Global Stability in a Delayed Ratio-dependent Predator-prey System with Feedback Controls*

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***Abstract*—The aim of this paper is to study the dynamical behavior of solutions for a class of 3-species ratio-dependent predator-prey model with feedback controls and delays. Firstly, the attractivity of positive solutions for the system is discussed by using the Lyapunov stability theory. Secondly, some conditions are established to guarantee the globally asymptotic stability of the unique positive periodic solution for the corresponding periodic system by constructing a suitable Poincare mapping and using the fixed point theory. Finally, some numerical simulations are provided to verify the results obtained in this paper.**

***Index Terms*—Ratio-dependent, Feedback control, Time delays, Global attractive, Periodic solution**

1. Introduction

The attractiveness and periodicity of biological systems are very important in the study of population dynamics. The traditional Lotka-Volterra predator-prey model has been studied by many scholars and has been achieved a lot of research results [1-4], but it is assumed that the average growth rate of predators depends only on the population density of the prey population. In recent years, with the deepening on research for ecological population models, especially considering predators which share or compete for food, scholars have found a more realistic and universal predator-prey model, that is, a "rate-dependent" predator-prey model [5-10]. This means that the average rate of predator growth should be a function of the ratio of predator population density to the predator population density. This theory has been proved by some scholars, but relatively few achievements have been made. Arditi and Ginzburg [5] firstly proposed the following ratio-based predator-prey model

$$\begin{cases} x' = x[a-bx] - \dfrac{cxy}{my+x}, \\ y' = y\left[-d + \dfrac{fx}{my+x}\right], \end{cases} \tag{1}$$

which incorporates mutual interference by predators, where $g(x) = cx/(my+x)$ is a Michaelis-Menten type functional response function. System (1) has been studied by many scholars, for detailed introduction and justifications of system (1), we refer the reader to (e.g., see [5, 6, 9, 10]). In 2004, Wang *et al.* [7] studied the following nonautonomous ratio-based predator-prey system with two competing prey predated by one predator

$$\begin{cases} x_1'(t) = x_1(t)[a_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) \\ \qquad - \dfrac{a_{13}(t)x_3(t)}{b_{13}(t)x_3(t) + x_1(t)}], \\ x_2'(t) = x_2(t)[a_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t) \\ \qquad - \dfrac{a_{23}(t)x_3(t)}{b_{23}(t)x_3(t) + x_2(t)}], \\ x_3'(t) = x_3(t)[a_3(t) + \dfrac{a_{31}(t)x_1(t)}{b_{13}(t)x_3(t) + x_1(t)} \\ \qquad + \dfrac{a_{32}(t)x_2(t)}{b_{23}(t)x_3(t) + x_2(t)}], \end{cases} \tag{2}$$

and showed that system (2) is permanent and globally asymptotically stable under some appropriate conditions. For the periodic and almost periodic case, they obtained some conditions for the existence, uniqueness and stability of a positive periodic and almost periodic solution respectively. In the real world, any biological or environmental parameter is naturally affected by temporal fluctuations. Consequently, more realistic models of population interactions should be

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taken into account the effect of time delays. See for example [11-14] and the references cited therein. In 2001, Xu and Chen [11] studied the following delayed nonautonomous three-species predator-prey Lotka-Volterra system

$$\begin{cases} \dot{x}_1 = x_1(t)[r_1(t) - a_{11}(t)x_1(t-\tau_{11}) \\ \quad -a_{12}(t)x_2(t-\tau_{12})], \\ \dot{x}_2 = x_2(t)[-r_2(t) + a_{21}(t)x_1(t-\tau_{21}) \\ \quad -a_{22}(t)x_2(t-\tau_{22}) - a_{23}(t)x_3(t-\tau_{23})], \\ \dot{x}_3 = x_3(t)[-r_3(t) + a_{32}(t)x_2(t-\tau_{32}) \\ \quad -a_{33}(t)x_3(t-\tau_{33})], \end{cases} \tag{3}$$

and proved that the system (3) is uniformly persistent under appropriate conditions. Moreover, by constructing a suitable Lyapunov functional, sufficient conditions are derived for the global stability of the system (3). In 2005, Muhammadhaji *et al.* in [14] discussed the following nonautonomous three-species Lotka-Volterra competitive-mutualism systems with time delays

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1(t) - a_{11}^1(t)x_1(t-\tau) \\ \quad -a_{11}^2(t)x_1(t-2\tau) - a_{12}(t)x_2(t-2\tau) \\ \quad +a_{13}(t)x_3(t-\tau)], \\ \dot{x}_2(t) = x_2(t)[r_2(t) - a_{21}(t)x_1(t-2\tau) \\ \quad -a_{22}^1(t)x_2(t-\tau) - a_{22}^2(t)x_2(t-2\tau) \\ \quad +a_{23}(t)x_3(t-\tau)], \\ \dot{x}_3(t) = x_3(t)[r_3(t) + a_{31}(t)x_1(t-\tau) \\ \quad +a_{32}(t)x_2(t-\tau) - a_{33}^1(t)x_3(t) \\ \quad -a_{33}^2(t)x_3(t-\tau)], \end{cases} \tag{4}$$

and obtained some sufficient conditions on the boundedness, permanence and global attractivity for the system (4). Moreover, Xu *et al.* in [15] studied the following three-species predator-prey food-chain model both with time delays and ratio-dependent functions

$$\begin{cases} \dot{x}_1 = x_1(t)[a_1 - a_{11}x_1(t-\tau_{11}) - \dfrac{a_{12}x_2(t)}{m_1 + x_1(t)}], \\ \dot{x}_2 = x_2(t)[-a_2 + \dfrac{a_{21}x_1(t-\tau_{21})}{m_1 + x_1(t-\tau_{21})} \\ \quad -a_{22}x_2(t-\tau_{22}) - \dfrac{a_{23}x_3(t)}{m_2 + x_2(t)}], \\ \dot{x}_3 = x_3(t)[-a_3 + \dfrac{a_{32}x_2(t-\tau_{32})}{m_2 + x_2(t-\tau_{32})} - a_{33}x_3(t-\tau_{33})], \end{cases} \tag{5}$$

and proved that the system (5) is uniformly persistent under some appropriate conditions. In addition, some sufficient conditions are derived for the global asymptotic stability of the positive equilibrium of the system (5) by means of constructing suitable Lyapunov functional.

On the other hand, in the real ecosystem, the biological population is often affected by some unpredictable factors that affect the biological parameters of the population, such as birth rate, mortality and other factors. Therefore, it is necessary to add feedback control to the model to study the influence of feedback control on the persistence and global stability of the system, and provide theoretical support for the protection of ecological population. More results see for example [16-20] and references cited therein. In 2003, K. Gopalsamy *et al.* in [16] studied the following two species competition system with feedback controls

$$\begin{cases} \dfrac{dx_1(t)}{dt} = x_1(t)[b_1 - a_{11}x_1(t) - a_{12}x_2(t) \\ \quad -\alpha_1 u_1(t)], \\ \dfrac{dx_2(t)}{dt} = x_2(t)[b_2 - a_{21}x_1(t) - a_{22}x_2(t) \\ \quad -\alpha_2 u_2(t)], \\ \dfrac{du_1(t)}{dt} = -\eta_1 u_1(t) + a_1 x_1(t), \\ \dfrac{du_2(t)}{dt} = -\eta_2 u_2(t) + a_2 x_2(t), \end{cases} \tag{6}$$

and obtained some conditions for the existence of a global attracting positive equilibrium point of the system. In [18], Nie *et al.* consider the following non-autonomous predator-prey Lotka-Volterra system with feedback controls

$$\begin{cases} \dot{x}_1(t) = x_1(t)[b_1(t) - a_{11}(t)x_1(t) \\ \quad -a_{12}(t)x_2(t) + c_1(t)u_1(t)], \\ \dot{x}_2(t) = x_2(t)[-b_2(t) + a_{21}(t)x_1(t) \\ \quad -a_{22}(t)x_2(t) - c_2(t)u_2(t)], \\ \dot{u}_1(t) = f_1(t) - e_1(t)u_1(t) - d_1(t)x_1(t), \\ \dot{u}_2(t) = -e_2(t)u_2(t) + d_2(t)x_2(t), \end{cases} \tag{7}$$

and studied whether the feedback controls have an influence on the permanence of a positive solution of the general non-autonomous predator-prey Lotka-Volterra type systems, at the same time established the general criteria on the permanence of system (7), which is independent of some feedback controls. In addition, by constructing a suitable Lyapunov function, some sufficient conditions are obtained for the global stability of any positive solution to system (7). In 2015, Xu and Chen [19] investigated the following Lotka-Volterra cooperative system with time delays and feedback controls

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1(t) - a_1(t)x_1(t) - a_{11}(t)x_1(t-\tau) \\ \qquad +a_{12}(t)x_2(t-\tau) - b_1(t)u_1(t-\sigma_1)], \\ \dot{x}_2(t) = x_2(t)[r_2(t) - a_2(t)x_2(t) + a_{21}(t)x_1(t) \\ \qquad -a_{22}(t)x_2(t-\tau) - b_2(t)u_2(t-\sigma_2)], \\ \dot{u}_1(t) = -c_1(t)u_1(t) + d_1(t)x_1(t-\eta_1), \\ \dot{u}_2(t) = -c_2(t)u_2(t) + d_2(t)x_2(t-\eta_2), \end{cases} \tag{8}$$

and obtained some new sufficient conditions for the permanence of system (8) by applying new inequalities. Their results showed that the feedback control variables had no influence on the permanence of the system (8). In 2016, Wang *et al.* [20] studied a ratio-dependent Lotka-Volterra predator-prey model with feedback control

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) \\ \qquad -\dfrac{a_{13}(t)x_3(t)}{a_{13}(t)x_3(t)+x_1(t)} - d_1(t)u_1(t)], \\ \dot{x}_2(t) = x_2(t)[r_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t) \\ \qquad -\dfrac{a_{23}(t)x_3(t)}{a_{23}(t)x_3(t)+x_2(t)} - d_2(t)u_2(t)], \\ \dot{x}_3(t) = x_3(t)[-r_3(t) + \dfrac{a_{31}(t)x_1(t)}{a_{13}(t)x_3(t)+x_1(t)} \\ \qquad +\dfrac{a_{32}(t)x_2(t)}{a_{23}(t)x_3(t)+x_2(t)} + d_3(t)u_3(t)], \\ \dot{u}_1(t) = e_1(t) - f_1(t)u_1(t) + q_1(t)x_1(t), \\ \dot{u}_2(t) = e_2(t) - f_2(t)u_2(t) + q_2(t)x_2(t), \\ \dot{u}_3(t) = e_3(t) - f_3(t)u_3(t) - q_3(t)x_3(t), \end{cases} \tag{9}$$

where $x_i(t)\ (i=1,2,3)$ denote the density of the *i*-th species, $r_i(t)\ (i=1,2)$ denote the intrinsic growth rate of the prey species, $r_3(t)$ are the death rate of the predators, $a_{11}(t), a_{21}(t)$ denote the internal competitive coefficient of the first and second species respectively. Authors obtained some sufficient conditions which guaranteed the global attractive of positive solution for the predator-prey model by constructing suitable Lyapunov function and developing some new analysis techniques. For more recent related work, please also refer to the references [21-24].

However, to the best of the authors knowledge, until today, there are no scholars still consider the general non-autonomous Lotka-Volterra system with time delays, ratio-dependent and feedback controls. Therefore, based on system (5) and motivated by the above works, in this paper, we propose and investigate the following 3-species multi-delay ratio-dependent biological chain predator-prey model with feedback controls

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1(t) - a_{11}(t)x_1(t) - \dfrac{a_{12}(t)x_2(t)}{b_{12}(t)x_2(t)+x_1(t)} \\ \qquad -\dfrac{a_{13}(t)x_3(t)}{b_{13}(t)x_3(t)+x_1(t)} - d_1(t)u_1(t)], \\ \dot{x}_2(t) = x_2(t)[-r_2(t) + \dfrac{a_{21}(t)x_1(t-\tau_1)}{b_{12}(t)x_2(t)+x_1(t-\tau_1)} \\ \qquad -\dfrac{a_{23}^1(t)x_3(t)}{b_{23}(t)x_3(t)+x_2(t)} - a_{23}^2(t)x_3(t-\tau_2) \\ \qquad +d_2(t)u_2(t)], \\ \dot{x}_3(t) = x_3(t)[-r_3(t) + \dfrac{a_{31}(t)x_1(t-\tau_3)}{b_{13}(t)x_3(t)+x_1(t-\tau_3)} \\ \qquad +\dfrac{a_{32}^1(t)x_2(t-\tau_3)}{b_{23}(t)x_3(t)+x_2(t-\tau_3)} - a_{32}^2(t)x_2(t-\tau_4) \\ \qquad +d_3(t)u_3(t)], \\ \dot{u}_1(t) = e_1(t) - f_1(t)u_1(t) + q_1(t)x_1(t), \\ \dot{u}_2(t) = e_2(t) - f_2(t)u_2(t) - q_2(t)x_2(t), \\ \dot{u}_3(t) = e_3(t) - f_3(t)u_3(t) - q_3(t)x_3(t), \end{cases} \tag{10}$$

where $x_i(t)\ (i=1,2,3)$ denote the density of the *i*-th species, $r_1(t)$ denote the intrinsic growth rate of the prey species, $r_i(t)\ (i=2,3)$ are the death rate of the predators, $a_{11}(t)$ denote the internal competitive coefficient of the first species, $a_{12}(t), a_{13}(t), a_{23}^1(t)$ shows the ratio of prey by predator, $a_{21}(t), a_{31}(t), a_{32}^1(t)$ represents the nutrient absorption ratio of predator after predation, $a_{23}^2(t), a_{32}^2(t)$ are the competitive coefficient of species $x_2(t)$ and $x_3(t)$, $u_i(t)\ (i=1,2,3)$ are the feedback control terms, and $\tau_1, \tau_2, \tau_3, \tau_4 > 0$ are constant delays. All of the coefficients in the model are continuous and positive bounded functions defined on $[0, +\infty)$. This system describes the predator-prey relationships among three species in which the second species prey on the first species, the third species prey on the second and the first species.

Due to the biological interpretation of the system (10), it is reasonable to consider only positive solution of (10), in other words, to take admissible initial conditions

$$\begin{aligned} &x_i(t) = \phi_i(t) \ge 0,\ t \in [-\tau, 0), \\ &\phi_i(0) > 0,\ u_i(0) = u_{i0} > 0,\ i = 1,2,3. \end{aligned} \tag{11}$$

where $\tau = \max\{\tau_1, \tau_2, \tau_3, \tau_4\}$. Obviously, the solutions of system (10) with the initial values (11) are positive for all $t \ge 0$.

The structure of this paper is as follows: In Section 2, the conditions ensuring the permanence of the system (10) are provided. In Section 3, some sufficient conditions are derived to guarantee the globally attractive of positive solution for the system (10) relying on a non-negative Lyapunov function . In Section 4, the global asymptotic stability of the unique positive periodic solution for the corresponding periodic system are discussed by constructing a suitable Poincare mapping and using the fixed point theory. Last but not least, some numerical simulations are provided to verify the results obtained in this paper.

2. Preliminaries and notations

To discuss the global attractivity of positive solution of the system (10), some definitions and lemmas are firstly introduced. For a continuous bounded function $g(t)$ defined on $[0,+\infty)$, we let

$$g^m = \sup\{g(t) \mid 0 \le t < \infty\}, g^l = \inf\{g(t) \mid 0 \le t < \infty\}.$$

Definition 1. System (10) is called permanent, if there exist positive constants $M_i, N_i, m_i, n_i (i = 1,2,3)$ and T such that $m_i \le x_i(t) \le M_i, n_i \le u_i(t) \le N_i$, $(i = 1,2,3)$ for any positive solution $Z(t) = (x_1(t), x_2(t), x_3(t), u_1(t), u_2(t), u_3(t))$ of system (10) as $t > T$.

As a direct corollary of **Lemma 2.1** of Chen [25], we have

Lemma 1. If $a > 0, b > 0$ and $\dot{x} \ge b - ax$, when $t \ge 0$ and $x(0) > 0$, we have $\liminf_{t\to+\infty} x(t) \ge b/a$.

If $a > 0, b > 0$ and $\dot{x} \le b - ax$, when $t \ge 0$ and $x(0) > 0$, we have $\limsup_{t\to+\infty} x(t) \le b/a$.

As a direct corollary of **Lemma 2.2** of Chen [25], we have

Lemma 2. If $a > 0, b > 0$ and $\dot{x} \ge x(b - ax)$, when $t \ge 0$ and $x(0) > 0$, we have $\liminf_{t\to+\infty} x(t) \ge b/a$.

If $a > 0, b > 0$ and $\dot{x} \le x(b - ax)$, when $t \ge 0$ and $x(0) > 0$, we have $\limsup_{t\to+\infty} x(t) \le b/a$.

For the system (10), we let

$$N_1 = \frac{e_1^m + q_1^m M_1}{f_1^l}, N_2 = \frac{e_2^m}{f_2^l},$$

$$N_3 = \frac{e_3^m}{f_3^l}, M_1 = \frac{r_1^m}{a_{11}^l},$$

$$M_2 = \frac{a_{31}^m + a_{32}^{1m} + d_3^m N_3 - r_3^l}{a_{32}^{2l}} \times \exp\left[\left(a_{21}^m + d_2^m N_2 - r_2^l\right)\tau_4\right],$$

$$M_3 = \frac{a_{21}^m + d_2^m N_2 - r_2^l}{a_{23}^{2l}} \times \exp[(a_{31}^m + a_{32}^{1m} + d_3^m N_3 - r_3^l)\tau_2],$$

$$m_1 = \frac{r_1^l - a_{12}^m / b_{12}^l - a_{13}^m / b_{13}^l - d_1^m N_1}{a_{11}^m},$$

$$n_1 = \frac{e_1^l + q_1^l m_1}{f_1^m}, \quad n_2 = \frac{e_2^l - q_2^m M_2}{f_2^m},$$

$$n_3 = \frac{e_3^l - q_3^m M_3}{f_3^m},$$

$$m_2 = \frac{d_3^l n_3 - r_3^m}{a_{32}^{2m}} \times \exp[(d_2^l n_2 - r_2^m - a_{23}^{1m} / b_{23}^l - a_{23}^{2m} M_3)\tau_4],$$

$$m_3 = \frac{d_2^l n_2 - r_2^m - a_{23}^{1m} / b_{23}^l}{a_{23}^{2m}} \times \exp[(d_3^l n_3 - r_3^m - a_{32}^{2m} M_2)\tau_2].$$

Lemma 3 [26]**.** Assume that the system (10) satisfies the initial conditions (11) and following conditions

(H_1) $r_3^l < a_{31}^m + a_{32}^{1m} + d_3^m N_3$,

(H_2) $r_2^l < a_{21}^m + d_2^m N_2$,

(H_3) $r_1^l > a_{12}^m / b_{12}^l + a_{13}^m / b_{13}^l + d_1^m N_1$,

(H_4) $e_2^l > q_2^m M_2$, (H_5) $e_3^l > q_3^m M_3$,

(H_6) $r_3^m < d_3^l n_3$, (H_7) $d_2^l n_2 > r_2^m + a_{23}^{1m} / b_{23}^l$,

then the system (10) is permanent.

3. Global Attractivity

In this section, we shall prove that the system (10) is global attractivity. To get the sufficient conditions for global attractivity of system (10), we give firstly the following definition and Lemma.

Definition 2. System (10) is said to be globally attractive, if there exists a positive solution $X(t) = (x_1(t), x_2(t), x_3(t), u_1(t), u_2(t), u_3(t))$ of the system (10) such that

$$\lim_{t\to+\infty} |x_i(t) - y_i(t)| = 0,$$
$$\lim_{t\to+\infty} |u_i(t) - v_i(t)| = 0, \ (i = 1,2,3),$$

for any other positive solution $Y(t) = (y_1(t), y_2(t), y_3(t), v_1(t), v_2(t), v_3(t))$ of the system (10).

Lemma 4 (see [27]**, Lemma 8.2).** If the function $f(t): R^+ \to R$ is uniformly continuous, and the limit $\lim_{t\to+\infty} \int_0^t f(s)ds$ exists and is finite, then $\lim_{t\to+\infty} f(t) = 0$.

Theorem 1. In addition to $(H_1)-(H_7)$, assume further that system (10) satisfies (H_8) $A_i>0,\ B_i>0,\ i=1,2,3,$ where

$$A_1=a_{11}^l-\frac{a_{12}^m M_2}{(b_{12}^l m_2+m_1)^2}-\frac{a_{13}^m M_3}{(b_{13}^l m_3+m_1)^2}-\frac{a_{21}^m b_{12}^m M_2}{(b_{12}^l m_2+m_1)^2}-\frac{a_{31}^m b_{13}^m M_3}{(b_{13}^l m_3+m_1)^2}-q_1^m,$$

$$A_2=\frac{a_{21}^l b_{12}^l m_1}{(b_{12}^m M_2+M_1)^2}-\frac{a_{23}^{1m} M_3}{(b_{23}^l m_3+m_2)^2}-\frac{a_{12}^m M_1}{(b_{12}^l m_2+m_1)^2}-\frac{a_{32}^{1m} b_{23}^m M_3}{(b_{23}^l m_3+m_2)^2}-a_{32}^{2m}-q_2^m,$$

$$A_3=\frac{a_{31}^l b_{13}^l m_1}{(b_{13}^m M_3+M_1)^2}+\frac{a_{32}^{1l} b_{23}^l m_2}{(b_{23}^m M_3+M_2)^2}-\frac{a_{13}^m M_1}{(b_{13}^l m_3+m_1)^2}-\frac{a_{23}^{1m} M_2}{(b_{23}^l m_3+m_2)^2}-a_{23}^{2m}-q_3^m,$$

$$B_1=f_1^l-d_1^m,\ B_2=f_2^l-d_2^m,\ B_3=f_3^l-d_3^m.$$

Then system (10) is globally attractive.

Proof. Suppose that $(x_1(t),x_2(t),x_3(t),u_1(t),u_2(t),u_3(t))$ and $(y_1(t),y_2(t),y_3(t),v_1(t),v_2(t),v_3(t))$ are any two positive solutions of the system (10). Then from **Lemma 3**, there exist positive constants M_i,N_i,m_i,n_i $(i=1,2,3)$ and T, such that $m_i\le x_i(t),y_i(t)\le M_i$, $n_i\le u_i(t),v_i(t)\le N_i$, $i=1,2,3,$ for all $t\ge T$.

We define

$$V_1(t)=\sum_{i=1}^{3}[|\ln x_i(t)-\ln y_i(t)|+|u_i(t)-v_i(t)|].$$

Calculating the upper right derivative of $V_1(t)$ along the solution of system (10), we get

$$\begin{aligned}
&D^+V_1(t)=\sum_{i=1}^{3}D^+[|\ln x_i(t)-\ln y_i(t)|+|u_i(t)-v_i(t)|]\\
&\le \operatorname{sgn}\{x_1(t)-y_1(t)\}[-a_{11}(t)(x_1(t)-y_1(t))\\
&-\frac{a_{12}(t)y_1(t)(x_2(t)-y_2(t))-a_{12}(t)y_2(t)(x_1(t)-y_1(t))}{(b_{12}(t)x_2(t)+x_1(t))(b_{12}(t)y_2(t)+y_1(t))}\\
&-\frac{a_{13}(t)y_1(t)(x_3(t)-y_3(t))-a_{13}(t)y_3(t)(x_1(t)-y_1(t))}{(b_{13}(t)x_3(t)+x_1(t))(b_{13}(t)y_3(t)+y_1(t))}\\
&-d_1(t)(u_1(t)-v_1(t))]+\operatorname{sgn}\{x_2(t)-y_2(t)\}\\
&\times[\frac{a_{21}(t)b_{12}(t)y_2(t)(x_1(t-\tau_1)-y_1(t-\tau_1))}{(b_{12}(t)x_2(t)+x_1(t-\tau_1))(b_{12}(t)y_2(t)+y_1(t-\tau_1))}\\
&-\frac{a_{21}(t)b_{12}(t)y_1(t-\tau_1)(x_2(t)-y_2(t))}{(b_{12}(t)x_2(t)+x_1(t-\tau_1))(b_{12}(t)y_2(t)+y_1(t-\tau_1))}\\
&-\frac{a_{23}^1(t)y_2(t)(x_3(t)-y_3(t))-a_{23}^1(t)y_3(t)(x_2(t)-y_2(t))}{(b_{23}(t)x_3(t)+x_2(t))(b_{23}(t)y_3(t)+y_2(t))}\\
&-a_{23}^2(t)(x_3(t-\tau_2)-y_3(t-\tau_2))+d_2(t)(u_2(t)-v_2(t))]\\
&+\operatorname{sgn}\{x_3(t)-y_3(t)\}\\
&\times[\frac{a_{31}(t)b_{13}(t)y_3(t)(x_1(t-\tau_3)-y_1(t-\tau_3))}{(b_{13}(t)x_3(t)+x_1(t-\tau_3))(b_{13}(t)y_3(t)+y_1(t-\tau_3))}\\
&-\frac{a_{31}(t)b_{13}(t)y_1(t-\tau_3)(x_3(t)-y_3(t))}{(b_{13}(t)x_3(t)+x_1(t-\tau_3))(b_{13}(t)y_3(t)+y_1(t-\tau_3))}\\
&+\frac{a_{32}^1(t)b_{23}(t)y_3(t)(x_2(t-\tau_3)-y_2(t-\tau_3))}{(b_{23}(t)x_3(t)+x_2(t-\tau_3))(b_{23}(t)y_3(t)+y_2(t-\tau_3))}\\
&-\frac{a_{32}^1(t)b_{23}(t)y_2(t-\tau_3)(x_3(t)-y_3(t))}{(b_{23}(t)x_3(t)+x_2(t-\tau_3))(b_{23}(t)y_3(t)+y_2(t-\tau_3))}\\
&-a_{32}^2(t)(x_2(t-\tau_4)-y_2(t-\tau_4))+d_3(t)(u_3(t)-y_3(t))]\\
&+\operatorname{sgn}\{u_1(t)-v_1(t)\}[-f_1(t)(u_1(t)-v_1(t))\\
&+q_1(t)(x_1(t)-y_1(t))]\\
&+\operatorname{sgn}\{u_2(t)-v_2(t)\}[-f_2(t)(u_2(t)-v_2(t))\\
&-q_2(t)(x_2(t)-y_2(t))]\\
&+\operatorname{sgn}\{u_3(t)-v_3(t)\}[-f_3(t)(u_3(t)-v_3(t))\\
&-q_3(t)(x_3(t)-y_3(t))]\\
&\le |x_1(t)-y_1(t)|[-a_{11}(t)\\
&+\frac{a_{12}(t)y_2(t)}{(b_{12}(t)x_2(t)+x_1(t))(b_{12}(t)y_2(t)+y_1(t))}\\
&+\frac{a_{13}(t)y_3(t)}{(b_{13}(t)x_3(t)+x_1(t))(b_{13}(t)y_3(t)+y_1(t))}\\
&+q_1(t)]+|x_2(t)-y_2(t)|\times\\
&[-\frac{a_{21}(t)b_{12}(t)y_1(t-\tau_1)}{(b_{12}(t)x_2(t)+x_1(t-\tau_1))(b_{12}(t)y_2(t)+y_1(t-\tau_1))}\\
&+\frac{a_{23}^1(t)y_3(t)}{(b_{23}(t)x_3(t)+x_2(t))(b_{23}(t)y_3(t)+y_2(t))}\\
&+\frac{a_{12}(t)y_1(t)}{(b_{12}(t)x_2(t)+x_1(t))(b_{12}(t)y_2(t)+y_1(t))}\\
&+q_2(t)]+|x_3(t)-y_3(t)|\\
&\times[-\frac{a_{31}(t)b_{13}(t)y_1(t-\tau_3)}{(b_{13}(t)x_3(t)+x_1(t-\tau_3))(b_{13}(t)y_3(t)+y_1(t-\tau_3))}\\
&-\frac{a_{32}^1(t)b_{23}(t)y_2(t-\tau_3)}{(b_{23}(t)x_3(t)+x_2(t-\tau_3))(b_{23}(t)y_3(t)+y_2(t-\tau_3))}\\
&+\frac{a_{13}(t)y_1(t)}{(b_{13}(t)x_3(t)+x_1(t))(b_{13}(t)y_3(t)+y_1(t))}\\
&+\frac{a_{23}^1(t)y_2(t)}{(b_{23}(t)x_3(t)+x_2(t))(b_{23}(t)y_3(t)+y_2(t))}
\end{aligned}$$

$$
\begin{aligned}
&+q_3(t)]+|x_1(t-\tau_1)-y_1(t-\tau_1)| \\
&\times[\frac{a_{21}(t)b_{12}(t)y_2(t)}{(b_{12}(t)x_2(t)+x_1(t-\tau_1))(b_{12}(t)y_2(t)+y_1(t-\tau_1))}] \\
&+|x_1(t-\tau_3)-y_1(t-\tau_3)| \\
&\times[\frac{a_{31}(t)b_{13}(t)y_3(t)}{(b_{13}(t)x_3(t)+x_1(t-\tau_3))(b_{13}(t)y_3(t)+y_1(t-\tau_3))}] \\
&+|x_2(t-\tau_3)-y_2(t-\tau_3)| \\
&\times[\frac{a_{32}^1(t)b_{23}(t)y_3(t)}{(b_{23}(t)x_3(t)+x_2(t-\tau_3))(b_{23}(t)y_3(t)+y_2(t-\tau_3))}] \\
&+|x_2(t-\tau_4)-y_2(t-\tau_4)|a_{32}^2(t) \\
&+|x_3(t-\tau_2)-y_3(t-\tau_2)|a_{23}^2(t) \\
&+|u_1(t)-v_1(t)|[-f_1(t)+d_1(t)] \\
&+|u_2(t)-v_2(t)|[-f_2(t)+d_2(t)] \\
&+|u_3(t)-v_3(t)|[-f_3(t)+d_3(t)] \\
&\le |x_1(t)-y_1(t)|[-a_{11}^l+\frac{a_{12}^m M_2}{(b_{12}^l m_2+m_1)^2} \\
&+\frac{a_{13}^m M_3}{(b_{13}^l m_3+m_1)^2}+q_1^m] \\
&+|x_2(t)-y_2(t)|[-\frac{a_{21}^l b_{12}^l m_1}{(b_{12}^m M_2+M_1)^2}+\frac{a_{23}^{1m} M_3}{(b_{23}^l m_3+m_2)^2} \\
&+\frac{a_{12}^m M_1}{(b_{12}^l m_2+m_1)^2}+q_2^m] \\
&+|x_3(t)-y_3(t)|[-\frac{a_{31}^l b_{13}^l m_1}{(b_{13}^m M_3+M_1)^2}-\frac{a_{32}^{1l} b_{23}^l m_2}{(b_{23}^m M_3+M_2)^2} \\
&+\frac{a_{13}^m M_1}{(b_{13}^l m_3+m_1)^2}+\frac{a_{23}^{1m} M_2}{(b_{23}^l m_3+m_2)^2}+q_3^m] \\
&+|x_1(t-\tau_1)-y_1(t-\tau_1)|[\frac{a_{21}^m b_{12}^m M_2}{(b_{12}^l m_2+m_1)^2}] \\
&+|x_1(t-\tau_3)-y_1(t-\tau_3)|[\frac{a_{31}^m b_{13}^m M_3}{(b_{13}^l m_3+m_1)^2}] \\
&+|x_2(t-\tau_3)-y_2(t-\tau_3)|[\frac{a_{32}^{1m} b_{23}^m M_3}{(b_{23}^l m_3+m_2)^2}] \\
&+|x_2(t-\tau_4)-y_2(t-\tau_4)|a_{32}^{2m} \\
&+|x_3(t-\tau_2)-y_3(t-\tau_2)|a_{23}^{2m} \\
&+|u_1(t)-v_1(t)|[-f_1^l+d_1^m] \\
&+|u_2(t)-v_2(t)|[-f_2^l+d_2^m] \\
&+|u_3(t)-v_3(t)|[-f_3^l+d_3^m].
\end{aligned} \tag{12}
$$

Now we define a Lyapunov function as follows

$$V(t)=V_1(t)+V_2(t),$$

where

$$
\begin{aligned}
V_2(t)=&\frac{a_{21}^m b_{12}^m M_2}{(b_{12}^l m_2+m_1)^2}\int_{t-\tau_1}^t |x_1(w)-y_1(w)|dw \\
&+\frac{a_{31}^m b_{13}^m M_3}{(b_{13}^l m_3+m_1)^2}\int_{t-\tau_3}^t |x_1(w)-y_1(w)|dw \\
&+\frac{a_{32}^{1m} b_{23}^m M_3}{(b_{23}^l m_3+m_2)^2}\int_{t-\tau_3}^t |x_2(w)-y_2(w)|dw \\
&+a_{32}^{2m}\int_{t-\tau_4}^t |x_2(w)-y_2(w)|dw \\
&+a_{23}^{2m}\int_{t-\tau_2}^t |x_3(w)-y_3(w)|dw.
\end{aligned}
$$

Calculating the upper right derivative of $V_2(t)$ and from (12), we have

$$
\begin{aligned}
D^+V(t)\le &-|x_1(t)-y_1(t)|[a_{11}^l-\frac{a_{12}^m M_2}{(b_{12}^l m_2+m_1)^2}-q_1^m \\
&-\frac{a_{13}^m M_3}{(b_{13}^l m_3+m_1)^2}-\frac{a_{21}^m b_{12}^m M_2}{(b_{12}^l m_2+m_1)^2}-\frac{a_{31}^m b_{13}^m M_3}{(b_{13}^l m_3+m_1)^2}] \\
&-|x_2(t)-y_2(t)|[\frac{a_{21}^l b_{12}^l m_1}{(b_{12}^m M_2+M_1)^2}-\frac{a_{23}^{1m} M_3}{(b_{23}^l m_3+m_2)^2} \\
&-\frac{a_{12}^m M_1}{(b_{12}^l m_2+m_1)^2}-\frac{a_{32}^{1m} b_{23}^m M_3}{(b_{23}^l m_3+m_2)^2}-a_{32}^{2m}-q_2^m] \\
&-|x_3(t)-y_3(t)|[\frac{a_{31}^l b_{13}^l m_1}{(b_{13}^m M_3+M_1)^2}+\frac{a_{32}^{1l} b_{23}^l m_2}{(b_{23}^m M_3+M_2)^2} \\
&-\frac{a_{13}^m M_1}{(b_{13}^l m_3+m_1)^2}-\frac{a_{23}^{1m} M_2}{(b_{23}^l m_3+m_2)^2}-a_{23}^{2m}-q_3^m] \\
&-|u_1(t)-v_1(t)|[f_1^l-d_1^m]-|u_2(t)-v_2(t)|[f_2^l-d_2^m] \\
&-|u_3(t)-v_3(t)|[f_3^l-d_3^m].
\end{aligned} \tag{13}
$$

Thus, for all $t\ge T+\tau$, we have

$$D^+V(t)\le -\sum_{i=1}^3 (A_i|x_i(t)-y_i(t)|+B_i|u_i(t)-v_i(t)|). \tag{14}$$

In view of the conditions (H_8) of **Theorem 1**, there exist a constant $\alpha>0$ and $T^*>T+\tau$ such that it holds that

$$A_i\ge\alpha>0,\ B_i\ge\alpha>0,\ (i=1,2,3), \tag{15}$$

for all $t\ge T^*$. Integrating from T^* to t on both sides of (14). Then use (15), we have

$$
\begin{aligned}
&V(t)+\alpha\int_{T^*}^t(\sum_{i=1}^3[|x_i(s)-y_i(s)| \\
&+|u_i(s)-v_i(s)|])ds\le V(T^*)<+\infty.
\end{aligned} \tag{16}
$$

Therefore, $V(t)$ bounded on $[T^*,+\infty)$, and we have

$$\int_{T^*}^\infty(\sum_{i=1}^3[|x_i(t)-y_i(t)|+|u_i(t)-v_i(t)|])ds\le\frac{V(T)}{\alpha}<+\infty. \tag{17}$$

By (17), we have

$$\sum_{i=1}^{3}(|x_i(t)-y_i(t)|+|u_i(t)-v_i(t)|)\in L^1(T,+\infty). \tag{18}$$

By **Lemma 3**, we can obtain that $|x_i(t)-y_i(t)|,|u_i(t)-v_i(t)|$, $i=1,2,3$ and their derivatives remain bounded on $[T^*,+\infty)$, and $|x_i(t)-y_i(t)|,|u_i(t)-v_i(t)|,\ i=1,2,3$ are uniformly continuous on $[T^*,+\infty)$. By **Lemma 4** , we can conclude that

$$\lim_{t\to+\infty}|x_i(t)-y_i(t)|=0,\ \lim_{t\to+\infty}|u_i(t)-v_i(t)|=0,\ i=1,2,3.$$

This completes the proof, and the solutions of system (10) are globally attractive.

4. Periodic solution

If all coefficients of the system (10) are positive continuous periodic functions, the system (10) becomes a periodic system. In this section, we will use the fixed point theory and some new analytical methods to obtain the existence, uniqueness and stability conditions of the positive period solution of the system (10). For the sake of convenience, a lemma is firstly given.

Lemma 5 (See [28]**).** Let $S\subset R_n$ be convex and compact. If mapping $T:S\to S$ is continuous, then there exists a fixed point. I.e., there exists $x^*\in S$ such that $T(x^*)=x^*$.

Theorem 2. Assume that the system (10) is a ω -periodic and satisfies conditions $(H_1)-(H_8)$, then the system (10) has a positive unique ω-periodic solution, which is globally asymptotically stable.

Proof. According to the existence and uniqueness theorem of solutions of functional differential equations, we can define a **Poincare** mapping $T:R_+^6\to R_+^6$ as follow

$$T(X_0)=X(t,\omega,X_0)$$

where

$$X(t,\omega,X_0)=(x_1(t),x_2(t),x_3(t),u_1(t),u_2(t),u_3(t))$$

be a positive solution of the system (10) with the initial conditions (11). And define

$$S=\begin{Bmatrix}(x_1,x_2,x_3,u_1,u_2,u_3)\in R_+^6, m_i\le x_i\le M_i,\\ n_i\le u_i\le N_i, i=1,2,3\end{Bmatrix}$$

then it is obvious that $S\subset R_+^6$ is a convex and compact set. By the **Lemma 3** and the continuity of solution of system (10) with respect to the initial conditions (11), the mapping $T:S\to S$ is continuous. Furthermore, it is not difficult to show that the system (10) has a positive unique ω -periodic solution, which is globally asymptotic stability by using the **Lemma 5** and **Theorem 1**.

5. Numerical Simulation

In this section, some numerical simulations are given to support our theoretical analysis. Due to the periodic growth of the population and the periodicity of the growth environment, we select the appropriate periodic function to verify the results obtained in this paper. As an example, we consider the following 3-species delayed ratio-dependent predator-prey system with feedback controls

$$\begin{cases}\dot{x}_1(t)=x_1(t)[(8.5+0.5\cos\pi t)\\ \quad-(5.5+0.5\sin\pi t)x_1(t)\\ \quad-\dfrac{(0.6+0.5\sin\pi t)x_2(t)}{(1.8+0.2\sin\pi t)x_2(t)+x_1(t)}\\ \quad-\dfrac{(1.5+0.5\sin\pi t)x_3(t)}{(2+0.5\sin\pi t)x_3(t)+x_1(t)}\\ \quad-(0.015+0.005\sin\pi t)u_1(t)],\\ \dot{x}_2(t)=x_2(t)[-(3.005+0.005\cos\pi t)\\ \quad+\dfrac{(7.5+0.05\sin\pi t)x_1(t-\tau_1)}{(1.8+0.2\sin\pi t)x_2(t)+x_1(t-\tau_1)}\\ \quad-\dfrac{(1.5+0.5\sin\pi t)x_3(t)}{(2.6+0.1\sin\pi t)x_3(t)+x_2(t)}\\ \quad-(0.15+0.05\sin\pi t)x_3(t-\tau_2)\\ \quad+(0.7+0.2\sin\pi t)u_2(t)],\\ \dot{x}_3(t)=x_3(t)[-(4.05+0.05\cos\pi t)\\ \quad+\dfrac{(7.6+0.1\sin\pi t)x_1(t-\tau_3)}{(2+0.5\sin\pi t)x_3(t)+x_1(t-\tau_3)}\\ \quad+\dfrac{(3.5+0.5\sin\pi t)x_2(t-\tau_3)}{(2.6+0.1\sin\pi t)x_3(t)+x_2(t-\tau_3)}\\ \quad-(0.25+0.05\sin\pi t)x_2(t-\tau_4)\\ \quad+(0.45+0.05\sin\pi t)u_3(t)],\\ \dot{u}_1(t)=(1.5+0.2\cos\pi t)-(0.7+0.2\sin\pi t)u_1(t)\\ \quad+(0.15+0.05\sin\pi t)x_1(t),\\ \dot{u}_2(t)=(4.5+0.5\cos\pi t)-(4.05+0.05\sin\pi t)u_2(t)\\ \quad-(1.5+0.5\sin\pi t)x_2(t),\\ \dot{u}_3(t)=(3+0.5\cos\pi t)-(5.45+0.05\sin\pi t)u_3(t)\\ \quad-(1.5+0.5\sin\pi t)x_3(t).\end{cases} \tag{19}$$

By calculating, we have

$$M_1 \approx 1.800,\ M_2 \approx 1.354,\ M_3 \approx 1.173,$$
$$N_1 \approx 4.120,\ N_2 \approx 1.250,\ N_3 \approx 0.648,$$
$$m_1 \approx 1.105,\ m_2 \approx 0.856,\ m_3 \approx 0.525,$$
$$n_1 \approx 1.5672,\ n_2 \approx 0.315,\ n_3 \approx 0.028,$$
$$a_{31}^m + a_{32}^{1m} + d_3^m N_3 - r_3^l \approx 8.024,$$
$$a_{21}^m + d_2^m N_2 - r_2^l \approx 5.675,$$
$$r_1^l - a_{12}^m / b_{12}^l - a_{13}^m / b_{13}^l - d_1^m N_1 \approx 6.231,$$
$$e_2^l - q_2^m M_2 \approx 1.292,$$
$$e_3^l - q_3^m M_3 \approx 0.154,\ d_3^l n_3 - r_3^m \approx 0.0112,$$
$$d_2^l n_2 - r_2^m - a_{23}^{1m} / b_{23}^l \approx 0.158.$$

It is easy to show that the systems (19) satisfy the conditions of **Theorem 1** and **Theorem 2.** It follows from **Theorem 1** that the system (19) is global attractivity. By **Theorem 2**, the system (19) has a positive unique ω-periodic solution, which is globally asymptotic stability. By employing the software package MATLAB 7.1, we can obtain the numerical solutions of the system (19) which are shown in **Fig.1** and **Fig.2**. From **Fig.1**, it is not difficult to find that the system (19) is globally attractive. **Fig.2** shows that the dynamic behavior of solutions for the systems (19).

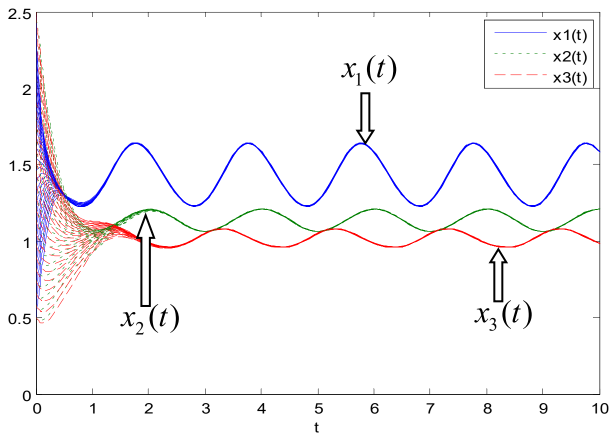
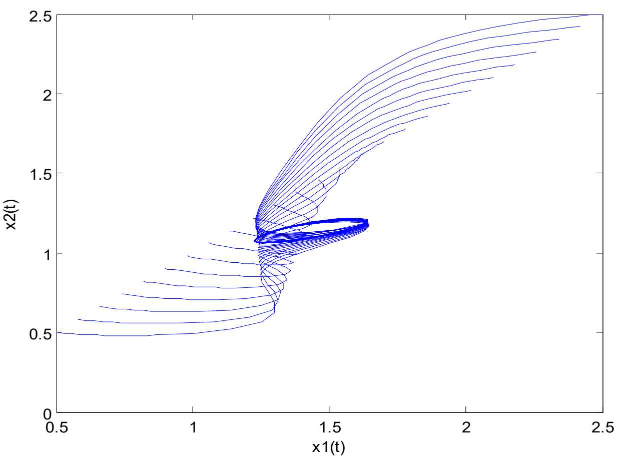


Fig. 1. The numerical solution of systems (19) with the different initial conditions



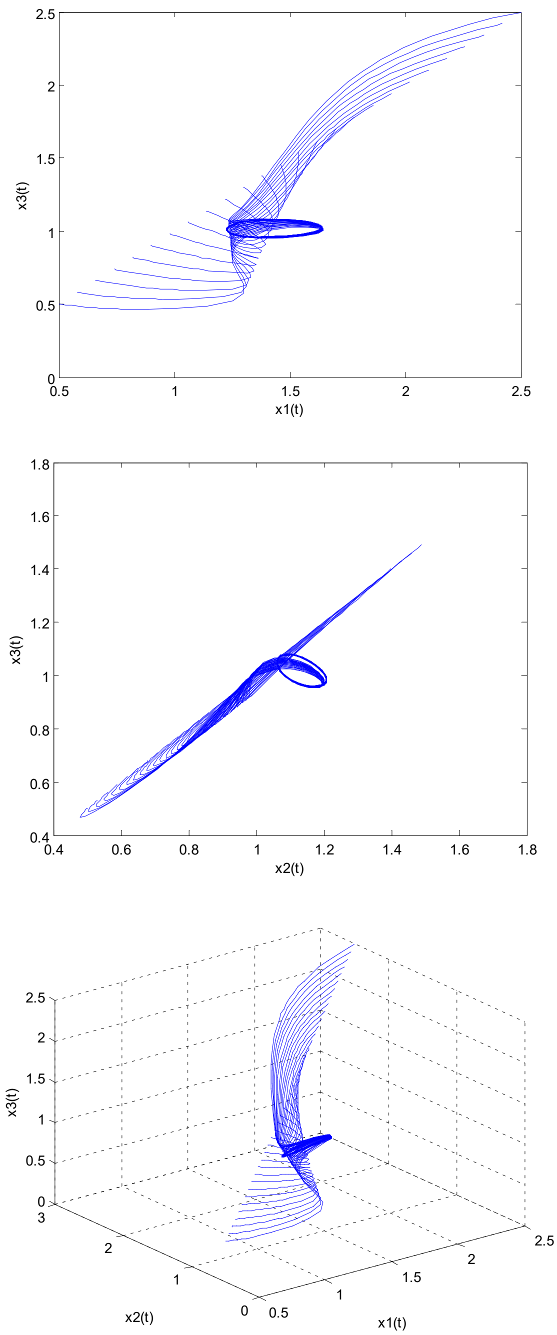


Fig. 2. The dynamic behavior of solutions for systems (19)

6. Conclusion

A class of ratio-dependent predator-prey system with feedback control and delays is proposed and studied. By using the comparison theorem and some new analytical methods, appropriate Lyapunov functions are constructed, and some sufficient conditions are obtained to ensure the global attractivity of positive solutions for the system. Furthermore, by defining Poincare mappings and using Brouwer fixed point theorem, some conditions for the existence, uniqueness and stability of positive periodic solutions for the corresponding periodic systems are obtained. Finally, some numerical solutions of the equations describing the system are given to confirm that the obtained criteria are new, general, and easily verifiable.

Time delay and feedback control are very common phenomena in ecosystem, but some scholars have not studied these problems profoundly. The time delay and feedback control terms added in this paper are relatively simple and cannot reflect more general ecosystems, but they are the focus of our future research, including the extension of multiple time delays to infinite time delays. By analyzing the delay system with the feedback control, it can be found that the feedback control item has influence on the stability of the original system. Consequently, some species in ecosystems can be controlled to maintain ecosystem balance and sustainable development, which is also the practical significance of this paper

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