A Stackelberg Game for Retailer-Led Supply Chains Considering the Selling Effort in a Fuzzy Environment

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***Abstract*—As it is difficult to describe economic activities with certainty, uncertain and ambiguous theories are often used to describe corporate behavior. This study uses fuzzy set theory to examine the optimal decision of each member of a two-stage supply chain, which includes a manufacturer and a retailer. In this supply chain, the retailer takes the leading position and makes sales efforts. By considering the market demand function, the manufacturer's manufacturing cost and the retailer's operating cost as fuzzy variables, and by employing sequential game, expected value, and opportunity constraint models, the optimal decision-making solutions are resolved. Finally, a numerical example demonstrates the effectiveness of the supply chain game model. In the equilibrium result, considering the cost of the sales effort undertaken by the retailer, their overall expected profit is lower than that of the manufacturer. However, the product unit has a higher marginal profit. The reason for this result is that the dominance of the retailers not only increases product sales through their sales efforts and increases the revenue of all members of the supply chain, but it also enables them to lower their wholesale prices and make themselves more profitable in the supply chain.**

***Index Terms*—Fuzzy variable, Sales effort, Sequential game, Two-stage supply chain**

I. Introduction

THE development of the socio-economic structure has led to a continuous shift of power among members of the supply chain. Furthermore, the rapid growth of large retail companies has made them more important. Large retailers are at the end of the sales channel and are in direct contact with customers. They can capture information about changes in customer needs easily and quickly, and can pass this information to manufacturers in an effective manner, thereby improving the performance and benefits of the entire supply chain. At the same time, large retailers continue to meet and create customer needs, form strong brand and channel advantages, continuously strengthen their position in the supply chain, and gradually become the leader of the industry chain [1]. Therefore, research on the coordination mechanism of retailer-led supply chains has attracted increased attention from domestic and foreign scholars. Hua and Li [2] assume that the price is determined by the market, and the retailer determines two other factors. The conditions of supply chain coordination are compared and analyzed, and the influence of the parameters on the dominance of the retailer is analyzed using an example. Chen et al. [3] constructed a supply chain in which a manufacturer, a leading retailer, and several small retailers co-existed and compared and analyzed the coordination effect between the quantity discount contract and the wholesale price contract. Zhang et al. [4] established a supply chain consisting of a manufacturer and a retailer, which is led by retailers and assumes they use dominant control to coordinate the supply chain. The study examined the revenue sharing contract when demand is affected by retail prices under both oversupply and undersupply conditions. In addition, scholars such as Santanu [5], Pan [6], and Zhao [7] analyzed the supply chain coordination and optimization mechanism that includes multiple retailers or manufacturers. In terms of bargaining power, Zhou and Shi [8] established a Rubinstein-Stahl bargaining model to analyze the relationship between manufacturers and retailers in a retail-led supply chain. Regarding the problem of profit distribution between the two, they argued that because retailers have greater market power, they are in a stronger position in terms of supply chain bargaining. Manufacturers' profit distribution ratio is also lower than that of retailers in terms of the value division of the supply chain.

With the rapid development of e-commerce, the scale and influence of large retailers have further increased. Against this background, the impact of large retailers on the structural adjustment of manufacturing, supply chain performance, and social welfare has received increased attention. However, the research conclusions on the impact of the purchasing power of retailers on welfare effects have not yet been unified. One view is that in the retailer-led supply chain, the retailer realizes the optimal allocation of social resources through terminal control, thereby promoting the improvement of overall efficiency. In contrast, scholars such as Zhao [9] and Battigalli [10] argued that the influence of the buyer on retailers would bring negative social welfare effects. Studies have suggested that such influence is uncertain and is determined by the size of the buyer's purchasing power. If the retail buyer's power is within a certain range, then their behavior can encourage the manufacturer to improve the quality of the product. If it is beyond a certain range, then their behavior will damage the interests of upstream and downstream enterprises and customers in the supply chain,

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thereby affecting their own interests and bringing about negative welfare effects. However, Li [11] argues that the influence of retail bargaining power on different nodes in the supply chain is different. Studies have shown that the increase in such power has a positive effect on consumer welfare and retailer profits. However, the effect on producer's profits and on those of the entire channel is uncertain. Irrespective of the conclusions of such research, there is a consensus that the overall value created by the supply chain can only increase through the cooperation of various links in an enterprise's supply chain [12].

It can be seen from the existing literature that although there are numerous studies on retailer-led supply chain cooperation and competition from multiple perspectives, few have considered the impact of members' sales efforts on prices and profits. However, in terms of market competition, the level of marketing effort required is directly proportional to competitiveness. To enhance the competitive advantage of the supply chain, members need to cooperate closely and highly leverage their sales efforts in relation to market development. Simultaneously, in the actual operation of the supply chain, an increasing number of uncertain factors affect the members and their overall revenue. Therefore, such uncertainty must be addressed to enhance the ability of the classical game theory model to explain realistic problems. This model is also extended to uncertain situations [13-15]. Based on previous research, this study introduces the fuzzy set theory, considers the retail sales effort level of the retailer, and takes the retailer-dominated two-stage supply chain as the research object to examine the game behavior of retailers and manufacturers in the supply chain. The sales efforts considered refer to the various measures retailers use to promote product sales such as publicity, advertising, and price adjustment.

II. Preliminaries and Model Construction

A. Preliminaries

Fuzzy theory is based on fuzzy sets. Its basic concept is to accept the fact that ambiguity exists, and to deal with things that have uncertain concepts as its research goal. In fuzzy theory, $Pos\{A\}$ is used to describe the possibility of event A. To ensure the rationality of $Pos\{A\}$ in practice, it needs to meet some mathematical properties. Assuming that Θ is a non-empty set and $P\{\Theta\}$ is the power set of Θ, then

Axiom 1. $Pos\{\Theta\}=1$.

Axiom 2. $Pos\{\varphi\}=0$.

Axiom 3. For any set $\{A_i\}$ of $P(\Theta)$, $Pos\{U_i A_i\}=\sup_i Pos\{A_i\}$.

If the above three axioms are met, it is called a possibility measure, and the triplet $(\Theta, P(\Theta), Pos)$ is called a possibility space.

In the subsequent analysis, the following definitions and properties will be used as the premise and basis for our research:

Definition 1. Assuming that a fuzzy variable is a function from a likelihood space $(\Theta, P(\Theta), Pos)$ to a solid line R, ξ is called a fuzzy variable defined in the likelihood space $(\Theta, P(\Theta), Pos)$.

Definition 2. A fuzzy variable ξ is a non-negative (or positive) variable if and only if $Pos\{\xi<0\}=0$ or $Pos\{\xi\le 0\}=0$.

Proposition 1. Suppose that ξ_i are fuzzy variables independent of each other, and $f_i: R\to R$, $i=1,2,...m$. Then, $f_1(\xi_1)$, $f_2(\xi_2)$, ..., $f_m(\xi_m)$ are also fuzzy variables independent of each other [16].

Definition 3. Suppose that ξ is a fuzzy variable of the likelihood space (Θ, $P(\Theta)$, Pos), and $\alpha\in(0,1]$. Then, the formula $\xi_\alpha^L=\inf\{r|Pos\{\xi\le r\}\ge\alpha\}$ and $\xi_\alpha^U=\sup\{r|Pos\{\xi\ge r\}\ge\alpha\}$ are called α pessimistic values and α optimistic values of the fuzzy variables, respectively.

In the aforementioned formula, r is the maximum value obtained by the fuzzy variable when the probability is α. A pessimistic value ξ_α^L is the lower bound of the value of ξ when the probability is α, and an optimistic value ξ_α^U is the upper bound of the value of ξ when the probability is α.

Example 1. The triangular fuzzy variable $\xi=(a,b,c)$ has α pessimistic values and α optimistic values

$\xi_\alpha^L=a+(b-a)\alpha$ and $\xi_\alpha^U=c-(c-b)\alpha$, respectively.

Proposition 2. There are two independent fuzzy variables, assuming ξ and η, respectively, the following conclusions will be obtained [17]:

For any $\alpha\in(0,1]$, $(\xi+\eta)_\alpha^L=\xi_\alpha^L+\eta_\alpha^L$.

For any $\alpha\in(0,1]$, $(\xi+\eta)_\alpha^U=\xi_\alpha^U+\eta_\alpha^U$.

If the sum are two non-negative independent fuzzy variables, then the following conclusions hold:

For any $\alpha\in(0,1]$, $(\xi\cdot\eta)_\alpha^L=\xi_\alpha^L\cdot\eta_\alpha^L$.

For any $\alpha\in(0,1]$, $(\xi\cdot\eta)_\alpha^U=\xi_\alpha^U\cdot\eta_\alpha^U$.

Definition 4. Suppose that ξ is a fuzzy variable and r_0 is a real number in ($-\infty$, ∞). Then, the expectation of ξ is

$$E[\xi]=\int_0^{+\infty} C_r\{\xi\ge r_0\}dr_0-\int_{-\infty}^0 C_r\{\xi\le r_0\}dr_0$$

Assume that at least one of the two integrals are finite assuming that ξ is a non-negative fuzzy variable. Then

$$E[\xi]=\int_0^{+\infty} C_r\{\xi\ge r_0\}dr_0$$

Example 2. The expectation of the triangular fuzzy variable $\xi=(a,b,c)$ is

$$E[\xi]=\frac{a+2b+c}{4}$$

Proposition 3. Assuming ξ is a fuzzy variable and has a finite expectation,

$$E[\xi]=\frac{1}{2}\int_0^1(\xi_\alpha^L+\xi_\alpha^U)d\alpha$$

Proposition 4. Assuming that ξ and η are fuzzy variables independent of each other and have a finite expectation, for any a and b, the following formula holds

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta]$$

B. Model Construction

This study adopts the two-stage supply chain as the research object. This consists of a manufacturer and a retailer, where the retailer occupies a dominant position in the supply chain. The behavior of the two companies is as follows. First, the manufacturer produces the product and sells it to the retailer at the wholesale price. Thereafter, the retailer sells the product to the consumer at the retail price. At the same time, to increase market demand and product sales, the sales chain is considered as a factor in the supply chain. The sales effort described herein refers to the sales policies or measures adopted to increase market demand, assuming that all sales efforts are undertaken by the retailer.

Manufacturers maximize profits by determining the optimal wholesale prices, and retailers maximize profits by determining the optimal retail prices and sales efforts. The two parties use non-cooperative methods to maximize their own profits, which is in line with the basic assumptions of the Stackelberg model. In the Stackelberg model, the dominant manufacturers make decisions first, and followers make decisions regarding their own best strategies based on the decisions of the leaders. Therefore, in the Stackelberg game of a two-tier supply chain dominated by retailers, the retailer first determines its unit profit margin (i.e., it determines the retail price) and the optimal level of the sales effort. After making this decision, the retailer set its optimal wholesale price [18].

Due to the uncertainty of the economic environment, this study uses the fuzzy theory to explain the problem of the two-stage supply chain game. Table I presents the symbols of the variables and related parameters used in the model and their various meanings.

TABLE I
PARAMETERS AND THEIR RELATED MEANINGS

Parameter	Meaning
w	The wholesale price per unit of the product
c_m	Manufacturing cost per unit of the product
m	Retailer's product sales margin
c_r	Operating costs of retailer unit products
p	Product price ($p = w + m$)
e	The retailer's sales effort
Π_m	The manufacturer's profit
Π_r	The retailer's profit

The subscripts m and r represent the manufacturer and the retailer, respectively.

Assume that the customer's demand function is a linear function of the wholesale price, the unit profit margin, and the degree of the sales effort, where the wholesale price and the unit profit margin decrease, and the sales effort increases. The customer's demand function is $D = a - bp + ke = a - b(w+m) + ke$, where a, b, and k are independent non-negative fuzzy variables. Here, a and b are two mutually independent non-negative fuzzy variables. a represents the maximum market capacity and b represents the demand to price change rate. Customer demand D is also a fuzzy variable. Since the demand in practice is positive, $Pos\{\mathrm{a} - b(w+m) + ke \le 0\} = 0$.

Suppose that the cost function of the retailer's effort is $g(e) = le^2$ Then, the manufacturer's and retailer's profit functions are respectively as follows:

$$\Pi_{\mathrm{M}}(w,m,e) = (w - c_m)D = (w - c_m)*[a - b(w+m) + ke] \quad (1)$$

$$\Pi_R(m,e) = (p - w - c_r)D - g(e)$$
$$= (m - c_r)*[a - b(w+m) + ke] - le^2 \quad (2)$$

III. STACKELBERG SUPPLY CHAIN GAME

This study analyzes a situation in which a manufacturer is dominant in the supply chain. Here, the manufacturer is the key enterprise in the supply chain, and the retailer is the follower. Provided that the information between the manufacturer and the retailer is symmetric, according to the Stackelberg game model, the manufacturer makes decisions first.

This study examines the two-stage supply chain in which the retailer plays a leading role. The leading position makes the retailer the core enterprise-also known as the leader-in the supply chain, and the manufacturer becomes the follower. We assume that the information between the two is symmetric. According to the Stackelberg model, the leading retailer will take the lead in making decisions. The decision variables are the marginal retail profit and the sales effort of the unit product. After observing the retailer's actions, the manufacturer will determine the unit wholesale price of its products based on the retailer's marginal retail profit and sales effort. Through this game process, the two will achieve their respective goals of maximizing profits.

A. Expected Profit Model

Since this study examines a supply chain game in a fuzzy environment, the profits of the the profits of both are uncertain and should be assumed to be expected profits. According to the aforementioned assumptions, when the retailer is dominant, the supply chain expectation model is as (3). In the two-level programming model, $E[\Pi_R(m,e,w)]$ is the retailer's expected profit, and $E[\Pi_R(w)]$ is the manufacturer's expected profit. By solving formula (3), we can obtain the following conclusions:

$$\begin{cases} \max\limits_{\{m,e\}} E[\Pi_r(m,e,w)] = \max\limits_{m} E\{(m - c_r)[a - b(w+m) + ke] - le^2\} \\ s.t. \\ m - c_r > 0 \\ w^* \text{ is the optimal solution of the model at the lower level} \\ \begin{cases} \max\limits_{w} E[\Pi_m(w)] = \max\limits_{w} E\{(w - c_m)[a - b(w+m) + ke]\} \\ s.t. \\ Pos\{a - b(w+m) + \mathrm{ke} \le 0\} = 0 \end{cases} \end{cases} \quad (3)$$

Theorem 1. Assume that the wholesale price of a unit product w is fixed, if

$$Pos\{a-b*[\frac{E(a)+E(c_m*b)}{2E(b)}+\frac{[E(a)+E(c_r*b)-E(c_m*b)]*E^2(k)-4E(b)*E(l)*[E(a)+E(c_r*b)-E(c_m*b)-2E(k)E(b)*\frac{1}{2}\int_0^1(c_{r\alpha}^U*k_\alpha^L+c_{r\alpha}^L*k_\alpha^U)d\alpha}{2E(b)*[8E(b)*E(l)-E^2(k)]}]\le 0\}=0$$

and $Pos\{c_m \ge \frac{4E(b)*E(l)*[E(a)-E(c_r*b)+3E(c_m*b)]+E^2(k)*[E(c_r*b)-2E(c_m*b)]+E(k)[4E(b)-E^2(k)]*\frac{1}{2}\int_0^1(c_{r\alpha}^U*k_\alpha^L+c_{r\alpha}^L*k_\alpha^U)d\alpha}{2E(b)*[8*E(b)*E(l)-E^2(k)]}\}=0$

Then, the manufacturer's optimal response function to the retailer's unit profit margin and sales effort is $w^*=-\frac{m}{2}+\frac{E(a)+e*E(k)+E(c_m*b)}{2E(b)}$.

Proposition 5. The optimal response function of the manufacturer's unit product wholesale price w^* is a strictly decreasing function of the retailer's unit product marginal profit m and a strictly increasing function of the sales effort e. This means that when the retailer increases its marginal profit per unit of product, the wholesale price will be reduced under the premise of keeping the sales price p unchanged. When a retailer makes sales efforts to expand market demand, the result of such demand means that the market price of unit products will be raised under the premise of constant supply, thereby increasing the wholesale price of unit products. This result is in line with the basic laws of supply and demand. The relevant proof is as follows:

$$\begin{aligned}
&E[\Pi_M(w)]\\
&=\frac{1}{2}\int_0^1\{[(w-c_m)*(a-b*(w+m)+ke)]_\alpha^L+[(w-c_m)*(a-b*(w+m)+ke)_\alpha^U\}d\alpha\\
&=\frac{1}{2}\int_0^1\{[(w-c_m)_\alpha^L*(a-b*(w+m)+ke)_\alpha^L+[(w-c_m)_\alpha^U*(a-b*(w+m)+ke)_\alpha^U\}d\alpha\\
&=\frac{1}{2}\int_0^1\{(w-c_{m\alpha}^U)*(a_\alpha^L-b_\alpha^U*(w+m)+k_\alpha^L e)+[(w-c_{m\alpha}^L)*(a_\alpha^U-b_\alpha^L*(w+m)+k_\alpha^U e)\}d\alpha\\
&=-w^2*E(b)+w*[E(a)-m*E(b)+e*E(k)+E(c_m*b)]-\frac{1}{2}\int_0^1(c_{m\alpha}^U*a_\alpha^L+c_{m\alpha}^L*a_\alpha^U)d\alpha
\end{aligned}$$

The first and second derivatives of the two ends of the above formula regarding w are respectively obtained as follows:

$$\begin{aligned}
&\frac{dE[\Pi_M(w)]}{dw}\\
&=-2*w*E(b)+E(a)-m*E(b)+e*E(k)+E(c_m*b)\\
&\frac{dE[\Pi_M(w)]}{dw}=-2*w*E(b)
\end{aligned}$$

So, $E[\prod_m(w)]$ is a concave function, and it is maximized at $w^*=-\frac{m}{2}+\frac{E(a)+e*E(k)+E(c_m b)}{2E(b)}$.

Obviously, w^* is a strongly decreasing function of m and an increasing function of e. This implies that, on the one hand, according to the assumption herein, there is a non-cooperative relationship between manufacturers and retailers, in which retailers play a leading role in the supply chain. Therefore, under the premise of keeping other factors unchanged, the increase of the retailer's unit profit (i.e., the increase of the retail price) will make the manufacturer's factory price drop. In contrast, the higher the retailer's sales efforts, the higher the manufacturer's ex-factory price. The possible reason for this is that when sales efforts increase, it stimulates social demand and causes the rise of the ex-factory price of goods under the condition that other factors remain unchanged. w^* is the manufacturer's optimal response function to the retailer's unit profit and sales effort.

Theorem 2. Suppose that $E[\prod_R(m,e)]$ is the expected value of the retailer's profit. According to the two-level programming model mentioned above, the following conclusions hold: if

$$Pos\{a-b*[\frac{E(a)+E(c_m*b)}{2E(b)}+\frac{[E(a)+E(c_r*b)-E(c_m*b)]*E^2(k)-4E(b)*E(l)*[E(a)+E(c_r*b)-E(c_m*b)-2E(k)E(b)*\frac{1}{2}\int_0^1(c_{r\alpha}^U*k_\alpha^L+c_{r\alpha}^L*k_\alpha^U)d\alpha}{2E(b)*[8E(b)*E(l)-E^2(k)]}]\le 0\}=0$$

and $Pos\{c_r \ge \frac{4E(b)*E(l)*[E(a)+E(c_r*b)-E(c_m*b)]+2E^2(k)*E(c_r*b)-4E(k)E(b)*\frac{1}{2}\int_0^1(c_{r\alpha}^U*k_\alpha^L+c_{r\alpha}^L*k_\alpha^U)d\alpha}{E(b)*[8E(b)*E(l)-E^2(k)]}\}=0$

Then, the retailer's optimal unit profit, optimal sales effort, and the manufacturer's optimal ex-factory price are:

$$m^*=\frac{4E(b)*E(l)*[E(a)+E(c_r*b)-E(c_m*b)]+2E^2(k)*E(c_r*b)-4E(k)E(b)*\frac{1}{2}\int_0^1(c_\alpha^U*k_\alpha^L+c_\alpha^L*k_\alpha^U)d\alpha}{E(b)*[8E(b)*E(l)-E^2(k)]}$$

$$e^*=\frac{E(k)*[E(a)+3E(c_r*b)-E(c_m*b)]-\frac{1}{2}\int_0^1(c_{r\alpha}^U*k_\alpha^L+c_{r\alpha}^L*k_\alpha^U)d\alpha}{8*E(b)*E(l)-E^2(k)}$$

$$w^*=\frac{4E(b)*E(l)*[E(a)-E(c_r*b)+3E(c_m*b)]+E^2(k)*[E(c_r*b)-2E(c_m*b)]+E(k)[4E(b)-E^2(k)]*\frac{1}{2}\int_0^1(c_{r\alpha}^U*k_\alpha^L+c_{r\alpha}^L*k_\alpha^U)d\alpha}{2E(b)*[8*E(b)*E(l)-E^2(k)]}$$

Theorem 3. At $(m^*,e^*,w^*(m^*,e^*))$, the retailer and the manufacturer achieved their maximum expected profit respectively,

$$\begin{aligned}
&E[\prod_R(m^*,e^*,w^*(m^*,e^*))]\\
&=\frac{1}{2}*\frac{4*E(k)*E(l)*[E(c_m*b)-E(a)-E(c_r*b)]-E^2(k)*E(c_r*b)}{E(b)*[8*E(b)*E(l)-E^2(k)]}\\
&*\frac{E^2(k)*[E(a)+2E(c_r*b)-E(c_m*b)]+4E(b)*E(l)*[E(c_m*b)-E(a)-E(c_r*b)]+E(a)+3E(c_r*b)-E(c_m*b)}{8*E(b)*E(l)-E^2(k)}\\
&+E(k)*\frac{E(a)+3E(c_r*b)-E(c_m*b)}{8*E(b)*E(l)-E^2(k)}*[\frac{E(k)}{2E(b)}*E(c_r*b)-\frac{E(a)+3E(c_r*b)-E(c_m*b)}{8*E(b)*E(l)-E^2(k)}*E(l)]+\frac{E(a)+E(c_m*b)}{2E(b)}*E(c_r*b)\\
&-\frac{1}{2}\int_0^1(c_{r\alpha}^U*a_\alpha^L+c_{r\alpha}^L*a_\alpha^U)d\alpha-\frac{E(a)+3E(c_r*b)-E(c_m*b)}{8*E(b)*E(l)-E^2(k)}*E(k)*\frac{1}{2}\int_0^1(c_{r\alpha}^U*k_\alpha^L+c_{r\alpha}^L*k_\alpha^U)d\alpha
\end{aligned} \tag{4}$$

$$
\begin{aligned}
&E[\prod_M(w^*,m^*,e^*)] \\
&=-\{\frac{E(b)*E(l)*[E(a)-E(c_r*b)+3E(c_m*b)]+E^2(k)*[E(c_r*b)-E(c_m*b)]}{8*E(b)*E(l)-E^2(k)}\}^2 \\
&+\frac{1}{E(b)}*\frac{E(b)*E(l)*[E(a)-E(c_r*b)+3E(c_m*b)]+E^2(k)*[E(c_r*b)-E(c_m*b)]}{8*E(b)*E(l)-E^2(k)} \\
&*\frac{E^2(k)*[E(a)-E(c_r*b)+E(c_m*b)]+4E(b)*E(l)*[E(c_r*b)-E(a)-3E(c_m*b)]+E(a)+3E(c_r*b)-E(c_m*b)}{8*E(b)*E(l)-E^2(k)} \\
&+\frac{1}{E(b)}*\frac{E(b)*E(l)*[E(c_m*b)-E(a)-E(c_r*b)]-E^2(k)-E(c_r*b)}{8*E(b)*E(l)-E^2(k)}*E(c_m*b) \\
&-\frac{1}{2}\int_0^1(c_{m\alpha}^U*a_\alpha^L+c_{m\alpha}^L*a_\alpha^U)d\alpha-\frac{E(a)+3E(c_r*b)-E(c_m*b)}{8*E(b)*E(l)-E^2(k)}*E(k)*\frac{1}{2}\int_0^1(c_{m\alpha}^U*k_\alpha^L+c_{m\alpha}^L*k_\alpha^U)d\alpha
\end{aligned}
\tag{5}
$$

Proof. The proof process is the same as in Proposition 5. Substituting w^* into the formula for the retailer's expected profit

$$
\begin{aligned}
&E[\prod_R(m,e,w^*(m,e))] \\
&=-\frac{1}{2}*m^2*E(b)+m*[E(a)+\frac{E(c_r*b)}{2}+\frac{e*E(k)-E(a)-E(c_m*b)}{2}] \\
&+\frac{e*E(k)+E(a)+E(c_m*b)}{2}*E(c_r*b)-e^2*E(l) \\
&-\frac{1}{2}\int_0^1(c_{r\alpha}^U*a_\alpha^L+c_{r\alpha}^L*a_\alpha^U)d\alpha-e*\frac{1}{2}\int_0^1(c_{r\alpha}^U*k_\alpha^L+c_{r\alpha}^L*k_\alpha^U)d\alpha
\end{aligned}
$$

We find the first, second, and the second partial derivative regarding, m and e, respectively:

$$
\frac{dE[\prod_r(m,e,w^*(m,e))]}{dm}=\frac{e*E(k)+E(a)+E(c_r*b)-E(c_m*b)}{2}-m*E(b)
$$

$$
\frac{d^2E[\prod_r(m,e,w^*(m,e))]}{dm^2}=-E(b)
$$

$$
\frac{d^2E[\prod_r(m,e,w^*(m,e))]}{dmde}=\frac{E(k)}{2}
$$

$$
\begin{aligned}
&\frac{dE[\prod_r(m,e,w^*(m,e))]}{de} \\
&=-2e*E(l)+\frac{1}{2}m*E(k)+\frac{E(k)}{2E(b)}*E(c_r*b) \\
&-\frac{1}{2}\int_0^1(c_{r\alpha}^U*k_\alpha^L+c_{r\alpha}^L*k_\alpha^U)d\alpha
\end{aligned}
$$

$$
\frac{d^2E[\prod_r(m,e,w^*(m,e))]}{de^2}=-2E(l)
$$

$$
\frac{d^2E[\prod_r(m,e,w^*(m,e))]}{dedm}=\frac{E(k)}{2}
$$

The Hessian matrix is as follows:

$$
H=\begin{bmatrix}-E(b) & \frac{E(k)}{2} \\ \frac{E(k)}{2} & -2E(l)\end{bmatrix}
$$

$$
|H|=\begin{vmatrix}-E(b) & \frac{E(k)}{2} \\ \frac{E(k)}{2} & -2E(l)\end{vmatrix}=2E(b)*E(l)-\frac{E^2(k)}{4}
$$

Generally speaking, demand is more sensitive to price than to sales effort, so the value of the above determinant is a positive number. Since $-E(b)\prec 0$, the retailer's profit function is concave, and the maximum value is obtained at $(m^*,e^*,w^*(m^*,e^*))$, as shown in formulas (4) and (5).

Therefore, the pricing strategy $(m^*,e^*,w^*(m^*,e^*))$ is the Stackelberg Nash equilibrium solution of the supply chain expectation model.

B. The Opportunity Constraint Model

In the two-stage supply chain, in addition to the expected value profit model, the maximum and the minimum opportunity constraint models minimax can be established respectively.

We first establish the maximum opportunity constraint model maximax as follows:

$$
\begin{cases}
\max\limits_m \prod_R \\
s.t \\
Pos\{(m-c_r)[a-b(w+m)+ke]-le^2\geq\prod_r\}\geq\alpha \\
m-c_r>0 \\
w^* \text{ is the optimal solution for lower-level planning} \\
\begin{cases}
\max\limits_w \prod_M \\
s.t \\
Pos\{(w-c_m)(a-b(w+m)+ke)\geq\prod_m\}\geq\alpha \\
Pos\{a-b(w+m)+ke\leq 0\}=0 \\
Pos\{w-c_m\leq 0\}=0
\end{cases}
\end{cases}
\tag{6}
$$

Where α is the pre-defined confidence level for manufacturers and retailers. For any given feasible strategy (w,m,e), $\max\limits_m\prod_r$ and $\max\limits_w\prod_m$ are the α optimistic values of the profits of retailers and manufacturers, respectively. Hence, model (6) can be equivalent to the following model:

$$
\begin{cases}
\max\limits_m((m-c_r)(a-b(w+m)+ke)-le^2)_\alpha^U \\
s.t \\
m-c_r>0 \\
w^* \text{ is the optimal solution for lower-level planning} \\
\begin{cases}
\max\limits_w((w-c_m)(a-b(w+m)+ke))_\alpha^U \\
s.t \\
Pos\{a-b(w+m)+ke\leq 0\}=0 \\
Pos\{w-c_m\leq 0\}=0
\end{cases}
\end{cases}
\tag{7}
$$

Where $(\prod_R(m,e,w(m,e)))_\alpha^U$ and $(\prod_M(w,m,e))_\alpha^U$ are the α optimistic values of the profits of retailers and manufacturers, respectively.

Proposition 6. If $Pos\{c_m\succ\frac{4_\alpha^L(a_\alpha^U+b_\alpha^L*c_{m\alpha}^L)-(k_\alpha^U)^2*c_{r\alpha}^L}{8b_\alpha^L*l_\alpha^L}+\frac{1}{2}*\frac{(k_\alpha^U)^2-4b_\alpha^L*l_\alpha^L}{8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2}*[\frac{a_\alpha^U}{b_\alpha^L}+c_{r\alpha}^L-c_{m\alpha}^L-\frac{(k_\alpha^U)^2*c_{r\alpha}^L}{4b_\alpha^L*l_\alpha^L}]\}=0$ and

$$
\begin{aligned}
&Pos\{a-b*(\frac{4_\alpha^L(a_\alpha^U+b_\alpha^L*c_{m\alpha}^L)-(k_\alpha^U)^2*c_{r\alpha}^L}{8b_\alpha^L*l_\alpha^L}+\frac{1}{2}*\frac{(k_\alpha^U)^2-4b_\alpha^L*l_\alpha^L}{8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2}*[\frac{a_\alpha^U}{b_\alpha^L}+c_{r\alpha}^L-c_{m\alpha}^L-\frac{(k_\alpha^U)^2*c_{r\alpha}^L}{4b_\alpha^L*l_\alpha^L})]+k*\frac{k_\alpha^U}{4l_\alpha^L}*[\frac{4b_\alpha^L*l_\alpha^L}{8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2}*(\frac{a_\alpha^U}{b_\alpha^L}+c_{r\alpha}^L-c_{m\alpha}^L-\frac{(k_\alpha^U)^2*c_{r\alpha}^L}{4b_\alpha^L*l_\alpha^L} \\
&-c_{r\alpha}^L]\}=0
\end{aligned}
$$

the above model has a unique α optimistic Stackelberg Nash equilibrium solution (m^*, e^*, w^*).

$$m^* = \frac{4b_\alpha^L * l_\alpha^L}{8b_\alpha^L * l_\alpha^L - (k_\alpha^U)^2} * [\frac{a_\alpha^U}{b_\alpha^L} + c_{r\alpha}^{\ L} - c_{m\alpha}^{\ L} - \frac{(k_\alpha^U)^2 * c_{r\alpha}^{\ L}}{4b_\alpha^L * l_\alpha^L}]$$

$$e^* = \frac{k_\alpha^U}{4l_\alpha^L} * [\frac{4b_\alpha^L * l_\alpha^L}{8b_\alpha^L * l_\alpha^L - (k_\alpha^U)^2} * (\frac{a_\alpha^U}{b_\alpha^L} + c_{r\alpha}^{\ L} - c_{m\alpha}^{\ L} - \frac{(k_\alpha^U)^2 * c_{r\alpha}^{\ L}}{4b_\alpha^L * l_\alpha^L}) - c_{r\alpha}^{\ L}]$$

$$w^* = \frac{4_\alpha^L (a_\alpha^U + b_\alpha^L * c_{m\alpha}^{\ L}) - (k_\alpha^U)^2 * c_{r\alpha}^{\ L}}{8b_\alpha^L * l_\alpha^L}$$

$$+ \frac{1}{2} * \frac{(k_\alpha^U)^2 - 4b_\alpha^L * l_\alpha^L}{8b_\alpha^L * l_\alpha^L - (k_\alpha^U)^2} * [\frac{a_\alpha^U}{b_\alpha^L} + c_{r\alpha}^{\ L} - c_{m\alpha}^{\ L} - \frac{(k_\alpha^U)^2 * c_{r\alpha}^{\ L}}{4b_\alpha^L * l_\alpha^L}]$$

Proof. The optimistic value function of the manufacturer's profit is

$$\max_w (\prod_M (w))_\alpha^U$$
$$= ((w - c_m)(a - b(w+m) + ke))_\alpha^U$$
$$= (w - c_{m\alpha}^{\ L}) * (a_\alpha^U - b_\alpha^L (w+m) + k_\alpha^U e)$$
$$= -w^2 b_\alpha^L + w(a_\alpha^U - m * b_\alpha^L + ek_\alpha^U + c_{m\alpha}^{\ L} b_\alpha^L)$$
$$+ m * c_{m\alpha}^{\ L} b_\alpha^L - c_{r\alpha}^{\ L} a_\alpha^U - e * c_{m\alpha}^{\ L} k_\alpha^U$$

For manufacturers, m and e are exogenous variables, so the above formula only needs to find the first and second derivatives, respectively, with respect to w:

$$\frac{d \max_w (\prod_M (w))_\alpha^U}{dw} = -2w * b_\alpha^L + a_\alpha^U - m * b_\alpha^L + e * k_\alpha^U + b_\alpha^L * c_{m\alpha}^{\ L}$$

$$\frac{d^2 \max_w (\prod_M (w))_\alpha^U}{dw^2} = -2b_\alpha^L \prec 0$$

Hence, $\max_w (\prod_M (w))_\alpha^U$ is a concave function, and obtains the maximum value at

$$w^* = \frac{a_\alpha^U - m * b_\alpha^L + e * k_\alpha^U + b_\alpha^L * c_{m\alpha}^{\ L}}{2b_\alpha^L}.$$

Obviously, w^* is a strictly decreasing function of m, and an increasing function of e, and its economic meaning is consistent with the previous part.

By substituting w^* into the retailer's profit optimizing function, we can obtain

$$\max_{\{m,e\}} (\prod_R (m,e))_\alpha^U$$
$$= ((m - c_r)(a - b(w+m) + ke) - le^2)_\alpha^U$$
$$= (m - c_{r\alpha}^{\ L}) * (a_\alpha^U - b_\alpha^L (w+m) + k_\alpha^U e - l_\alpha^L e^2)$$
$$= -\frac{b_\alpha^L}{2} * m^2 + \frac{a_\alpha^U + e * k_\alpha^U - c_{m\alpha}^{\ L} b_\alpha^L}{2} * m$$
$$- \frac{a_\alpha^U + e * k_\alpha^U - c_{m\alpha}^{\ L} b_\alpha^L}{2} * c_{r\alpha}^{\ L} b_\alpha^L - e^2 * l_\alpha^L$$

Since m and e are both decision variables of retailers, we find the first, second, and the second partial derivative of the above formula with m and e, respectively:

$$\frac{d \max_m (\prod_R (m,e))_\alpha^U}{dm} = -m * b_\alpha^L + \frac{a_\alpha^U + ek_\alpha^U - c_{m\alpha}^{\ L} b_\alpha^L + c_{r\alpha}^{\ L} b_\alpha^L}{2}$$

$$\frac{d^2 \max_m (\prod_R (m,e))_\alpha^U}{dm^2} = -b_\alpha^L \prec 0$$

$$\frac{\partial^2 \max_m (\prod_R (m,e))_\alpha^U}{\partial m \partial e} = \frac{k_\alpha^U}{2}$$

$$\frac{d \max_e (\prod_R (m,e))_\alpha^U}{de} = -2e * l_\alpha^L + \frac{k_\alpha^U}{2} * m - \frac{k_\alpha^U * c_{r\alpha}^{\ L}}{2}$$

$$\frac{d^2 \max_e (\prod_R (m,e))_\alpha^U}{de^2} = -2l_\alpha^L$$

$$\frac{\partial^2 \max_e (\prod_R (m,e))_\alpha^U}{\partial e \partial m} = \frac{k_\alpha^U}{2}$$

According to the first and second derivatives of the retailer's optimistic profit function, the value of the corresponding Hessian matrix and its corresponding determinant can be obtained as follows:

$$H = \begin{bmatrix} -b_\alpha^L & \frac{k_\alpha^U}{2} \\ \frac{k_\alpha^U}{2} & -2l_\alpha^L \end{bmatrix}, \quad |H| = \begin{vmatrix} -b_\alpha^L & \frac{k_\alpha^U}{2} \\ \frac{k_\alpha^U}{2} & -2l_\alpha^L \end{vmatrix} = 2b_\alpha^L l_\alpha^L - \frac{(k_\alpha^U)^2}{4}$$

Similarly, demand is in general more sensitive to price than to sales effort, so the value of the above determinant is a positive number greater than zero. Because $-b_\alpha^L \prec 0$, the retailer's optimistic profit function is a concave function and obtains the maximum value at $(m^*, e^*, w^*(m^*, e^*))$.

Therefore, the most optimistic profits of the manufacturer and retailer are respectively as follow:

$$\max_w (\prod_m (w))_\alpha^U$$

$$= -b_\alpha^L * \{\frac{a_\alpha^U + b_\alpha^L * c_{m\alpha}^{\ L}}{2b_\alpha^L} - \frac{1}{2} * \frac{4a_\alpha^U * l_\alpha^L - 4b_\alpha^U * l_\alpha^L * c_{m\alpha}^{\ L} - (k_\alpha^U)^2 * c_{r\alpha}^{\ L}}{8b_\alpha^L * l_\alpha^L - (k_\alpha^U)^2} + \frac{k_\alpha^U}{2b_\alpha^L} * \frac{k_\alpha^U * [4a_\alpha^U * l_\alpha^L - 4b_\alpha^U * l_\alpha^L * c_{m\alpha}^{\ L} - (k_\alpha^U)^2 * c_{r\alpha}^{\ L} - c_{r\alpha}^{\ L}]}{4l_\alpha^L * (8b_\alpha^L * l_\alpha^L - (k_\alpha^U)^2)}\}^2$$

$$+ \{\frac{a_\alpha^U + b_\alpha^L * c_{m\alpha}^{\ L}}{2b_\alpha^L} - \frac{1}{2} * \frac{4a_\alpha^U * l_\alpha^L - 4b_\alpha^U * l_\alpha^L * c_{m\alpha}^{\ L} - (k_\alpha^U)^2 * c_{r\alpha}^{\ L}}{8b_\alpha^L * l_\alpha^L - (k_\alpha^U)^2} + \frac{k_\alpha^U}{2b_\alpha^L} * \frac{k_\alpha^U * [4a_\alpha^U * l_\alpha^L - 4b_\alpha^U * l_\alpha^L * c_{m\alpha}^{\ L} - (k_\alpha^U)^2 * c_{r\alpha}^{\ L} - c_{r\alpha}^{\ L}]}{4l_\alpha^L * (8b_\alpha^L * l_\alpha^L - (k_\alpha^U)^2)}\}$$

$$* \{a_\alpha^U - \frac{4a_\alpha^U * l_\alpha^L - 4b_\alpha^U * l_\alpha^L * c_{m\alpha}^{\ L} - (k_\alpha^U)^2 * c_{r\alpha}^{\ L}}{8b_\alpha^L * l_\alpha^L - (k_\alpha^U)^2} * b_\alpha^L + \frac{(k_\alpha^U)^2 * [4a_\alpha^U * l_\alpha^L - 4b_\alpha^U * l_\alpha^L * c_{m\alpha}^{\ L} - (k_\alpha^U)^2 * c_{r\alpha}^{\ L} - c_{r\alpha}^{\ L}]}{4l_\alpha^L * (8b_\alpha^L * l_\alpha^L - (k_\alpha^U)^2)} + b_\alpha^L c_{m\alpha}^{\ L}\}$$

$$- a_\alpha^U c_{m\alpha}^{\ L} + \frac{4a_\alpha^U * l_\alpha^L - 4b_\alpha^U * l_\alpha^L * c_{m\alpha}^{\ L} - (k_\alpha^U)^2 * c_{r\alpha}^{\ L}}{8b_\alpha^L * l_\alpha^L - (k_\alpha^U)^2} * b_\alpha^L c_{m\alpha}^{\ L} - \frac{(k_\alpha^U)^2 * c_{m\alpha}^{\ L} * [4a_\alpha^U * l_\alpha^L - 4b_\alpha^U * l_\alpha^L * c_{m\alpha}^{\ L} - (k_\alpha^U)^2 * c_{r\alpha}^{\ L} - c_{r\alpha}^{\ L}]}{4l_\alpha^L * (8b_\alpha^L * l_\alpha^L - (k_\alpha^U)^2)}$$

$$\max_{\{m,e\}} (\prod_r (m,e))_\alpha^U$$

$$= -\frac{b_\alpha^L}{2} * \{\frac{4a_\alpha^U * l_\alpha^L - 4b_\alpha^U * l_\alpha^L * c_{m\alpha}^{\ L} - (k_\alpha^U)^2 * c_{r\alpha}^{\ L}}{8b_\alpha^L * l_\alpha^L - (k_\alpha^U)^2}\}^2$$

$$+ (\frac{a_\alpha^U - b_\alpha^L c_{m\alpha}^{\ L}}{2} + \frac{k_\alpha^U}{2} * \frac{k_\alpha^U * [4a_\alpha^U * l_\alpha^L - 4b_\alpha^U * l_\alpha^L * c_{m\alpha}^{\ L} - (k_\alpha^U)^2 * c_{r\alpha}^{\ L} - c_{r\alpha}^{\ L}]}{4l_\alpha^L * (8b_\alpha^L * l_\alpha^L - (k_\alpha^U)^2)}) * (\frac{4a_\alpha^U * l_\alpha^L - 4b_\alpha^U * l_\alpha^L * c_{m\alpha}^{\ L} - (k_\alpha^U)^2 * c_{r\alpha}^{\ L}}{8b_\alpha^L * l_\alpha^L - (k_\alpha^U)^2} - c_{r\alpha}^{\ L})$$

$$- l_\alpha^L * \{\frac{k_\alpha^U * [4a_\alpha^U * l_\alpha^L - 4b_\alpha^U * l_\alpha^L * c_{m\alpha}^{\ L} - (k_\alpha^U)^2 * c_{r\alpha}^{\ L} - c_{r\alpha}^{\ L}]}{4l_\alpha^L * (8b_\alpha^L * l_\alpha^L - (k_\alpha^U)^2)}\}^2$$

From the above analysis, we can see that the pricing strategy (m^*, e^*, w^*) is the only equilibrium solution for manufacturers and retailers to achieve the α optimistic value.

In contrast, an opportunity constraint model minimax can be established as follows:

$$\begin{cases} \max\limits_{m} \min\limits_{\Pi_r} \Pi_r \\ s.t \\ Pos\{(m-c_r)[a-b(w+m)+ke]-le^2 \le \Pi_r\} \ge \alpha \\ m-c_r>0 \\ w^* \text{ is the optimal solution for lower-level planning} \\ \begin{cases} \max\limits_{w} \min\limits_{\Pi_m} \Pi_m \\ s.t \\ Pos\{(w-c_m)(a-b(w+m)+ke) \le \Pi_m\} \ge \alpha \\ Pos\{a-b(w+m)+ke \le 0\}=0 \\ Pos\{w-c_m \le 0\}=0 \end{cases} \end{cases} \tag{8}$$

Where α is the pre-defined confidence level for manufacturers and retailers. For any given and feasible strategy (w,m,e), $\max\limits_{m}\min\limits_{\Pi_r}\Pi_r$ and $\max\limits_{w}\min\limits_{\Pi_m}\Pi_m$ are the α pessimistic values of the profits of retailers and manufacturers, respectively, so model (8) can be equivalent to the following model:

$$\begin{cases} \max\limits_{m} ((m-c_r)(a-b(w+m)+ke)-le^2)_\alpha^L \\ s.t \\ m-c_r>0 \\ w^* \text{ is the optimal solution of the model in the lower level} \\ \begin{cases} \max\limits_{w}((w-c_m)(a-b(w+m)+ke))_\alpha^L \\ s.t \\ Pos\{a-b(w+m)+ke \le 0\}=0 \\ Pos\{w-c_m \le 0\}=0 \end{cases} \end{cases} \tag{9}$$

Where $(\Pi_r(m,e,w(m,e)))_\alpha^L$ and $(\Pi_m(w,m,e))_\alpha^L$ are the α pessimistic values of the profits of retailers and manufacturers, respectively.

Regarding the above model (9), the following conclusions are established:

Proposition 7. If $Pos\{c_m \succ \frac{4b_\alpha^U(a_\alpha^L+b_\alpha^U*c_{m\alpha}^U)-(k_\alpha^L)^2*c_{r\alpha}^U}{8b_\alpha^U*l_\alpha^U}+\frac{1}{2}*\frac{(k_\alpha^L)^2-4b_\alpha^U*l_\alpha^U}{8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2}*[\frac{a_\alpha^L}{b_\alpha^U}+c_{r\alpha}^U-c_{m\alpha}^U-\frac{(k_\alpha^L)^2*c_{r\alpha}^U}{4b_\alpha^U*l_\alpha^U}]\}=0$

and

$Pos\{a-b*(\frac{4_\alpha^U(a_\alpha^L+b_\alpha^U*c_{m\alpha}^U)-(k_\alpha^L)^2*c_{r\alpha}^U}{8b_\alpha^U*l_\alpha^U}+\frac{1}{2}*\frac{(k_\alpha^L)^2-4b_\alpha^U*l_\alpha^U}{8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2}*[\frac{a_\alpha^L}{b_\alpha^U}+c_{r\alpha}^U-c_{m\alpha}^U-\frac{(k_\alpha^L)^2*c_{r\alpha}^U}{4b_\alpha^U*l_\alpha^U})]+k*\frac{k_\alpha^L}{4l_\alpha^U}*[\frac{4b_\alpha^U*l_\alpha^U}{8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2}*(\frac{a_\alpha^L}{b_\alpha^U}+c_{r\alpha}^U-c_{m\alpha}^U$
$-\frac{(k_\alpha^L)^2*c_{r\alpha}^U}{4b_\alpha^U*l_\alpha^U})-c_{r\alpha}^U]\}=0$

Then, model (9) has a unique α optimistic Stackelberg Nash equilibrium solution (m^*, e^*, w^*).

$$m^*=\frac{4b_\alpha^U*l_\alpha^U}{8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2}*[\frac{a_\alpha^L}{b_\alpha^U}+c_{r\alpha}^U-c_{m\alpha}^U-\frac{(k_\alpha^L)^2*c_{r\alpha}^U}{4b_\alpha^U*l_\alpha^U}]$$

$$e^*=\frac{k_\alpha^L}{4l_\alpha^U}*[\frac{4b_\alpha^U*l_\alpha^U}{8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2}*(\frac{a_\alpha^L}{b_\alpha^U}+c_{r\alpha}^U-c_{m\alpha}^U-\frac{(k_\alpha^L)^2*c_{r\alpha}^U}{4b_\alpha^U*l_\alpha^U})-c_{r\alpha}^U]$$

$$w^*=\frac{4_\alpha^U(a_\alpha^L+b_\alpha^U*c_{m\alpha}^U)-(k_\alpha^L)^2*c_{r\alpha}^U}{8b_\alpha^U*l_\alpha^U}+\frac{1}{2}*\frac{(k_\alpha^L)^2-4b_\alpha^U*l_\alpha^U}{8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2}*[\frac{a_\alpha^L}{b_\alpha^U}+c_{r\alpha}^U-c_{m\alpha}^U-\frac{(k_\alpha^L)^2*c_{r\alpha}^U}{4b_\alpha^U*l_\alpha^U}]$$

Proof. The proof process is the same as in Proposition 6. According to the above analysis process, we can summarize the game equilibrium of the two-stage supply chain, as shown in Tables II and III below.

TABLE II
GAME EQUILIBRIUM SOLUTION OF A RETAILER-DOMINATED TWO-STAGE SUPPLY CHAIN UNDER THE FUZZY ENVIRONMENT CONSIDERING SALES EFFORT

Ranking criterion	Optimal unit product profit w^*	Optimal wholesale price m^*	Optimal sales effort degree e^*
Expectation criterion	$\frac{4E(b)*E(l)*[E(a)-E(c_r*b)+3E(c_m*b)]}{2E(b)*[8*E(b)*E(l)-E^2(k)]}+\frac{E^2(k)*[E(c_r*b)-2E(c_m*b)]}{2E(b)*[8*E(b)*E(l)-E^2(k)]}+\frac{E(k)[4E(b)-E^2(k)]*\frac{1}{2}\int_0^1(c_{r\alpha}^U*k_\alpha^L+c_{r\alpha}^L*k_\alpha^U)d\alpha}{2E(b)*[8*E(b)*E(l)-E^2(k)]}$	$\frac{4E(b)*E(l)*[E(a)+E(c_r*b)-E(c_m*b)]}{E(b)*[8E(b)*E(l)-E^2(k)]}+\frac{2E^2(k)*E(c_r*b)}{E(b)*[8E(b)*E(l)-E^2(k)]}-\frac{4E(k)E(b)*\frac{1}{2}\int_0^1(c_{r\alpha}^U*k_\alpha^L+c_{r\alpha}^L*k_\alpha^U)d\alpha}{E(b)*[8E(b)*E(l)-E^2(k)]}$	$\frac{E(k)*[E(a)+3E(c_r*b)-E(c_m*b)]}{8*E(b)*E(l)-E^2(k)}-\frac{\frac{1}{2}\int_0^1(c_{r\alpha}^U*k_\alpha^L+c_{r\alpha}^L*k_\alpha^U)d\alpha}{8*E(b)*E(l)-E^2(k)}$
α -optimistic criterion	$\frac{4_\alpha^L(a_\alpha^U+b_\alpha^L*c_{m\alpha}^L)-(k_\alpha^U)^2*c_{r\alpha}^L}{8b_\alpha^L*l_\alpha^L}+\frac{1}{2}*\frac{(k_\alpha^U)^2-4b_\alpha^L*l_\alpha^L}{8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2}*[\frac{a_\alpha^U}{b_\alpha^L}+c_{r\alpha}^L-c_{m\alpha}^L-\frac{(k_\alpha^U)^2*c_{r\alpha}^L}{4b_\alpha^L*l_\alpha^L}]$	$\frac{4b_\alpha^L*l_\alpha^L}{8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2}*[\frac{a_\alpha^U}{b_\alpha^L}+c_{r\alpha}^L-c_{m\alpha}^L-\frac{(k_\alpha^U)^2*c_{r\alpha}^L}{4b_\alpha^L*l_\alpha^L}]$	$\frac{k_\alpha^U}{4l_\alpha^L}*\frac{4b_\alpha^L*l_\alpha^L}{8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2}*(\frac{a_\alpha^U}{b_\alpha^L}+c_{r\alpha}^L-c_{m\alpha}^L)-\frac{k_\alpha^U}{4l_\alpha^L}*[\frac{4b_\alpha^L*l_\alpha^L}{8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2}*\frac{(k_\alpha^U)^2*c_{r\alpha}^L}{4b_\alpha^L*l_\alpha^L}-c_{r\alpha}^L]$
α -pessimist criterion	$\frac{4_\alpha^U(a_\alpha^L+b_\alpha^U*c_{m\alpha}^U)-(k_\alpha^L)^2*c_{r\alpha}^U}{8b_\alpha^U*l_\alpha^U}+\frac{1}{2}*\frac{(k_\alpha^L)^2-4b_\alpha^U*l_\alpha^U}{8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2}*[\frac{a_\alpha^L}{b_\alpha^U}+c_{r\alpha}^U-c_{m\alpha}^U-\frac{(k_\alpha^L)^2*c_{r\alpha}^U}{4b_\alpha^U*l_\alpha^U}]$	$\frac{4b_\alpha^U*l_\alpha^U}{8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2}*[\frac{a_\alpha^L}{b_\alpha^U}+c_{r\alpha}^U-c_{m\alpha}^U-\frac{(k_\alpha^L)^2*c_{r\alpha}^U}{4b_\alpha^U*l_\alpha^U}]$	$\frac{k_\alpha^L}{4l_\alpha^U}*\frac{4b_\alpha^U*l_\alpha^U}{8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2}*(\frac{a_\alpha^L}{b_\alpha^U}+c_{r\alpha}^U-c_{m\alpha}^U)-\frac{k_\alpha^L}{4l_\alpha^U}*[\frac{4b_\alpha^U*l_\alpha^U}{8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2}*\frac{(k_\alpha^L)^2*c_{r\alpha}^U}{4b_\alpha^U*l_\alpha^U}-c_{r\alpha}^U]$

TABLE III
THE MAXIMUM PROFIT OF THE TWO-STAGE SUPPLY CHAIN LED BY THE RETAILER UNDER THE FUZZY ENVIRONMENT CONSIDERING THE SELLING EFFORT

Ranking criterion	Manufacturer's maximum profit
Expectation criterion	$\begin{aligned} &-\{\frac{E(b)*E(l)*[E(a)-E(c_r*b)+3E(c_m*b)]+E^2(k)*[E(c_r*b)-E(c_m*b)]}{8*E(b)*E(l)-E^2(k)}\}^2+\frac{1}{E(b)}*\frac{E(b)*E(l)*[E(a)-E(c_r*b)+3E(c_m*b)]+E^2(k)*[E(c_r*b)-E(c_m*b)]}{8*E(b)*E(l)-E^2(k)} \\ &*\frac{E^2(k)*[E(a)-E(c_r*b)+E(c_m*b)]+4E(b)*E(l)*[E(c_r*b)-E(a)-3E(c_m*b)]+E(a)+3E(c_r*b)-E(c_m*b)}{8*E(b)*E(l)-E^2(k)} \\ &+\frac{1}{E(b)}*\frac{E(b)*E(l)*[E(c_m*b)-E(a)-E(c_r*b)]-E^2(k)-E(c_r*b)}{8*E(b)*E(l)-E^2(k)}*E(c_m*b)-\frac{1}{2}\int_0^1(c_{m\alpha}^U*a_\alpha^L+c_{m\alpha}^L*a_\alpha^U)d\alpha-\frac{E(a)+3E(c_r*b)-E(c_m*b)}{8*E(b)*E(l)-E^2(k)}*E(k)*\frac{1}{2}\int_0^1(c_{m\alpha}^U*k_\alpha^L+c_{m\alpha}^L*k_\alpha^U)d\alpha \end{aligned}$
α -optimistic	$\begin{aligned} &-b_\alpha^L*\{\frac{a_\alpha^U+b_\alpha^L*c_{m\alpha}^L}{2b_\alpha^L}-\frac{1}{2}*\frac{4a_\alpha^U*l_\alpha^L-4b_\alpha^U*l_\alpha^L*c_{m\alpha}^L-(k_\alpha^U)^2*c_{r\alpha}^L}{8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2}+\frac{k_\alpha^U}{2b_\alpha^L}*\frac{k_\alpha^U*[4a_\alpha^U*l_\alpha^L-4b_\alpha^U*l_\alpha^L*c_{m\alpha}^L-(k_\alpha^U)^2*c_{r\alpha}^L-c_{r\alpha}^L]}{4l_\alpha^L*(8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2)}\}^2 \\ &+\{\frac{a_\alpha^U+b_\alpha^L*c_{m\alpha}^L}{2b_\alpha^L}-\frac{1}{2}*\frac{4a_\alpha^U*l_\alpha^L-4b_\alpha^U*l_\alpha^L*c_{m\alpha}^L-(k_\alpha^U)^2*c_{r\alpha}^L}{8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2}+\frac{k_\alpha^U}{2b_\alpha^L}*\frac{k_\alpha^U*[4a_\alpha^U*l_\alpha^L-4b_\alpha^U*l_\alpha^L*c_{m\alpha}^L-(k_\alpha^U)^2*c_{r\alpha}^L-c_{r\alpha}^L]}{4l_\alpha^L*(8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2)}\} \\ &*\{a_\alpha^U-\frac{4a_\alpha^U*l_\alpha^L-4b_\alpha^U*l_\alpha^L*c_{m\alpha}^L-(k_\alpha^U)^2*c_{r\alpha}^L}{8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2}*b_\alpha^L+\frac{(k_\alpha^U)^2*[4a_\alpha^U*l_\alpha^L-4b_\alpha^U*l_\alpha^L*c_{m\alpha}^L-(k_\alpha^U)^2*c_{r\alpha}^L-c_{r\alpha}^L]}{4l_\alpha^L*(8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2)}+b_\alpha^L c_{m\alpha}^L\} \\ &-a_\alpha^U c_{m\alpha}^L+\frac{4a_\alpha^U*l_\alpha^L-4b_\alpha^U*l_\alpha^L*c_{m\alpha}^L-(k_\alpha^U)^2*c_{r\alpha}^L}{8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2}*b_\alpha^L c_{m\alpha}^L-\frac{(k_\alpha^U)^2*c_{m\alpha}^L*[4a_\alpha^U*l_\alpha^L-4b_\alpha^U*l_\alpha^L*c_{m\alpha}^L-(k_\alpha^U)^2*c_{r\alpha}^L-c_{r\alpha}^L]}{4l_\alpha^L*(8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2)} \\ &-a_\alpha^U c_{m\alpha}^L+\frac{4a_\alpha^U*l_\alpha^L-4b_\alpha^U*l_\alpha^L*c_{m\alpha}^L-(k_\alpha^U)^2*c_{r\alpha}^L}{8b_\alpha^U*l_\alpha^L-k_\alpha^U}*b_\alpha^L c_{m\alpha}^L-\frac{(k_\alpha^U)^2*c_{m\alpha}^L*[4a_\alpha^U*l_\alpha^L-4b_\alpha^U*l_\alpha^L*c_{m\alpha}^L-(k_\alpha^U)^2*c_{r\alpha}^L-c_{r\alpha}^L]}{4l_\alpha^L*(8b_\alpha^U*l_\alpha^L-k_\alpha^U)} \end{aligned}$
α -pessimistic	$\begin{aligned} &-b_\alpha^U*\{\frac{aL+b_\alpha^U*c_{m\alpha}^U}{2b_\alpha^U}-\frac{1}{2}*\frac{4a_\alpha^L*l_\alpha^U-4b_\alpha^L*l_\alpha^U*c_{m\alpha}^U-(k_\alpha^L)^2*c_{r\alpha}^U}{8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2}+\frac{k_\alpha^L}{2b_\alpha^U}*\frac{k_\alpha^L*[4a_\alpha^L*l_\alpha^U-4b_\alpha^L*l_\alpha^U*c_{m\alpha}^U-(k_\alpha^L)^2*c_{r\alpha}^U-c_{r\alpha}^U]}{4l_\alpha^U*(8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2)}\}^2 \\ &+\{\frac{a_\alpha^U+b_\alpha^U*c_{m\alpha}^U}{2b_\alpha^U}-\frac{1}{2}*\frac{4a_\alpha^L*l_\alpha^U-4b_\alpha^L*l_\alpha^U*c_{m\alpha}^U-(k_\alpha^L)^2*c_{r\alpha}^U}{8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2}+\frac{k_\alpha^L}{2b_\alpha^U}*\frac{k_\alpha^L*[4a_\alpha^L*l_\alpha^U-4b_\alpha^L*l_\alpha^U*c_{m\alpha}^U-(k_\alpha^L)^2*c_{r\alpha}^U-c_{r\alpha}^U]}{4l_\alpha^U*(8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2)}\} \\ &*\{a_\alpha^L-\frac{4a_\alpha^L*l_\alpha^U-4b_\alpha^L*l_\alpha^U*c_{m\alpha}^U-(k_\alpha^L)^2*c_{r\alpha}^U}{8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2}*b_\alpha^U+\frac{(k_\alpha^L)^2*[4a_\alpha^L*l_\alpha^U-4b_\alpha^L*l_\alpha^U*c_{m\alpha}^U-(k_\alpha^L)^2*c_{r\alpha}^U-c_{r\alpha}^U]}{4l_\alpha^U*(8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2)}+b_\alpha^U c_{m\alpha}^U\} \\ &-a_\alpha^L c_{m\alpha}^U+\frac{4a_\alpha^L*l_\alpha^U-4b_\alpha^L*l_\alpha^U*c_{m\alpha}^U-(k_\alpha^L)^2*c_{r\alpha}^U}{8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2}*b_\alpha^U c_{m\alpha}^U-\frac{(k_\alpha^L)^2*c_{m\alpha}^U*[4a_\alpha^L*l_\alpha^U-4b_\alpha^L*l_\alpha^U*c_{m\alpha}^U-(k_\alpha^L)^2*c_{r\alpha}^U-c_{r\alpha}^U]}{4l_\alpha^U*(8b_\alpha^U*l_\alpha^U-(k_\alpha^L)^2)}-a_\alpha^L c_{m\alpha}^U \\ &+\frac{4a_\alpha^L*l_\alpha^U-4b_\alpha^L*l_\alpha^U*c_{m\alpha}^U-(k_\alpha^L)^2*c_{r\alpha}^U}{8b_\alpha^U*l_\alpha^L-k_\alpha^U}*b_\alpha^U c_{m\alpha}^U-\frac{(k_\alpha^L)^2*c_{m\alpha}^U*[4a_\alpha^L*l_\alpha^U-4b_\alpha^L*l_\alpha^U*c_{m\alpha}^U-(k_\alpha^L)^2*c_{r\alpha}^U-c_{r\alpha}^U]}{4l_\alpha^U*(8b_\alpha^L*l_\alpha^U-k_\alpha^L)} \end{aligned}$
Ranking criterion	Retailer's maximum profit
Expectation criterion	$\begin{aligned} &\frac{1}{2}*\frac{4*E(k)*E(l)*[E(c_m*b)-E(a)-E(c_r*b)]-E^2(k)*E(c_r*b)}{E(b)*[8*E(b)*E(l)-E^2(k)]}*\{\frac{E^2(k)*[E(a)+2E(c_r*b)-E(c_m*b)]+4E(b)*E(l)*[E(c_m*b)-E(a)-E(c_r*b)]}{8*E(b)*E(l)-E^2(k)} \\ &\frac{E(a)+3E(c_r*b)-E(c_m*b)}{8*E(b)*E(l)-E^2(k)}\}+E(k)*\frac{E(a)+3E(c_r*b)-E(c_m*b)}{8*E(b)*E(l)-E^2(k)}*[\frac{E(k)}{2E(b)}*E(c_r*b)-\frac{E(a)+3E(c_r*b)-E(c_m*b)}{8*E(b)*E(l)-E^2(k)}*E(l)] \\ &+\frac{E(a)+E(c_m*b)}{2E(b)}*E(c_r*b)-\frac{1}{2}\int_0^1(c_{r\alpha}^U*a_\alpha^L+c_{r\alpha}^L*a_\alpha^U)d\alpha-\frac{E(a)+3E(c_r*b)-E(c_m*b)}{8*E(b)*E(l)-E^2(k)}*E(k)*\frac{1}{2}\int_0^1(c_{r\alpha}^U*k_\alpha^L+c_{r\alpha}^L*k_\alpha^U)d\alpha \end{aligned}$
α -optimistic	$\begin{aligned} &-\frac{b_\alpha^L}{2}*\{\frac{4a_\alpha^U*l_\alpha^L-4b_\alpha^U*l_\alpha^L*c_{m\alpha}^L-(k_\alpha^U)^2*c_{r\alpha}^L}{8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2}\}^2 \\ &+(\frac{a_\alpha^U-b_\alpha^L c_{m\alpha}^L}{2}+\frac{k_\alpha^U}{2}*\frac{k_\alpha^U*[4a_\alpha^U*l_\alpha^L-4b_\alpha^U*l_\alpha^L*c_{m\alpha}^L-(k_\alpha^U)^2*c_{r\alpha}^L-c_{r\alpha}^L]}{4l_\alpha^L*(8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2)})*(\frac{4a_\alpha^U*l_\alpha^L-4b_\alpha^U*l_\alpha^L*c_{m\alpha}^L-(k_\alpha^U)^2*c_{r\alpha}^L}{8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2}-c_{r\alpha}^L) \\ &-l_\alpha^L*\{\frac{k_\alpha^U*[4a_\alpha^U*l_\alpha^L-4b_\alpha^U*l_\alpha^L*c_{m\alpha}^L-(k_\alpha^U)^2*c_{r\alpha}^L-c_{r\alpha}^L]}{4l_\alpha^L*(8b_\alpha^L*l_\alpha^L-(k_\alpha^U)^2)}\}^2 \end{aligned}$
α -pessimistic	$\begin{aligned} &-\frac{b_\alpha^U}{2}*\{\frac{4a_\alpha^L*l_\alpha^U-4b_\alpha^L*l_\alpha^U*c_{m\alpha}^U-(k_\alpha^L)^2*c_{r\alpha}^U}{8b_\alpha^L*l_\alpha^U-(k_\alpha^L)^2}\}^2 \\ &+(\frac{a_\alpha^L-b_\alpha^U c_{m\alpha}^U}{2}+\frac{(k_\alpha^L)^2*[4a_\alpha^L*l_\alpha^U-4b_\alpha^L*l_\alpha^U*c_{m\alpha}^U-(k_\alpha^L)^2*c_{r\alpha}^U-c_{r\alpha}^U]}{8l_\alpha^U*[8b_\alpha^L*l_\alpha^U-(k_\alpha^L)^2]})*(\frac{4a_\alpha^L*l_\alpha^U-4b_\alpha^L*l_\alpha^U*c_{m\alpha}^U-(k_\alpha^L)^2*c_{r\alpha}^U}{8b_\alpha^L*l_\alpha^U-(k_\alpha^L)^2}-c_{r\alpha}^U) \\ &-l_\alpha^U*\{\frac{k_\alpha^L*[4a_\alpha^L*l_\alpha^U-4b_\alpha^L*l_\alpha^U*c_{m\alpha}^U-(k_\alpha^L)^2*c_{r\alpha}^U-c_{r\alpha}^U]}{4l_\alpha^U*[8b_\alpha^L*l_\alpha^U-(k_\alpha^L)^2]}\}^2 \end{aligned}$

IV. NUMERICAL EXAMPLE

The above content solves the pricing strategy of each manufacturer in the two-stage supply chain led by the retailer. Next, a numerical example is used to illustrate the effectiveness of the game model.

Example. Assume that the manufacturing cost is c_m, operating cost is c_r, market capacity is a, the elasticity of demand to price is b, and elasticity of demand to sales effort is k. Then, the elasticity of sales cost to sales effort l is usually estimated by management decision-makers and experts. When making estimations, expressions such as "low cost" "large market capacity," and "sensitive demand change rate" are often used to make oral estimates. The estimator determines the relationship between fuzzy linguistic variables and triangular fuzzy numbers based on experience, as shown in Table IV.

TABLE IV
FUZZY LANGUAGE VARIABLES AND THEIR TRIANGULAR FUZZY NUMBERS

	Language variable	Triangle fuzzy value
Manufacturing costs	Lower (approximately 3)	(2,3,4)
	Medium (approximately 5)	(4,5,6)
	Higher (approximately 8)	(7,8,9)
Operational costs	Lower (approximately 2)	(1,2,3)
	Medium (approximately 3)	(2,3,4)
	Higher (approximately 6)	(5,6,7)
Market capacity	Very large (approximately 5000)	(4900,5000,5100)
	Small (approximately 3000)	(2500,3000,3500)
Elasticity of demand to price	Very sensitive (approximately 500)	(450,500,550)
	Sensitive (approximately 300)	(200,300,400)
Elasticity of demand to the sales effort	Very sensitive (approximately 100)	(90,100,110)
	Sensitive (approximately 60)	(50,60,70)
Elasticity of sales cost to the sales effort	Very sensitive (approximately 5)	(4,5,6)
	Sensitive (approximately 3)	(2,3,4)

TABLE V
THE OPTIMAL STRATEGY OF A TWO-STAGE SUPPLY CHAIN LED BY THE RETAILER UNDER THE FUZZY ENVIRONMENT CONSIDERING SALES EFFORT

Ranking criterion	Optimal wholesale price w^*	Optimal unit product profit m^*	Optimal sales effort degree e^*
Expectation criterion	7.9	9.033	50
	Retailer's maximum profit	Manufacturer's maximum profit	
	1104.444	3235.556	

TABLE VI
SENSITIVITY ANALYSIS OF THE OPTIMAL STRATEGY WITH THE CHANGE OF α

α value	Optimistic criterion			Pessimism criterion		
	w^*	m^*	e^*	w^*	m^*	e^*
α=1	12.719	23.157	167.978	8	9	50
α=0.95	14.991	27.824	211.852	7.439	7.829	38.976
α=0.9	18.549	35.064	280.046	7.033	6.966	30.867
α=0.85	24.865	47.820	400.391	6.727	6.304	24.659
α=0.8	39.043	76.301	669.384	6.490	5.781	19.759

TABLE VII
SENSITIVITY ANALYSIS OF THE MAXIMUM PROFIT WITH THE CHANGE OF α

α value	Optimistic criterion		Pessimism criterion	
	Manufacturer's maximum profit	Retailer's maximum profit	Manufacturer's maximum profit	Retailer's maximum profit
α=1	191567.794	77294.199	68500.000	8850.000
α=0.95	273772.161	123633.394	58491.748	5619.306
α=0.9	432845.182	216495.034	51846.077	3678.512
α=0.85	806285.138	442144.522	47196.005	2447.097
α=0.8	2068381.639	1231386.158	43816.759	1635.160

Suppose that the current situation under consideration is as follows. The market capacity of the product is estimated to be large (approximately 5000), demand is very sensitive to price changes (approximately 500) and to the change in the degree of sales effort (approximately 100), the sales cost is sensitive to the change in the degree of sales effort (approximately 3), the manufacturer's manufacturing cost is moderate (approximately 5), and the retailer's operating cost is moderate (approximately 3). According to the expected value model, fuzzy variables, and other relevant formulas, we can draw the conclusions shown in Tables V, VI, and VII.

Table V exhibits that in the Stackelberg game of the two-stage supply chain dominated by retailers when considering sales effort, the dominant retailers obtain a higher marginal profit per unit product. However, the expected profit is smaller than that of manufacturers. The possible reason is that retailers take the initiative in price setting by their dominance in the supply chain. Meanwhile, retailers expand the market demand for products and improve their profits through various sales efforts, which also enables manufacturers' profits to be improved due to the expansion of market demand.

It can be seen from Table VI and Table VII that the optimal strategy and maximum profit of the Stackelberg game will change with the different pre-defined confidence degrees of retailers and manufacturers. Under optimistic value criterion, with the decrease of the confidence degree, the optimal wholesale price gradually increases, and the retailer's marginal profit per unit product and the optimal degree of the sales effort also gradually increase. From the perspective of profit, with the decrease of the confidence degree, the manufacturer's and retailer's maximum profit also gradually increases. Under the pessimistic value criterion, with the decrease of the confidence degree, the optimal wholesale price of the manufacturer, the optimal unit marginal profit of the retailer, and the optimal sales effort degree gradually decrease. From the perspective of profit, the maximum pessimistic value profit of the manufacturer and the retailer gradually decreases.

V. CONCLUSION

In the case of market demand, the manufacturer's manufacturing cost and the retailer's operating cost are fuzzy variables. This study assumes that sales efforts will affect the market demand of the product and the cost for the company making the effort. Therefore, the degree of sales effort is introduced into the model as a fuzzy variable, which fully considers the influencing factors of the supply chain game, and enriches the expected value model, the opportunity constraint programming model, and the corresponding α optimistic and α pessimistic models of a two-stage supply chain that is dominated by the retailer under the fuzzy environment.

From the perspective of economic reality, in a two-stage supply chain composed of a manufacturer and a retailer, it is common for the retailer to make sales efforts. The retailer pays the cost of the sales effort. If the retailer can reap the corresponding benefits, their sales effort will not only improve the sales volume of the enterprise's products, but it will also increase the demand for the manufacturer's products due to the stimulation of consumption and the expansion of the market demand for the products. However, the cost of this action does not increase the cost of the manufacturing enterprise, so the manufacturer's profits will increase quicker than the retailer's. If the sales effort does not achieve the expected results, then the retailer will bear the cost. This mechanism once again proves the conclusion that the manufacturer's profit is higher than the retailer's profit.

This study used the Stackelberg game model to analyze the optimal pricing strategy of manufacturers and retailers and the maximum profit of participating manufacturers when retailers play a dominant role and make sales efforts. In the equilibrium result, although the retailer's total profit is lower than that of the manufacturer, the marginal profit per unit product is higher. It can be inferred that this is mainly due to the retailer's dominant position in the supply chain. This enables the retailer not only to improve product sales through sales efforts but also to find a way to lower the wholesale price to capture more unit profit. However, considering the cost of sales effort and the particularity of numerical examples, it is reasonable that the retailer's expected profit is lower than that of the manufacturer. From the numerical simulation analysis, we can see that the optimal pricing strategy and the optimal sales effort level of manufacturers and retailers are also related to the pre-determined confidence level. Under the optimistic value criterion, the value of each decision variable and manufacturers' profit will increase with the decrease of the confidence level. Under the pessimistic value criterion, the value of each decision variable and manufacturers' profit will decrease with the decrease in the confidence level. Theoretically, this conclusion is an effective extension of the conclusion under the fuzzy environment [19].

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