Structurally Parametric Synthesis and Position Analysis of a RoboMech Class Parallel Manipulator with Two End-Effectors

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***Abstract*— In this paper, the methods of structurally parametric synthesis and position analysis of a RoboMech class parallel manipulator with two end-effectors are presented. This parallel manipulator is formed by connecting the two moving output objects with the fixed base by two passive, one active and two negative closing kinematic chains. Geometrical parameters of the active and negative closing kinematic chains are determined by the Chebyshev and least-square approximations. Position analysis is made on base of the conditional generalized coordinates method.**

***Index Terms*—Parallel manipulator, end-effector, structurally parametric synthesis, position analysis**

I. Introduction

Depending on the type of technological operation, the robot manipulator can operate in two modes: a simultaneous manipulation of two objects and a sequential manipulation of one object.

In the simultaneous manipulation of two objects, two serial manipulators *ABC* and *DEF* handle two objects in the initial positions P_1 and P_2 (Fig. 1a), then two objects are moved to the specified position P_3 (Fig. 1b). Further, the manipulators return to their initial positions.

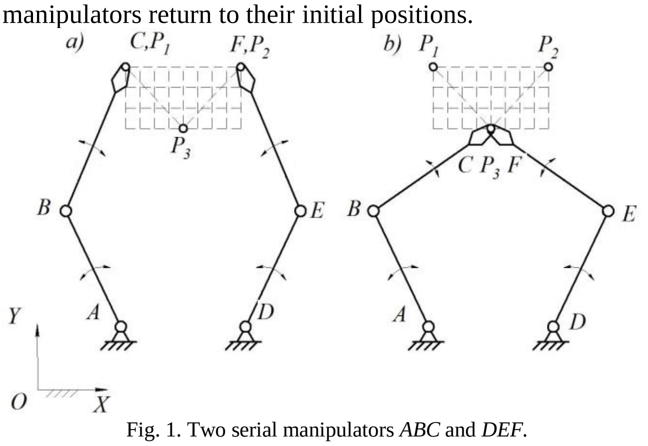

Fig. 1. Two serial manipulators *ABC* and *DEF*.

In the sequential manipulation of one object, the first serial manipulator *ABC* handles the object in the position P_1 (Fig. 1a), then the object is moved to the intermediate position P_3, where the object is transferred to the gripper of the second serial manipulator *DEF* (Fig. 1b). Further, the object is moved by the second serial manipulator *DEF* to the specified position P_2 (Fig. 1b).

For example, a printing machine operates in the mode of sequential manipulation of one object. In this machine, a blank sheet of paper is fed by the first manipulator onto the printing table, and the second manipulator picks up the sheet after printing. This cyclical process occurs in a short period of time. Therefore, in such automatic machines, instead of two serial manipulators, it is advisable to use one manipulator (mechanism) with two end-effectors and one DOF. The parallel manipulators (PM) of a class RoboMech belong to such manipulators. PM having the property of manipulation robots such as a reproducing the specified laws of motions of the end-effectors, and the property of mechanisms such as a setting the laws of motions the actuators which simplify the control system and increase speed, are called PM of a class RoboMech [1-3].

The methods of structural and parametric synthesis of a RoboMech Class PM are presented in [4]. In this paper, the structurally parametric optimization and position analysis of a RoboMech class PM with two end-effectors are developed. There are many methods of structural and kinematic (parametric or dimensional) synthesis of mechanisms [5-7], where the kinematic synthesis of mechanisms is carried out for their given structural schemes. In this case, it is possible that a given structural scheme of the mechanism may not provide the specified laws of motions of the end-effectors. Therefore, it is necessary to carry out the kinematic synthesis together with the structural synthesis. The methods of structurally parametric synthesis and position analysis allow to simultaneously determine the optimal structural schemes of PM and the geometrical parameters of their links according to the given laws of motions of the end-effectors and actuators. Many works are dedicated for kinematic analysis of planar mechanisms [8-11]. There are works [12,13] on kinematic analysis of the third class mechanism, but they can't applicable for kinematic analysis of the fourth class mechanism.

II. Structural Optimization

According to the developed principle of forming mechanisms and manipulators [1,2], the PM with two end-effectors is formed by connecting two output objects with a fixed base using closing kinematic chains (CKC), which can be active, passive and negative. If we connect these two output objects with the fixed base by two passive CKC *ABC* and *DEF*, having zero DOF, we obtain two serial manipulators (Fig. 2). In the paper [14], a PM of the fifth class with two end-effectors and two DOF (Fig. 2) was

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formed from these two serial manipulators by connecting the links 2 and 4 by the negative CKC *GH* of type **RR**, then by connecting the link *GH* with a fixed base by the negative CKC *IK* of type **RR**, and by connecting the links *IK* and *DE* by the negative CKC *LM* of type **RR**, where **R** is a revolute kinematic pair. Each of the binary links of type **RR** has one negative DOF. The disadvantages of this PM is a small workspace because the links 2 and 4 of two serial manipulators *ABC* and *DEF* are connected by one link *GH*.

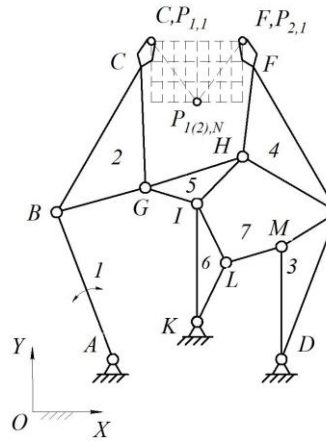

Fig. 2. PM with two end-effectors of the fifth class.

The workspace of the PM with two end-effectors can be increased by connecting the links 2 and 4 of the serial manipulators *ABC* and *DEF* by the active CKC *GHKI* with active kinematic pair *K*. As a result, we obtain PM *ABGHKIED* with three DOF, where the links *AB*, *KH* and *DE* are input links (Fig. 3). For formation of a RoboMech class PM with one DOF, we connect the links 1 and 5, as well as the links 3 and 6 by the negative CKC *ML* and *NQ* of type **RR**. As a result, we obtain a structural scheme of a RoboMech class PM with two end-effectors, which has the following structural formula

$$\text{IV}(1,2,5,8) \leftarrow \text{I}(0,8) \rightarrow \text{IV}(3,4,6,9). \tag{1}$$

Therefore, the formed RoboMech class PM consists of an input link 7 and two fourth class Assur groups, or two Stephenson II mechanisms with the common input link 7.

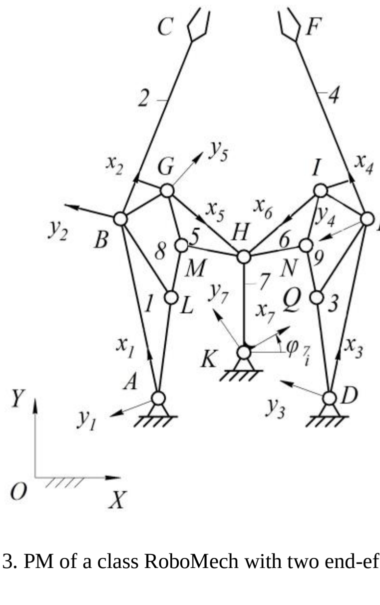

Fig. 3. PM of a class RoboMech with two end-effectors.

Thus, this RoboMech class PM with two end-effectors is formed by connecting of two output objects with a fixed base by two passive CKC *ABC* and *DEF*, one active CKC *GHKI* and two negatives CKC *LM* and *NQ*.

Since the active and negative CKC impose geometrical constraints on the motions of the output objects, then the formed PM of a RoboMech class with two end-effectors works at certain values of the geometrical parameters (synthesis parameters) of the links. Passive CKC do not impose geometrical constraints on the motions of output objects, therefore, their synthesis parameters vary taking into account the imposed geometrical constraints of the connecting active and negative CKC. Consequently, the problem of parametric synthesis of the whole RoboMech class PM with two end-effectors is reduced to the subproblems of parametric synthesis of its structural modules (passive, active and negative CKC). Such modular representation of structural - parametric synthesis simplifies the problem of designing of PM. Let consider the parametric synthesis of the structural modules of the RoboMech class PM with two end-effectors.

III. Parametric Optimization of Structural Modules

Given *N* discrete values of the grippers centers *C* and *F* coordinates X_{C_i}, Y_{C_i} and X_{F_i}, X_{F_i} $(i = 1, 2, ..., N)$.

The synthesis parameters of two passive CKC *ABC* and *DEF* (or serial manipulators) are X_A, Y_A, l_{AB}, l_{BC} and X_D, Y_D, l_{DE}, l_{EF}, where X_A, Y_A and X_D, Y_D are coordinates of the pivot joints *A* and *D* in the absolute coordinate system *OXY*; $l_{AB}, l_{BC}, l_{DE}, l_{EF}$ are lengths of the links *AB*,*BC*,*DE*,*EF*. The synthesis parameters of the passive CKC are varied using the « LP_τ sequence» [15].

The synthesis parameters of the active CKC *GHKI* are $x_G^{(2)}, y_G^{(2)}, x_I^{(4)}, y_I^{(4)}, x_H^{(7)}, y_H^{(7)}, X_K, Y_K, l_{HG}, l_{HI}$, where $x_G^{(2)}, y_G^{(2)}, x_I^{(4)}, y_I^{(4)}, x_H^{(7)}, y_H^{(7)}$ are coordinates of the joints *G*, *I*, *H* in the moving coordinate systems $Bx_2y_2, Ex_4y_4, Kx_7y_7$, fixed to the links *BC*, *EF*, *KH*, respectively; X_K, Y_K are coordinates of the pivot joint *K* in the absolute coordinate system *OXY*; l_{HG}, l_{HI} are lengths of the links *HG*, *HI*.

Write the vector loop-closure equations of *OKHGBO* and *OKHIEO*

$$\mathbf{R}_K + \mathbf{\Gamma}(\varphi_{7i})\mathbf{r}_H^{(7)} + \mathbf{l}_{(HG)_i} = \mathbf{R}_{B_i} + \mathbf{\Gamma}(\varphi_{2i})\mathbf{r}_G^{(2)}, \tag{2}$$

$$\mathbf{R}_K + \mathbf{\Gamma}(\varphi_{7i})\mathbf{r}_H^{(7)} + \mathbf{l}_{(HI)_i} = \mathbf{R}_{E_i} + \mathbf{\Gamma}(\varphi_{4i})\mathbf{r}_I^{(4)}, \tag{3}$$

where $\mathbf{R}_K = \left[X_K, Y_K\right]^T$, $\mathbf{r}_H^{(7)} = \left[x_H^{(7)}, y_H^{(7)}\right]^T$,

$$\mathbf{l}_{(HG)_i} = \left[l_{HG}\cos\varphi_{(HG)_i}, l_{HG}\sin\varphi_{(HG)_i}\right]^T,$$

$$\mathbf{R}_{Bi} = \left[X_{Bi}, Y_{Bi}\right]^T,$$

$$\mathbf{r}_G^{(2)} = \left[x_G^{(2)}, y_G^{(2)}\right]^T, \mathbf{r}_H^{(7)} = \left[x_H^{(7)}, y_H^{(7)}\right]^T,$$

$$\mathbf{l}_{(HI)_i} == \left[l_{HI}\cos\varphi_{(HI)_i}, l_{HI}\sin\varphi_{(HI)_i} \right]^T,$$

$$\mathbf{R}_{E_i} = \left[X_{E_i}, Y_{E_i} \right]^T, \mathbf{r}_I^{(4)} = \left[x_I^{(4)}, y_I^{(4)} \right]^T,$$

$\boldsymbol{\Gamma}(\alpha)$ is an orthogonal rotation matrix

$$\boldsymbol{\Gamma}(\alpha) = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix}.$$

The angles φ_{2i} and φ_{4i} in the Eqs (2) and (3), which determine the positions of the links BC and EF of the passive CKC ABC and DEF, are calculated from the analysis of positions of these CKC by the expressions

$$\varphi_{2i} = \mathrm{tg}^{-1}\frac{Y_{C_i} - Y_{B_i}}{X_{C_i} - X_{B_i}}, \tag{4}$$

$$\varphi_{4i} = \mathrm{tg}^{-1}\frac{Y_{F_i} - Y_{E_i}}{X_{F_i} - X_{E_i}}, \tag{5}$$

where

$$\begin{bmatrix} X_{Bi} \\ Y_{Bi} \end{bmatrix} = \begin{bmatrix} X_A \\ Y_A \end{bmatrix} + l_{AB}\begin{bmatrix} \cos\varphi_{1i} \\ \sin\varphi_{1i} \end{bmatrix}, \tag{6}$$

$$\begin{bmatrix} X_{Ei} \\ Y_{Ei} \end{bmatrix} = \begin{bmatrix} X_D \\ Y_D \end{bmatrix} + l_{DE}\begin{bmatrix} \cos\varphi_{3i} \\ \sin\varphi_{3i} \end{bmatrix}. \tag{7}$$

The angles φ_{1i} and φ_{3i} in Eqs (6) and (7) are determined by the expressions

$$\varphi_{1i} = \mathrm{tg}^{-1}\frac{Y_{C_i} - Y_A}{X_{C_i} - X_A} \pm \cos^{-1}\frac{l_{AC_i}^2 + l_{AB}^2 - l_{BC}^2}{2l_{AC_i}\cdot l_{AB}}, \tag{8}$$

$$\varphi_{3i} = \mathrm{tg}^{-1}\frac{Y_{F_i} - Y_D}{X_{F_i} - X_D} \pm \cos^{-1}\frac{l_{DF_i}^2 + l_{DE}^2 - l_{EF}^2}{2l_{DF_i}\cdot l_{DE}}, \tag{9}$$

where

$$l_{AC_i} = \left[\left(X_{C_i} - X_A \right)^2 + \left(Y_{C_i} - Y_A \right)^2 \right]^{\frac{1}{2}},$$

$$l_{DF_i} = \left[\left(X_{F_i} - X_D \right)^2 + \left(Y_{F_i} - Y_D \right)^2 \right]^{\frac{1}{2}}.$$

Eliminating the unknown angles $\varphi_{(HG)i}$ and $\varphi_{(HI)i}$, from Eqs (2) and (3) yields

$$\left[\mathbf{R}_K + \boldsymbol{\Gamma}(\varphi_{7i})\mathbf{r}_H^{(7)} - \mathbf{R}_{B_i} - \boldsymbol{\Gamma}(\varphi_{2i})\mathbf{r}_G^{(2)} \right]^2 - l_{HG}^2 = 0, \tag{10}$$

$$\left[\mathbf{R}_K + \boldsymbol{\Gamma}(\varphi_{7i})\mathbf{r}_H^{(7)} - \mathbf{R}_{E_i} - \boldsymbol{\Gamma}(\varphi_{4i})\mathbf{r}_I^{(4)} \right]^2 - l_{HI}^2 = 0, \tag{11}$$

Eqs (10) and (11) are the equations of geometrical constraints imposed on the motion of two output objects. The geometric meaning of Eqs (10) and (11) are the equations of two circles with radiuses l_{HG} and l_{HI} in relative motions of the planes Bx_2y_2 and Bx_4y_4 relative to the plane Kx_7y_7. The problem of determining the geometrical parameters of the links at which such geometrical constraints are approximately realized is the problem of parametric synthesis of the active CKC *GHKI*.

The left parts of Eqs (10) and (11) are denoted by $\Delta q_{1i}^{(1)}$ and $\Delta q_{2i}^{(2)}$, which are functions of weighted differences

$$\Delta q_{1i}^{(1)} = \left[\mathbf{R}_K + \boldsymbol{\Gamma}(\varphi_{7i})\mathbf{r}_H^{(7)} - \mathbf{R}_{B_i} - \boldsymbol{\Gamma}(\varphi_{2i})\mathbf{r}_G^{(2)} \right]^2 - l_{HG}^2, \tag{12}$$

$$\Delta q_{2i}^{(2)} = \left[\mathbf{R}_K + \boldsymbol{\Gamma}(\varphi_{7i})\mathbf{r}_H^{(7)} - \mathbf{R}_{E_i} - \boldsymbol{\Gamma}(\varphi_{4i})\mathbf{r}_I^{(4)} \right]^2 - l_{HI}^2 = 0. \tag{13}$$

After converting these equations and the following change of variables

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} X_K \\ Y_K \end{bmatrix}, \begin{bmatrix} p_4 \\ p_5 \end{bmatrix} = \begin{bmatrix} x_G^{(2)} \\ y_G^{(2)} \end{bmatrix}, \begin{bmatrix} p_6 \\ p_7 \end{bmatrix} = \begin{bmatrix} x_H^{(7)} \\ y_H^{(7)} \end{bmatrix},$$

$$p_3 = \frac{1}{2}(X_K^2 + Y_K^2 + x_H^{(7)^2} + y_H^{(7)^2} + x_G^{(2)^2} + y_G^{(2)^2} - l_{HG}^2),$$

$$\begin{bmatrix} p_8 \\ p_9 \end{bmatrix} = \begin{bmatrix} x_I^{(4)} \\ y_I^{(4)} \end{bmatrix},$$

$$p_{10} = \frac{1}{2}(X_K^2 + Y_K^2 + x_H^{(7)^2} + y_H^{(7)^2} + x_I^{(4)^2} + y_I^{(4)^2} - l_{HI}^2)$$

the functions Δq_{1i} and Δq_{2i} are represented as linear forms by groups $\mathbf{p}_1^{(j)}$ and $\mathbf{p}_2^{(k)}$ of synthesis parameters

$$\Delta q_{1i}^{(j)} = 2\left(\mathbf{g}_{1i}^{(j)^T} \cdot \mathbf{p}_1^{(j)} - g_{01i}^{(j)} \right), \ (j = 1, 2, 3), \tag{14}$$

$$\Delta q_{2i}^{(k)} = 2\left(\mathbf{g}_{2i}^{(k)^T} \cdot \mathbf{p}_2^{(k)} - g_{02i}^{(k)} \right), \ (k = 1, 2, 3), \tag{15}$$

where

$$\mathbf{g}_{1i}^{(1)} = \begin{bmatrix} -X_{B_i} \\ -Y_{B_i} \\ 1 \end{bmatrix} - \left[\begin{array}{c:c} \boldsymbol{\Gamma}(\varphi_{2i}) & \begin{matrix} 0 \\ 0 \end{matrix} \\ \hdashline \begin{matrix} 0 & 0 \end{matrix} & 1 \end{array} \right] \cdot \begin{bmatrix} p_4 \\ p_5 \\ 0 \end{bmatrix} + \left[\begin{array}{c:c} \boldsymbol{\Gamma}(\varphi_{7i}) & \begin{matrix} 0 \\ 0 \end{matrix} \\ \hdashline \begin{matrix} 0 & 0 \end{matrix} & 1 \end{array} \right] \cdot \begin{bmatrix} p_6 \\ p_7 \\ 0 \end{bmatrix},$$

$$\mathbf{g}_{2i}^{(2)} = \left[\begin{array}{c:c} \boldsymbol{\Gamma}^T(\varphi_{2i}) & \begin{matrix} 0 \\ 0 \end{matrix} \\ \hdashline \begin{matrix} 0 & 0 \end{matrix} & 1 \end{array} \right] \cdot \begin{bmatrix} X_{B_i} - p_1 \\ Y_{B_i} - p_2 \\ 1 \end{bmatrix} - \left[\begin{array}{c:c} \boldsymbol{\Gamma}(\varphi_{7i} - \varphi_{2i}) & \begin{matrix} 0 \\ 0 \end{matrix} \\ \hdashline \begin{matrix} 0 & 0 \end{matrix} & 1 \end{array} \right] \cdot \begin{bmatrix} p_6 \\ p_7 \\ 0 \end{bmatrix},$$

$$\mathbf{g}_{3i}^{(3)}=\left[\begin{array}{c:c}\mathbf{\Gamma}^T(\varphi_{7i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline 0\quad 0 & 1\end{array}\right]\cdot\begin{bmatrix}p_1-X_{B_i}\\p_2-Y_{B_i}\\1\end{bmatrix}-\left[\begin{array}{c:c}\mathbf{\Gamma}(\varphi_{7i}-\varphi_{2i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline 0\quad 0 & 1\end{array}\right]\cdot\begin{bmatrix}p_4\\p_5\\0\end{bmatrix},$$

$$g_{01i}^{(1)}=-\frac{1}{2}\left(X_{B_i}^2+Y_{B_i}^2\right)-[X_{B_i},Y_{B_i}]\cdot\mathbf{\Gamma}(\varphi_{2i})\cdot\begin{bmatrix}p_4\\p_5\end{bmatrix}+$$
$$+[X_{B_i},Y_{B_i}]\cdot\mathbf{\Gamma}(\varphi_{7i})\cdot\begin{bmatrix}p_6\\p_7\end{bmatrix}+\left[p_4,p_5\right]\cdot\mathbf{\Gamma}(\varphi_{7i}-\varphi_{2i})\cdot\begin{bmatrix}p_6\\p_7\end{bmatrix},$$

$$g_{01i}^{(2)}=-\frac{1}{2}\left(X_{B_i}^2-Y_{B_i}^2\right)-[p_1-X_{B_i},p_2-Y_{B_i}]\cdot\mathbf{\Gamma}(\varphi_{7i})\cdot\begin{bmatrix}p_6\\p_7\end{bmatrix},$$

$$g_{01i}^{(3)}=-\frac{1}{2}\left(X_{B_i}^2-Y_{B_i}^2\right)-[X_{B_i}-p_1,Y_{B_i}-p_2]\cdot\mathbf{\Gamma}(\varphi_{7i})\cdot\begin{bmatrix}p_4\\p_5\end{bmatrix},$$

$$\mathbf{g}_{2i}^{(1)}=\begin{bmatrix}-X_{E_i}\\-Y_{E_i}\\1\end{bmatrix}-\left[\begin{array}{c:c}\mathbf{\Gamma}(\varphi_{4i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline 0\quad 0 & 1\end{array}\right]\cdot\begin{bmatrix}p_8\\p_9\\0\end{bmatrix}+\left[\begin{array}{c:c}\mathbf{\Gamma}(\varphi_{7i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline 0\quad 0 & 1\end{array}\right]\cdot\begin{bmatrix}p_6\\p_7\\0\end{bmatrix},$$

$$\mathbf{g}_{2i}^{(2)}=\left[\begin{array}{c:c}\mathbf{\Gamma}^T(\varphi_{4i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline 0\quad 0 & 1\end{array}\right]\cdot\begin{bmatrix}X_{E_i}-p_1\\Y_{E_i}-p_2\\1\end{bmatrix}-\left[\begin{array}{c:c}\mathbf{\Gamma}(\varphi_{7i}-\varphi_{4i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline 0\quad 0 & 1\end{array}\right]\cdot\begin{bmatrix}p_6\\p_7\\0\end{bmatrix},$$

$$\mathbf{g}_{3i}^{(3)}=\left[\begin{array}{c:c}\mathbf{\Gamma}^T(\varphi_{7i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline 0\quad 0 & 1\end{array}\right]\cdot\begin{bmatrix}p_1-X_{E_i}\\p_2-Y_{E_i}\\1\end{bmatrix}-\left[\begin{array}{c:c}\mathbf{\Gamma}(\varphi_{7i}-\varphi_{4i}) & \begin{matrix}0\\0\end{matrix}\\ \hdashline 0\quad 0 & 1\end{array}\right]\cdot\begin{bmatrix}p_8\\p_9\\0\end{bmatrix},$$

$$g_{02i}^{(1)}=-\frac{1}{2}\left(X_{E_i}^2+Y_{E_i}^2\right)-[X_{E_i},Y_{E_i}]\cdot\mathbf{\Gamma}(\varphi_{4i})\cdot\begin{bmatrix}p_8\\p_9\end{bmatrix}+$$
$$+[X_{E_i},Y_{E_i}]\cdot\mathbf{\Gamma}(\varphi_{7i})\cdot\begin{bmatrix}p_6\\p_7\end{bmatrix}+\left[p_8,p_9\right]\cdot\mathbf{\Gamma}(\varphi_{7i}-\varphi_{4i})\cdot\begin{bmatrix}p_6\\p_7\end{bmatrix},$$

$$g_{02i}^{(2)}=-\frac{1}{2}\left(X_{E_i}^2-Y_{E_i}^2\right)-[p_1-X_{E_i},p_2-Y_{E_i}]\cdot\mathbf{\Gamma}(\varphi_{7i})\cdot\begin{bmatrix}p_6\\p_7\end{bmatrix},$$

$$g_{02i}^{(3)}=-\frac{1}{2}\left(X_{E_i}^2-Y_{E_i}^2\right)-[X_{E_i}-p_1,Y_{E_i}-p_2]\cdot\mathbf{\Gamma}(\varphi_{7i})\cdot\begin{bmatrix}p_8\\p_9\end{bmatrix}.$$

The linear representability of the geometrical constraints Eqs (14) and (15) with respect to the groups $\mathbf{p}_1^{(j)}$ and $\mathbf{p}_2^{(k)}$ of synthesis parameters allows to formulate the following approximation problems of parametric synthesis:

-Chebyshev approximation,

- least-square approximation

to determine the groups $\mathbf{p}_1^{(j)}$ and $\mathbf{p}_2^{(k)}$ of synthesis parameters.

In the Chebyshev approximation problem, the vectors of synthesis parameters are determined from the minimum of the functionals

$$S_1^{(j)}(\mathbf{p}_1^{(j)})=\max_{i=1,N}\left|\Delta q_{1i}^{(j)}(\mathbf{p}_1^{(j)})\right|\to\min_{\mathbf{p}_1^{(j)}}S_1^{(j)}(\mathbf{p}_1^{(j)}),\qquad(16)$$

$$S_2^{(k)}(\mathbf{p}_2^{(k)})=\max_{i=1,N}\left|\Delta q_{2i}^{(k)}(\mathbf{p}_2^{(k)})\right|\to\min_{\mathbf{p}_2^{(k)}}S_2^{(k)}(\mathbf{p}_2^{(k)}).\qquad(17)$$

In the least-square approximation problem, the vectors of synthesis parameters are determined from the minimum of the functionals

$$S_1^{(j)}(\mathbf{p}_1^{(j)})=\sum_{i=1}^{N}\left(\Delta q_{1i}^{(j)}\right)^2\to\min_{\mathbf{p}_1^{(j)}}S_1^{(j)}(\mathbf{p}_1^{(j)}),\qquad(18)$$

$$S_2^{(k)}(\mathbf{p}_2^{(k)})=\sum_{i=1}^{N}\left(\Delta q_{2i}^{(k)}\right)^2\to\min_{\mathbf{p}_2^{(k)}}S_2^{(k)}(\mathbf{p}_2^{(k)}),\qquad(19)$$

Since the synthesis parameters of the active CKC *GHKI* are simultaneously included in functionals (16-19), their values are determined by joint consideration of the functionals (16) and (17), and also (18) and (19).

The linear representability of Eqs (12) and (13) in the forms (14) and (15) allows for solving the Chebyshev approximation problem (16) and (17), to apply the kinematic inversion method, which is an iterative process, at each step of which one group of synthesis of the parameters $\mathbf{p}_1^{(j)}$ and $\mathbf{p}_2^{(k)}$ is defined. In this case, the problem of linear programming is solved by four parameters. To do this, we introduce a new variable $p_{11}=\varepsilon$, where ε is a required accuracy of the approximation. Then the minimax problems (16) and (17) are reduced to the following linear programming problem: determine the minimum of the sum

$$\sigma=\mathbf{c}^T\cdot\mathbf{x}\to\min_{\mathbf{x}}\sigma,\qquad(20)$$

where $\mathbf{c}=\left[0,...,0,1\right]^T$, $\mathbf{x}=\left[\mathbf{p}^{j(k)},p_{11}\right]^T$ with the following restrictions

$$\left.\begin{aligned}\left[\mathbf{g}_{1(2)i}^{(j(k))},-\frac{1}{2}\right]\cdot\begin{bmatrix}\mathbf{p}^{j(k)}\\p_{11}\end{bmatrix}=\mathbf{g}_{01(2)i}^{(j(k))}\\ \left[\mathbf{g}_{1(2)i}^{(j(k))},\frac{1}{2}\right]\cdot\begin{bmatrix}\mathbf{p}^{j(k)}\\p_{11}\end{bmatrix}=\mathbf{g}_{01(2)i}^{(j(k))}\end{aligned}\right\}\qquad(21)$$

The sequence of the obtained values of the functions $S_{1(2)}^{(j(k))}(\mathbf{p}^{(j(k))})$ will decrease and have a limit as a sequence bounded below, because $S_{1(2)}^{(j(k))}(\mathbf{p}^{(j(k))})\geq 0$ for any $\mathbf{p}^{(j(k))}$.

Let consider the solution of the least-square approximation problem (18) and (19) for the synthesis of the considered active CKC *GHKI*. From the necessary conditions for the minimum of functions $S_{1(2)}^{(j(k))}$ by groups $\mathbf{p}_{1(2)}^{(j(k))}$ of synthesis parameters

$$\frac{\partial S_{1(2)}^{(j(k))}}{\partial \mathbf{p}_{1(2)}^{(j(k))}} = 0 \qquad (22)$$

we obtain the systems of linear equations in the forms

$$\mathbf{H}_{1(2)}^{(j(k))} \cdot \mathbf{p}_{1(2)}^{(j(k))} = \mathbf{h}_{1(2)}^{(j(k))}, (j,k = 1,2,3). \qquad (23)$$

Solving the systems of equations (23) for each group of synthesis parameters for given values of the remaining parameter groups, we determine their values

$$\mathbf{p}_{1(2)}^{(j(k))} = \mathbf{H}_{1(2)}^{(j(k))-1} \cdot \mathbf{h}_{1(2)}^{(j(k))}. \qquad (24)$$

It is not difficult to show that the Hessian $\mathbf{H}_{1(2)}^{(j(k))}$ is positively defined together with the main minors. Then the solutions of the systems (23) correspond to the minimum of the functions $S_{1(2)}^{(j(k))}$. Consequently, the least-square approximation problem for parametric synthesis is reduced to the linear iteration method, at each step of which the systems of linear equations are solved.

Let consider the solution of parametric synthesis problem of the negative CKC *LM* and *NQ*. For this, we preliminarily determine the positions of the synthesized active CKC links *HG* and *HI*

$$\varphi_{6i} = \mathrm{tg}^{-1}\frac{Y_{I_i} - Y_{G_i}}{X_{I_i} - X_{G_i}} + \cos^{-1}\frac{l_{(GI)_i}^2 + l_{HI}^2 - l_{HG}^2}{2 l_{(GI)_i} \cdot l_{HI}}, \qquad (25)$$

$$\varphi_{5i} = \mathrm{tg}^{-1}\frac{Y_{G_i} - Y_{H_i}}{X_{G_i} - X_{H_i}}, \qquad (26)$$

where

$$\begin{bmatrix} X_{G_i} \\ Y_{G_i} \end{bmatrix} = \begin{bmatrix} X_{B_i} \\ Y_{B_i} \end{bmatrix} + \begin{bmatrix} \cos\varphi_{2i} & -\sin\varphi_{2i} \\ \sin\varphi_{2i} & \cos\varphi_{2i} \end{bmatrix} \cdot \begin{bmatrix} x_G^{(2)} \\ y_G^{(2)} \end{bmatrix},$$

$$\begin{bmatrix} X_{I_i} \\ Y_{I_i} \end{bmatrix} = \begin{bmatrix} X_{E_i} \\ Y_{E_i} \end{bmatrix} + \begin{bmatrix} \cos\varphi_{4i} & -\sin\varphi_{4i} \\ \sin\varphi_{4i} & \cos\varphi_{4i} \end{bmatrix} \cdot \begin{bmatrix} x_I^{(4)} \\ y_I^{(4)} \end{bmatrix},$$

$$l_{(GI)_i} = \left[\left(X_{I_i} - X_{G_i} \right)^2 + \left(Y_{I_i} - Y_{G_i} \right)^2 \right]^{\frac{1}{2}},$$

$$\begin{bmatrix} X_{H_i} \\ Y_{H_i} \end{bmatrix} = \begin{bmatrix} X_{I_i} \\ Y_{I_i} \end{bmatrix} - l_{HI} \begin{bmatrix} \cos\varphi_{6i} \\ \sin\varphi_{6i} \end{bmatrix}.$$

Write the vector loop-closure equations of *OBGMLAO* and *OEINQDO*

$$\mathbf{R}_{G_i} + \mathbf{\Gamma}(\varphi_{5i})\mathbf{r}_M^{(5)} + \mathbf{l}_{ML} = \mathbf{R}_{A_i} + \mathbf{\Gamma}(\varphi_{1i})\mathbf{r}_L^{(1)}, \qquad (27)$$

$$\mathbf{R}_{I_i} + \mathbf{\Gamma}(\varphi_{6i})\mathbf{r}_N^{(6)} + \mathbf{l}_{NQ} = \mathbf{R}_{D_i} + \mathbf{\Gamma}(\varphi_{3i})\mathbf{r}_Q^{(3)}, \qquad (28)$$

where $\mathbf{R}_{G_i} = \left[X_{G_i}, Y_{G_i} \right]^T$, $\mathbf{r}_M^{(5)} = \left[x_M^{(5)}, y_M^{(5)} \right]^T$,

$\mathbf{l}_{(ML)_i} = \left[l_{ML}\cos\varphi_{(ML)_i}, l_{ML}\sin\varphi_{(ML)_i} \right]^T$, $\mathbf{R}_A = \left[X_A, Y_A \right]^T$,

$\mathbf{r}_L^{(1)} = \left[x_L^{(1)}, y_L^{(1)} \right]^T$, $\mathbf{R}_{I_i} = \left[X_{I_i}, Y_{I_i} \right]^T$, $\mathbf{r}_N^{(6)} = \left[x_N^{(6)}, y_N^{(6)} \right]^T$,

$\mathbf{l}_{(NQ)_i} \left[l_{NQ}\cos\varphi_{(NQ)_i}, l_{NQ}\sin\varphi_{(NQ)_i} \right]^T$,

$\mathbf{R}_D = \left[X_D, Y_D \right]^T$, $\mathbf{r}_Q^{(3)} = \left[x_Q^{(3)}, y_Q^{(3)} \right]^T$.

Eliminating the unknown angles $\varphi_{(ML)_i}$ and $\varphi_{(NQ)_i}$ from Eqs (27) and (28) yields

$$\left[\mathbf{R}_{G_i} + \mathbf{\Gamma}(\varphi_{5i})\mathbf{r}_M^{(5)} - \mathbf{R}_A - \mathbf{\Gamma}(\varphi_{1i})\mathbf{r}_I^{(1)} \right]^2 - l_{ML}^2 = 0, \qquad (29)$$

$$\left[\mathbf{R}_{I_i} + \mathbf{\Gamma}(\varphi_{6i})\mathbf{r}_N^{(6)} - \mathbf{R}_D - \mathbf{\Gamma}(\varphi_{3i})\mathbf{r}_Q^{(3)} \right]^2 - l_{NQ}^2 = 0. \qquad (30)$$

Eqs (29) and (30) are the equations of geometrical constraints imposed on the motions of links 1 and 5, 3 and 6 by the negative CKC *ML* and *NQ*. The geometric meanings of these constraints are the equations of two circles in the relative motions of the planes of links 1 and 5, 3 and 6 with radiuses l_{ML} and l_{NQ}. The problem of determining the geometrical parameters of the links, at which such geometric constraints are approximately realized, is the problem of parametric synthesis of two negative CKC *ML* and *NQ*.

The left parts of Eqs (29) and (30) are denoted by Δq_{3i} and Δq_{4i}, which are functions of weighted differences

$$\Delta q_{3i} = \left[\mathbf{R}_{G_i} + \mathbf{\Gamma}(\varphi_{5i})\mathbf{r}_M^{(5)} - \mathbf{R}_A - \mathbf{\Gamma}(\varphi_{1i})\mathbf{r}_{1i}^{(1)} \right]^2 - l_{ML}^2, \qquad (31)$$

$$\Delta q_{4i} = \left[\mathbf{R}_{I_i} + \mathbf{\Gamma}(\varphi_{6i})\mathbf{r}_H^{(6)} - \mathbf{R}_D - \mathbf{\Gamma}(\varphi_{3i})\mathbf{r}_Q^{(3)} \right]^2 - l_{NQ}^2. \qquad (32)$$

After converting these equations and the following change of variables

$$\begin{bmatrix} p_{11} \\ p_{12} \end{bmatrix} = \begin{bmatrix} x_L^{(1)} \\ y_L^{(1)} \end{bmatrix}, \begin{bmatrix} p_{14} \\ p_{15} \end{bmatrix} = \begin{bmatrix} x_M^{(5)} \\ y_M^{(5)} \end{bmatrix},$$

$$p_{13} = \frac{1}{2}(x_L^{(1)^2} + y_L^{(1)^2} + x_M^{(5)^2} + y_M^{(5)^2} - l_{LM}^2),$$

$$\begin{bmatrix} p_{16} \\ p_{17} \end{bmatrix} = \begin{bmatrix} x_Q^{(3)} \\ y_Q^{(3)} \end{bmatrix}, \begin{bmatrix} p_{19} \\ p_{20} \end{bmatrix} = \begin{bmatrix} x_N^{(5)} \\ y_N^{(5)} \end{bmatrix},$$

$$p_{18} = \frac{1}{2}(x_Q^{(3)^2} + y_Q^{(3)^2} + x_N^{(6)^2} + y_N^{(6)^2} - l_{QN}^2)$$

the functions (31) and (32) are expressed linearly by two groups of synthesis parameters

$$\mathbf{p}_3^{(1)} = [p_{11}, p_{12}, p_{13}]^T, \ \mathbf{p}_3^{(2)} = [p_{14}, p_{15}, p_{13}]^T,$$
$$\mathbf{p}_4^{(1)} = [p_{16}, p_{17}, p_{18}]^T, \ \mathbf{p}_4^{(2)} = [p_{19}, p_{20}, p_{18}]^T$$

in the forms

$$\Delta q_{3i}^{(j)} = 2\left(\mathbf{g}_{3i}^{(j)} \cdot \mathbf{p}_3^{(j)} - g_{03i}^{(j)}\right), (j = 1, 2), \tag{33}$$

$$\Delta q_{4i}^{(k)} = 2\left(\mathbf{g}_{4i}^{(k)^T} \cdot \mathbf{p}_4^{(k)} - g_{04i}^{(k)}\right), (k = 1, 2), \tag{34}$$

where

$$\mathbf{g}_{3i}^{(1)} = \left[\begin{array}{cc|c} \multicolumn{2}{c|}{\mathbf{\Gamma}^{-1}(\varphi_{1i})} & 0 \\ & & 0 \\ \hline 0 & 0 & 1 \end{array}\right] \cdot \begin{bmatrix} X_{G_i} - X_A \\ Y_{G_i} - Y_A \\ 1 \end{bmatrix} - \left[\begin{array}{cc|c} \multicolumn{2}{c|}{\mathbf{\Gamma}(\varphi_{2i} - \varphi_{1i})} & 0 \\ & & 0 \\ \hline 0 & 0 & 1 \end{array}\right] \cdot \begin{bmatrix} p_{14} \\ p_{15} \\ 0 \end{bmatrix},$$

$$\mathbf{g}_{3i}^{(2)} = \left[\begin{array}{cc|c} \multicolumn{2}{c|}{\mathbf{\Gamma}^{-1}(\varphi_{2i})} & 0 \\ & & 0 \\ \hline 0 & 0 & 1 \end{array}\right] \cdot \begin{bmatrix} X_{G_i} - X_A \\ Y_{G_i} - Y_A \\ 1 \end{bmatrix} - \left[\begin{array}{cc|c} \multicolumn{2}{c|}{\mathbf{\Gamma}^{-1}(\varphi_{2i} - \varphi_{1i})} & 0 \\ & & 0 \\ \hline 0 & 0 & 1 \end{array}\right] \cdot \begin{bmatrix} p_{11} \\ p_{12} \\ 0 \end{bmatrix},$$

$$g_{03i}^{(1)} = -\frac{1}{2}\left[\left(X_{G_i} - X_A\right)^2 + \left(Y_{G_i} - Y_A\right)^2\right] +$$
$$+\left[X_{G_i} - X_A, Y_{G_i} - Y_A\right] \cdot \mathbf{\Gamma}(\varphi_{2i}) \cdot \begin{bmatrix} p_{14} \\ p_{15} \end{bmatrix},$$

$$g_{03i}^{(2)} = -\frac{1}{2}\left[\left(X_{G_i} - X_A\right)^2 + \left(Y_{G_i} - Y_A\right)^2\right] -$$
$$+\left[X_{G_i} - X_A, Y_{G_i} - Y_A\right] \cdot \mathbf{\Gamma}(\varphi_{1i}) \cdot \begin{bmatrix} p_{11} \\ p_{12} \end{bmatrix},$$

$$\mathbf{g}_{4i}^{(1)} = -\left[\begin{array}{cc|c} \multicolumn{2}{c|}{\mathbf{\Gamma}^{-1}(\varphi_{3i})} & 0 \\ & & 0 \\ \hline 0 & 0 & 1 \end{array}\right] \cdot \begin{bmatrix} X_{I_i} - X_D \\ Y_{I_i} - Y_D \\ 1 \end{bmatrix} - \left[\begin{array}{cc|c} \multicolumn{2}{c|}{\mathbf{\Gamma}(\varphi_{4i} - \varphi_{3i})} & 0 \\ & & 0 \\ \hline 0 & 0 & 1 \end{array}\right] \cdot \begin{bmatrix} p_{19} \\ p_{20} \\ 0 \end{bmatrix},$$

$$\mathbf{g}_{4i}^{(2)} = \left[\begin{array}{cc|c} \multicolumn{2}{c|}{\mathbf{\Gamma}^{-1}(\varphi_{4i})} & 0 \\ & & 0 \\ \hline 0 & 0 & 1 \end{array}\right] \cdot \begin{bmatrix} X_{I_i} - X_D \\ Y_{I_i} - Y_D \\ 1 \end{bmatrix} - \left[\begin{array}{cc|c} \multicolumn{2}{c|}{\mathbf{\Gamma}^{-1}(\varphi_{4i} - \varphi_{3i})} & 0 \\ & & 0 \\ \hline 0 & 0 & 1 \end{array}\right] \cdot \begin{bmatrix} p_{16} \\ p_{17} \\ 0 \end{bmatrix},$$

$$g_{04i}^{(1)} = -\frac{1}{2}\left[\left(X_{I_i} - X_D\right)^2 + \left(Y_{I_i} - Y_D\right)^2\right] +$$
$$+\left[X_{I_i} - X_D, Y_{I_i} - Y_D\right] \cdot \mathbf{\Gamma}(\varphi_{4i}) \cdot \begin{bmatrix} p_{19} \\ p_{20} \end{bmatrix},$$

$$g_{04i}^{(2)} = -\frac{1}{2}\left[\left(X_{I_i} - X_D\right)^2 + \left(Y_{I_i} - Y_D\right)^2\right] -$$
$$+\left[X_{I_i} - X_D, Y_{I_i} - Y_D\right] \cdot \mathbf{\Gamma}(\varphi_{3i}) \cdot \begin{bmatrix} p_{16} \\ p_{17} \end{bmatrix}.$$

Further, on the basis of the approximation problems of the Chebyshev and least-square approximations, outlined above, the parametric synthesis of the considered CKC *LM* and *QN* separately is carried out.

IV. Position Analysis

It is known that any Assur group has zero DOF defining by Chebyshev's formula [16]

$$W = 3n - 2p_5, \tag{35}$$

where n is number of links, p_5 is number of kinematic pairs of the fifth class. Class of kinematic pair is determined by number of constraints imposed on relative motion of its elements. For example, the revolute and prismatic kinematic pairs are kinematic pairs of the fifth class.

The basic conditions for the existence of Assur groups are that their DOF number should be equal to zero and they should not split into several other groups of the lower classes. Assur groups having n=2 and p_5=3 belong to Assur groups of the second class (Fig.4a). Assur groups having n=4 and p_5=6 belong to Assur groups of the third (Fig.4b) or fourth class (Fig .4c).

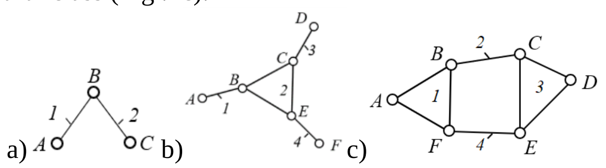

Fig.4. Assur groups of the a) second, b) third and c) fourth classes.

Assur groups having n = 6 and p_5 = 9 belong to Assur groups of the fourth (Fig.5a), fifth (Fig.5b) or sixth (Fig.5c) classes.

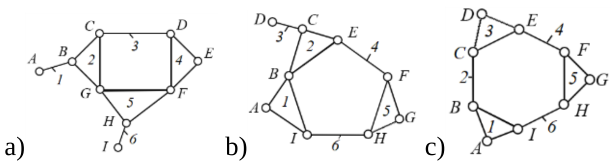

Fig.5 Assur groups of the a) fourth, b) fifth and c) sixth classes.

Classes of the Assur groups from the fourth and higher are determined by the number of sides of the variable closed loop, for example, the contours *CDFG, BEFHI* and *BCEFHI* in the Assur groups of the fourth, fifth and sixth classes are four-, five- and six-sided, respectively.

Assur groups of the fourth and higher classes belong to the Assur groups of high classes. Each high class Assur group also has an order, which is determined by the number of external joints. For example, Assur groups of the fourth class shown in Figures 4c and 5a are Assur groups of the fourth class of the second and third order, respectively, since the first group (Fig. 4c) has two external kinematic pairs *A* and *D*, and the second group (Fig. 5a) has three external kinematic pairs *A, E, I*.

In Assur groups of the fourth (Fig. 4c), fifth (Fig. 5b) and sixth (Fig. 5c) classes with a different arrangement of links, as shown in Fig. 5, they fall into Assur groups of the second class with structural formulas

$$\text{II}(1,4) \rightarrow \text{II}(2,3), \tag{36}$$

$$\begin{array}{ccc} & \nearrow & \mathrm{II}(2,3) \\ \mathrm{II}(1,6) & & \downarrow \\ & \searrow & \mathrm{II}(4,5) \end{array}, \qquad (37)$$

$$\begin{array}{ccc} & \nearrow & \mathrm{II}(4,5) \\ \mathrm{II}(1,6) & & \downarrow \\ & \searrow & \mathrm{II}(2,3) \end{array}. \qquad (38)$$

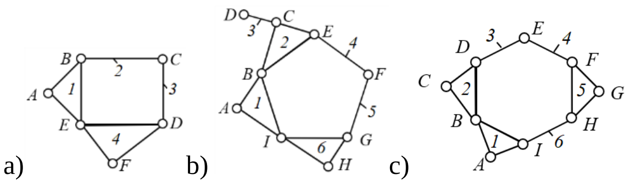

Fig.6. Assur groups of the second class

The class of the mechanism or manipulator is determined by the highest class of Assur groups that are part of the mechanism or manipulator. Position analysis of the mechanisms of the second class are solved analytically. Position analysis of mechanisms of the third and higher classes are solved numerically. There are known methods of position analysis of the third class mechanism [12], which are reduced to solving the 6th degree polynomials. There are no methods in the literature for positions analysis of mechanisms of the fourth and higher classes.

For position analysis of any class mechanisms including the mechanisms of high classes we have proposed a simple numerical method called the method of conditional generalized coordinates. According to this method after remove one link by disconnecting the elements of kinematic pairs in the analyzing group one DOF is appeared, because a binary link with two revolute kinematic pairs imposes one geometrical constraint. Indeed, the binary link with two revolute kinematic pairs has n=1 and p_5=2 then according to the formula (35) we obtain W= -1. If we choose one link of the analyzing group as a conditional input link due to appear one DOF then this group is transformed into the mechanism of the second class the positions of which are determined analytically relative the variable parameter of the conditional input link called the conditional generalized coordinate. In variation of the value of the generalized coordinate a distance between the centers of the removed joints is changed. A function of the difference between this variable distance and the length of the removed link is derived which is function of one variable parameter – the conditional generalized coordinate. Then minimizing the derived function by conventional generalized coordinate its value is determined. Values of parameters defining the positions of other links of the analyzing group are determined simultaneously with the conditional generalized coordinate.

According to the conditional generalized coordinates method, after removing the link 4 (Fig.4c), link 3 (Fig.5a), link 4 (Fig.5b), link 6 (Fig.5c), one DOF appears in the Assur groups. If we choose the link 1 (Fig. 4c), link 4 (Fig. 5a), link 1 (Fig. 5b), link 1 (Fig. 5c) as the conditional input links due to the DOF that appears, these Assur groups are transformed into second class mechanisms with structural formulas

$$\mathrm{I}(1) \to \mathrm{II}(2{,}3)\,, \qquad (39)$$

$$\mathrm{I}(4) \to \mathrm{II}(5{,}6) \to \mathrm{II}(1{,}2)\,, \qquad (40)$$

$$\mathrm{I}(1) \to \mathrm{II}(2{,}3) \to \mathrm{II}(4{,}5)\,, \qquad (41)$$

$$\mathrm{I}(1) \to \mathrm{II}(2{,}3) \to \mathrm{II}(4{,}5). \qquad (42)$$

Let consider the position analysis of the fourth class PM with two end-effectors (Fig. 3) with the structural formula (1). After removing the links 8 and 9 of the Assur groups of the fourth class and the second order IV (1,2,5,8) and IV (3,4,6,9) and choosing the conditional input links 1 and 3, this manipulator is converted into a second class mechanism with the structural formula

$$\mathrm{I}(0,1) \to \mathrm{II}(2{,}5) \leftarrow \mathrm{I}(0,7) \to \mathrm{II}(4,6) \leftarrow \mathrm{I}(0,3). \qquad (43)$$

For a given value of the angle φ_{7i} of the input link 7, when the values of the conditional generalized coordinates φ_{1i} and φ_{3i} are changed, the distances $\tilde{l}_{LM}$ and $\tilde{l}_{NQ}$ between the centers of the disconnected joints *L* and *M*, as well as *N* and *Q*, are changed. Let derive the functions

$$f(\varphi_{1i}) = \tilde{l}_{(LM)_i} - l_{(LM)_i}\,, \qquad (44)$$

$$f(\varphi_{3i}) = \tilde{l}_{(NQ)_i} - l_{(NQ)_i}\,, \qquad (45)$$

where l_{LM} and l_{NQ} are the lengths of the removed links 8 and 9.

The variable distances $\tilde{l}_{(NQ)_i}$ and $\tilde{l}_{(LM)_i}$ in the equations (44) and (45) are determined by the expressions

$$\tilde{l}_{(LM)_i} = \left[(X_{M_i} - X_{L_i})^2 + (Y_{M_i} - Y_{L_i})^2 \right]^{\frac{1}{2}}, \qquad (46)$$

$$\tilde{l}_{(NQ)_i} = \left[(X_{N_i} - X_{Q_i})^2 + (Y_{N_i} - Y_{Q_i})^2 \right]^{\frac{1}{2}}, \qquad (47)$$

where the coordinates of the joints *M*, *L*, *Q*, *N* in the absolute coordinate system *OXY* are calculated by the equations

$$\begin{bmatrix} X_{M_i} \\ Y_{M_i} \end{bmatrix} = \begin{bmatrix} X_{G_i} \\ Y_{G_i} \end{bmatrix} + \begin{bmatrix} \cos\varphi_{5i} & -\sin\varphi_{5i} \\ \sin\varphi_{5i} & \cos\varphi_{5i} \end{bmatrix} \cdot \begin{bmatrix} x_M^{(5)} \\ y_M^{(5)} \end{bmatrix}, \qquad (48)$$

$$\begin{bmatrix} X_{L_i} \\ Y_{L_i} \end{bmatrix} = \begin{bmatrix} X_A \\ Y_A \end{bmatrix} + \begin{bmatrix} \cos\varphi_{1i} & -\sin\varphi_{5i} \\ \sin\varphi_{5i} & \cos\varphi_{5i} \end{bmatrix} \cdot \begin{bmatrix} x_L^{(1)} \\ y_L^{(1)} \end{bmatrix}, \qquad (49)$$

$$\begin{bmatrix} X_{Q_i} \\ Y_{Q_i} \end{bmatrix} = \begin{bmatrix} X_D \\ Y_D \end{bmatrix} + \begin{bmatrix} \cos\varphi_{3i} & -\sin\varphi_{3i} \\ \sin\varphi_{3i} & \cos\varphi_{3i} \end{bmatrix} \cdot \begin{bmatrix} x_Q^{(3)} \\ y_Q^{(3)} \end{bmatrix}, \qquad (50)$$

$$\begin{bmatrix} X_{N_i} \\ Y_{N_i} \end{bmatrix} = \begin{bmatrix} X_{I_i} \\ Y_{I_i} \end{bmatrix} + \begin{bmatrix} \cos\varphi_{6i} & -\sin\varphi_{6i} \\ \sin\varphi_{6i} & \cos\varphi_{6i} \end{bmatrix} \cdot \begin{bmatrix} x_N^{(6)} \\ y_N^{(6)} \end{bmatrix}. \quad (51)$$

The coordinates of the joints *G* and *I* in the equations (48) and (51) are determined by the expressions

$$\begin{bmatrix} X_{G_i} \\ Y_{G_i} \end{bmatrix} = \begin{bmatrix} X_A \\ Y_A \end{bmatrix} + l_{AB} \begin{bmatrix} \cos\varphi_{1i} \\ \sin\varphi_{1i} \end{bmatrix} + \begin{bmatrix} \cos\varphi_{2i} & -\sin\varphi_{2i} \\ \sin\varphi_{2i} & \cos\varphi_{2i} \end{bmatrix} \begin{bmatrix} x_G^{(2)} \\ y_G^{(2)} \end{bmatrix}, \quad (52)$$

$$\begin{bmatrix} X_{I_i} \\ Y_{I_i} \end{bmatrix} = \begin{bmatrix} X_D \\ Y_D \end{bmatrix} + l_{DE} \begin{bmatrix} \cos\varphi_{3i} \\ \sin\varphi_{3i} \end{bmatrix} + \begin{bmatrix} \cos\varphi_{4i} & -\sin\varphi_{4i} \\ \sin\varphi_{4i} & \cos\varphi_{4i} \end{bmatrix} \begin{bmatrix} x_I^{(4)} \\ y_I^{(4)} \end{bmatrix}. \quad (53)$$

The angles $\varphi_{2i}, \varphi_{5i}$ and $\varphi_{4i}, \varphi_{6i}$ in the expressions (52), (49) and (53), (51) are determined by solving the position analysis of the groups II (2,5) and II (4.6) for given values of the conditional generalized coordinates φ_{1i} and φ_{3i}. To do this, we derive the vector loop-closure equations *BGH* and *EIH*

$$l_{BG}\mathbf{e}_{(BG)_i} + l_{GH}\mathbf{e}_{5i} - l_{(BH)_i}\mathbf{e}_{(BH)_i} = 0, \quad (54)$$

$$l_{EI}\mathbf{e}_{(EI)_i} + l_{IH}\mathbf{e}_{6i} - l_{(EH)_i}\mathbf{e}_{(EH)_i} = 0, \quad (55)$$

where the modules of vectors are denoted by *l*, and their unit vectors are denoted by $\mathbf{e}$. In the equations (54) and (55), the modules and directions of the vectors $(\overline{BH})_i$ and $(\overline{EH})_i$, as well as the modules of the vectors $\overline{BG}$ and $\overline{EI}$ are determined by the expressions

$$l_{(BH)_i} = \left[(X_{H_i} - X_{B_i})^2 + (Y_{H_i} - Y_{B_i})^2 \right]^{\frac{1}{2}}, \quad (56)$$

$$l_{(EH)_i} = \left[(X_{H_i} - X_{E_i})^2 + (Y_{H_i} - Y_{E_i})^2 \right]^{\frac{1}{2}}, \quad (57)$$

$$\varphi_{(BH)_i} = \text{tg}^{-1} \frac{Y_{H_i} - Y_{B_i}}{X_{H_i} - X_{B_i}}, \quad (58)$$

$$\varphi_{(EH)_i} = \text{tg}^{-1} \frac{Y_{H_i} - Y_{E_i}}{X_{H_i} - X_{E_i}}. \quad (59)$$

In the equations (56-59), the coordinates of the joints *B* and *E* in the absolute coordinate system *OXY* are determined by the expression (6), (7), and the coordinates of the joint *H* are determined by the expression

$$\begin{bmatrix} X_{H_i} \\ Y_{H_i} \end{bmatrix} = \begin{bmatrix} X_K \\ Y_K \end{bmatrix} + \begin{bmatrix} \cos\varphi_{7i} & -\sin\varphi_{7i} \\ \sin\varphi_{7i} & \cos\varphi_{7i} \end{bmatrix} \cdot \begin{bmatrix} x_H^{(7)} \\ y_H^{(7)} \end{bmatrix}. \quad (60)$$

To determine the unknown directions of the vectors $l_{BG}(\mathbf{e}_{BG})_i$ and $l_{EI}(\mathbf{e}_{EI})_i$, we transfer the vectors $l_{GH}\mathbf{e}_{5i}$ and $l_{IH}\mathbf{e}_{6i}$ to the right-hand sides of the equations (54), (55) and square there both sides

$$l_{BG}^2 + l_{(BH)_i}^2 - 2l_{BG}l_{(BH)_i}\cos(\varphi_{(BG)_i} - \varphi_{(BH)_i}) = l_{GH}, \quad (61)$$

$$l_{EI}^2 + l_{(EH)_i}^2 - 2l_{EI}l_{(EH)_i}\cos(\varphi_{(EH)_i} - \varphi_{(EI)_i}) = l_{IH}, \quad (62)$$

and obtain

$$\varphi_{(BG)_i} = \varphi_{(BH)_i} + \cos^{-1}\frac{l_{BG}^2 + l_{(BH)_i}^2 - l_{GH}^2}{2l_{BG}l_{(BH)_i}}, \quad (63)$$

$$\varphi_{(EI)_i} = \varphi_{(EH)_i} - \cos^{-1}\frac{l_{EI}^2 + l_{(EH)_i}^2 - l_{IH}^2}{2l_{EI}l_{(EH)_i}}. \quad (64)$$

The directions of the vectors $\overline{l}_{(GH)_i}$ and $\overline{l}_{(IH)_i}$ are determined by the equations

$$\varphi_{5i} = \text{tg}^{-1}\frac{Y_{H_i} - Y_{G_i}}{X_{H_i} - X_{G_i}}, \quad (65)$$

$$\varphi_{6i} = \text{tg}^{-1}\frac{Y_{H_i} - Y_{I_i}}{X_{H_i} - X_{I_i}}, \quad (66)$$

where

$$\begin{bmatrix} X_{G_i} \\ Y_{G_i} \end{bmatrix} = \begin{bmatrix} X_{B_i} \\ Y_{B_i} \end{bmatrix} + l_{BG} \begin{bmatrix} \cos\varphi_{(BG)_i} \\ \sin\varphi_{(BG)_i} \end{bmatrix}, \quad (67)$$

$$\begin{bmatrix} X_{I_i} \\ Y_{I_i} \end{bmatrix} = \begin{bmatrix} X_{E_i} \\ Y_{E_i} \end{bmatrix} + l_{EI} \begin{bmatrix} \cos\varphi_{(EI)_i} \\ \sin\varphi_{(EI)_i} \end{bmatrix}. \quad (68)$$

Thus, the functions (44) and (45) are functions of the single variable φ_{1i} and φ_{3i}, respectively. Therefore, minimizing these functions with respect to the variables φ_{1i} and φ_{3i}, their values have been determined. In this case, the angles $\varphi_{(BG)_i}$, $\varphi_{(EI)_i}$, φ_{5i}, φ_{6i} are simultaneously determined. Then the coordinates of the output points *C* and *F* (centers of grippers) are determined by the equations,

$$\begin{bmatrix} X_{C_i} \\ Y_{C_i} \end{bmatrix} = \begin{bmatrix} X_{B_i} \\ Y_{B_i} \end{bmatrix} + l_{BC} \begin{bmatrix} \cos\varphi_{2i} \\ \sin\varphi_{2i} \end{bmatrix}, \quad (69)$$

$$\begin{bmatrix} X_{F_i} \\ Y_{F_i} \end{bmatrix} = \begin{bmatrix} X_{E_i} \\ Y_{E_i} \end{bmatrix} + l_{EF} \begin{bmatrix} \cos\varphi_{4i} \\ \sin\varphi_{4i} \end{bmatrix}, \quad (70)$$

where

$$\varphi_{2i} = \varphi_{(BG)_i} + \mathrm{tg}^{-1}\frac{y_G^{(2)}}{x_G^{(2)}}, \quad (71)$$

$$\varphi_{4i} = \varphi_{(EI)_i} - \mathrm{tg}^{-1}\frac{y_I^{(4)}}{x_I^{(4)}}. \quad (72)$$

V. VELOCITIES AND ACCELERATIONS

To determine the velocities and accelerations of the links and output points of the considered manipulator, let determine the independent vector contours and differentiate them by time. The number of independent vector contours is determined by dividing into two numbers of the Assur groups links. The number of the Assur groups links of the manipulator is eight, therefore, the number of independent vector contours is four. As the independent vector contours, we choose the contours *KHGBA, KHMLA, KHIED, KHNQD* and derive their vector closure-loop equations

$$\left.\begin{aligned}
&l_{KH}\mathbf{e}_{(KH)_i} + l_{GH}\mathbf{e}_{(HG)_i} - l_{BG}\mathbf{e}_{(BG)_i} - l_{AB}\mathbf{e}_{(AB)_i} - l_{KA}\mathbf{e}_{KA} = 0\\
&l_{KH}\mathbf{e}_{(KH)_i} + l_{HM}\mathbf{e}_{(HM)_i} - l_{LM}\mathbf{e}_{(LM)_i} - l_{AL}\mathbf{e}_{(AL)_i} - l_{KA}\mathbf{e}_{KA} = 0\\
&l_{KH}\mathbf{e}_{(KH)_i} + l_{HI}\mathbf{e}_{(HI)_i} - l_{EI}\mathbf{e}_{(EI)_i} - l_{DE}\mathbf{e}_{(DE)_i} - l_{KD}\mathbf{e}_{KD} = 0\\
&l_{KH}\mathbf{e}_{(KH)_i} + l_{HN}\mathbf{e}_{(HN)_i} - l_{QN}\mathbf{e}_{(QN)_i} - l_{DQ}\mathbf{e}_{(DQ)_i} - l_{KD}\mathbf{e}_{KD} = 0
\end{aligned}\right\}(73)$$

Let project the system of equations (73) on the *OX* and *OY* axes of the absolute coordinate system *OXY*

$$\left.\begin{aligned}
&l_{KH}\cos\varphi_{(KH)_i} + l_{HG}\cos\varphi_{(HG)_i} - l_{BG}\cos\varphi_{(BG)_i} -\\
&- l_{AB}\cos\varphi_{(AB)_i} - l_{KA}\cos\varphi_{KA} = 0\\
&l_{KH}\sin\varphi_{(KH)_i} + l_{HG}\sin\varphi_{(HG)_i} - l_{BG}\sin\varphi_{(BG)_i} -\\
&- l_{AB}\sin\varphi_{(AB)_i} - l_{KA}\sin\varphi_{KA} = 0\\
&l_{KH}\cos\varphi_{(KH)_i} + l_{HM}\cos\varphi_{(HM)_i} - l_{LM}\cos\varphi_{(LM)_i} -\\
&- l_{AL}\cos\varphi_{(AL)_i} - l_{KA}\cos\varphi_{KA} = 0\\
&l_{KH}\sin\varphi_{(KH)_i} + l_{HM}\sin\varphi_{(HM)_i} - l_{LM}\sin\varphi_{(LM)_i} -\\
&- l_{AL}\sin\varphi_{(AL)_i} - l_{KA}\sin\varphi_{KA} = 0\\
&l_{KH}\cos\varphi_{(KH)_i} + l_{HI}\cos\varphi_{(HI)_i} - l_{EI}\cos\varphi_{(EI)_i} -\\
&- l_{DE}\cos\varphi_{(DE)_i} - l_{KD}\cos\varphi_{KD} = 0\\
&l_{KH}\sin\varphi_{(KH)_i} + l_{HI}\sin\varphi_{(HI)_i} - l_{EI}\sin\varphi_{(EI)_i} -\\
&- l_{DE}\sin\varphi_{(DE)_i} - l_{KD}\sin\varphi_{KD} = 0\\
&l_{KH}\cos\varphi_{(KH)_i} + l_{HN}\cos\varphi_{(HN)_i} - l_{QN}\cos\varphi_{(QN)_i} -\\
&- l_{DQ}\cos\varphi_{(DQ)_i} - l_{KD}\cos\varphi_{KD} = 0\\
&l_{KH}\sin\varphi_{(KH)_i} + l_{HN}\sin\varphi_{(HN)_i} - l_{QN}\sin\varphi_{(QN)_i} -\\
&- l_{DQ}\sin\varphi_{(DQ)_i} - l_{KD}\sin\varphi_{KD} = 0
\end{aligned}\right\}(74)$$

and differentiate by time

$$\left.\begin{aligned}
&-l_{KH}\sin\varphi_{(KH)_i}\cdot\omega_{7i} - l_{HG}\sin\varphi_{(HG)_i}\cdot\omega_{5i} +\\
&+ l_{BG}\sin\varphi_{(BG)_i}\cdot\omega_{2i} + l_{AB}\sin\varphi_{(AB)_i}\cdot\omega_{1i} = 0\\
&l_{KH}\cos\varphi_{(KH)_i}\cdot\omega_{7i} + l_{HG}\cos\varphi_{(HG)_i}\cdot\omega_{5i} -\\
&- l_{BG}\cos\varphi_{(BG)_i}\cdot\omega_{2i} - l_{AB}\cos\varphi_{(AB)_i}\cdot\omega_{1i} = 0\\
&-l_{KH}\sin\varphi_{(KH)_i}\cdot\omega_{7i} - l_{HM}\sin\varphi_{(HM)_i}\cdot\omega_{5i} +\\
&+ l_{LM}\sin\varphi_{(LM)_i}\cdot\omega_{8i} + l_{AL}\sin\varphi_{(AL)_i}\cdot\omega_{1i} = 0\\
&l_{KH}\cos\varphi_{(KH)_i}\cdot\omega_{7i} + l_{HM}\cos\varphi_{(HM)_i}\cdot\omega_{5i} -\\
&- l_{LM}\cos\varphi_{(LM)_i}\cdot\omega_{8i} - l_{AL}\cos\varphi_{(AL)_i}\cdot\omega_{1i} = 0\\
&-l_{KH}\sin\varphi_{(KH)_i}\cdot\omega_{7i} - l_{HI}\sin\varphi_{(HI)_i}\cdot\omega_{6i} +\\
&+ l_{EI}\sin\varphi_{(EI)_i}\cdot\omega_{4i} + l_{DE}\sin\varphi_{(DE)_i}\cdot\omega_{3i} = 0\\
&l_{KH}\cos\varphi_{(KH)_i}\cdot\omega_{7i} + l_{HI}\cos\varphi_{(HI)_i}\cdot\omega_{6i} -\\
&- l_{EI}\cos\varphi_{(EI)_i}\cdot\omega_{4i} - l_{DE}\cos\varphi_{(DE)_i}\cdot\omega_{3i} = 0\\
&-l_{KH}\sin\varphi_{(KH)_i}\cdot\omega_{7i} - l_{HN}\sin\varphi_{(HN)_i}\cdot\omega_{6i} +\\
&+ l_{QN}\sin\varphi_{(QN)_i}\cdot\omega_{9i} + l_{DQ}\sin\varphi_{(DQ)_i}\cdot\omega_{3i} = 0\\
&l_{KH}\cos\varphi_{(KH)_i}\cdot\omega_{7i} + l_{HN}\cos\varphi_{(HN)_i}\cdot\omega_{6i} -\\
&- l_{QN}\cos\varphi_{(QN)_i}\cdot\omega_{9i} - l_{DQ}\cos\varphi_{(DQ)_i}\cdot\omega_{3i} = 0
\end{aligned}\right\}(75)$$

or

$$\left.\begin{aligned}
&(Y_K - Y_{H_i})\omega_{7i} + (Y_{H_i} - Y_{G_i})\omega_{5i} + (Y_{G_i} - Y_{B_i})\omega_{2i} +\\
&+ (Y_{B_i} - Y_A)\omega_{1i} = 0\\
&(X_{H_i} - X_K)\omega_{7i} + (X_{G_i} - X_{H_i})\omega_{5i} + (X_{B_i} - X_{G_i})\omega_{2i} +\\
&+ (X_A - X_{B_i})\omega_{1i} = 0\\
&(Y_K - Y_{H_i})\omega_{7i} + (Y_{H_i} - Y_{M_i})\omega_{5i} + (Y_{M_i} - Y_{L_i})\omega_{8i} +\\
&+ (Y_{L_i} - Y_A)\omega_{1i} = 0\\
&(X_{H_i} - X_K)\omega_{7i} + (X_{M_i} - X_{H_i})\omega_{5i} + (X_{L_i} - X_{M_i})\omega_{8i} +\\
&+ (X_A - X_{L_i})\omega_{1i} = 0\\
&(Y_K - Y_{H_i})\omega_{7i} + (Y_{H_i} - Y_{I_i})\omega_{6i} + (Y_{I_i} - Y_{E_i})\omega_{4i} +\\
&+ (Y_{E_i} - Y_D)\omega_{3i} = 0\\
&(X_{H_i} - X_K)\omega_{7i} + (X_{I_i} - X_{H_i})\omega_{6i} + (X_{E_i} - X_{I_i})\omega_{4i} +\\
&+ (X_D - X_{E_i})\omega_{3i} = 0\\
&(Y_K - Y_{H_i})\omega_{7i} + (Y_{H_i} - Y_{N_i})\omega_{6i} + (Y_{N_i} - Y_{Q_i})\omega_{9i} +\\
&+ (Y_{Q_i} - Y_D)\omega_{3i} = 0\\
&(X_{H_i} - X_K)\omega_{7i} + (X_{N_i} - X_{H_i})\omega_{6i} + (X_{Q_i} - X_{N_i})\omega_{9i} +\\
&+ (X_D - X_{Q_i})\omega_{3i} = 0
\end{aligned}\right\}(76)$$

From the system of linear equations (76) we determine the angular velocities of the links by the equation

$$\bar{X} = \mathbf{A}^{-1} \cdot \bar{B}, \qquad (77)$$

when $\det|A| \neq 0$, where

$$\mathbf{A} = \begin{bmatrix} Y_{H_i} - Y_{G_i} & Y_{G_i} - Y_{B_i} & Y_{B_i} - Y_A & 0 & 0 & 0 & 0 & 0 \\ X_{G_i} - X_{H_i} & X_{B_i} - X_{G_i} & X_A - X_{B_i} & 0 & 0 & 0 & 0 & 0 \\ Y_{H_i} - Y_{M_i} & 0 & Y_{L_i} - Y_A & Y_{M_i} - Y_{L_i} & 0 & 0 & 0 & 0 \\ X_{M_i} - X_{H_i} & 0 & X_A - X_{L_i} & X_{L_i} - X_{M_i} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_{H_i} - Y_{I_i} & Y_{I_i} - Y_{E_i} & Y_{E_i} - Y_D & 0 \\ 0 & 0 & 0 & 0 & X_{I_i} - X_{H_i} & X_{E_i} - X_{I_i} & X_D - X_{E_i} & 0 \\ 0 & 0 & 0 & 0 & Y_{H_i} - Y_{N_i} & 0 & Y_{Q_i} - Y_D & Y_{N_i} - Y_{Q_i} \\ 0 & 0 & 0 & 0 & X_{N_i} - X_{H_i} & 0 & X_D - X_{Q_i} & X_{Q_i} - X_{N_i} \end{bmatrix}, \qquad (78)$$

$$\bar{X} = \begin{bmatrix} \omega_{5i} \\ \omega_{2i} \\ \omega_{1i} \\ \omega_{8i} \\ \omega_{6i} \\ \omega_{4i} \\ \omega_{3i} \\ \omega_{9i} \end{bmatrix}, \quad \bar{B} = \omega_{7i} \begin{bmatrix} Y_{K_i} - Y_{H_i} \\ X_{H_i} - X_{K_i} \\ Y_K - Y_{H_i} \\ X_{H_i} - X_K \\ Y_K - Y_{H_i} \\ X_{H_i} - X_K \\ Y_K - Y_{H_i} \\ X_{H_i} - X_K \end{bmatrix}. \qquad (79)$$

Velocities of the output points *C* and *F* are determined by differentiating the equations (69) and (70) with respect to time

$$\begin{bmatrix} v_{C_i}^X \\ v_{C_i}^Y \end{bmatrix} = \begin{bmatrix} v_{B_i}^X \\ v_{B_i}^Y \end{bmatrix} + l_{BC}\omega_{2i} \begin{bmatrix} -\sin\varphi_{2i} \\ \cos\varphi_{2i} \end{bmatrix}, \qquad (80)$$

$$\begin{bmatrix} v_{F_i}^X \\ v_{F_i}^Y \end{bmatrix} = \begin{bmatrix} v_{E_i}^X \\ v_{E_i}^Y \end{bmatrix} + l_{EF}\omega_{4i} \begin{bmatrix} -\sin\varphi_{4i} \\ \cos\varphi_{4i} \end{bmatrix}, \qquad (81)$$

where the velocities of the joints *B* and *E* are determined by differentiating the equations (6) and (7) with respect to time

$$\begin{bmatrix} v_{B_i}^X \\ v_{B_i}^Y \end{bmatrix} = l_{AB}\omega_{1i} \begin{bmatrix} -\sin\varphi_{1i} \\ \cos\varphi_{1i} \end{bmatrix}, \qquad (82)$$

$$\begin{bmatrix} v_{E_i}^X \\ v_{E_i}^Y \end{bmatrix} = l_{DE}\omega_{3i} \begin{bmatrix} -\sin\varphi_{3i} \\ \cos\varphi_{3i} \end{bmatrix}. \qquad (83)$$

To determine the angular accelerations of the manipulator links, let differentiate the system of equations (74) with respect to time

$$\left.\begin{array}{c}
-l_{KH}\cos\varphi_{(KH)_i}\cdot\omega_{7i}^2 - l_{KH}\sin\varphi_{(KH)_i}\cdot\varepsilon_{7i} - l_{HG}\cos\varphi_{(HG)_i}\cdot\omega_{5i}^2 - \\
-l_{HG}\sin\varphi_{(HG)_i}\cdot\varepsilon_{5i} + l_{BG}\cos\varphi_{(BG)_i}\cdot\omega_{2i}^2 + l_{BG}\sin\varphi_{(BG)_i}\cdot\varepsilon_{2i} + \\
+l_{AB}\cos\varphi_{(AB)_i}\cdot\omega_{1i}^2 + l_{AB}\sin\varphi_{(AB)_i}\cdot\varepsilon_{1i} = 0 \\
-l_{KH}\sin\varphi_{(KH)}\cdot\omega_{7i}^2 + l_{KH}\cos\varphi_{(KH)_i}\cdot\varepsilon_{7i} - l_{HG}\sin\varphi_{(HG)_i}\cdot\omega_{5i}^2 + \\
+l_{HG}\cos\varphi_{(HG)_i}\cdot\varepsilon_{5i} + l_{BG}\sin\varphi_{(BG)_i}\cdot\omega_{2i}^2 - l_{BG}\cos\varphi_{(BG)_i}\cdot\varepsilon_{2i} \\
+l_{AB}\sin\varphi_{(AB)_i}\cdot\omega_{1i}^2 - l_{AB}\cos\varphi_{(AB)_i}\cdot\varepsilon_{1i} = 0 \\
-l_{KH}\cos\varphi_{(KH)_i}\cdot\omega_{7i}^2 - l_{KH}\sin\varphi_{(KH)_i}\cdot\varepsilon_{7i} - l_{HM}\cos\varphi_{(HM)_i}\cdot\omega_{5i}^2 - \\
-l_{HM}\sin\varphi_{(HM)_i}\cdot\varepsilon_{5i} + l_{LM}\cos\varphi_{(LM)_i}\cdot\omega_{8i}^2 + l_{LM}\sin\varphi_{(LM)_i}\cdot\varepsilon_{8i} + \\
+l_{AL}\cos\varphi_{(AL)_i}\cdot\omega_{1i}^2 + l_{AL}\sin\varphi_{(AL)_i}\cdot\varepsilon_{1i} = 0 \\
-l_{KH}\sin\varphi_{(KH)_i}\cdot\omega_{7i}^2 + l_{KH}\cos\varphi_{(KH)_i}\cdot\varepsilon_{7i} - l_{HM}\sin\varphi_{(HM)_i}\cdot\omega_{5i}^2 + \\
+l_{HM}\cos\varphi_{(HM)_i}\cdot\varepsilon_{5i} + l_{LM}\sin\varphi_{(LM)_i}\cdot\omega_{8i}^2 - l_{LM}\cos\varphi_{(LM)_i}\cdot\varepsilon_{8i} + \\
+l_{AL}\sin\varphi_{(AL)_i}\cdot\omega_{1i}^2 - l_{AL}\cos\varphi_{(AL)_i}\cdot\varepsilon_{1i} = 0 \\
-l_{KH}\cos\varphi_{(KH)_i}\cdot\omega_{7i}^2 - l_{KH}\sin\varphi_{(KH)_i}\cdot\varepsilon_{7i} - l_{HI}\cos\varphi_{(HI)_i}\cdot\omega_{6i}^2 - \\
-l_{HI}\sin\varphi_{(HI)_i}\cdot\varepsilon_{6i} + l_{EI}\cos\varphi_{(EI)_i}\cdot\omega_{4i}^2 + l_{EI}\sin\varphi_{(EI)_i}\cdot\varepsilon_{4i} + \\
+l_{DE}\cos\varphi_{(DE)_i}\cdot\omega_{3i}^2 + l_{DE}\sin\varphi_{(DE)_i}\cdot\varepsilon_{3i} = 0 \\
-l_{KH}\sin\varphi_{(KH)_i}\cdot\omega_{7i}^2 + l_{KH}\cos\varphi_{(KH)_i}\cdot\varepsilon_{7i} - l_{HI}\sin\varphi_{(HI)_i}\cdot\omega_{6i}^2 + \\
+l_{HI}\cos\varphi_{(HI)_i}\cdot\varepsilon_{6i} + l_{EI}\sin\varphi_{(EI)_i}\cdot\omega_{4i}^2 - l_{EI}\cos\varphi_{(EI)_i}\cdot\varepsilon_{4i} + \\
+l_{DE}\sin\varphi_{(DE)_i}\cdot\omega_{3i}^2 - l_{DE}\cos\varphi_{(DE)_i}\cdot\varepsilon_{3i} = 0 \\
-l_{KH}\cos\varphi_{(KH)_i}\cdot\omega_{7i}^2 - l_{KH}\sin\varphi_{(KH)_i}\cdot\varepsilon_{7i} - l_{HN}\cos\varphi_{(HN)_i}\cdot\omega_{6i}^2 - \\
-l_{HN}\sin\varphi_{(HN)_i}\cdot\varepsilon_{6i} + l_{QN}\cos\varphi_{(QN)_i}\cdot\omega_{9i}^2 + l_{QN}\sin\varphi_{(QN)_i}\cdot\varepsilon_{9i} + \\
+l_{DQ}\cos\varphi_{(DQ)_i}\cdot\omega_{3i}^2 + l_{DQ}\sin\varphi_{(DQ)_i}\cdot\varepsilon_{3i} = 0 \\
-l_{KH}\sin\varphi_{(KH)_i}\cdot\omega_{7i}^2 + l_{KH}\cos\varphi_{(KH)_i}\cdot\varepsilon_{7i} - l_{HN}\sin\varphi_{(HN)_i}\cdot\omega_{6i}^2 + \\
+l_{HN}\cos\varphi_{(HN)_i}\cdot\varepsilon_{6i} + l_{QN}\sin\varphi_{(QN)_i}\cdot\omega_{9i}^2 - l_{QN}\cos\varphi_{(QN)_i}\cdot\varepsilon_{9i} \\
+l_{DQ}\sin\varphi_{(DQ)_i}\cdot\omega_{3i}^2 - l_{DQ}\cos\varphi_{(DQ)_i}\cdot\varepsilon_{3i} = 0
\end{array}\right\} \qquad (84)$$

From the systems of linear equations (83), let determine the angular accelerations of the manipulator links by the equation

$$\bar{Y} = \mathbf{A}^{-1} \cdot \bar{C}, \qquad (85)$$

where

$$\bar{Y}=\begin{bmatrix}\varepsilon_{5i}\\ \varepsilon_{2i}\\ \varepsilon_{1i}\\ \varepsilon_{8i}\\ \varepsilon_{6i}\\ \varepsilon_{4i}\\ \varepsilon_{3i}\\ \varepsilon_{9i}\end{bmatrix},\bar{C}=\begin{bmatrix}(X_{H_i}-X_K)\omega_{7i}^2+(Y_{H_i}-Y_K)\varepsilon_{7i}+(X_{G_i}-X_{H_i})\omega_{5i}^2+\\ +(X_{B_i}-X_{G_i})\omega_{2i}^2+(X_A-X_{B_i})\omega_{1i}^2\\ (Y_{H_i}-Y_K)\omega_{7i}^2+(X_K-X_{H_i})\varepsilon_{7i}+(Y_{G_i}-Y_{H_i})\omega_{5i}^2+\\ +(Y_{B_i}-Y_{G_i})\omega_{2i}^2+(Y_A-Y_{B_i})\omega_{1i}^2\\ (X_{H_i}-X_K)\omega_{7i}^2+(Y_{H_i}-Y_K)\varepsilon_{7i}+(X_{M_i}-X_{H_i})\omega_{5i}^2+\\ +(X_{L_i}-X_{M_i})\omega_{8i}^2+(X_A-X_{L_i})\omega_{1i}^2\\ (Y_{H_i}-Y_K)\omega_{7i}^2+(X_K-X_{H_i})\varepsilon_{7i}+(Y_{M_i}-Y_{H_i})\omega_{5i}^2+\\ +(Y_{L_i}-Y_{M_i})\omega_{8i}^2+(Y_A-Y_{L_i})\omega_{1i}^2\\ (X_{H_i}-X_K)\omega_{7i}^2+(Y_{H_i}-Y_K)\varepsilon_{7i}+(X_{I_i}-X_{H_i})\omega_{6i}^2+\\ +(X_{E_i}-X_{I_i})\omega_{4i}^2+(X_D-X_{E_i})\omega_{3i}^2\\ (Y_{H_i}-Y_K)\omega_{7i}^2+(X_K-X_{H_i})\varepsilon_{7i}+(Y_{I_i}-Y_{H_i})\omega_{6i}^2+\\ +(Y_{E_i}-Y_{I_i})\omega_{4i}^2+(Y_D-Y_{E_i})\omega_{3i}^2\\ (X_{H_i}-X_K)\omega_{7i}^2+(Y_{H_i}-Y_K)\varepsilon_{7i}+(X_{N_i}-X_{H_i})\omega_{6i}^2+\\ +(X_{Q_i}-X_{N_i})\omega_{9i}^2+(X_D-X_{Q_i})\omega_{3i}^2\\ (Y_{H_i}-Y_K)\omega_{7i}^2+(X_K-X_{H_i})\varepsilon_{7i}+(Y_{N_i}-Y_{H_i})\omega_{6i}^2+\\ +(Y_{Q_i}-Y_{N_i})\omega_{9i}^2+(Y_D-Y_{Q_i})\omega_{3i}^2\end{bmatrix} \tag{86}$$

Angular velocities and accelerations of the manipulator links can also be determined by independent solution of the first and second four equations of systems (76), i.e. a separate consideration of groups IV (1,2,5,8) and IV (3,4,6,9).

Accelerations of the output points *C* and *F* are determined by differentiating the equations (80) and (81) with respect to time

$$\begin{bmatrix}a_{C_i}^X\\ a_{C_i}^Y\end{bmatrix}=\begin{bmatrix}a_{B_i}^X\\ a_{B_i}^Y\end{bmatrix}+l_{BC}\omega_{2i}^2\begin{bmatrix}-\cos\varphi_{2i}\\ -\sin\varphi_{2i}\end{bmatrix}+l_{BC}\varepsilon_{2i}\begin{bmatrix}-\sin\varphi_{2i}\\ \cos\varphi_{2i}\end{bmatrix}, \tag{87}$$

$$\begin{bmatrix}a_{F_i}^X\\ a_{F_i}^Y\end{bmatrix}=\begin{bmatrix}a_{E_i}^X\\ a_{E_i}^Y\end{bmatrix}+l_{EF}\omega_{4i}^2\begin{bmatrix}-\cos\varphi_{4i}\\ -\sin\varphi_{4i}\end{bmatrix}+l_{EF}\varepsilon_{4i}\begin{bmatrix}-\sin\varphi_{4i}\\ \cos\varphi_{4i}\end{bmatrix}, \tag{88}$$

where the accelerations of the joints *B* and *E* are determined by differentiating the equations (82) and (83) with respect to time

$$\begin{bmatrix}a_{B_i}^X\\ a_{B_i}^Y\end{bmatrix}=l_{AB}\omega_{1i}^2\begin{bmatrix}-\cos\varphi_{1i}\\ -\sin\varphi_{1i}\end{bmatrix}+l_{AB}\varepsilon_{1i}\begin{bmatrix}-\sin\varphi_{1i}\\ \cos\varphi_{1i}\end{bmatrix}, \tag{89}$$

$$\begin{bmatrix}a_{E_i}^X\\ a_{E_i}^Y\end{bmatrix}=l_{DE}\omega_{3i}^2\begin{bmatrix}-\cos\varphi_{3i}\\ -\sin\varphi_{3i}\end{bmatrix}+l_{DE}\varepsilon_{3i}\begin{bmatrix}-\sin\varphi_{3i}\\ \cos\varphi_{3i}\end{bmatrix}. \tag{90}$$

VI. CONCLUSION

The methods of structurally parametric optimization and position analysis of a novel RoboMech class PM with two end-effectors are developed. The investigated PM is formed by connecting the two moving output objects with the fixed base by two passive, one active and two negative CKC. The active and negative CKC impose the geometrical constraints on the motions of the output objects, and they work with certain geometrical parameters of links. Geometrical parameters of the active and negative CKC links are determined on the base of Chebyshev and least-square approximations, and position analysis is solved on the base of conditional generalized coordinates method.

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