Fixed-Time Stabilization of Spatial Constrained Wheeled Mobile Robot via Nonlinear Mapping

Yanling Shang and Jiacai Huang

***Abstract*—The problem of fixed-time stabilizing control for wheeled mobile robot subject to spatial constraint is studied in this paper. A nonlinear mapping is first introduced to transform the constrained system into a new unconstrained one. Then, by employing the adding a power integrator technique and switching control strategy, a state feedback controller is successfully constructed to guarantee that the states of closed-loop system are regulated to zero in a given fixed time without violation of the constraint. Finally, simulation results are given to confirm the efficacy of the presented control scheme.**

***Index Terms*—wheeled mobile robot, spatial constraint, adding a power integrator, fixed-time stabilization.**

I. Introduction

THE Wheeled mobile robot (WMR) has attracted a great deal of attention during the past decades because it wide applications in entertainment, security, war, rescue missions, spacial missions, assistant health-care, etc [1-3]. An important feature of WMR is that the number of control inputs is less than the number of degree of freedom, which leads to the control of WMR challenging. As pointed out by Brockett in [4], there is not any smooth (or even continuous) time-invariant state feedback to stabilize such category of nonlinear systems. To give this difficulty a solution, a number of control approaches have been proposed, which mainly are time-varying feedback [5-7] and discontinuous time-invariant feedback [8,9] Mainly thanks to these valid approaches, a number of interesting results on asymptotic stabilization have been established over the last years, see, e.g., [10-16] and the references therein.

In practical applications, the closed-loop system is desired to possess the property that trajectories converge to the equilibrium in finite time rather than merely asymptotically since system with finite-time convergence may retain not only faster convergence, but also better robustness and disturbance rejection properties [17]. Motivated by this, the finite-time control of nonlinear systems has attained significant amount of interests and efforts over the last years [18-20]. Particularly, by using state feedback, the authors in [21] first addressed the finite-time stabilization of nonholonomic systems with weak drifts, and then the adaptive finite-time stabilization problems were considered for nonholonomic systems with linear parameterization in [22] and nonlinear parameterization [23], respectively. By relaxed the restriction on system growth, [24] and [25] respectively studied the finite-time control for a class of nonholonomic systems by state feedback and output feedback. An output feedback controller was developed in [26] to finite-time stabilize a class of nonholonomic systems in feedforward-like form. Later, this result is further extended to the high order case in [27]. However, a common drawback of the above-mentioned studies is that the convergence time seriously relies on the initial condition of the considered systems, which renders that they cannot achieve the desired performance in an exact preset time. Recently, to remove the limitation of finite-time algorithm, a novel finite-time stability concept that requires the convergence time of a global finite-time stable system being bounded independent of initial conditions, was introduced in [28]. Such stability, usually called fixed-time stability, offers a new perspective to study the finite-time control problems and has stimulated some interesting results [29-31]. However, the effect of the constraints is omitted in the above-mentioned results.

As a matter that the constraints which can represent not only physical limitations but also performance requirements are common in practical systems. Violation of the constraints may cause performance degradation or system damage. In recent years, driven by practical needs and theoretical challenges, the control design for constrained nonlinear systems has become an important research topic [32-35]. However, less attention has been paid to the space-constrained nonholonomic mobile robots.

Motivated by the above observations, this paper focuses on solving the fixed-time stabilization problem of nonholonomic WMR subject to spatial constraint. The contributions are highlighted as follows. (i) The fixed-time stabilization problem of nonholonomic WMR subject to spatial constraint is studied. (ii) A nonlinear mapping is introduced, under which the constrained interval is mapped to the whole Euclidean space, and then the constrained control problem is transformed into an unconstrained one. (iii) Based on a switching strategy to eliminate the phenomenon of uncontrollability of $u_0 = 0$, and by using backstepping technique, a systematic state feedback control design procedure is proposed to force the states of the closed-loop system to zero for any given fixed time while the state constraints are not violated.

II. Problem Statement and Preliminaries

Consider a tricycle-type WMR shown in Fig.1. The kinematic equations of this robot are represented by

$$\begin{aligned} \dot{x}_c &= v\cos\theta, \\ \dot{y}_c &= v\sin\theta, \\ \dot{\theta} &= \omega, \end{aligned} \tag{1}$$

where (x_c, y_c) denotes the position of the center of mass of the robot, θ is the heading angle of the robot, v is the forward

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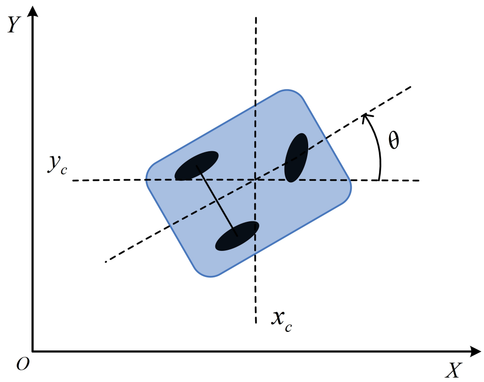

Fig. 1. The planar graph of a mobile robot.

velocity while ω is the angular velocity of the robot.

Introducing the following change of coordinates

$$\begin{aligned} &x_0 = x_c, \quad x_1 = y_c, \quad x_2 = \tan\theta, \\ &u_0 = v\cos\theta, \quad u_1 = w\sec^2\theta, \end{aligned} \tag{2}$$

system (1) is transformed into the chained form as

$$\begin{aligned} \dot{x}_0 &= u_0, \\ \dot{x}_1 &= u_0 x_2, \\ \dot{x}_2 &= u_1. \end{aligned} \tag{3}$$

Note that the state (x_0, x_1) can be see as the displacement from the parking position. As we all know, when the robots initial position is far away from the parking position, it usually can move directly to the parking position. The robots body angle can be aligned without difficulties and no more maneuvers are needed. However, when the robots initial position is close to the parking position, it might not be feasible to get to the parking position while aligning the robots body angle at the same time. Therefore it is very necessary to develop control techniques for spatial constrained WMR for giving this difficulty a straightforward solution.

Due to physical limitations, in this paper we assume that the states x_0 and x_1 are constrained in the compact sets

$$\Omega_{x_i} = \{-k_i < x_i < k_i\}, \quad i = 0, 1, \tag{4}$$

where k_i's are positive constants.

The objective of this paper is to present a state feedback control design strategy which stabilizes the system (3) for any given fixed time with the constraint being not violated.

Remark 1. Although great progress on constrained control design has been made, for the constrained nonholonomic system (3), how to construct a fixed-time stabilizer is still very difficult problem. The crucial obstacle is that the time-varying coefficient u_0 makes the x-subsystem uncontrollable in the case of $u_0 = 0$, and thus the existing constrained control methods mainly based on barrier Lyapunov function are highly difficult to the control problem of the system (3) or even inapplicable. Thereby, how to overcome this obstacle and design a fixed-time stabilizer for the constrained system (3) is main work of this paper.

The following definitions and lemmas will serve as the basis of the coming control design and performance analysis.

Definition 1[17]**.** Consider the nonlinear system

$$\dot{x} = f(t, x) \;\; with \; f(t, 0) = 0, \quad x \in R^n, \tag{5}$$

where $f : R^+ \times U_0 \to R^n$ is continuous with respect to x on an open neighborhood U_0 of the origin $x = 0$. The equilibrium $x = 0$ of the system is (locally) uniformly finite-time stable if it is uniformly Lyapunov stable and finite-time convergent in a neighborhood $U \subseteq U_0$ of the origin. By "finite-time convergence," we mean: If, for any initial condition $x(t_0) \in U$ at any given initial time $t_0 \geq 0$, there is a settling time $T > 0$, such that every $x(t, t_0, x(t_0))$ of system (5) is defined with $x(t, t_0, x(t_0)) \in U/\{0\}$ for $t \in [t_0, T)$ and satisfies $\lim_{t\to T} x(t, t_0, x(t_0)) = 0$ and $x(t, t_0, x(t_0)) = 0$ for any $t \geq T$. If $U = U_0 = R^n$, the origin is a globally uniformly finite-time stable equilibrium.

Lemma 1[17]**.** Consider the nonlinear system described in (5). Suppose there is a C^1 function $V(t, x)$ defined on $\hat{U} \subseteq U_0 \times R$, where $\hat{U}$ is a neighborhood of the origin, class K functions π_1 and π_2, real numbers $c > 0$ and $0 < \alpha < 1$, for $t \in [t_0, T)$ and $x \in \hat{U}$ such that

$$\pi_1(|x|) \leq V(t, x) \leq \pi_2(|x|), \forall t \geq t_0, \forall x \in \hat{U},$$

and

$$\dot{V}(t, x) + cV^{\alpha}(t, x) \leq 0, \forall t \geq t_0 \;, \forall x \in \hat{U}.$$

Then, the origin of (5) is uniformly finite-time stable with $T \leq \frac{V^{1-\alpha}(t_0, x(t_0))}{c(1-\alpha)}$ for initial condition $x(t_0)$ in some open neighborhood $\hat{U}$ of the origin at initial time t_0. If $\hat{U} = U_0 = R^n$ and π_1 and π_2 are class K_∞ functions, the origin of system (5) is globally uniformly finite-time stable.

Definition 2[31]**.** The origin of system (5) is said to be globally fixed-time stable if it is globally finite-time stable and the settling time function $T(x_0)$ is bounded, that is, there exists a positive constant T_{max} such that $T(x_0) \leq T_{max}$, $\forall x_0 \in \mathbb{R}^n$.

Lemma 2[31]**.** Consider the nonlinear system (5). Suppose there exist a C^1, positive definite and radially unbounded function $V(x) : \mathbb{R}^n \to \mathbb{R}$ and real numbers $c > 0$, $d > 0$, $0 < \alpha < 1$, $\gamma > 1$, such that

$$\dot{V}(x) \leq -cV^{\alpha}(x) - dV^{\gamma}(x), \;\; \forall x \in \mathbb{R}^n.$$

Then, the origin of system (5) is globally fixed-time stable and the settling time $T(x_0)$ satisfies

$$T(x_0) \leq T_{max} := \frac{1}{c(1-\alpha)} + \frac{1}{d(\gamma-1)}, \;\; \forall x_0 \in \mathbb{R}^n.$$

Lemma 3[36]**.** For $x \in R$, $y \in R$, $p \geq 1$ and $c > 0$ are constants, the following inequalities hold: (i) $|x+y|^p \leq 2^{p-1}|x^p + y^p|$, (ii) $(|x| + |y|)^{1/p} \leq |x|^{1/p} + |y|^{1/p} \leq 2^{(p-1)/p}(|x| + |y|)^{1/p}$, (iii) $||x| - |y||^p \leq ||x|^p - |y|^p|$, (iv) $|x|^p + |y|^p \leq (|x| + |y|)^p$, (v) $|[x]^{1/p} - [y]^{1/p}| \leq 2^{1-1/p}|x - y|^{1/p}$, (vi) $|[x]^p - [y]^p| \leq c|x - y|||x - y|^{p-1} + |y|^{p-1}|$.

Lemma 4[36]**.** For any positive real numbers c, d and any real-valued function $\pi(x, y) > 0$, $|x|^c|y|^d \leq \frac{c}{c+d}\pi(x, y)|x|^{c+d} + \frac{d}{c+d}\pi^{-c/d}(x, y)|y|^{c+d}$.

III. Fixed-time control design

In this section, we give a constructive procedure for the finite-time stabilizer design of system (3) for any given settling time $T > 0$. The overall controller design consists of two steps: (i) Choose an appropriate nonzero constant input u_0^* for u_0. In this way, the x-subsystem can be interpreted as a linear-like system, for which the fixed-time stabilization

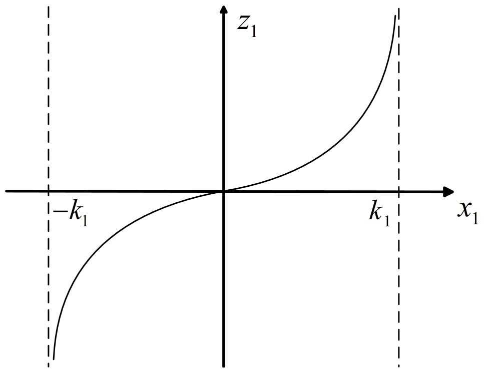

Fig. 2. Schematic illustration of the nonlinear mapping $\mathscr{H}_1$.

controller can be proposed; (ii) After x arrives at zero before a fixed time and remains zero afterwards, we design a new fixed-time stabilization controller u_0 to stabilize the x_0-subsystem.

A. Fixed-time stabilization of the x-subsystem

For the x_0-subsystem, we take the following control law

$$u_0 = \begin{cases} u_0^*, & x_0(0) < 0, \\ -u_0^*, & x_0(0) \geq 0, \end{cases} \\ := (|\operatorname{sign}(x_0(0))| - \operatorname{sign}(x_0(0)) - 1)u_0^*, \tag{6}$$

where u_0^* is a positive constant satisfying $u_0^* < k_0/(\theta T)$ with $\theta \in (0,1) \cap \mathbb{R}$. As a result, the following lemma can be established by some simple derivations.

Lemma 5. For any initial condition $x_0(0) \in \Omega_{x_0}$, the corresponding solution $x_0(t)$ is well defined on $[0, \theta T)$ and satisfies $x_0(t) \in \Omega_{x_0}$.

Under the control law (6), the x-subsystem can be rewritten as

$$\begin{aligned} \dot{x}_1 &= d_1 x_2, \\ \dot{x}_2 &= d_2 u_1, \end{aligned} \tag{7}$$

where $d_1 = (|\operatorname{sign}(x_0(0))| - \operatorname{sign}(x_0(0)) - 1)u_0^*$ and $d_2 = 1$.

Next, we will stabilize the system (7) within the settling time θT. To prevent the state x_1 from violating the constraint, we introduce a one-to-one nonlinear mapping $\mathscr{M}_1 : \Omega_{x_1} \to \mathbb{R}$ as follows:

$$z_1 = \mathscr{M}_1(x_1) = \ln\Big(\frac{k_1 + x_1}{k_1 - x_1}\Big), \tag{8}$$

where $\mathscr{M}_1$ is shown in Fig. 2, from which, it is clear that the function $\mathscr{M}_1$ has a continuous inverse, see Remark 1. Based on (8), we can obtain

$$\dot{z}_1 = \frac{1}{2k_1}(e^{z_1} + e^{-z_1} + 2)d_1 x_2. \tag{9}$$

Furthermore, by denoting

$$z_2 = x_2, \tag{10}$$

we can rewrite the system (7) as

$$\begin{aligned} \dot{z}_1 &= \tilde{d}_1 z_2, \\ \dot{z}_2 &= \tilde{d}_2 u_1, \end{aligned} \tag{11}$$

where $\tilde{d}_1 = d_1(e^{z_1} + e^{-z_1} + 2)/2k_1$ and $\tilde{d}_2 = d_2$.

Remark 2. From the nonlinear mapping $\mathscr{M}_1$, the state z_1 is defined in the whole real number field $\mathbb{R}$ and thus it is an unconstrained variable. Moreover, based on the inverse mapping

$$x_1 = \mathscr{M}_1^{-1} = k_1\Big(1 - \frac{2}{e^{z_1} + 1}\Big), \tag{12}$$

and (10), we know that $x \to 0$ if and only if $z \to 0$ and that x_1 will stay in the constraint interval $|x_1| < k_1$ regardless of the value of z_1. Therefore, the control design for the constrained system (7) is equivalent to the control design for the unconstrained system (11).

With the aid of (11), a fixed-time stabilization controller will be designed for u_1 by employing recursive technique. Our design procedure consists of n steps. Before proceeding, we take $r_1 = 1$ and $r_{i+1} = r_i + \tau > 0, \quad i = 1, 2, 3$ with $\tau \in (-\frac{1}{n}, 0)$ being a negative number, and introduce the following coordinate transformation:

$$\begin{aligned} \xi_i &= [z_i]^{\frac{1}{r_i}} - [\alpha_{i-1}]^{\frac{1}{r_i}}, \\ \alpha_i &= -g_i^{r_{i+1}}(\bar{z}_i)[\xi_i]^{r_{i+1}}, \quad i = 1, 2, \end{aligned} \tag{13}$$

where $\alpha_0 = 0$, $\alpha_2 = u_1$ and $g_i(\bar{z}_i) > 0$ is a C^1 function to be specified later.

We further define $W_i : \mathbb{R}^i \to \mathbb{R}$ as follows:

$$W_i(\bar{z}_i) = \int_{\alpha_{i-1}}^{z_i} \Big[[s]^{\frac{1}{r_i}} - [\alpha_{i-1}]^{\frac{1}{r_i}}\Big]^{2-r_{i+1}} ds. \tag{14}$$

In the following, the detailed design procedure is elaborated.

Step 1. For the z_1-subsystem of (11), take the state variable z_2 as a virtual control input. Choose $V_1 = W_1$ and $g_1 = ((1 + l_1 + l_2|\xi_1|^p)/\tilde{d}_1)^{\frac{1}{r_2}}$ with design parameters $l_1 > 0$, $l_2 > 0$ and $p > -\tau$ to be determined later, we have

$$\dot{V}_1 \leq -(1 + l_1)|\xi_1|^2 - l_2|\xi_1|^{2+p} + \tilde{d}_1[\xi_1]^{2-r_2}(z_2 - \alpha_1). \tag{15}$$

Step 2 . Consider the 2 ed Lyapunov function $V_2 = V_1 + W_2$. It can be deduced from (15) that

$$\begin{aligned} \dot{V}_2 \leq& -(1 + l_1)|\xi_1|^2 - l_2|\xi_1|^{2+p} \\ &+ \tilde{d}_1[\xi_1]^{2-r_2}(z_2 - \alpha_1) + \tilde{d}_2[\xi_2]^{2-r_3}u_1 + \frac{\partial W_2}{\partial z_1}\tilde{d}_1 z_2. \end{aligned} \tag{16}$$

First, we observe from Lemmas 3 and 4 that

$$\begin{aligned} \tilde{d}_1[\xi_1]^{2-r_2}(z_2 - \alpha_1) &\leq 2\tilde{d}_1|\xi_1|^{2-r_2}|\xi_2|^{r_2} \\ &\leq \frac{1}{2}|\xi_1|^2 + \varphi_{21}|\xi_2|^2, \end{aligned} \tag{17}$$

where $\varphi_{21} \geq 0$ is a C^1 function.

Then, by using Lemmas 3 and 4, we have

$$\frac{\partial W_2}{\partial z_1}\tilde{d}_1 z_2 \leq \frac{1}{2}|\xi_1|^2 + \varphi_{22}|\xi_2|^2, \tag{18}$$

where $\varphi_{22} \geq 0$ is a C^1 function.

Choosing

$$g_2 = \Big(\frac{l_1 + \varphi_{21} + \varphi_{22} + l_2|\xi_2|^p}{\tilde{d}_2}\Big)^{\frac{1}{r_3}}, \tag{19}$$

and substituting (17), (18 and (19) into (16), we have

$$\dot{V}_2 \leq -l_1\sum_{j=1}^{2}|\xi_j|^2 - l_2\sum_{j=1}^{2}|\xi_j|^{2+p}. \tag{20}$$

So far, the inductive design steps are completed. Therefore, there exists a continuous state feedback controller of the form

$$u_1 = \alpha_2 = -g_2^{r_3}[\xi_2]^{r_3}, \tag{21}$$

such that

$$\dot{V}_2 \leq -l_1 \sum_{j=1}^{2} |\xi_j|^2 - l_2 \sum_{j=1}^{2} |\xi_j|^{2+p}, \tag{22}$$

where $V_2 = \sum_{j=1}^{2} W_j$.

Consequently, the following result is obtained.

Lemma 6. If the controller u_1 of system (11) is specified by (21) with design parameters $l_1 > 0$, $l_2 > 0$ and $p > -\tau$ satisfying

$$\frac{2(\tau-2)}{\theta l_1 \tau} + \frac{(2-\tau)2^{\frac{2+p}{2-\tau}} n^{\frac{p+\tau}{2-\tau}}}{\theta l_2 (p+\tau)} < T, \tag{23}$$

then the equilibrium $z = 0$ of closed-loop system is globally fixed-time stable and all the trajectories converge to zero before a fixed time θT.

Proof. According to $(z_i - \alpha_{i-1})([z_i]^{\frac{1}{r_i}} - [\alpha_{i-1}]^{\frac{1}{r_i}}) \geq 0$, we easily verify that $V_2 = \sum_{j=1}^{2} W_j$ is positive definite and radially unbounded. Moreover, we have the following estimation for V_2.

$$V_2 = \sum_{j=1}^{2} W_j \leq 2 \sum_{j=1}^{2} |\xi_j|^{2-\tau}. \tag{24}$$

Letting $\alpha = 2/(2-\tau)$, it is not difficult to obtain that

$$-\sum_{j=1}^{2} |\xi_j|^2 \leq -\frac{1}{2} V_2^{\alpha}. \tag{25}$$

On the other hand, taking (24) into account, it can be deduced that

$$\begin{aligned} -\sum_{j=1}^{2} |\xi_j|^{2+p} &= -\sum_{j=1}^{2} \left(|\xi_j|^{2-\tau}\right)^{\frac{2+p}{2-\tau}} \\ &\leq -2^{1-\frac{2+p}{2-\tau}} \left(\sum_{j=1}^{n} |\xi_j|^{2-\tau}\right)^{\frac{2+p}{2-\tau}} \\ &\leq -2^{-\gamma} 2^{1-\gamma} V_2^{\gamma}, \end{aligned} \tag{26}$$

where $\gamma = (2+p)/(2-\tau)$.

Therefore, by considering (22), (25) and (26), it follows that

$$\dot{V}_2 \leq -\frac{1}{2} l_1 V_2^{\alpha} - l_2 2^{-\gamma} 2^{1-\gamma} V_2^{\gamma}. \tag{27}$$

Since $\alpha < 1$ and $\gamma > 1$, from Lemma 2, we conclude that the equilibrium $z = 0$ of the closed-loop system is globally fixed-time stable and the settling time function T_1 satisfies

$$\begin{aligned} T_1 &\leq \frac{2}{l_1(1-\alpha)} + \frac{2^{\gamma} n^{\gamma-1}}{l_2(\gamma-1)} \\ &= \frac{2(\tau-2)}{l_1 \tau} + \frac{(2-\tau)2^{\frac{2+p}{2-\tau}} n^{\frac{p+\tau}{2-\tau}}}{l_2(p+\tau)} \\ &< \theta T. \end{aligned} \tag{28}$$

With the help of Lemma 6, we are ready to state the main result of this subsection.

Lemma 7. If the proposed control design procedure with appropriate design parameters is applied to system (7), then, for any initial condition $x(0) \in \Theta_1 = \{x \in \mathbb{R}^n | -k_1 < x_1(0) < k_1\}$, the following properties hold.

(i) The state x_1 remains in the set $\Omega_{x_1} = \{-k_1 < x_1(t) < k_1\}$, $\forall t \geq 0$.

(ii) All the states of closed-loop system are regulated to zero within a fixed settling time θT.

Proof. From Lemma 6, we can easily see that the states $z_i(t)$, $i = 1, 2$ are bounded, and satisfy $\lim_{t \to \frac{T}{2}} z_i(t) = 0$. The bounded state $z_1(t)$ together with the nonlinear mapping (8) leads to

$$|x_1(t)| = k_1 \left|1 - \frac{2}{e^{z_1(t)} + 1}\right| < k_1, \tag{29}$$

that is, the state x_1 will remains in the set Ω_{x_1} and never violates the constraint. Furthermore, $\lim_{t \to \theta T} z_2(t) = 0$ and (10), (12) imply that $\lim_{t \to \theta T} x_2(t) = 0$, and

$$\begin{aligned} \lim_{t \to \theta T} x_1(t) &= \lim_{t \to \theta T} k_1 \left(1 - \frac{2}{e^{z_1(t)} + 1}\right) \\ &= k_1 \left(1 - \frac{2}{e^{\lim_{t \to \theta T} z_1(t)} + 1}\right) \\ &= 0. \end{aligned} \tag{30}$$

Thus, the proof is completed.

B. Fixed-time stabilization of the x_0-subsystem

From Lemma 7, we know that $x(t) \equiv 0$ when $t \geq \theta T$. Since the time derivative of $x(t)$ is identically zero, $x(t)$ will always keep zero for $t \geq \theta T$ in spite that a new controller will be designed for u_0 when $t \geq \theta T$. Therefore, we just need to stabilize the x_0-subsystem in a fixed time θT. In this case, for the x_0-subsystem, we can take the control u_0 as

$$u_0 = -(m_0 + m_1 |x_0|^q)[x_0]^{\sigma}, \tag{31}$$

where $0 < \sigma < 1$, $m_0 > 0$, $m_1 > 0$ and $q > 1 - \sigma$ are design parameters to be determined later.

Lemma 8. If design parameters $0 < \sigma < 1$, $m_0 > 0$, $m_1 > 0$ and $q > 1 - \sigma$ in (31) satisfy

$$\frac{2}{m_0(1-\sigma)(1-\theta)} + \frac{2}{m_1(\sigma+q-1)(1-\theta)} < T, \tag{32}$$

then, for any initial condition $x_0(0) \in \{-k_0 < x_0(0) < k_0\}$, the following properties hold.

(i) The state x_0 remains in the set $\Omega_{x_0} = \{-k_0 < x_0(t) < k_0\}$, $\forall t \geq 0$ and never violates the constraint.

(ii) The state x_0 is regulated to zero within a fixed settling time $(1-\theta)T$.

Proof. The proof of Lemma 8 follows the same line of the proofs of Lemmas 6 and 7.

Up to now, we have finished the fixed-time state feedback stabilizing controller design of the system (**??**). Consequently, the following theorem can be obtained to summarize the main result of the paper.

Theorem 1. If the following switching control strategy with an appropriate choice of the design parameters is applied to system (3) subject to constraints (4),

$$u_0 = \begin{cases} u_0^*, & t < \theta T, \\ -\dfrac{1}{d_0}(m_0 + m_1 |z_0|^q)[z_0]^{\sigma}, & t \geq \theta T, \end{cases} \tag{33}$$

$$u_1 = -\beta_n^{r_{n+1}} [\xi_n]^{r_{n+1}}, \tag{34}$$

then the states of the closed-loop system are regulated to zero within any given settling time T while, at the same time constraints (4) are met.

IV. Simulation results

In this section, we illustrate the effectiveness of the proposed approach with the boundedness of $k_i = 1$, i.e., $|x_0| < 1$ and $|x_1| < 1$. Then, for the x_0-subsystem, we can choose the control law

$$u_0 = \begin{cases} u_0^*, & x_0(0) \le 0, \\ -u_0^*, & x_0(0) > 0, \end{cases} \quad := (|\operatorname{sign}(x_0(0))| - \operatorname{sign}(x_0(0)) - 1)u_0^*, \tag{35}$$

where u_0^* is a positive constant satisfying $u_0^* < 2/T$. Choosing $\tau = -1/3$ and following the design procedure shown in Section III, we can explicitly construct a state feedback controller

$$\alpha_1 = -\frac{1}{\tilde{d}_1}(1 + l_1 + l_2|\xi_1|^p)[\xi_1]^{\frac{2}{3}} := -g_1^{\frac{2}{3}}[\xi_1]^{\frac{2}{3}}, \quad u_1 = -(l_1 + \varphi_{21} + \varphi_{22} + l_2|\xi_2|^p)[\xi_2]^{\frac{1}{3}}, \tag{36}$$

with $\xi_1 = [z_1]$, $\xi_2 = [z_2]^{\frac{3}{2}} - [\alpha_1]^{\frac{3}{2}}$, $\varphi_{21} = 1.1852\tilde{d}_1^3$, $\varphi_{22} = 2.0999|\frac{\partial([\alpha_1]^{\frac{3}{2}})}{\partial z_1}| + 1.3999|\frac{\partial([\alpha_1]^{\frac{3}{2}})}{\partial z_1}|^{\frac{3}{2}}\tilde{d}_1^{\frac{3}{2}} g_1$ and appropriate positive constants l_1, l_2 and p such that the states of the x-subsystem of (3) are globally regulated to zero within a fixed settling time $T/2$ without violation of the constraints.

Then, when $t \ge T/2$, for the x_0-subsystem, we switch the control input u_0 to

$$u_0 = -\frac{1}{d_0}(m_0 + m_1|z_0|^q)[z_0]^\sigma, \tag{37}$$

with $z_0 = \ln(\frac{1+x_0}{1-x_0})$, $d_0 = (e^{z_0} + e^{-z_0} + 2)/2$ and some suitable positive constants σ, m_0, m_1, q, under which, the state x_0 can be regulated to zero within a fixed settling time $T/2$ without violation of the constraints.

In the simulation, by choosing the fixed time $T = 10$ and the gains for the control laws as $u_0^* = 0.19$, $l_1 = 4$, $l_2 = 5$, $p = 2$, $\sigma = 0.5$ and $m_0 = m_1 = q = 2$, Fig. 3 is obtained to exhibit the responses of the closed-loop system with $(x_c(0), y_c(0), \theta(0))= (-0.8, 0.9, \pi/4)$. From the figure, it can be seen that the mobile robot moves to the desired location in a given fixed time and the state constraints are never violated, which accords with the main result established in Theorem 1 and demonstrates the effectiveness of the control method proposed in this paper.

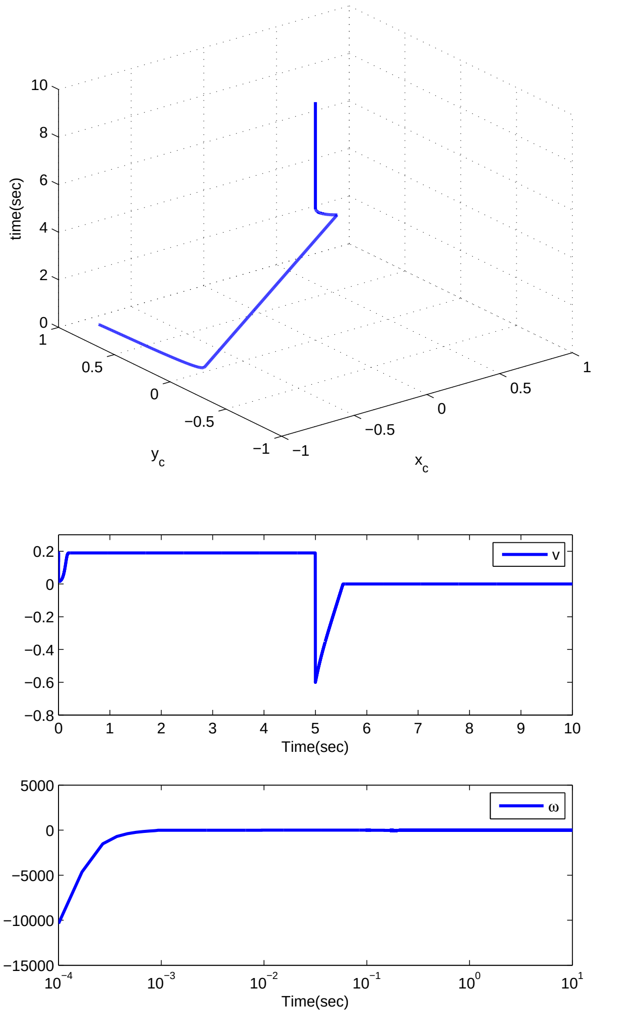

Fig. 3. The responses of the closed-loop system with $(x_c(0), y_c(0), \theta(0))= (-0.8, 0.9, \pi/4)$.

V. Conclusion

This paper has studied the problem of Fixed time stabilization by state feedback for nonholonomic WMR subject to spatial constraint. Based on the nonlinear mapping, and by skillfully using recursive technique, a constructive design procedure for state feedback control is given. Together with a novel switching control strategy,a constructive design procedure for state feedback control is given. Together with a novel switching control strategy, the designed controller can guarantee that the closed-loop system states are regulated to zero for any given fixed time while the constraint is not violated.

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