theta\partial w}\\ \frac{\partial^2\Pi_M}{\partial w\partial\theta} & \frac{\partial^2\Pi_M}{\partial w^2}\end{bmatrix}=\begin{bmatrix}-(1-\Phi)\eta & \gamma\\ \gamma & -\beta\end{bmatrix}$$

Note that the Hessian matrix of Π_M is negative definite, since $\beta>0$, $\gamma>0$, $\eta>0$ and $0<\Phi<1-\frac{\gamma^2}{\beta\eta}$. Consequently, Π_M is strictly jointly concave in θ and w. Hence, let the first order conditions be zero, we get the optimal solutions of the manufacturer as follows

$$\theta^{***}(p)=\frac{\gamma(\alpha-\beta p)}{(1-\Phi)\beta\eta-\gamma^2}\tag{30}$$

$$w^{***}(p)=\frac{(1-\Phi)\eta(\alpha-\beta p)}{(1-\Phi)\beta\eta-\gamma^2}+c\tag{31}$$

Next we solve the profit function of the retailer

$$\max_{p}\Pi_R=(p-w)(\alpha-\beta p+\gamma\theta)-\frac{1}{2}\Phi\eta\theta^2\tag{32}$$

The first order condition is

$$\frac{\mathrm{d}\,\Pi_R}{\mathrm{d}\,p}=-2\beta p+\alpha+\beta w+\gamma\theta$$

Then the second order condition is

$$\frac{\mathrm{d}^2\,\Pi_R}{\mathrm{d}\,p^2}=-2\beta$$

Note that the second order condition of Π_R is negative definite, since $\beta>0$. Consequently, Π_R is strictly concave in p.

Hence, let the first order condition be zero, we can get the optimal response function of the retailer as

$$p^{***}(\theta,w)=\frac{\alpha+\beta w+\gamma\theta}{2\beta}\tag{33}$$

Substituting $\theta^{***}(p)$ and $w^{***}(p)$ into (33), we get

$$p^{***}=\frac{2(1-\Phi)\eta(\alpha-\beta c)}{3(1-\Phi)\beta\eta-\gamma^2}+c$$

Substituting p^{***} into (30) and (31), we get

$$\theta^{***}=\frac{\gamma(\alpha-\beta c)}{3(1-\Phi)\beta\eta-\gamma^2}$$

$$w^{***}=\frac{(1-\Phi)\eta(\alpha-\beta c)}{3(1-\Phi)\beta\eta-\gamma^2}+c$$

The proof of Theorem 3 is completed.

By combining (26), (27) and (28) with (2) and (3), we derive the optimal profits of the manufacturer and the retailer in the VN game model as follows

$$\Pi_M{}^{***}=\frac{\left[2(1-\Phi)\beta\eta-\gamma^2\right](1-\Phi)\eta(\alpha-\beta c)^2}{2\left[3(1-\Phi)\beta\eta-\gamma^2\right]^2} \tag{34}$$

$$\Pi_R{}^{***}=\frac{\left[2(1-\Phi)^2\beta\eta-\Phi\gamma^2\right]\eta(\alpha-\beta c)^2}{2\left[3(1-\Phi)\beta\eta-\gamma^2\right]^2} \tag{35}$$

Proposition 3. In the VN game model

1) The greening level, the wholesale price and the retail price increase with Φ.
2) The profit of the manufacturer increases with Φ.
3) If $0<\Phi<\frac{1}{7}+\frac{\gamma^2}{7\beta\eta}$, then the profit of the retailer increases with Φ, and if $\frac{1}{7}+\frac{\gamma^2}{7\beta\eta}<\Phi<1-\frac{\gamma^2}{\beta\eta}$, then the profit of the retailer decreases with Φ

Proof. The first derivatives of the optimal greening level, wholesale price, retail price, manufacturer's profit and retailer's profit are as follows

$$\frac{\mathrm{d}\theta^{***}}{\mathrm{d}\Phi}=\frac{3\beta\gamma\eta(\alpha-\beta c)}{\left[3(1-\Phi)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\mathrm{d}w^{***}}{\mathrm{d}\Phi}=\frac{\gamma^2\eta(\alpha-\beta c)}{\left[3(1-\Phi)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\mathrm{d}p^{**}}{\mathrm{d}\Phi}=\frac{2\gamma^2\eta(\alpha-\beta c)}{\left[3(1-\Phi)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\mathrm{d}\Pi_M{}^{**}}{\mathrm{d}\Phi}=\frac{\left[(1-\Phi)\beta\eta-\gamma^2\right]\gamma^2\eta(\alpha-\beta c)^2}{2\left[3(1-\Phi)\beta\eta-\gamma^2\right]^3}>0$$

$$\frac{\mathrm{d}\Pi_R{}^{**}}{\mathrm{d}\Phi}=\frac{\left[\gamma^2-(7\Phi-1)\beta\eta\right]\gamma^2\eta(\alpha-\beta c)^2}{2\left[3(1-\Phi)\beta\eta-\gamma^2\right]^3}$$

When $0<\Phi<\frac{1}{7}+\frac{\gamma^2}{7\beta\eta}$, then $\frac{\mathrm{d}\Pi_R{}^{***}}{\mathrm{d}\Phi}>0$, and when $\frac{1}{7}+\frac{\gamma^2}{7\beta\eta}<\Phi<1-\frac{\gamma^2}{\beta\eta}$, then $\frac{\mathrm{d}\Pi_R{}^{***}}{\mathrm{d}\Phi}<0$.

The proof of Proposition 3 is completed.

Remark 3. If Φ=0, then the cost sharing contract is not considered, and the optimal policies of the manufacturer and the retailer under the VN game are as follows

$$\theta^{***}=\frac{\gamma(\alpha-\beta c)}{3\beta\eta-\gamma^2}$$

$$w^{***}=\frac{\eta(\alpha-\beta c)}{3\beta\eta-\gamma^2}+c$$

$$p^{***}=\frac{2\eta(\alpha-\beta c)}{3\beta\eta-\gamma^2}+c$$

$$\Pi_M{}^{***}=\frac{\left(2\beta\eta-\gamma^2\right)\eta(\alpha-\beta c)^2}{2\left(3\beta\eta-\gamma^2\right)^2}$$

$$\Pi_R{}^{***}=\frac{\beta\eta^2(\alpha-\beta c)^2}{\left(3\beta\eta-\gamma^2\right)^2}$$

D. Models comparison

On the basis of the above three models, the optimal solutions of the manufacturer and the retailer are compared, and the following three propositions are proposed.

Proposition 4. The optimal greening level meets the condition that $\theta^{***}>\theta^{**}>\theta^{*}$.

Proof. It is easy to verify that

$$\theta^{***}-\theta^{**}=\frac{\gamma(\alpha-\beta c)}{3(1-\Phi)\beta\eta-\gamma^2}-\frac{\gamma(\alpha-\beta c)}{2\left[2(1-\Phi)\beta\eta-\gamma^2\right]}$$
$$=\frac{\left[(1-\Phi)\beta\eta-\gamma^2\right]\gamma(\alpha-\beta c)}{2\left[3(1-\Phi)\beta\eta-\gamma^2\right]\left[2(1-\Phi)\beta\eta-\gamma^2\right]}>0$$

$$\theta^{**}-\theta^{*}=\frac{\gamma(\alpha-\beta c)}{2\left[2(1-\Phi)\beta\eta-\gamma^2\right]}-\frac{\gamma(\alpha-\beta c)}{4(1-\Phi)\beta\eta-\gamma^2}$$
$$=\frac{\gamma^3(\alpha-\beta c)}{2\left[4(1-\Phi)\beta\eta-\gamma^2\right]\left[2(1-\Phi)\beta\eta-\gamma^2\right]}>0$$

The proof of Proposition 4 is completed.

Proposition 5. The optimal wholesale price meets the condition that $w^{*}>w^{***}>w^{**}$.

Proof. It is easy to verify that

$$w^{*}-w^{***}=\left[\frac{2(1-\Phi)\eta(\alpha-\beta c)}{4(1-\Phi)\beta\eta-\gamma^2}+c\right]-\left[\frac{(1-\Phi)\eta(\alpha-\beta c)}{3(1-\Phi)\beta\eta-\gamma^2}+c\right]$$
$$=\frac{\left[2(1-\Phi)\beta\eta-\gamma^2\right](1-\Phi)\eta(\alpha-\beta c)}{\left[4(1-\Phi)\beta\eta-\gamma^2\right]\left[3(1-\Phi)\beta\eta-\gamma^2\right]}>0$$

$$w^{***}-w^{**}=\left[\frac{(1-\Phi)\eta(\alpha-\beta c)}{3(1-\Phi)\beta\eta-\gamma^2}+c\right]-\left[\frac{(1-\Phi)\eta(\alpha-\beta c)}{2\left[2(1-\Phi)\beta\eta-\gamma^2\right]}+c\right]$$
$$=\frac{\left[(1-\Phi)\beta\eta-\gamma^2\right](1-\Phi)\eta(\alpha-\beta c)}{2\left[3(1-\Phi)\beta\eta-\gamma^2\right]\left[2(1-\Phi)\beta\eta-\gamma^2\right]}>0$$

The proof of Proposition 5 is completed.

Proposition 6. The optimal retail price meets the condition that $p^{*}>p^{**}>p^{***}$.

Proof. It is easy to verify that

$$p^{*}-p^{**}=\left[\frac{3(1-\Phi)\eta(\alpha-\beta c)}{4(1-\Phi)\beta\eta-\gamma^2}+c\right]$$
$$-\left[\frac{\left[3(1-\Phi)\beta\eta-\gamma^2\right](\alpha-\beta c)}{2\beta\left[2(1-\Phi)\beta\eta-\gamma^2\right]}+c\right]$$
$$=\frac{\left[(1-\Phi)\beta\eta-\gamma^2\right]\gamma^2(\alpha-\beta c)}{2\beta\left[4(1-\Phi)\beta\eta-\gamma^2\right]\left[2(1-\Phi)\beta\eta-\gamma^2\right]}>0$$

$$p^{**}-p^{***}=\left[\frac{\left[3(1-\Phi)\beta\eta-\gamma^2\right](\alpha-\beta c)}{2\beta\left[2(1-\Phi)\beta\eta-\gamma^2\right]}+c\right]$$
$$-\left[\frac{2(1-\Phi)\eta(\alpha-\beta c)}{3(1-\Phi)\beta\eta-\gamma^2}+c\right]$$

$$=\frac{\left[(1-\Phi)\beta\eta-\gamma^{2}\right]^{2}(\alpha-\beta c)}{2\beta\left[3(1-\Phi)\beta\eta-\gamma^{2}\right]\left[2(1-\Phi)\beta\eta-\gamma^{2}\right]}>0$$

The proof of Proposition 6 is completed.

IV. Numerical Example

In this section, we tend to further elucidate the proposed three game models with a numerical example. We will analyze that the effective of the cost sharing proportion of investments Φ on the optimal solutions. The other parameter values are α=1000, β=50, γ=40, η=80 and c=6. The cost sharing proportion of the retailer is set to be $\Phi \in (0, 0.6)$, this ensures our analysis within the feasible region.

The optimal solutions with different of the cost sharing proportion of investments Φ are listed in Table II.

TABLE II
THE OPTIMAL POLICIES WITH DIFFERENT Φ

	Φ	θ	w	p	Π_M	Π_R	Π_{SC}
MS	0.10	2.19	13.88	17.81	1378.13	756.05	2134.18
	0.20	2.50	14.00	18.00	1400.00	750.00	2150.00
	0.30	2.92	14.17	18.25	1429.17	731.60	2160.77
	0.40	3.50	14.40	18.60	1470.00	686.00	2156.00
	0.50	4.38	14.75	19.13	1531.25	574.22	2105.47
RS	0.10	2.50	10.50	17.50	787.50	1550.00	2337.50
	0.20	2.92	10.67	17.67	816.67	1565.28	2381.95
	0.30	3.50	10.90	17.90	857.50	1568.00	2425.50
	0.40	4.38	11.25	18.25	918.75	1531.25	2450.00
	0.50	5.83	11.83	18.83	1020.83	1361.11	2381.94
VN	0.10	3.04	11.48	16.96	1167.11	1463.52	2630.63
	0.20	3.50	11.60	17.20	1176.00	1470.00	2646.00
	0.30	4.12	11.76	17.53	1186.85	1458.13	2644.98
	0.40	5.00	12.00	18.00	1200.00	1400.00	2600.00
	0.50	6.36	12.36	18.73	1214.88	1214.88	2429.76

Based on the results showed in Table II, we find:

1) The optimal greening level θ is the highest in the VN game when no actor is a pricing leader, followed by the RS and then the MS games. The wholesale price w is the highest in the MS game, which is a result of the manufacturer being the leader in pricing of the item, followed by the VN and then the RS games. The retail price p is the highest in the MS game, because under this game the manufacturer charges a high wholesale price, followed by the RS and then the VN games. These results are in accordance with Propositions 4, 5, and 6.
2) The manufacturer makes his largest profits in the MS game, and the smallest in the RS game. The retailer makes his largest profits in the RS game, and the smallest in the MS case. It shows that the actor who has the leadership in the green supply chain takes advantage in obtaining the higher profits. That is, the manufacturer's profit is the largest when the manufacturer is the leader, and the retailer's profit is largest when the retailer is the leader. In addition, the profit of the manufacturer is larger than that of the retailer in the MS game, and the profit of retailer is larger than that of the manufacturer in the RS game. The profit of the whole supply chain denoted by Π_{SC} is the largest in the VN game when the manufacturer and the retailer have the same bargaining power.
3) The VN game is a preferred policy for the customers this is because under this game the greening level θ is highest and the retail price p is lowest.
4) The optimal greening level, wholesale price, retail price, and profit of the manufacturer all increase as the cost sharing proportion of the retailer Φ increases, which in accordance with Propositions 1, 2, and 3. In this numerable example, when the cost sharing proportion of the retailer Φ increases, the profit of the retailer decreases in the RS game, while increases first and then decreases in the MS and VN games, and the profit of the whole supply chain increases first and then decreases in the three games.

V. Conclusion

This paper considers a green supply chain including a manufacturer and a retailer in order to analyze the effect of the cost sharing contract. We studied the conditions in which the manufacturer and the retailer pursue three different kinds of games: Manufacturer-Stackelberg game, Retailer -Stackelberg game and Vertical-Nash game. We also analyze the effect of the cost sharing proportion of the retailer on the optimal greening level, wholesale price, retail price, and profits of the manufacturer, the retailer and the whole supply chain.

Based on the discussions above, three main findings can be obtained. First, the supply chain actor who has the leadership in the green supply chain takes advantage in obtaining the higher profits. Second, the VN game is a preferred policy for the customers this is because under this game the greening level is highest and the retail price is lowest. Third, with the cost sharing proportion of the retailer increasing, the manufacturer's profit increases, and the retailer's profit decreases in the RS game, while increases first and then decreases in the MS and VN games.

This paper has some limitations. First, we only consider one manufacturer and one retailer. Therefore, one possible extension work is to study the greening level and pricing decisions with multiple competing manufacturers or retailers with the cost sharing contract. Second, we assume that the demand function is a linear demand function. Then the other types of the demand function can be considered. Third, we only consider the cost sharing contract in this paper. In future, the other coordination contracts such as two-part tariff or revenue sharing contract can be employed.

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