Dynamic Behaviors of a Lotka-Volterra Commensal Symbiosis Model with Non-selective Michaelis-Menten Type Harvesting

Zhenliang Zhu Runxin Wu Fengde Chen* Zhong Li

***Abstract*—In this paper, we study the following Lotka-Volterra commensal symbiosis model with non-selective Michaelis-Menten type harvesting**

$$\begin{aligned}\frac{dx}{dt} &= r_1x\Big(1-\frac{x}{K_1}+\alpha\frac{y}{K_1}\Big)-\frac{q_1Ex}{m_1E+m_2x},\\ \frac{dy}{dt} &= r_2y\Big(1-\frac{y}{K_2}\Big)-\frac{q_2Ey}{m_3E+m_4y},\end{aligned}$$

where r_1, r_2, K_1, K_2, α, q_1, q_2, E, m_1, m_2, m_3 and m_4 are all positive constants. Extinction, partial survival and global attractivity of the positive equilibrium are investigated, respectively. The results obtained here essentially improve and generalize the main results of Baoguo Chen (The influence of commensalism to a Lotka-Volterra commensal symbiosis model with Michaelis-Menten type harvesting, Advances in Difference Equations, 2019, 2019: 43).

***Index Terms*—Commensal symbiosis model, Michaelis-Menten type harvesting, Non-selective harvesting, Global attractivity.**

I. Introduction

DURING the last decade, many scholars ([1]-[16]) investigated the dynamic behaviors of the mutualism model, some substantial progress on persistent, extinction and stability of the mutualism system has been made. However, only recently did scholars ([17]-[25]) paid attention to the commensalism model. By means of commensalism, it means that in a long-term biological interaction in which members of one species gain benefits while those of the other species neither are benefited nor are harmed. Sun and Sun ([17]) first time proposed the following commensalism system

$$\begin{aligned}\frac{dx}{dt} &= r_1x\Big(1-\frac{x}{K_1}+\alpha\frac{y}{K_1}\Big),\\ \frac{dy}{dt} &= r_2y\Big(1-\frac{y}{K_2}\Big),\end{aligned}\tag{1.1}$$

where r_1, r_2, K_1, K_2, α are all positive constants. One could refer to [17] for more detail about the biological meaning of the coefficients. The system admits four equilibria:

$$E_1(0,0), E_2(K_1,0), E_3(0,K_2), E_4(K_1+\alpha K_2, K_2).$$

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The authors showed that E_1, E_2 and E_3 are all unstable equilibria, and E_4 is a stable node.

It is well known that harvesting of species is necessary for human being to obtain a resource. Recently, many scholars ([23],[32],[39],[40]) argued that the nonlinear harvesting (or named as Michealis-Menten type harvesting is more realistic. In [23], Baoguo Chen tried to study the influence of human harvesting to the commensalism model, and he incorporated the Michaelis-Menten type harvesting term to the first species in system (1.1), this leads to the following model:

$$\begin{aligned}\frac{dx}{dt} &= r_1x\Big(1-\frac{x}{K_1}+\alpha\frac{y}{K_1}\Big)-\frac{qEx}{m_1E+m_2x},\\ \frac{dy}{dt} &= r_2y\Big(1-\frac{y}{K_2}\Big),\end{aligned}\tag{1.2}$$

where r_1, r_2, K_1, K_2, α, q, E, m_1, m_2 are all positive constants, r_1, r_2, K_1, K_2, α, have the same meaning as that of the system (1.1), E is the fishing effort used to harvest and q is the catchablity coefficient, m_1 and m_2 are suitable constants. One could refer to [23], [25]-[38] for more detail discussion about the influence of harvesting to the ecological model.

Concerned with the stability property of the positive equilibrium, Baoguo Chen obtained the following result (Theorem 3.1 in [23]):

Theorem A. *Assume that*

$$r_1\Big(1+\alpha\frac{K_2}{K_1}\Big)>\frac{q}{m_1}\tag{1.3}$$

and

$$\frac{qm_2}{Em_1^2}<\frac{r_1}{K_1}\tag{1.4}$$

hold, then the positive equilibrium $E_4(x^*, y^*)$ of system (1.2) is globally stable.

Now let's consider the following example.

Example 1.1.

$$\begin{aligned}\frac{dx}{dt} &= x\Big(1-x+y\Big)-\frac{x}{1+3x},\\ \frac{dy}{dt} &= y\Big(1-y\Big).\end{aligned}\tag{1.5}$$

Here, corresponding to system (1.2), we take $r_1=1, r_2=1, E=1, q=1, \alpha=K_1=K_2=m_1=1, m_2=3$. In this case, by simple computation, it follows that

$$2=r_1\Big(1+\alpha\frac{K_2}{K_1}\Big)>\frac{q}{m_1}=\frac{1}{1}\tag{1.6}$$

and

$$\frac{3}{1}=\frac{qm_2}{Em_1^2}>\frac{r_1}{K_1}=1\tag{1.7}$$

hold, that is, condition (1.3) in Theorem A holds, however, condition (1.4) does not hold. Numeric simulation (Fig. 1) shows that the positive equilibrium of system (1.5) is globally stable.

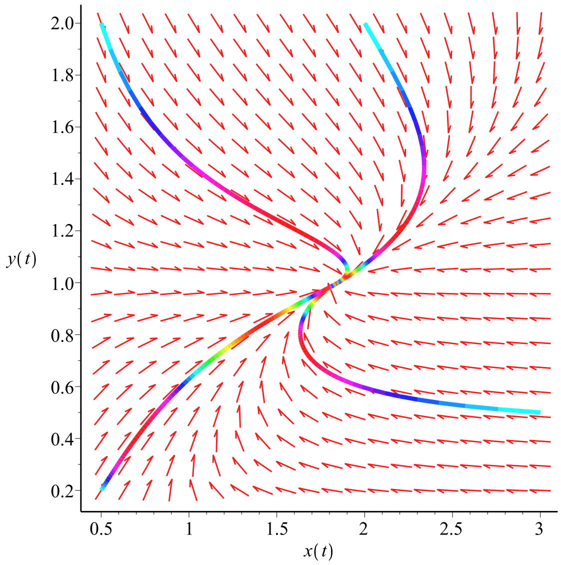

Fig. 1. Dynamic behaviors of the system (1.5), the initial condition $(x(0), y(0)) = (3, 0.5), (2, 2), (0.5, 0.2)$ and $(0.5, 2)$, respectively.

Example 1.1 implies that condition (1.4) may have room to improve or maybe it is not a necessary one. It is necessary to revisit the stability property of the positive equilibrium of system (1.2).

On the other hand, in system (1.2), the author assume that the second species is not of commercial importance, this, generally speaking, is not the real case. If we further assume that both species x and y are of commercial importance, then, we will establish the following Lotka-Volterra commensal symbiosis model with non-selective Michaelis-Menten type harvesting.

$$\begin{aligned} \frac{dx}{dt} &= r_1x\Big(1-\frac{x}{K_1}+\alpha\frac{y}{K_1}\Big)-\frac{q_1Ex}{m_1E+m_2x}, \\ \frac{dy}{dt} &= r_2y\Big(1-\frac{y}{K_2}\Big)-\frac{q_2Ey}{m_3E+m_4y}, \end{aligned} \tag{1.8}$$

where r_1, r_2, K_1, K_2, α, q_1, q_2, E, m_1, m_2, m_3 and m_4 are all positive constants. One could easily see that if $q_2 = 0$, then system (1.8) is degenerate to system (1.2). It is natural to investigate the dynamic behaviors of the system (1.8) and to find out the difference between the selective harvesting and non-selective harvesting.

The paper is arranged as follows. We will investigate the dynamic behaviors of the second species in the section 2, and then, depending on the different assumption, we will investigate the dynamic behaviors of the system (1.8) in section 3 and section 4, respectively. Some examples together with their numeric simulations are presented in Section 5 to show the feasibility of the main results. We end this paper with a brief discussion.

II. Dynamic Behaviors of the Second Species

Before we begin to study the dynamic behaviors of the system (1.8), we would like to investigate the positivity of the solutions of system (1.8). Indeed, we have the following result.

Lemma 2.1. *The solutions of system (1.8) with positive initial value is positive for all $t \geq 0$.*

Proof. Let $(x(t), y(t))$ be any solution of system (1.8) with $x(0) > 0, y(0) > 0$, then from (1.8), we have

$$\begin{aligned} x(t) &= x(0)\exp\Big\{\int_0^t r_1\Big(1-\frac{x(s)}{K_1}+\alpha\frac{y(s)}{K_1}\Big) \\ &\quad -\frac{q_1E}{m_1E+m_2x(s)}ds\Big\} > 0, \\ y(t) &= y(0)\exp\Big\{\int_0^t r_2\Big(1-\frac{y(s)}{K_2}\Big) \\ &\quad -\frac{q_2E}{m_3E+m_4y(s)}ds\Big\} > 0. \end{aligned}$$

This ends the proof of Lemma 2.1.

As a direct corollary of Lemma 2.2 of Chen[33], we have

Lemma 2.2. *If $a > 0, b > 0$ and $\dot{x} \geq x(b-ax)$, when $t \geq 0$ and $x(0) > 0$, we have*

$$\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}.$$

If $a > 0, b > 0$ and $\dot{x} \leq x(b-ax)$, when $t \geq 0$ and $x(0) > 0$, we have

$$\limsup_{t\to+\infty} x(t) \leq \frac{b}{a}.$$

The aim of this section is to investigate the dynamic behaviors of the second species in system (1.8), since the second equation of system (1.8) is independent of the first species, it may be easily to investigate.

For the sake of convenience, we restate the second equation of system (1.8) again.

$$\frac{dy}{dt} = r_2y\Big(1-\frac{y}{K_2}\Big)-\frac{q_2Ey}{m_3E+m_4y}. \tag{2.1}$$

Lemma 2.3. *Assume that*

$$r_2 > \frac{q_2}{m_3} \tag{2.2}$$

holds, then

$$F_1(y) = r_2\Big(1-\frac{y}{K_2}\Big)-\frac{q_2E}{m_3E+m_4y} = 0 \tag{2.3}$$

admits a unique positive solution

$$y_1 = \frac{-A_2+\sqrt{A_2^2-4A_1A_3}}{2A_1}, \tag{2.4}$$

where

$$\begin{aligned} A_1 &= m_4r_2, \\ A_2 &= Em_3r_2-K_2m_4r_2, \\ A_3 &= EK_2q_2-EK_2m_3r_2. \end{aligned} \tag{2.5}$$

Proof. Since

$$\begin{aligned} F_1(y) &= r_2\Big(1-\frac{y}{K_2}\Big)-\frac{q_2E}{m_3E+m_4y} \\ &= -\frac{G_1(y)}{K_2(Em_3+m_4y)}, \end{aligned} \tag{2.6}$$

where

$$G_1(y) = A_1 y^2 + A_2 y + A_3.$$

Noting that $G_1(y)$ is the quadratic function, and under the assumption of Lemma 2.2, $G_1(0) = A_3 < 0$. Hence, from the properties of quadratic function, one has $G_1(y) < 0$ for $y \in (0, y_1)$ and $G_1(y) > 0$ for $y \in (y_1, +\infty)$. That is, $G_1(y) = 0$ admits unique positive solution $y_1 \in (0, +\infty)$. From (2.6) one could see that $F_1(y) = 0$ also admits unique positive solution $y_1 \in (0, +\infty)$, $F_1(y) > 0$ for $y \in (0, y_1)$ and $F_1(y) < 0$ for $y \in (y_1, +\infty)$. This ends the proof of Lemma 2.2.

Theorem 2.1. *Assume that*

$$r_2 < \frac{q_2 E}{m_3 E + m_4 K_2} \tag{2.7}$$

holds, then in system (2.1), species y will finally be driven to extinction, i.e.,

$$\lim_{t \to +\infty} y(t) = 0. \tag{2.8}$$

Proof. From (2.7), for any enough small positive constant $\varepsilon > 0$, the inequality

$$r_2 < \frac{q_2 E}{m_3 E + m_4 (K_2 + \varepsilon)} \tag{2.9}$$

holds. From (2.1) we have

$$\frac{dy}{dt} \le r_2 y \Big(1 - \frac{y}{K_2}\Big). \tag{2.10}$$

Applying Lemma 2.1 to (2.10) leads to

$$\lim_{t \to +\infty} y(t) \le K_2. \tag{2.11}$$

For $\varepsilon > 0$ enough small which satisfies (2.9), it follows from (2.11) that there exists an enough large $T_1 > 0$ such that

$$y(t) < K_2 + \varepsilon \ \text{ for all } \ t \ge T_1. \tag{2.12}$$

For $t \ge T_1$, from (2.1) and (2.12), one has

$$\frac{dy}{dt} \le r_2 y \Big(1 - \frac{q_2 E}{r_2 \big(m_3 E + m_4 (K_2 + \varepsilon)\big)}\Big). \tag{2.13}$$

Hence,

$$y(t) \le y(T_1) \exp\Big\{\Gamma (t - T_1)\Big\}, \tag{2.14}$$

where

$$\Gamma = r_2 \Big(1 - \frac{q_2 E}{r_2 \big(m_3 E + m_4 (K_2 + \varepsilon)\big)}\Big).$$

This, together with (2.9) leads to

$$\lim_{t \to +\infty} y(t) = 0. \tag{2.15}$$

This ends the proof of Theorem 2.1.

Theorem 2.2 *Assume that (2.2) holds, then system (2.1) admits a unique positive equilibrium which is globally stable.*

Proof. Let

$$F_2(y) = r_2 \Big(1 - \frac{y}{K_2}\Big) - \frac{q_2 E}{m_3 E + m_4 y} \tag{2.16}$$

From the proof of Lemma 2.2, one could easily see that
(1)There is a unique y_1, such that $F_2(y_1) = 0$, where y_1 is defined by (2.4);
(2)For all $y_1 > y > 0, F_2(y) > 0$;
(3)For all $y > y_1 > 0, F_2(y) < 0$.
Hence, it immediately follows from Theorem 2.1 in [30] that the unique positive equilibrium y_1 of system (2.1) is globally stable.

The proof of Theorem 2.2 is finished.

III. Dynamic behaviors of system (1.8), Case: $r_2 < \frac{q_2 E}{m_3 E + m_4 K_2}$

The aim of this section is to investigate the dynamic behaviors of system (1.8) under the assumption

$$r_2 < \frac{q_2 E}{m_3 E + m_4 K_2} \tag{3.1}$$

holds. In this case, as was shown in Theorem 2.1, the second species in system (1.8) will finally be driven to extinction.

Lemma 3.1. *Assume that*

$$r_1 > \frac{q_1}{m_1} \tag{3.2}$$

holds, then for any enough small positive constant ϵ,

$$F_3(x) = r_1 \Big(1 - \frac{x}{K_1} + \alpha \frac{\epsilon}{K_1}\Big) - \frac{q_1 E}{m_1 E + m_2 x} = 0 \tag{3.3}$$

admits a unique positive solution

$$x_1 = \frac{-B_2 + \sqrt{B_2^2 - 4 B_1 B_3}}{2 B_1}, \tag{3.4}$$

where

$$\begin{aligned}
B_1 &= m_2 r_2, \\
B_2 &= -\alpha \, \epsilon \, m_2 \, r_1 + E m_1 \, r_1 - K_1 \, m_2 \, r_1, \\
B_3 &= -E \alpha \, m_1 \, r_1 \, \epsilon - E K_1 \, m_1 \, r_1 + q_1 \, E K_1.
\end{aligned} \tag{3.5}$$

Proof. Since

$$\begin{aligned}
F_3(x) &= r_1 \Big(1 - \frac{x}{K_1} + \alpha \frac{\epsilon}{K_1}\Big) - \frac{q_1 E}{m_1 E + m_2 x} \\
&= -\frac{G_2(x)}{K_1 (E m_1 + m_2 x)},
\end{aligned} \tag{3.6}$$

where

$$G_2(x) = B_1 x^2 + B_2 x + B_3. \tag{3.7}$$

Noting that inequality (3.2) implies that

$$r_1 \Big(1 + \alpha \frac{\varepsilon}{K_1}\Big) > \frac{q_1}{m_1} \tag{3.8}$$

holds, therefore,

$$B_3 = -E \alpha \, m_1 \, r_1 \, \epsilon - E K_1 \, m_1 \, r_1 + q_1 \, E K_1 < 0. \tag{3.9}$$

Noting that $G_2(x)$ is the quadratic function, and under the assumption of Lemma 3.1, $G_2(0) = B_3 < 0$. Hence, from the properties of quadratic function, one has $G_2(x) < 0$ for $x \in (0, x_1)$ and $G_2(x) > 0$ for $x \in (x_1, +\infty)$. That is, $G_2(x) = 0$ admits unique positive solution $x_1 \in (0, +\infty)$. From (3.6) one could see that $F_3(x) = 0$ also admits unique positive solution $x_1 \in (0, +\infty)$, $F_3(x) > 0$ for $x \in (0, x_1)$ and $F_3(x) < 0$ for $x \in (x_1, +\infty)$. This ends the proof of Lemma 3.1.

Lemma 3.2. *Assume that (3.2) holds, then*

$$F_4(x) = r_1 \Big(1 - \frac{x}{K_1}\Big) - \frac{q_1 E}{m_1 E + m_2 x} = 0 \tag{3.10}$$

admits a unique positive solution

$$x_2 = \frac{-C_2 + \sqrt{C_2^2 - 4C_1C_3}}{2C_1}, \tag{3.11}$$

where

$$\begin{aligned} C_1 &= m_2 r_1, \\ C_2 &= Em_1 r_1 - K_1 m_2 r_1, \\ C_3 &= -EK_1 m_1 r_1 + q_1 EK_1. \end{aligned} \tag{3.12}$$

Proof. The proof of Lemma 3.2 is similar to that of the proof of Lemma 3.1, we omit the detail here. Set

$$F_4(x) = r_1\Big(1 - \frac{x}{K_1}\Big) - \frac{q_1 E}{m_1 E + m_2 x}.$$

We only mention here that $F_4(x) = 0$ also admits unique positive solution $x_2 \in (0, +\infty)$, also, $F_4(x) > 0$ for $x \in (0, x_2)$ and $F_4(x) < 0$ for $x \in (x_2, +\infty)$. This ends the proof of Lemma 3.2.

Concerned with the partial survival of the system (1.8), we have

Theorem 3.1. *Assume that (3.1) and (3.2) hold, then* $(x_2, 0)$ *of system (1.8) is globally stable, i.e.,*

$$\lim_{t\to+\infty} x(t) = x_2, \ \lim_{t\to+\infty} y(t) = 0,$$

where x_2 *is defined by (3.11).*

Proof. Under the assumption (3.1), Theorem 2.1 shows that the second species will be driven to extinction, i.e.,

$$\lim_{t\to+\infty} y(t) = 0. \tag{3.13}$$

For any enough small positive constant $\varepsilon > 0$, it follows from (3.13) that there exists a $T_1 > 0$ such that

$$y(t) < \varepsilon \ \text{ for all } \ t \geq T_1. \tag{3.14}$$

For $t \geq T_1$, form the first equation of (1.8) and (3.14), one has

$$\frac{dx}{dt} \leq r_1 x\Big(1 - \frac{x}{K_1} + \alpha\frac{\varepsilon}{K_1}\Big) - \frac{q_1 E x}{m_1 E + m_2 x}. \tag{3.15}$$

Now let's consider the equation

$$\frac{du}{dt} = r_1 u\Big(1 - \frac{u}{K_1} + \alpha\frac{\varepsilon}{K_1}\Big) - \frac{q_1 E u}{m_1 E + m_2 u}, \tag{3.16}$$

It follows from Lemma 3.1 that system (3.16) admits a unique positive equilibrium $u = x_1$, also, let

$$F_3(u) = r_1\Big(1 - \frac{u}{K_1} + \alpha\frac{\varepsilon}{K_1}\Big) - \frac{q_1 E}{m_1 E + m_2 u},$$

then $F_3(u) > 0$ for $u \in (0, x_1)$ and $F_3(u) < 0$ for $u \in (x_1, +\infty)$. Hence, it immediately follows from Theorem 2.1 in [30] that the unique positive equilibrium $u = x_1$ of system (3.16) is globally stable. By applying the comparison principle, it follows from (3.15) and (3.16) that

$$\limsup_{t\to+\infty} x(t) \leq x_1. \tag{3.17}$$

Noting that from (3.4), (3.5), (3.11) and (3.12), one could see that

$$x_1 \to x_2 \ \text{ as } \ \varepsilon \to 0.$$

Since $\varepsilon > 0$ is an arbitrary small positive constant, letting $\varepsilon \to 0$ in (3.17) leads to

$$\limsup_{t\to+\infty} x(t) \leq x_2. \tag{3.18}$$

From the first equation of system (1.8), we also have

$$\frac{dx}{dt} \geq r_1 x\Big(1 - \frac{x}{K_1}\Big) - \frac{q_1 E x}{m_1 E + m_2 x}.$$

From this, by applying Lemma 3.2 and the comparison principle, similarly to the analysis of (3.15)-(3.18), we have

$$\liminf_{t\to+\infty} x(t) \geq x_2. \tag{3.19}$$

(3.18) together with (3.19) leads to

$$\lim_{t\to+\infty} x(t) = x_2. \tag{3.20}$$

(3.13) and (3.20) shows that the conclusion of Theorem 3.1 holds. This ends the proof of Theorem 3.1.

Concerned with the extinction of the system (1.8), we have

Theorem 3.2. *In addition to (3.1), assume further that*

$$r_1 < \frac{q_1 E}{m_1 E + m_2 K_1} \tag{3.21}$$

holds, then the boundary equilibrium $(0, 0)$ *of system (1.1) is globally stable. That is, both species* x *and* y *will be driven to extinction.*

Proof. It follows from (3.21) that for enough small ε, the following inequality holds

$$r_1\Big(1 + \frac{\alpha\varepsilon}{K_1}\Big) < \frac{q_1 E}{m_1 E + m_2(K_1 + \varepsilon)}. \tag{3.22}$$

It follows from (3.1) and Theorem 2.1 that

$$\lim_{t\to+\infty} y(t) = 0. \tag{3.23}$$

For $\varepsilon > 0$ enough small, which satisfies (3.22), there exists a $T_2 > T_1$ such that

$$y(t) < \varepsilon \ \text{ for all } \ t \geq T_2. \tag{3.24}$$

For $t \geq T_2$, it follows from the first equation of (1.8) that

$$\frac{dx}{dt} \leq r_1 x\Big(1 - \frac{x}{K_1} + \alpha\frac{\varepsilon}{K_1}\Big). \tag{3.25}$$

Applying Lemma 2.1 to (3.25) leads to

$$\limsup_{t\to+\infty} x(t) \leq K_1\Big(1 + \alpha\frac{\varepsilon}{K_1}\Big). \tag{3.26}$$

Setting $\varepsilon \to 0$ in (3.26) leads to

$$\limsup_{t\to+\infty} x(t) \leq K_1. \tag{3.27}$$

Hence, for $\varepsilon > 0$ which satisfies (3.22), there exists a $T_3 > T_2$ such that

$$x(t) < K_1 + \varepsilon \ \text{ for all } \ t \geq T_3. \tag{3.28}$$

Again, for $t > T_3$, from (3.28) and the first equation of system (1.8), we have

$$\begin{aligned} \frac{dx}{dt} &\leq r_1 x\Big(1 - \frac{x}{K_1} + \alpha\frac{\varepsilon}{K_1}\Big) - \frac{q_1 E x}{m_1 E + m_2(K_1 + \varepsilon)} \\ &< r_1 x\Big(1 + \alpha\frac{\varepsilon}{K_1} - \frac{q_1 E}{m_1 E + m_2(K_1 + \varepsilon)}\Big). \end{aligned} \tag{3.29}$$

Hence

$$x(t) < x(T_3)\exp\Big\{\Delta(t-T_3)\Big\}, \tag{3.30}$$

where

$$\Delta = r_1\Big(1+\alpha\frac{\varepsilon}{K_1}\Big) - \frac{q_1E}{m_1E+m_2(K_1+\varepsilon)}.$$

Therefore, from (3.22) one has

$$\lim_{t\to+\infty} x(t) = 0. \tag{3.31}$$

(3.23) together with (3.31) shows that both species x and y will be driven to extinction. This ends the proof of Theorem 3.2.

IV. Dynamic behaviors of system (1.8), Case: $r_2 > \dfrac{q_2}{m_3}$

Under the assumption

$$r_2 > \frac{q_2}{m_3} \tag{4.1}$$

holds, it follows from Theorem 2.2 that system (2.1) admits a unique positive equilibrium $y = y_1$, which is globally stable, where y_1 is defined by (2.4).

Consider the equation

$$\frac{du_1}{dt} = r_1u_1\Big(1-\frac{u_1}{K_1}+\alpha\frac{y_1+\varepsilon}{K_1}\Big) - \frac{q_1Eu_1}{m_1E+m_2u_1}. \tag{4.2}$$

where $\varepsilon > 0$ is enough small positive constant.

Lemma 4.1. *Assume that*

$$r_1\Big(1+\alpha\frac{y_1}{K_1}\Big) > \frac{q_1}{m_1} \tag{4.3}$$

holds. Then system (4.2) admits a unique positive equilibrium $u^*_{1\varepsilon}$, which is globally asymptotically stable, where

$$u^*_{1\varepsilon} = \frac{-D_2+\sqrt{D_2^2-4D_1D_3}}{2D_1}. \tag{4.4}$$

$$\begin{aligned} D_1 &= m_2r_1,\\ D_2 &= Em_1r_1-(y_1+\varepsilon)\alpha m_2r_1-K_1m_2r_1,\\ D_3 &= EK_1q_1-EK_1m_1r_1-E(y_1+\varepsilon)\alpha m_1r_1. \end{aligned} \tag{4.5}$$

Proof. It follows from (4.3) that for enough small positive constant $\varepsilon > 0$, the inequality

$$r_1\Big(1+\alpha\frac{y_1+\varepsilon}{K_1}\Big) > \frac{q_1}{m_1} \tag{4.6}$$

holds, by applying (4.6), the rest of the proof of Lemma 4.1 is similar to the proof of Lemma 2.2 and Theorem 2.2, we omit the detail here.

Consider the equation

$$\frac{dv_1}{dt} = r_1v_1\Big(1-\frac{v_1}{K_1}+\alpha\frac{y_1-\varepsilon}{K_1}\Big) - \frac{q_1Ev_1}{m_1E+m_2v_1}. \tag{4.7}$$

where $\varepsilon > 0$ is enough small positive constant.

Lemma 4.2. *Assume that*

$$r_1\Big(1+\alpha\frac{y_1}{K_1}\Big) > \frac{q_1}{m_1} \tag{4.8}$$

holds. Then system (4.7) admits a unique positive equilibrium $v^*_{1\varepsilon}$, which is globally asymptotically stable, where

$$v^*_{1\varepsilon} = \frac{-E_2+\sqrt{E_2^2-4E_1E_3}}{2E_1}. \tag{4.9}$$

$$\begin{aligned} E_1 &= m_2r_1,\ \ E_2 = Em_1r_1-(y_1-\varepsilon)\alpha m_2r_1-K_1m_2r_1,\\ E_3 &= EK_1q-EK_1m_1r_1-E(y_1-\varepsilon)\alpha m_1r_1. \end{aligned} \tag{4.10}$$

Proof. It follows from (4.8) that for enough small positive constant $\varepsilon > 0$, the inequality

$$r_1\Big(1+\alpha\frac{y_1-\varepsilon}{K_1}\Big) > \frac{q_1}{m_1} \tag{4.11}$$

holds, by applying (4.11), the rest of the proof of Lemma 4.2 is similar to the proof of Lemma 2.2 and Theorem 2.2, we omit the detail here.

Concerned with the global stability of the positive equilibrium of system (1.8), we have the following result.

Theorem 4.1. *Assume that (4.1) and*

$$r_1\Big(1+\alpha\frac{y_1}{K_1}\Big) > \frac{q_1}{m_1} \tag{4.12}$$

hold, then system (1.8) admits a unique positive equilibrium (x^*, y_1), which is globally stable, here

$$x^* = \frac{-G_2+\sqrt{G_2^2-4G_1G_3}}{2G_1}. \tag{4.13}$$

$$\begin{aligned} G_1 &= m_2r_1,\ \ G_2 = Em_1r_1-y_1\alpha m_2r_1-K_1m_2r_1,\\ G_3 &= EK_1q-EK_1m_1r_1-Ey_1\alpha m_1r_1. \end{aligned} \tag{4.14}$$

Proof. Condition (4.12) implies that for enough small positive constant $\varepsilon > 0$, the inequalities

$$r_1\Big(1+\alpha\frac{y_1+\varepsilon}{K_1}\Big) > \frac{q_1}{m_1} \tag{4.15}$$

and

$$r_1\Big(1+\alpha\frac{y_1-\varepsilon}{K_1}\Big) > \frac{q_1}{m_1} \tag{4.16}$$

hold. It follows from (4.1) and Theorem 2.2 that the second equation of system (1.8) admits a unique positive equilibrium y_1, which is globally stable, that is,

$$\lim_{t\to+\infty} y(t) = y_1. \tag{4.17}$$

Hence, for $\varepsilon > 0$ enough small ($\varepsilon < \frac{1}{2}y_1$) which satisfies (4.15) and (4.16), there exists a $T_4 > 0$ such that

$$y_1-\varepsilon < y(t) < y_1+\varepsilon \ \text{ for all } \ t \ge T_4. \tag{4.18}$$

(4.18) together with the first equation of system (1.8) leads to

$$\frac{dx}{dt} \le r_1x\Big(1-\frac{x}{K_1}+\alpha\frac{y_1+\varepsilon}{K_1}\Big) - \frac{q_1Ex}{m_1E+m_2x}, \tag{4.19}$$

and

$$\frac{dx}{dt} \ge r_1x\Big(1-\frac{x}{K_1}+\alpha\frac{y_1-\varepsilon}{K_1}\Big) - \frac{q_1Ex}{m_1E+m_2x}. \tag{4.20}$$

Hence, by using the comparison principle, it follows from Lemma 4.1 and 4.2 that

$$v_{1\varepsilon}^* \le \liminf_{t\to+\infty} x(t) \le \limsup_{t\to+\infty} x(t) \le u_{1\varepsilon}^*. \tag{4.21}$$

Noting that

$$v_{1\varepsilon}^* \to x^*, \quad u_{1\varepsilon}^* \to x^* \quad \text{as} \quad \varepsilon \to 0. \tag{4.22}$$

Setting $\varepsilon \to 0$ in (4.21) leads to

$$\lim_{t\to+\infty} x(t) = x^*. \tag{4.23}$$

(4.17) and (4.23) show that (x^*, y_1) is globally attractive. This ends the proof of Theorem 4.1.

Concerned with the partial survival of the system (1.8), we have the following result.

Theorem 4.2. *In addition to (4.1), further assume that*

$$r_1\Big(1 + \frac{\alpha y_1}{K_1}\Big) < \frac{q_1 E}{m_1 E + m_2(K_1 + \alpha y_1)}, \tag{4.24}$$

holds, then

$$\lim_{t\to+\infty} x(t) = 0, \quad \lim_{t\to+\infty} y(t) = y_1.$$

i. e., the first species will be driven to extinction due to the over harvesting, while the second species is permanent.

Proof. From (4.24) we could choose $\varepsilon > 0$ small enough, such that

$$r_1\Big(1 + \frac{\alpha(y_1+\varepsilon)}{K_1}\Big) < \frac{q_1 E}{m_1 E + m_2\big(K_1 + \alpha(y_1+\varepsilon) + \varepsilon\big)}. \tag{4.25}$$

It follows from (4.1) and Theorem 2.2 that

$$\lim_{t\to+\infty} y(t) = y_1. \tag{4.26}$$

Hence, for $\varepsilon > 0$ enough small, which satisfies (4.25), there exists an enough large T_5 such that

$$y(t) < y_1 + \varepsilon \ \text{ for all } \ t \ge T_5. \tag{4.27}$$

By applying (4.27), similarly to the analysis of (3.25) and (3.31), we can finally show that

$$\lim_{t\to+\infty} x(t) = 0. \tag{4.28}$$

This ends the proof of Theorem 4.2.

V. Numerical simulations

Example 5.1. Let's take $r_1 = 2, E = 1, q_1 = q_2 = 1, \alpha = K_1 = K_2 = m_2 = m_1 = m_3 = m_4, r_2 = \frac{1}{4}$. In this case, by simple computation, one could easily see that

$$2 = r_1 > \frac{q_1}{m_1} = \frac{1}{2} \tag{5.1}$$

and

$$\frac{1}{4} = r_2 < \frac{q_2 E}{m_3 E + m_4 K_2} = \frac{1}{2} \tag{5.2}$$

hold, that is, condition (3.1) and (3.2) in Theorem 3.1 hold, and so, it follows from Theorem 3.1 that the boundary equilibrium $(x_2, 0) = (0.7808, 0)$ of the system is globally stable. Numeric simulations (Fig. 2, Fig. 3) support this assertion.

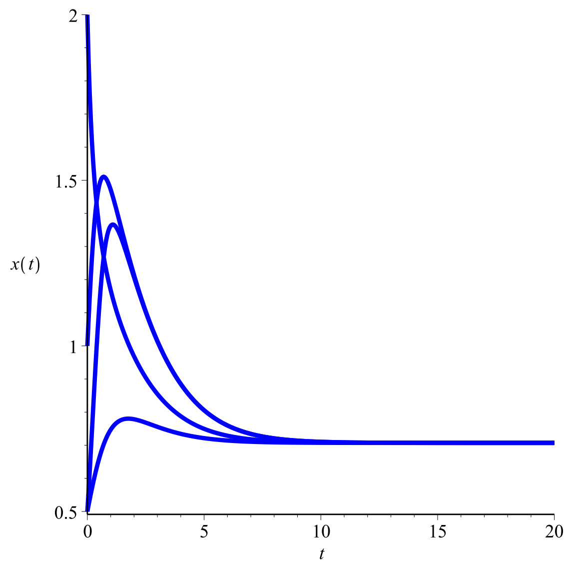

Fig. 2. Dynamic behaviors of the first species of Example 5.1, the initial condition $(x(0), y(0)) = (2, 0.5), (1, 1), (0.5, 0.2)$ and $(0.5, 1)$, respectively.

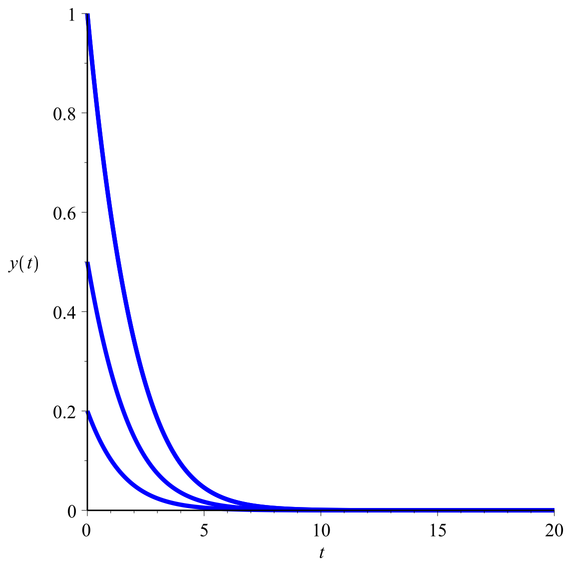

Fig. 3. Dynamic behaviors of the second species of Example 5.1, the initial condition $(x(0), y(0)) = (2, 0.5), (1, 1), (0.5, 0.2)$ and $(0.5, 1)$, respectively.

Example 5.2. Let's take $r_1 = \frac{1}{4}, E = 1, q_1 = q_2 = 1, \alpha = K_1 = K_2 = m_2 = m_1 = m_3 = m_4, r_2 = \frac{1}{4}$. In this case, by simple computation, one could easily see that

$$\frac{1}{4} = r_1 < \frac{q_1 E}{m_1 E + m_2 K_1} = \frac{1}{2} \tag{5.3}$$

and

$$\frac{1}{4} = r_2 < \frac{q_2 E}{m_3 E + m_4 K_2} = \frac{1}{2} \tag{5.4}$$

hold, that is, condition (3.1) and (3.21) in Theorem 3.2 hold, and so, it follows from Theorem 3.2 that the boundary equilibrium $(0, 0)$ of the system is globally stable. Numeric simulations (Fig. 4, Fig. 5) support this assertion.

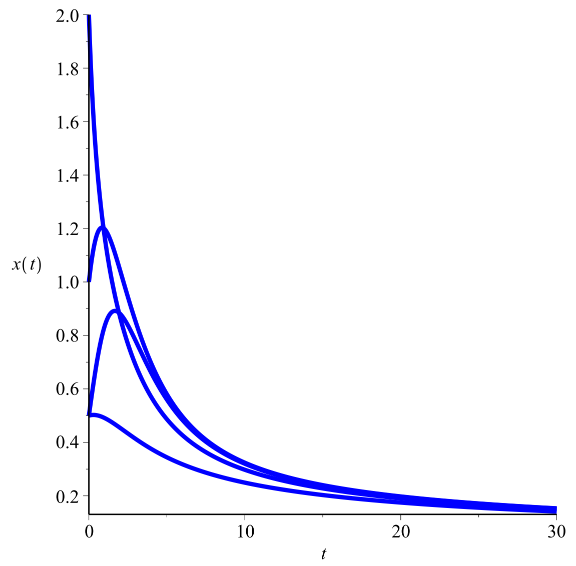

Fig. 4. Dynamic behaviors of the first species of Example 5.2, the initial condition $(x(0), y(0)) = (2, 0.5), (1, 1), (0.5, 0.2)$ and $(0.5, 1)$, respectively.

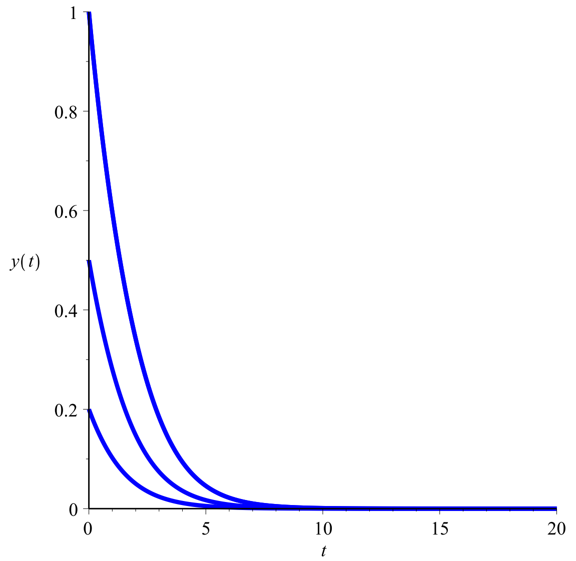

Fig. 5. Dynamic behaviors of the second species of Example 5.2, the initial condition $(x(0), y(0)) = (2, 0.5), (1, 1), (0.5, 0.2)$ and $(0.5, 1)$, respectively.

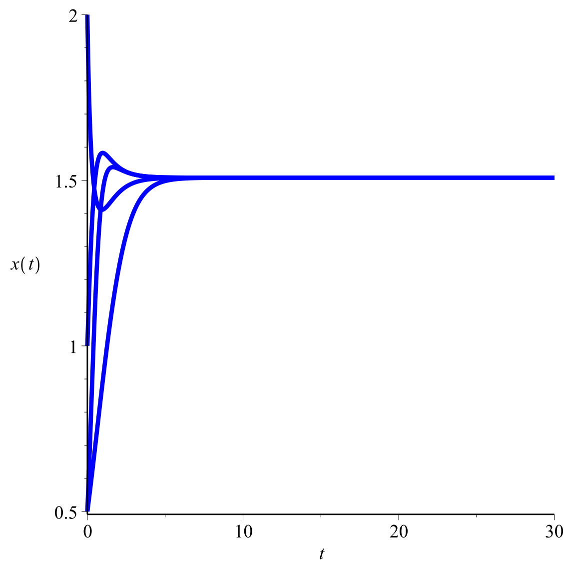

Fig. 6. Dynamic behaviors of the first species of Example 5.3, the initial condition $(x(0), y(0)) = (2, 0.5), (1, 1), (0.5, 0.2)$ and $(0.5, 1)$, respectively.

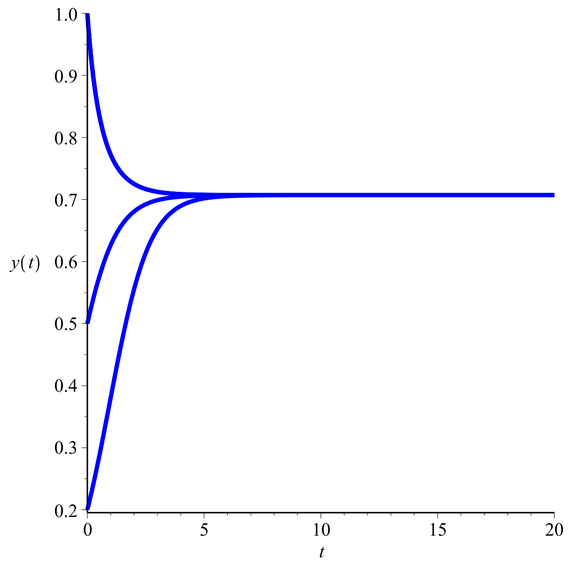

Fig. 7. Dynamic behaviors of the second species of Example 5.3, the initial condition $(x(0), y(0)) = (2, 0.5), (1, 1), (0.5, 0.2)$ and $(0.5, 1)$, respectively.

Example 5.3. Let's take $r_1 = 2, E = 1, q_1 = q_2 = 1, \alpha = K_1 = K_2 = m_2 = m_1 = m_3 = m_4, r_2 = 2$. In this case, by simple computation, one could easily see that

$$2 = r_1 > \frac{q_1}{m_1} = 1 \tag{5.5}$$

and

$$2 = r_2 > \frac{q_2}{m_3} = 1 \tag{5.6}$$

hold, that is, condition (4.1) and (4.12) in Theorem 4.1 hold, and so, it follows from Theorem 4.1 that the positive equilibrium $(1.508, 0.7071)$ of the system is globally stable. Numeric simulations (Fig. 6, Fig. 7) support this assertion.

Example 5.4. Let's take $r_1 = 0.1, E = 1, q_1 = q_2 = 1, \alpha = K_1 = K_2 = m_2 = m_1 = m_3 = m_4, r_2 = 2$. In this case, by simple computation, one could easily see that

$$0.17 \approx r_1\left(1 + \frac{\alpha y_1}{K_1}\right) < \frac{q_1 E}{m_1 E + m_2(K_1 + \alpha y_1)} \approx 0.37 \tag{5.7}$$

and

$$2 = r_2 > \frac{q_2}{m_3} = 1 \tag{5.8}$$

holds, that is, condition (4.1) and (4.24) in Theorem 4.2 hold, and so, it follows from Theorem 4.2 that the boundary equilibrium $(0, 0.7071)$ of the system is globally stable. Numeric simulations (Fig. 8, Fig. 9) support this assertion.

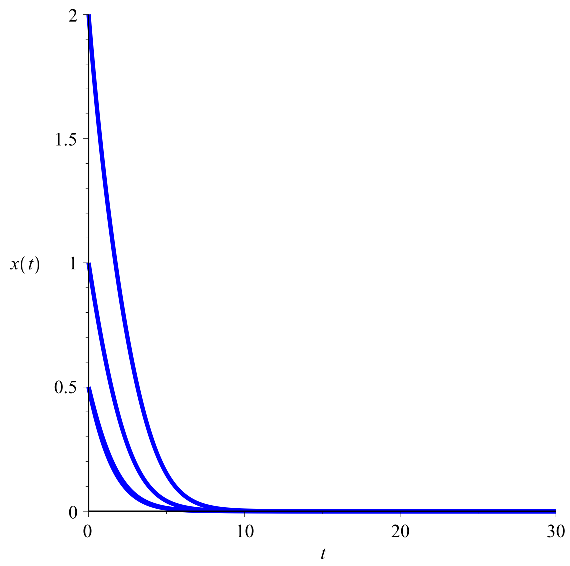

Fig. 8. Dynamic behaviors of the first species of Example 5.4, the initial condition $(x(0), y(0)) = (2, 0.5), (1, 1), (0.5, 0.2)$ and $(0.5, 1)$, respectively.

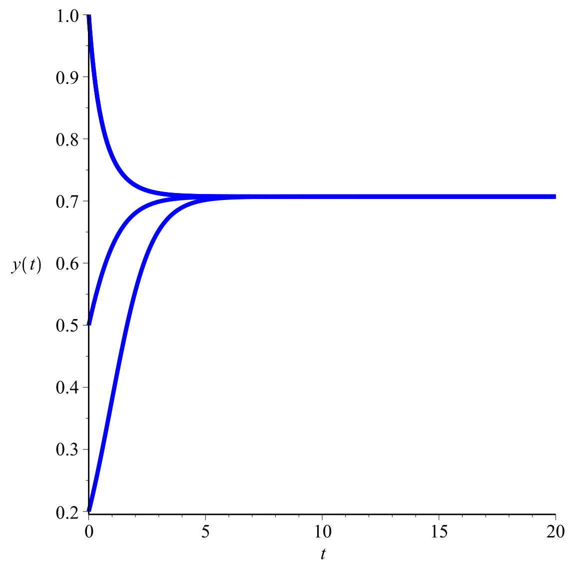

Fig. 9. Dynamic behaviors of the second species of Example 5.4, the initial condition $(x(0), y(0)) = (2, 0.5), (1, 1), (0.5, 0.2)$ and $(0.5, 1)$, respectively.

VI. Discussion

There are many scholars ([25]-[38]) investigated the influence of harvesting. Among those works, two directions becomes very popular. The first one is to incorporate the Michaelis-Menten type harvesting, see [23], [28], [29], [32]. They argued that the nonlinear one is more suitable, which could overcome the drawback of the traditional linear ones. The second one is concerned with the non-selective harvesting, see [31], [34]-[38]. They thought it is necessary to consider the harvesting on both species, since both species may commercial importance. However, to the best of our knowledge, to this day, still no scholars propose an ecosystem with non-selective Michaelis-Menten type harvesting, this stimulated us to propose the system (1.8).

From Theorem 3.1, 3.2, 4.1 and 4.2, we show that under some suitable assumption on harvesting coefficients, all of the forth equilibria $E_0(0,0)$, $E_1(x_2,0)$, $E_2(0,y_1)$ and $E_4(x^*,y_1)$ are possible globally attractive, such a phenomenon is quite different to the dynamic behaviors of the commensalism system without harvesting or only with harvesting on the first species, one could refer to the dynamic behaviors of the system (1.1) and (1.2) for more detail information on this direction.

The direct motivation of our paper comes from recent work of Baoguo Chen[23], in[23], he proposed the system (1.2), investigated the local and global stability property of the equilibria, however, as was shown in the introduction section, their results still have room to improve. In this paper, by establishing the new lemmas (Lemma 2.2, 3.1, 3.2, 4.1 and 4.2), we finally obtain the results to ensure the global attractivity of all of the possible equilibria of the system (1.8). One could easily see that, if $q_2 = 0$, i.e, without the harvesting of the second species, then Theorem 4.1 and 4.2 gives the conditions to ensure the existence of the global attractivity positive equilibrium and the extinction of the first species, respectively. Theorem 4.1 essentially improve and generalize the corresponding result of Baoguo Chen[23], by means of dropping the unnecessary condition (1.4).

To sum up, by introducing the non-selective Michaelis-Menten type harvesting, the dynamic behaviors of the system (1.8) becomes complicated. Overfishing may lead to the extinction of the both species or the extinction of the second species. Harvesting plays important role on determining the dynamic behaviors of the system. To ensure the system be permanent, one needs to limit the capture to a certain range of intense.

VII. Declarations

Competing interests

The authors declare that there is no conflict of interests.

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Authors' Contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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