Two repairmen Machine Repairable System with Flexible Repair Policy

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***Abstract*—This paper studies machine repairable system with flexible service policy. There are two repairmen and finite quantity repairable machines in the system, The machines do some kind of production work and may fail at any time. Two repairmen are responsible for repairing the failure machines. When at least two machines are failure in the system, every repairman repair one failure machine separately. On the other hand, when only one machine is the failure, the two repairmen repair the one failure machine collectively at the same time. For such a machine repairable system, we derive the steady-state and transient-state indexes of the system. Numerical experiments have been done to show the system performances.**

***Index Terms*—machine repairable system, Markov process, flexible repair policy, steady-state availability, reliability.**

I. Introduction

THE machine repairing system can be applied to a variety of real situations, such as computer network, telecommunications, aircraft maintenance, and many others[1]. A multi-server machine repair problem with warm standbys under synchronous multiple vacation policies was investigated by Ke and Wu[2]. Wang, Liou, and Lin [3] studied the issue of the M/M/R machine repair problem with imperfect coverage and service pressure condition. Liou, Wang, and Liou [4] considered the controllable M/M/2 machine repair problem under the triadic (0, Q, N, M) policy. Wang et al. [5] utilized a recursive method based on the supplementary variable technique to develop steady-state analytical solutions in the M/G/1 machine repair problem with multiple imperfect coverage. The recursive and supplementary variable technique was used by Ke et al. [6] to analyze the steady-state behavior of machine repairable system with switching failure and warm standby support. Ye and Liu [7] considered an M/M/1 queue with two vacation policies. Kafhali and Hanini [8] make two mathematical models based on queueing theory to evaluate the performance of VoIP traffic in a single cell IEEE 802.16e Networks. Recently, Li and Li [9] considered an M/M/1 retrial queue with working vacation, orbit search and balking. They obtained the necessary and sufficient condition for the system to be stable, the stationary probability distribution and the performance measures of the system.

An unreliable multi-server queue with a controllable repair policy was considered by Wu et al. [10]. Fitouhi et al. [11] developed a two-machine continuous flow manufacturing system with a buffer of finite capacity. The queueing analysis of a multi-component machine repairable system comprising of operating as well as standby machines and a skilled repairman has been investigated by Meena et al. [12]. Chen et al. [13] analyzed the system reliability of the retrial machine repairable system with M operating units, S warm standby units and a single repair server with N-policy. Wang [14] deals with a modified M/M/R machine repair problem of M operating units with two types of spares, and R servers in the repair facility under steady-state conditions. Spares are considered to be either cold-standby, warm-standby or hot-standby. The servers have two service rates for repair of slow and fast. Ramasamy et al. [15] discussed the steady-state analysis of a heterogeneous server queuing system called the Geo/G/2 queue. Tsai et al. [16] studied an open queueing network with operating service stations suffering breakdowns.

In most of the existing studies of machine repairable systems with multi-repairman, the repair policy is only one repairman at a time can work on one failure machine, that means one repairman can repair only one failure machine, and one failure machine can be repaired by only one repairman at the same time. We call this repair policy as One-to-One (OTO) repair policy. Nevertheless, Many-to-One (MTO) repair policy is used in many multi-repairman machine repairable systems for efficiency reasons. MTO repair policy is that one repairman can repair only one failure machine, but one failure machine can be repaired by several repairmen at the same time. Actually, MTO service policy is common in realistic multi-repairman machine repairable system. For instance, a large transport logistics centre may has many forklifts, a truck can be serviced by two or more forklifts at the same time. In addition, multi-core processor technology has been used in computer technology widely, two or more computing cores may work together on one task at the same time. These cases indicate MTO service or repair policy is used in many service or repair systems, but until now it is difficult to find any study that deals with MTO repair policy in a multi-repairman machine repairable system.

II. Model description

We consider a machine repairable system which has two repairmen and $N(\geq 2)$ identical machines. Every machine may fail and the failure rate is λ. The two repairmen are responsible for repairing the failure machines. One repairman can repair only one failure machine at the same time, but one failure machine can be repaired by two repairmen together at the same time. This repair policy is called as Two-to-One (TTO) repair policy.

When the system has at least two failure machines, each repairman repair one failure machine respectively. Otherwise, when single one machine is failure in the system, the two repairmen repair the one failure machine together at the same

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time. If another machine breaks down before the completion of the repair for the single one failure machine, one of the two repairmen turns to repairing the new coming failure machine immediately.

When a failure machine is repaired by one repairmen, the repair time is exponential distribution with the parameter of μ. On the other hand, when a failure machine is repaired by two repairmen together, the repair time density function is

$$f(x) = \begin{cases} 2\mu q e^{-2\mu q t}, & t > 0, \\ 0, & \text{other}, \end{cases}$$

where $q(> 0)$ is a constant value. Generally, it is faster for two repairmen to repair one failure machine at the same time compared to a single repairman to repair one failure at the same time. Though the repair with two repairmen for one failure machine at the same time is faster, the two repairmen may produce interaction when they work together, so we introduce the interactional parameter q.

III. Stead-state performances

Let $X(t)$ be the number of failure machines in the system at time t, then $\{X(t), t \geq 0\}$ is a Markov process with state space

$$\Omega = \{i, 0 \leq i \leq N\}.$$

We let $P\{X(t) = i\} \equiv p_i(t)$ denote the transient-state probability that the system state is i at time t moment, and let p_i denote the steady-state probability that the system state is i. Then we have

$$p_i = \begin{cases} \lim\limits_{t\to\infty} p_i(t), & i = 0, 1, 2, \cdots, N, \\ 0, & \text{others}, \end{cases} \tag{1}$$

then

$$\sum_{i=0}^{N} p_i = 1. \tag{2}$$

The system state transfer rate matrix is as follows [14]:

$$Q = \begin{bmatrix} -N\lambda & N\lambda & & \\ 2\mu q & -(N-1)\lambda - 2\mu q & (N-1)\lambda & \\ & \ddots & \ddots & \ddots \\ & & 2\mu & -2\mu \end{bmatrix},$$

then the balance equations of system are as follows [17]:

$$\begin{cases} N\lambda p_0 - 2\mu q p_1 = 0, \\ N\lambda p_0 - [(N-1)\lambda + 2\mu q]p_1 + 2\mu p_2 = 0, \\ \quad \cdots \quad \cdots \\ 2\lambda p_{N-2} - (\lambda + 2\mu)p_{N-1} + 2\mu p_N = 0, \\ -\lambda p_{N-1} + 2\mu p_N = 0. \end{cases} \tag{3}$$

Solving Eq. (3) yields

$$p_k = \frac{N!}{q(N-k)!}(\frac{\lambda}{2\mu})^k p_0, (1 \geq k \geq N). \tag{4}$$

Using normalizing condition of Eq. (2) we obtain

$$p_0 = \frac{q}{q + \sum\limits_{i=1}^{N} \frac{N!}{(N-i)!}(\frac{\lambda}{2\mu})^i}.$$

Thus, we have

$$p_k = \frac{\frac{N!}{(N-k)!}(\frac{\lambda}{2\mu})^k}{q + \sum\limits_{i=1}^{N} \frac{N!}{(N-i)!}(\frac{\lambda}{2\mu})^i}, (1 \geq k \geq N)$$

Then, the steady-state probability that the repairmen are idle denoted by P_{RI} is

$$P_{RI} = p_0 = \frac{q}{q + \sum\limits_{i=1}^{N} \frac{N!}{(N-i)!}(\frac{\lambda}{2\mu})^i}.$$

The steady-state probability of no failure machine is waiting denoted by P_{NW} is

$$P_{NW} = p_0 + p_1 + p_2 = \frac{q + \frac{N\lambda}{2\mu} + N(N-1)(\frac{\lambda}{2\mu})^2}{q + \sum\limits_{i=1}^{N} \frac{N!}{(N-i)!}(\frac{\lambda}{2\mu})^i}.$$

The steady-state availability of the system denoted by P_{AV} is

$$P_{AV} = 1 - p_N = 1 - \frac{N!(\frac{\lambda}{2\mu})^N}{q + \sum\limits_{i=1}^{N} \frac{N!}{(N-i)!}(\frac{\lambda}{2\mu})^i}.$$

The expected number of the failure machines denoted by $E[FM] \equiv$ is

$$E[FM] = \sum_{k=1}^{N} k p_k = \frac{\sum\limits_{k=1}^{N} k \frac{N!}{(N-k)!}(\frac{\lambda}{2\mu})^k}{q + \sum\limits_{i=1}^{N} \frac{N!}{(N-i)!}(\frac{\lambda}{2\mu})^i}.$$

IV. Transient-state performances

We define

$$P(t) = (p_0(t), p_1(t), \cdots, p_N(t)),$$

and

$$P'(t) = (p'_0(t), p'_1(t), \cdots, p'_N(t)),$$

where $p'_i(t)$ is the differential function of $p_i(t)$. The transient-state probability differential equations in matrix form and the initial distribution are as follows:

$$\begin{cases} P'(t) = P(t)Q, \\ P(0) = (p_0(0), p_1(0), \cdots, p_N(0)), \end{cases}$$

where $0 \leq p_i(0) \leq 1$, and $\sum\limits_{i=0}^{N} p_i(0) = 1$.

We assume that all machines are available at the initial time $t = 0$. The transient-state reliability of the machines denoted by $R_M(t)$, and $R_M(t))$ is the probability that there is at least one machine is available till time $t(> 0)$ from the initial time. Letting the state of all machines are failure is absorbing state, we obtain a new Markov process, and its transition rate matrix is

$$\widehat{Q} = \begin{bmatrix} -N\lambda & N\lambda & & \\ 2\mu q & -(N-1)\lambda - 2\mu q & (N-1)\lambda & \\ & \ddots & \ddots & \ddots \\ & & 0 & 0 \end{bmatrix}.$$

Under the initial distribution is $\widehat{p}_0(0) = 1, \widehat{p}_1(0) = 0, \cdots, \widehat{p}_N(0) = 0$, the machine transient-state reliability is

$$R_M(t) = \sum_{i=0}^{N-1} \widehat{p}_i(t),$$

where $\widehat{p}_0(t), \widehat{p}_2(t), \cdots, \widehat{p}_{N-1}(t)$ are solutions of equations as follows:

$$\begin{cases} \widehat{p}_0'(t) = -N\lambda \widehat{p}_0(t) + 2\mu q \widehat{p}_1(t), \\ \widehat{p}_1'(t) = N\lambda \widehat{p}_0(t) - [(N-1)\lambda + 2\mu q)]\widehat{p}_1(t) + 2\mu \widehat{p}_2(t), \\ \quad \cdots \quad \cdots \\ \widehat{p}_{N-1}'(t) = 2\lambda p_{N-2}(t) - (\lambda + 2\mu) p_{N-1}(t) \\ \widehat{p}_0(0) = 1, \widehat{p}_2(0) = 0, \cdots, \widehat{p}_{N-1}(0) = 0. \end{cases} \tag{5}$$

V. THE INTENSIVE ANALYSIS OF SPECIFIC CASES

A. $N = 2$

1) OTO $(N = 2)$ system: For OTO system of $N = 2$, the state space is $\{0, 1, 2\}$, and the system state transfer rate matrix of OTO $(N = 2)$ system is as follows:

$$Q = \begin{bmatrix} -2\lambda & 2\lambda & 0 \\ \mu & -\lambda - \mu & \lambda \\ 0 & 2\mu & -2\mu \end{bmatrix}.$$

Then, the transient-state probability differential equations of OTO $(N = 2)$ system is

$$\begin{cases} p_0'(t) = -2\lambda p_0(t) + \mu p_1(t), \\ p_1'(t) = 2\lambda p_0(t) - (\lambda + \mu) p_1(t) + 2\mu p_2(t), \\ p_2'(t) = \lambda p_1(t) - 2\mu p_2(t). \end{cases} \tag{6}$$

Setting the initial distribution as $p_0(0) = 1, p_1(0) = 0$ and $p_2(0) = 0$, and letting $\lambda = 1, \mu = 1.2$, the solutions of Eq. (6) are as follows:

$$\begin{cases} p_0(t) = 0.297521 + 0.206612e^{-4.4t} + 0.495868e^{-2.2t}, \\ p_1(t) = 0.495868 - 0.413223e^{-4.4t} - 0.0826446e^{-2.2t}, \\ p_2(t) = 0.206612 + 0.206612e^{-4.4t} - 0.413223e^{-2.2t}. \end{cases} \tag{7}$$

Figure 1 is the curves of the solutions of Eq. (7). From Eq. (1) and Eq. (7) we obtain

$$p_0 = 0.297521, p_1 = 0.495868, p_2 = 0.206612.$$

Then we obtain the performances of OTO $(N = 2)$ system as follows:

$$P_{RI} = 0.298, P_{NW} = 1, P_{AV} = 0.793, E[FM] = 0.909.$$

Here, we derive the transient-state reliability of the machines denoted by $R_{2M}(t)$. We assume that all machines are available at the initial time $t = 0$, $R_{2M}(t))$ is the probability that there is at least one machine is available till time $t (> 0)$ from the initial time. Letting the state of all machines are failure is an absorbing state, we obtain a new Markov process, and its transition rate matrix is

$$\widehat{Q} = \begin{bmatrix} -2\lambda & 2\lambda & 0 \\ \mu & -\lambda - \mu & \lambda \\ 0 & 0 & 0 \end{bmatrix}.$$

Under the initial distribution of $\widehat{p}_0(0) = 1, \widehat{p}_1(0) = 0$ and $\widehat{p}_2(0) = 0$, the machine transient-state reliability is

$$R_{2M}(t) = \widehat{p}_0(t) + \widehat{p}_1(t),$$

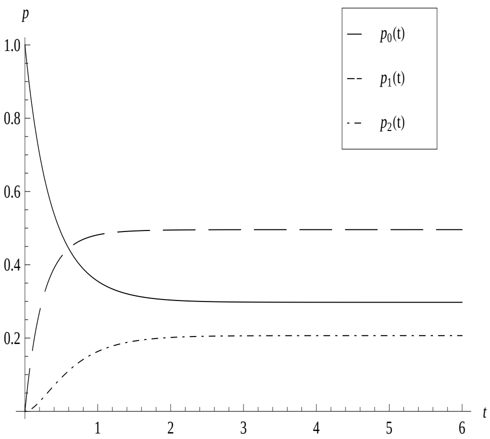

Figure 1. The transient-state probability of OTO $(N = 2)$ system for $N = 2, \lambda = 1$ and $\mu = 1.2$.

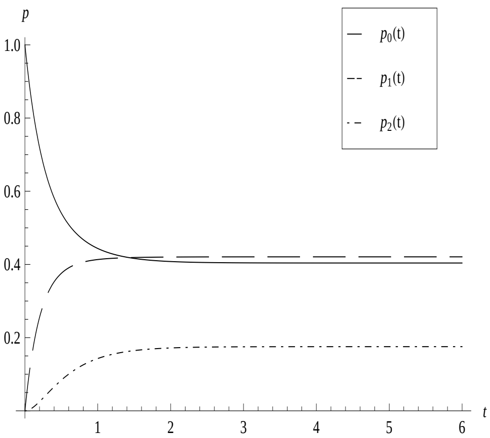

Figure 2. The transient-state probability of TTO $(N = 2)$ system for $N = 2, q = 0.8, \lambda = 1$ and $\mu = 1.2$.

where $\widehat{p}_0(t)$ and $\widehat{p}_1(t)$ are solutions of the equations as follows:

$$\begin{cases} \widehat{p}_0'(t) = -2\lambda \widehat{p}_0(t) + \mu \widehat{p}_1(t), \\ \widehat{p}_1'(t) = 2\lambda \widehat{p}_0(t) - (\lambda + \mu)\widehat{p}_1(t), \\ \widehat{p}_0(0) = 1, \widehat{p}_1(0) = 0. \end{cases} \tag{8}$$

Solving Eq. (8) by mathematic calculate software, we obtain

$$\widehat{p}_0(t) = \frac{\lambda - e^{t\Psi}\lambda - \mu + \Psi + e^{t\Psi}(\mu + \Psi)}{2\Psi e^{\frac{1}{2}t(3\lambda + \mu + \Psi)}},$$

$$\widehat{p}_1(t) = \frac{2\left(-1 + e^{t\Psi}\right)\lambda}{\Psi e^{\frac{1}{2}t(3\lambda + \mu + \Psi)}},$$

where $\Psi = \sqrt{\lambda^2 + 6\lambda\mu + \mu^2}$. Then

$$R_{2M}(t) = \frac{-3\lambda - \mu + \Psi + e^{t\Psi}(3\lambda + \mu + \Psi)}{2\Psi e^{\frac{1}{2}t(3\lambda + \mu + \Psi)}}.$$

2) TTO $(N = 2)$ system: For TTO system of $N = 2$, the state space is $\{0, 1, 2\}$, and the system state transfer rate matrix of TTO $(N = 2)$ system is as follows:

$$Q = \begin{bmatrix} -2\lambda & 2\lambda & 0 \\ 2q\mu & -\lambda - 2q\mu & \lambda \\ 0 & 2\mu & -2\mu \end{bmatrix}.$$

Then, the transient-state probability differential equations of TTO $(N=2)$ system is

$$\begin{cases} p_0'(t) = -2\lambda p_0(t) + 2q\mu p_1(t), \\ p_1'(t) = 2\lambda p_0(t) - (\lambda+2q\mu)p_1(t) + 2\mu p_2(t), \\ p_2'(t) = \lambda p_1(t) - 2\mu p_2(t). \end{cases} \tag{9}$$

Setting the initial distribution as $p_0(0)=1, p_1(0)=0$ and $p_2(0)=0$, and letting $\lambda=1, \mu=1.2$, and $q=0.8$, the solutions of Eq. (9) are as follows:

$$\begin{cases} p_0(t) = 0.403927 + 0.233622e^{-5.07t} + 0.362451e^{-2.25t}, \\ p_1(t) = 0.420757 - 0.37353e^{-5.07t} - 0.0472277e^{-2.25t}, \\ p_2(t) = 0.175316 + 0.139908e^{-5.07t} - 0.315224e^{-2.25t}. \end{cases} \tag{10}$$

From Eq. (1) and Eq. (10) we obtain

$$p_0 = 0.403927, p_1 = 0.420757, p_2 = 0.175316.$$

Then we obtain the performances of TTO $(N=2)$ system as follows:

$$P_{RI} = 0.404, P_{NW} = 1, P_{AV} = 0.825, E[FM] = 0.771.$$

Comparing with OTO (N=2) system, both P_{RI} and P_{AV} increase, but $E[FM]$ decreases.

Figure 2 is the curves of the solutions of Eq. (10). Comparing Figure 1 and Figure 2, $p_0(t)$ of TTO $(N=2)$ system is significantly greater than that of OTO $(N=2)$ system.

As the above OTO $(N=2)$ system, we derive the transient-state reliability of the machines denoted by $\widetilde{R}_{2M}(t)$. We assume that all machines are available at the initial time $t=0$, $\widetilde{R}_{2M}(t))$ is the probability that there is at least one machine is available till time $t(>0)$ from the initial time. Setting the state of all machines are failure as an absorbing state, we obtain a new Markov process, and its transition rate matrix is

$$\widetilde{Q} = \begin{bmatrix} -2\lambda & 2\lambda & 0 \\ 2\mu q & -\lambda-2\mu q & \lambda \\ 0 & 0 & 0 \end{bmatrix}.$$

Under the initial distribution of $\widetilde{p}_0(0)=1, \widetilde{p}_1(0)=0$ and $\widetilde{p}_2(0)=0$, the machine transient-state reliability is

$$\widetilde{R}_{2M}(t) = \widetilde{p}_0(t) + \widetilde{p}_1(t),$$

where $\widetilde{p}_0(t)$ and $\widetilde{p}_1(t)$ are solutions of equations as follows:

$$\begin{cases} \widetilde{p}_0'(t) = -2\lambda\widetilde{p}_0(t) + 2\mu q\widetilde{p}_1(t), \\ \widetilde{p}_1'(t) = 2\lambda\widetilde{p}_0(t) - (\lambda+2\mu q)\widetilde{p}_1(t), \\ \widetilde{p}_0(0) = 1, \widetilde{p}_1(0) = 0. \end{cases} \tag{11}$$

Solving Eq. (11) by mathematic calculate software, we obtain

$$\widetilde{p}_0(t) = \frac{\Phi Cosh\left[\frac{1}{2}t\Phi\right] - (\lambda-2q\mu)Sinh\left[\frac{1}{2}t\Phi\right]}{\Phi e^{\frac{1}{2}t(3\lambda+2q\mu)}},$$

$$\widetilde{p}_1(t) = \frac{2\left(-1+e^{t\Phi}\right)\lambda}{\Phi e^{\frac{1}{2}t(3\lambda+2q\mu+\Phi)}},$$

where $\Phi = \sqrt{\lambda^2 + 12q\lambda\mu + 4q^2\mu^2}$. Then

$$\widetilde{R}_{2M}(t) = \frac{-3\lambda - 2q\mu + \Phi + e^{t\Phi}(3\lambda+2q\mu+\Phi)}{2\Phi e^{\frac{1}{2}t(3\lambda+2q\mu+\Phi)}}.$$

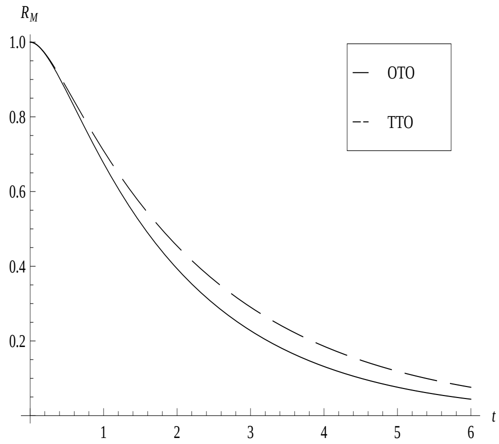

Figure 3. The machine reliability of OTO $(N=2)$ system $(R_{2M}(t))$ and TTO $(N=2)$ system $(\widetilde{R}_{2M}(t))$ $(N=2, \lambda=1, \mu=1.2, q=0.8)$.

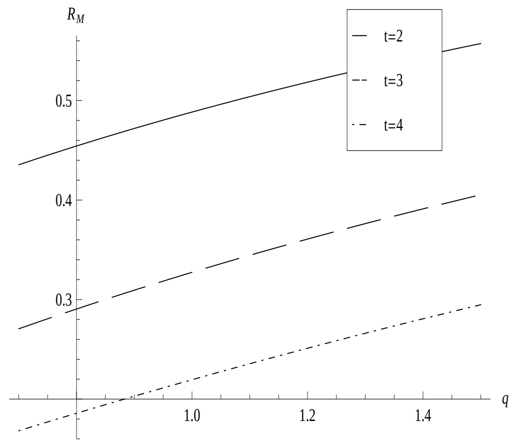

Figure 4. Effect of parameter q with t on the machine reliability $(\widetilde{R}_{2M}(t))$ of TTO $(N=2)$ system $(N=2, \lambda=1, \mu=1.2)$.

Figure 3 is the curves of the machine reliability of OTO $(N=2)$ system $(R_{2M}(t))$ and TTO $(N=2)$ system $(\widetilde{R}_{2M}(t))$ with the parameters $\lambda=1, \mu=1.2$. We note that the machine reliability of TTO $(N=2)$ system $(\widetilde{R}_{2M}(t))$ is greater than that of OTO $(N=2)$ system $(R_{2M}(t))$. Figure 4 displays the effect of the parameter q with three different values of t on the machine reliability of TTO $(N=2)$ system. We see that the machine reliability increases with q increases, but decreases with t increases.

B. $N=3$

1) OTO $(N=3)$system: For OTO $(N=3)$ system of $N=3$, the state space is $\{0,1,2,3\}$, and the system state transfer rate matrix is as follows:

$$Q = \begin{bmatrix} -3\lambda & 3\lambda & & \\ \mu & -2\lambda-\mu & 2\lambda & \\ & 2\mu & -\lambda-2\mu & \lambda \\ & & 2\mu & -2\mu \end{bmatrix}.$$

Then, the transient-state probability differential equations of OTO system is

$$\begin{cases} p_0'(t) = -3\lambda p_0(t) + 2\mu p_1(t), \\ p_1'(t) = 3\lambda p_0(t) - (2\lambda + \mu)p_1(t) + 2\mu p_2(t), \\ p_2'(t) = 2\lambda p_1(t) - (\lambda + 2\mu)p_2(t) + 2\mu p_3(t), \\ p_3'(t) = \lambda p_2(t) - 2\mu p_3(t). \end{cases} \tag{12}$$

Setting the initial distribution as $p_0(0) = 1, p_1(0) = 0, p_2(0) = 0$ and $p_3(0) = 0$, and letting $\lambda = 1$ and $\mu = 1.2$, the solutions of Eq. (12) are as follows:

$$\begin{cases} p_0(t) = 0.16 + 0.14e^{-6.33t} + 0.42e^{-3.82t} + 0.29e^{-1.84t}, \\ p_1(t) = 0.39 - 0.38e^{-6.34t} - 0.28e^{-3.82t} + 0.28e^{-1.84t}, \\ p_2(t) = 0.32 + 0.33e^{-6.34t} - 0.45e^{-3.82t} - 0.20e^{-1.84t}, \\ p_3(t) = 0.13 - 0.08e^{-6.34t} + 0.32e^{-3.82t} - 0.37e^{-1.84t}. \end{cases} \tag{13}$$

From Eq. (1) and Eq. (13) we obtain

$$p_0 = 0.16, p_1 = 0.39, p_2 = 0.32, p_3 = 0.13.$$

Then we obtain the performances of OTO $(N = 3)$ system are as follows:

$$P_{RI} = 0.16, P_{NW} = P_{AV} = 0.87, E[FM] = 1.42.$$

For the machine transient-state reliability, we set the state of all machines are failure as an absorbing state, then we obtain a new Markov process, and its transition rate matrix is as follows:

$$\widehat{Q} = \begin{bmatrix} -3\lambda & 3\lambda & & \\ \mu & -2\lambda - \mu & 2\lambda & \\ & 2\mu & -\lambda - 2\mu & \lambda \\ & & 0 & 0 \end{bmatrix}.$$

Under the initial distribution of $\widehat{p}_0(0) = 1$, $\widehat{p}_1(0) = 0$, $\widehat{p}_2(0) = 0$ and $\widehat{p}_3(0) = 0$, the machine transient-state reliability is

$$R_{3M}(t) = \widehat{p}_0(t) + \widehat{p}_1(t) + \widehat{p}_2(t),$$

where $\widehat{p}_0(t), \widehat{p}_1(t)$ and $\widehat{p}_2(t)$ are solutions of equations as follows:

$$\begin{cases} \widehat{p}_0'(t) = -3\lambda \widehat{p}_0(t) + \mu \widehat{p}_1(t), \\ \widehat{p}_1'(t) = 3\lambda \widehat{p}_0(t) - (2\lambda + \mu)\widehat{p}_1(t) + 2\mu \widehat{p}_2(t), \\ \widehat{p}_2'(t) = 2\lambda \widehat{p}_1(t) - (\lambda + 2\mu)\widehat{p}_2(t), \\ \widehat{p}_0(0) = 1, \widehat{p}_1(0) = 0, \widehat{p}_2(0) = 0. \end{cases} \tag{14}$$

Letting $\lambda = 1$ and $\mu = 1.2$, and solving Eq. (14) by mathematic calculate software, we obtain

$$\widehat{p}_0(t) = 0.18e^{-6.12t} + 0.57e^{-3.17t} + 0.25e^{-0.31t},$$

$$\widehat{p}_1(t) = -0.48e^{-6.12t} - 0.081e^{-3.17t} + 0.56e^{-0.31t},$$

$$\widehat{p}_2(t) = 0.35e^{-6.12t} - 0.71e^{-3.17t} + 0.36e^{-0.31t},$$

Then

$$R_{3M}(t) = 0.06e^{-6.12t} - 0.23e^{-3.12t} + 1.17e^{-0.31t}.$$

2) TTO $(N = 3)$ system: For TTO $(N = 3)$ system, the system state space is $\{0, 1, 2, 3\}$, and the system state transfer rate matrix is as follows:

$$Q = \begin{bmatrix} -3\lambda & 3\lambda & & \\ 2\mu q & -2\lambda - 2\mu q & 2\lambda & \\ & 2\mu & -\lambda - 2\mu & \lambda \\ & & 2\mu & -2\mu \end{bmatrix}.$$

Then, the transient-state probability differential equations are as follows:

$$\begin{cases} p_0'(t) = -3\lambda p_0(t) + 2\mu q p_1(t), \\ p_1'(t) = 3\lambda p_0(t) - (2\lambda + 2\mu q)p_1(t) + 2\mu p_2(t), \\ p_2'(t) = 2\lambda p_1(t) - (\lambda + 2\mu)p_2(t) + 2\mu p_3(t), \\ p_3'(t) = \lambda p_2(t) - 2\mu p_3(t). \end{cases} \tag{15}$$

Setting the initial distribution as $p_0(0) = 1, p_1(0) = 0, p_2(0) = 0$ and $p_3(0) = 0$, and letting $\lambda = 1, \mu = 1.2$ and $q = 0.8$, the solutions of Eq. (15) are as follows:

$$\begin{cases} p_0(t) = 0.23 + 0.19e^{-6.96t} + 0.29e^{-4.02t} + 0.29e^{-1.74t}, \\ p_1(t) = 0.35 - 0.39e^{-6.96t} - 0.16e^{-4.02t} + 0.19e^{-1.74t}, \\ p_2(t) = 0.30 + 0.26e^{-6.96t} - 0.36e^{-4.02t} - 0.19e^{-1.74t}, \\ p_3(t) = 0.12 - 0.06e^{-6.96t} + 0.22e^{-4.02t} - 0.29e^{-1.74t}. \end{cases} \tag{16}$$

From Eq. (1) and Eq. (16) we obtain

$$p_0 = 0.23, p_1 = 0.35, p_2 = 0.30, p_3 = 0.12.$$

Then we obtain the performances of TTO $(N = 3)$ system as follows:

$$P_{RI} = 0.23, P_{NW} = P_{AV} = 0.88, E[FM] = 1.31.$$

Comparing with OTO (N=3) system, both P_{RI} and P_{NW} increase, but $E[FM]$ decreases. Further, Comparing with TTO (N=2) system, P_{RI} decreases, but both P_{NW} and $E[FM]$ increase.

For the machine transient-state reliability, we set the state of all machines are failure as an absorbing state, then we obtain a new Markov process, and its transition rate matrix is

$$\widetilde{Q} = \begin{bmatrix} -3\lambda & 3\lambda & & \\ 2\mu q & -2\lambda - 2\mu q & 2\lambda & \\ & 2\mu & -\lambda - 2\mu & \lambda \\ & & 0 & 0 \end{bmatrix}.$$

Under the initial distribution of $\widetilde{p}_0(0) = 1, \widetilde{p}_1(0) = 0$ and $\widetilde{p}_2(0) = 0$, the machine transient-state reliability is as follow:

$$\widetilde{R}_{3M}(t) = \widetilde{p}_0(t) + \widetilde{p}_1(t) + \widetilde{p}_2(t),$$

where $\widetilde{p}_0(t), \widetilde{p}_1(t)$ and $\widetilde{p}_2(t)$ are solutions of the equations as follows:

$$\begin{cases} \widetilde{p}_0'(t) = -3\lambda \widetilde{p}_0(t) + 2\mu q \widetilde{p}_1(t), \\ \widetilde{p}_1'(t) = 3\lambda \widetilde{p}_0(t) - (2\lambda + 2\mu q)\widetilde{p}_1(t) + 2\mu \widetilde{p}_2(t), \\ \widetilde{p}_2'(t) = 2\lambda \widetilde{p}_1(t) - (\lambda + 2\mu)\widetilde{p}_2(t), \\ \widetilde{p}_0(0) = 1, \widetilde{p}_1(0) = 0. \end{cases} \tag{17}$$

Letting $\lambda = 1, \mu = 1.2$ and $q = 0.8$, and solving Eq. (17) by mathematic calculate software, we obtain

$$\widetilde{p}_0(t) = 0.22e^{-6.83t} + 0.44e^{-3.22t} + 0.34e^{-0.27t},$$

$$\widetilde{p}_1(t) = -0.44e^{-6.83t} - 0.05e^{-3.22t} + 0.49e^{-0.27t},$$

$$\widetilde{p}_2(t) = 0.25e^{-6.83t} - 0.56e^{-3.22t} + 0.31e^{-0.27t}.$$

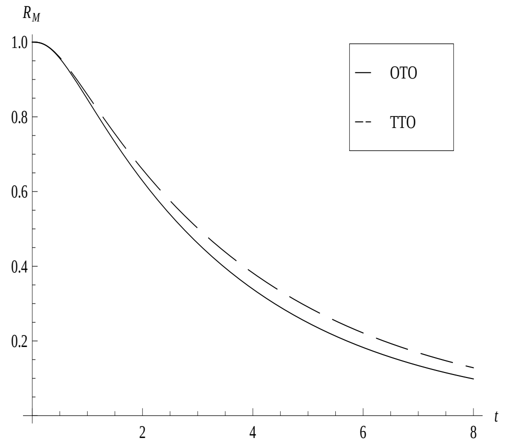

Figure 5. The machine reliability of OTO $(N = 3)$ system $(R_{3M}(t))$ and TTO $(N = 3)$ system $(\widetilde{R}_{3M}(t))$ $(N = 3, \lambda = 1, \mu = 1.2, q = 0.8)$.

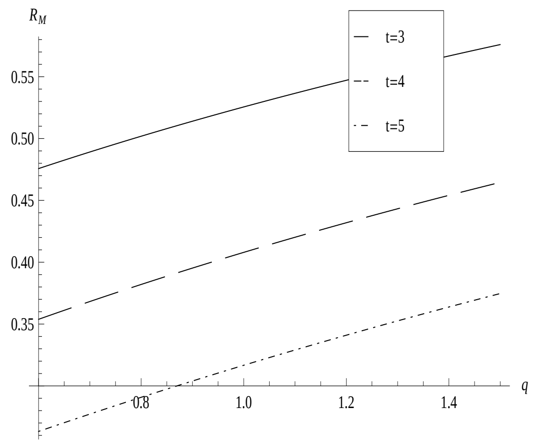

Figure 6. Effect of parameter q with t on the machine reliability $(\widetilde{R}_{3M}(t))$ of TTO $(N = 3)$ system $(N = 3, \lambda = 1, \mu = 1.2)$.

Then

$$\widetilde{R}_{3M}(t) = 0.04e^{-6.83t} - 0.18e^{-3.22t} + 1.14e^{-0.27t}.$$

Figure 5 is the curves of the machine reliability of OTO $(N = 3)$ system $(R_{3M}(t))$ and TTO $(N = 3)$ system $(\widetilde{R}_{3M}(t))$ with the parameters $\lambda = 1, \mu = 1.2$. We note that the machine reliability of TTO $(N = 3)$ system $(\widetilde{R}_{3M}(t))$ is greater than that of OTO $(N = 3)$ system $(R_{3M}(t))$. Figure 6 displays the effect of the parameter q with three different values of t on the machine reliability of TTO $(N = 3)$ system. We see that the machine reliability increases with q increases, but decreases with t increases.

VI. Conclusions

In this paper, we introduce a flexible repair policy to the machine repair system with two same repairmen and $N(> 1)$ identical repairable machines. the stead-state and transient-state performances have been derived in general form for the models. We intensively analyse two cases of $N = 2$ and $N = 3$. In every case, the performances of the machine repairable systems with TTO repair policy are given and compared with the regular system with OTO repair policy. The numeric results indicate that the performances of the system with the flexible repair policy (TTO repair policy) are better than that of the system with the ordinary repair policy (OTO repair policy). Further, the numeric results indicate the interactional parameter q has a significant effect on the performances of the machine repairable system with TTO repair policy.

More comprehensive analysis, for the case of OTO $(N = 2)$, $p_0 = 0.297520$, for the case of TTO $(N = 2), p_0 = 0.403927$, the difference between the two numbers is 0.106407. On the other hand, for the case of OTO $(N = 3)$, $p_0 = 0.16$, for the case of TTO $(N = 3), p_0 = 0.23$, the difference between the two numbers is 0.07. Since the other parameters are the same, we can say that the flexible repair policy has a greater influence on the system of $(N = 2)$, that means the smaller of N the greater of influence of the the flexible repair policy.

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