Predictive Indicator of Domestic Violence: Questionnaire and Mathematical Model

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Abstract—Mathematical models aimed at predicting Intimate Partner Violence (IPV) levels are being developed nowadays. However, most of them do not prescribe how to use them systematically by either care centers or experts. This article proposes a detailed questionnaire to identify probable domestic violence scenarios through a mathematical model whose key factor is the loss of control by the perpetrator. Based on the results collected, a table of indicators and graphs have been generated which could be used by care centers and experts to help in their process at making decisions.

Index Terms—predictive indicators, violence questionnaire, mathematical model, assistance to victims.

I. INTRODUCTION

Violence, in any of its forms, particularly Intimate Partner Violence (IPV), impacts the well-being and health of societies [1]. Statistical studies suggest that the perpetrators are men while the victims are women in about 75% of the cases, and that support social programs after the incidents take place as an average that ranges from one to two years. [2].

Violence is often persistent and cumulative in some cases (see study on physical violence [5]). According to the World Health Organization, IPV is one of the main causes of death in women: “Violence against women is a global health problem of epidemic proportions”, see [1].

In order to provide victims with help, the determination of their IPV status is required in the first place. A standard measure generally employed by public and private institutions consists of using risk assessment and monitoring programs which have a prevalence of between one and two years on average (several studies have measured the average level of violence in monthly intervals during an observation period of one year[2]). [2], [6]. The number of victims who seek help is relatively moderate, so it is very important that care centers provide them with good care, emotional support, advice and information on how to solve conflicts with their intimate partners (the percentage of perpetrators and victims seeking help varies according to age, race and social class, among others, for example, the percentage of adolescents looking for help is around 21% [7]); for example, the percentage of teenagers who do not seek help is around 79% [7].

In view of the above, it is important to have useful quantitative and qualitative tools to make favorable decisions for victims (a cycle of domestic violence basically comprehends the following steps: violent incidents (physical / sexual / emotional), tension accumulation (violent outbursts) and honeymoon phase [8]). As well as finding preventive methods to determine the factors that contribute to accelerate crimes due to the loss of control by the perpetrator. [9]. Such tools are basically a questionnaire designed to establish the IPV status, and a model aimed at establishing a quantitative and/or qualitative indicator which represents the status of the victim or risk level[10].

Both predictive tools and Early Warning Systems (EWS) are currently being developed. The present work presents a mathematical model designed to calculate the potential levels of violence that victims may experience in a period of twelve months following the identification of IPV [11], [12], [13], [14].

These tools could bring benefits for victims by showing them the possible violence scenarios that they might experience in the absence of either help or treatment [13], [7]; all the more so when we consider that most victims do not have a clear perception of the approaching danger [14].

The mathematical model proposed by Leal-Enríquez E. [13] was designed to calculate the probable IPV levels that may occur in a period of twelve months following the first IPV diagnosis of the victim at the care center (in order to estimate the initial level of violence (statistics), questionnaires are applied [10], [15], [16]).

Nonetheless, this model does not clearly state how to record both the frequency and severity of violent attacks, nor how the expert on violence could use the information to provide assistance to help the victim (the estimation of the initial level of violence can be quantified by applying a questionnaire, in which each query is assigned a severity weight [15]).

Therefore, a complete example of how to use Leal-Enríquez’s model [13] is developed in this work by analyzing both the victim’s responses to the questionnaire at the moment of arrival at the care center and the factor of loss of control by the perpetrator as manifested by his violent acts against his partner [13].

The premises of the model and key variables are as follows [13]: that violence can be measured by assessing both the severity weight of violent attacks over time, the loss of control by the perpetrator (treatments for loss of control contribute to reduce violence levels, some studies report a 53%, percent reduction for patients under treatment for a one-year period of time [17]), as well as the frequency of the attacks (measured by statistical tools [15]). With
these premises and added variables, Leal-Enríquez’s model is enabled to simulate the probable violent scenarios that the victim may suffer in the next twelve-month period.

Domestic violence treatments are socially justified because, besides providing care to the victims, they can help the perpetrators themselves in controlling their own violent behavior. Thus, contributing to diminish domestic violence levels (help programs regularly measure their own effectiveness in monthly periods [18]) [17], [18].

II. QUESTIONNAIRE

In this section, Leal-Enríquez’s model is broadened [13] by administering the victim a risk questionnaire (methodological proposal for measuring IPV in women [14], [15]), which measures the dimensions of psychological, sexual, physical and severe physical violence.

The questionnaire incorporates 18 items selected from the “Index of Spouse Abuse (ISA)” and the “Severity of Violence Against Women Scale (SVAWS)” [19], [20], both of which have proven themselves useful to quantify male domestic violence against women (see Table I).

The variable \( \omega_n \) (weight score) has been assigned for the severity degree pertaining to each of the factors (the method for assigning the weight of severity depends on the judgment on the subject of violence by experts [16], [15]) and the variable \( f_n \) for the frequency of violent actions suffered by women in recent months, whose values are: 0 = never, 1 = some times, 2 = several times and 3 = many times. For violence as such, the following forms or factors have been included: \( I \) = Psychological violence, \( II \) = Physical violence, \( III \) = Severe physical violence and \( IV \) = Sexual violence. To assign the corresponding factor to each of the factors, a factorial analysis is performed (for more details, see [14]).

Then a frequency \( f_n \), and a weight \( \omega_n \) are assigned to each questionnaire item; therefore, the level of violence \( \Omega_n \) for each question is calculated as follows [13], [15]:

\[
\Omega_n(0) = \omega_n(0) \times f_n(0),
\]

where \( f_n(0) = 0, 1, 2, 3 \) and \( \omega_n(0) \) take the values from Table I. For example, if a woman reports that she has been beaten or slapped on her face \((n = 8)\) frequently (\( f_8 = 3 \) and \( \omega_8 = 7 \)), the following is obtained [15]:

\[
\Omega_8(0) = \omega_8(0) \times f_8(0) = 7 \times 3 = 21.
\]

Hence, the initial level of violence against the victim \( \Omega(0) \), taking into account I, II, III and IV factors (where each of the questions has already been related to its proper dimension (so as to identify the classification of variables that best explains the dimension of each type of violence, a factorial analysis is usually performed [15]), is represented by the following equation [13], [15]:

\[
\Omega(0) = \sum_{n=1}^{4} \omega_n(0) f_n(0),
\]

Equation (3) can be separated into each of its four dimensions. For example, for the questionnaire shown in Table I, the outcome is as follows:

\[
\Omega(0) = \sum_{n=1}^{5} \omega_n(0) f_n(0) + \sum_{n=6}^{10} \omega_n(0) f_n(0) + \sum_{n=11}^{15} \omega_n(0) f_n(0) + \sum_{n=16}^{18} \omega_n(0) f_n(0)
\]

\[
\Omega_I(0) + \Omega_{II}(0) + \Omega_{III}(0) + \Omega_{IV}(0).
\]

The violence indicator assigned in (3) is not specific and indicates the initial condition of the victim when she arrives at the care center (this indicator comprehends the psychological, physical, severe physical and sexual dimensions [15], or else it might be determined by the assessment of a health worker). The calculation of this initial condition is shown in Table I (see (3) [15], as well as, in the same Table, the calculation of the initial global condition is performed (see (3))).

A. Categorization

To verbally categorize the IPV initial condition, the following categories are proposed (the procedure to categorize IPV is based on the minimum, maximum and average values of the data obtained for each factor of domestic violence. For more details see [15], see Table I [15]):

- Factor I, \( \Omega_I(0) \): “Psychological Violence”.
- Factor II, \( \Omega_{II}(0) \): “Physical Violence”.
- Factor III, \( \Omega_{III}(0) \): “Severe Physical Violence”.
- Factor IV, \( \Omega_{IV}(0) \): “Sexual Violence”.

III. THE MODEL: \( \Omega(k) \)

Once the questionnaire has been applied, the probable levels of violence that the victim may experience over the next twelve months can be calculated. This can be achieved by using Leal-Enríquez’s model, which is represented as follows [13]:

\[
\Omega(t) = (X^+Y^-(t)),
\]

where

\[
(X^+Y^-)(t) = (X^+Y^-)(0)e^{(\xi\sigma_-(k) - \sigma_+(k))t}.
\]

Notice that substituting (6) in (5), provided that the initial condition \((X^+Y^-)(0) = \Omega(0)\), the outcomes goes as (this equation for Leal-Enríquez’s model can be logically linked in a particular way to each of the questionnaires usually applied by care centers for victims of domestic violence [13]):

\[
\Omega(t) = \Omega(0)e^{\beta(k)t},
\]

where

\[
\beta(k) = (\xi\sigma_-(k) - \sigma_+(k))
\]

and

\[
[\sigma_+(k) \sigma_-(k)] = [\sigma_+(0) \sigma_-(0)] [1 - \lambda \mu (1 - \mu)]^k.
\]

The \( t \) parameter stands for the period measured in months, taken in the interval \((k - 1) \leq t < k\) with \( k = [1, 2, \ldots, l], \)

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TABLE I: INITIAL CONDITION OF VIOLENCE $\Omega(0)$ BY FREQUENCY $f_n$, WEIGHT $\omega_n$ AND DIMENSION: I=PSYCHOLOGICAL VIOLENCE, II=PHYSICAL VIOLENCE, III=SEVERE PHYSICAL VIOLENCE AND IV=SEXUAL VIOLENCE.

<table>
<thead>
<tr>
<th>n</th>
<th>Question</th>
<th>Factor</th>
<th>Weight</th>
<th>$f_n$</th>
<th>$\omega_n$</th>
<th>$\Omega_n(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Has he ever told you that you are not attractive or that you are ugly?</td>
<td>I</td>
<td>4.5</td>
<td>0</td>
<td>4.5</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>Has he ever shown jealousy towards you or become suspicious of your friends?</td>
<td>I</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>Has he ever rejected you?</td>
<td>I</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>Has he ever offended you?</td>
<td>I</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>Has he ever made you feel less in front of other people?</td>
<td>I</td>
<td>5.5</td>
<td>0</td>
<td>5.5</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>Has he ever kicked you?</td>
<td>II</td>
<td>8</td>
<td>0</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>Has he ever pushed you intentionally?</td>
<td>II</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>Has he ever beaten you or slapped on your face?</td>
<td>II</td>
<td>7</td>
<td>0</td>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>Has he ever twisted your arm?</td>
<td>II</td>
<td>6.5</td>
<td>0</td>
<td>6.5</td>
<td>13</td>
</tr>
<tr>
<td>10</td>
<td>Has he ever pulled you forcefully?</td>
<td>II</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>Has he put out a cigarette on your body or burned you with any other substances?</td>
<td>III</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>Has he ever threatened you with a gun or a firearm?</td>
<td>III</td>
<td>6.5</td>
<td>0</td>
<td>6.5</td>
<td>13</td>
</tr>
<tr>
<td>13</td>
<td>Has he ever shot at you with a gun or another firearm?</td>
<td>III</td>
<td>9.5</td>
<td>0</td>
<td>9.5</td>
<td>19</td>
</tr>
<tr>
<td>14</td>
<td>Has he ever threatened you with a knife?</td>
<td>III</td>
<td>7</td>
<td>0</td>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td>15</td>
<td>Has he ever tried to drown you or suffocate you?</td>
<td>III</td>
<td>9.5</td>
<td>0</td>
<td>9.5</td>
<td>19</td>
</tr>
<tr>
<td>16</td>
<td>Has he ever forced you to engage in sexual intercourse?</td>
<td>IV</td>
<td>6</td>
<td>0</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>17</td>
<td>Has he ever used physical force to have sex?</td>
<td>IV</td>
<td>9</td>
<td>0</td>
<td>9</td>
<td>18</td>
</tr>
<tr>
<td>18</td>
<td>Has he ever threatened you with leaving you for other women if you do not agree to engage in sexual intercourse?</td>
<td>IV</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Indicator of psychological violence</td>
<td>$\Omega_{I}(0)$</td>
<td>0</td>
<td>31.5</td>
<td>63</td>
<td>94.5</td>
</tr>
<tr>
<td></td>
<td>Indicator of physical violence</td>
<td>$\Omega_{II}(0)$</td>
<td>0</td>
<td>38.5</td>
<td>77</td>
<td>115.5</td>
</tr>
<tr>
<td></td>
<td>Indicator of severe physical violence</td>
<td>$\Omega_{III}(0)$</td>
<td>0</td>
<td>107</td>
<td>193</td>
<td>304</td>
</tr>
<tr>
<td></td>
<td>Initial condition of sexual violence</td>
<td>$\Omega_{IV}(0)$</td>
<td>0</td>
<td>112</td>
<td>224</td>
<td>336</td>
</tr>
</tbody>
</table>

i.e. the period $l$ is divided into $k$ monthly intervals$^1$. For example, a semester will be $l = 6$. The probability $\lambda \in (0\ 1]$ and $\mu \in [0\ 1]$, which is assumed to be constant during the given period (first month $k = 1$, second $k = 2$, ..., etc; see (9)); that is to say, the execution of persistent IPV against the victim is assumed$^2$.

The probabilities $(1 - \mu)$ and $(1 - \lambda)$ are parameters associated with the prevalence of both the violence and self-control states by the perpetrator, respectively. The vector $[\sigma_+(-) \sigma_-(\cdot)]$ stands for the probabilities that the perpetrator is either in a state of self-control or in one of loss of control, both values being assigned at the beginning of the victim’s risk assessment through a finite Markov chain. For more details see [13].

$\xi(k) \in [0\ 1]$ is the proportion that takes into account the perpetrator’s loss of control percentage$^2$ in the form of violent acts, including injuries against the victim (see dimensions $I$, $II$, $III$ and $IV$ of Table I).

Notice that the violence indicator $\Omega(t)$ is greater than zero with values in the range of $(0\ \infty)$ (see (6)), which can increase if $\sigma_-(\cdot) > \sigma_+(-)$ or decrease in the case $\sigma_+(-) > \sigma_-(\cdot)$ or remain at the value $\Omega(t_w)$ if $\sigma_+(-) = \sigma_-(\cdot)$. In this last case the indicator $\Omega(t)$ (see (5)) would represent a probable scenario of stationary violence. Note that $\Omega(t)$ is greater than 0 for all time $t$. In equations (5)-(9) it is assumed that the probability $\sigma_+(-)$ will be a factor for violence reduction$^{24}$.

To show the application of the equations (7)-(9), the data of the indicator $\Omega_{I}(0)$ is used for the case of physical violence acts (the value of $(1 - \mu)$ was approximated by the prevalence of physical violence reported as 23.4%, according to a study applied to IMSS workers in the state of Morelos, México. The value of the probability of loss of control by the perpetrator is equal to 1 because the woman is already a victim of domestic violence$^{13}$, encompassed in the questions ranging from $n = 6$ to $n = 10$, for a frequency of “many times”, $f_n = 3$ (see Table I). These data are complemented with those proposed in [13]. Therefore, we have: $\sigma_-(0) = 1$, $\sigma_+(0) = 0$, $1 - \mu = 0.234$, $\mu = 0.7660$,

$^1$In statistical studies, the degrees of Intimate Partner Violence are measured by frequencies $(f_n)$ of violent acts, taking into account their severity $\Omega_n$, which is measured in time intervals (usually months). Examples of these measurements can be seen in Olaiz et al. [22].

$^2$Mathematical models are currently being developed to notice root causes to identify factors that accelerate crimes (loss of control manifested in a perpetrated crime) by using an intelligent hybrid artificial neural network [9].
TABLE II: INDICATORS OF VIOLENCE LEVEL SCENARIOS.

<table>
<thead>
<tr>
<th>Month</th>
<th>$\sigma_-(k)$</th>
<th>$\beta(k)$</th>
<th>$\Omega_f(k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2340</td>
<td>-0.692641</td>
<td>57.77923933</td>
</tr>
<tr>
<td>2</td>
<td>0.284556</td>
<td>-0.693732377</td>
<td>28.84122466</td>
</tr>
<tr>
<td>3</td>
<td>0.281219304</td>
<td>-0.662452469</td>
<td>15.8300989</td>
</tr>
<tr>
<td>4</td>
<td>0.281439526</td>
<td>-0.534048721</td>
<td>13.64904111</td>
</tr>
<tr>
<td>5</td>
<td>0.281424991</td>
<td>-0.47637757</td>
<td>11.71748002</td>
</tr>
<tr>
<td>6</td>
<td>0.281425951</td>
<td>-0.482007395</td>
<td>6.405943029</td>
</tr>
<tr>
<td>7</td>
<td>0.281425887</td>
<td>-0.539643534</td>
<td>2.642606643</td>
</tr>
<tr>
<td>8</td>
<td>0.281425891</td>
<td>-0.622213883</td>
<td>6.405943029</td>
</tr>
<tr>
<td>9</td>
<td>0.281425891</td>
<td>-0.470834897</td>
<td>1.68621878</td>
</tr>
<tr>
<td>10</td>
<td>0.281425891</td>
<td>-0.705909944</td>
<td>0.099278236</td>
</tr>
<tr>
<td>11</td>
<td>0.281425891</td>
<td>-0.701153846</td>
<td>0.051641907</td>
</tr>
<tr>
<td>12</td>
<td>0.281425891</td>
<td>-0.696228893</td>
<td>0.027174502</td>
</tr>
</tbody>
</table>

$\Omega_f(0) = 94.5$ and $\lambda$ random.

Because victims often turn for help to the care center after violent acts have already occurred, in measuring the proportion of the loss of control by the perpetrator $\xi(k)$ that has caused injuries or the violent acts being inflicted over a given period, the following values for the next twelve months are considered starting in the month in which the questionnaire to assess the risk of violence was applied (data distribution for $\xi(k)$) was performed considering the triggering of a cycle of violence between the perpetrator and victim [8], [13].

$$\xi(k) = [0.3135 \ 0.0763 \ 0.2003 \ 0.6556 \ 0.9272 \ 0.8406 \ 0.6385 \ 0.3424 \ 0.8803 \ 0.0450 \ 0.0619 \ 0.0794].$$ (10)

When substituting these values in (8) and (9) and by assuming a (random) control-prevalence probability of $\lambda = 0.3$, the factor $\beta(k)$ is obtained (see (8)) from $k = 1$ to $k = 12$.

Finally, with the resulting values from (8) and the initial condition of violence $\Omega_f(0) = 94.5$ (see Table I), the IPV $\Omega(k)$ (see (7)) for $k = 1, 2, 12$ can be obtained. The results of this procedure are shown in Table II.

A. Simulations

A mathematical model simulation of human behavior is useful in exploring probable consequences of behavior patterns on the basis of certain hypotheses [9], [25].

The assumed hypothesis in our study is that the perpetrator is prone to violence (the likelihood of a man being violent can be determined by clinical studies conducted by experts. For an example, see Mossman [28]); it is also assumed that the state of self-control of the aggressor does change monthly on average, and that the victim is submissive and weak in character. This implies that for $k = 0$ the value of $\sigma_-(0) = 1$ [11], [13], [26], [27].

The simulations (the programming of the mathematical model (7)-(9) was executed in MatLab [29]. The calculations can be done by using appropriate software such as Excel), were performed by taking the values from section III for five probable prevalences of the state of control of the perpetrator (the health worker must assign these values based on the number of simulated scenarios to observe the likely levels of violence that the victim could experience [13]).

Finally, with the resulting values from (8) and the initial condition of violence $\sigma_-(0)$ (see (9)) can be obtained, as such the selection of the five probable scenarios was performed heuristically since our objective is to observe the behavior of the perpetrator when he is in a state of loss of control.

1) Simulation: tension-outburst-honeymoon: The five likely scenarios of loss of control by the perpetrator $\sigma_-(k)$ (see (9)) are shown in Fig. 1, where their evolution through time can be observed in monthly intervals.

Fig. 2 shows $\xi(k)$ for a period of one year, derived from the cycle of violence, going from the tension-outburst of violent acts to honeymoon. Note that this percentage of control loss is the same for the five scenarios of loss of control by the perpetrator [13]. The coefficients, $\beta(k) + t$, a proportion that can be manifested as violent acts against the victim, considering the probable self-control factor of the perpetrator (see (8)) at weekly intervals, are recorded in Figure 3. The probable values of the IPV indicator $\Omega_f(k)$ are shown in Figure 4, where the probable evolution of the indicator is represented (the values in Figures 1-8, were joined by means of lines to facilitate their monitoring throughout monthly intervals).

2) Simulation: outburst-honeymoon-tension: Finally, a simulation taking the following values of $\xi(k)$ is developed:

$$\xi(k) = [1 \ 0.6 \ 0.4 \ 0.7 \ 0.8 \ 0.35 \ 0.01 \ 0.02 \ 0.2 \ 0.3 \ 0.1]$$ (11)

which represent a situation where the victim is experiencing a cycle of violent events [13].

The possible scenarios of loss of control by the perpetrator $\sigma_-(k)$ (see (9)) and the factor $\xi(k)$ for a period of one year derived from the cycle of violence are shown in Figures 5-8 [13].

The coefficients $\beta(k) + t$, (see (11)) are represented by weekly intervals for a whole year.

The simulations were performed for only two probable cycles of violence because victims usually go to care centers for help after having suffered some kind of violent aggression [115], [17], [30].

IV. DISCUSSION AND CONCLUSIONS

This article shows the use of a quantitative tool which can contribute to Intimate Partner Violence risk assessment, as well as helping both the care centers and the victims to break the cycle of domestic violence (see Table II).
specific situations (see Table I), and assigns a severity weight to each of the inquiries in the questionnaire and a frequency of violent acts as well.

Once the value of $\Omega(0)$ has been determined, either globally or for each violence dimension (Factor I, II, III, or IV; see section II-A), and using this value in the mathematical model proposed by Leal-Enríquez E. [13] (see equations (7)-(9)), it can be noted that this model can be used to simulate the probable violence scenarios that the victim may experience in the hands of her intimate partner.

The probable scenarios of loss of control by the perpetrator which are generated by the model (see Table II), can be used by health experts to ask the victim which of them best suits the behavior of the aggressor (also the expert him/herself can identify it based on his/her own experience [27]). That is to say, when observing the values of $\Omega(k)$ in Table II, it can be interpreted that starting in the fourth month, the perpetrator shows a behavior of uniform loss of control along the ensuing months. So the expert, when interviewing the victim, manages to establish the kind of violence that the perpetrator uses to inflict against her on a regular basis. For instance, the aggressor usually slaps his intimate partner on
the face once a month (see Table I, question \( n = 6 \)).

Once the type of loss-of-control behavior by the perpetrator is identified, the expert can select the scenarios and likely levels of violence \( \Omega_{II}(k) \) that may occur over the next twelve months. With this information, the expert would establish an idea about the probable danger that the victim may experience. See for example graphs 1–4 and Table III, which depict a scenario where the loss of control by the perpetrator culminates in violent attacks against his intimate partner in the sixth month \( \Omega_{II}(6) \simeq 391 \), a value that exceeds the initial \( \Omega(0)_{II} \) condition of the victim. This could be interpreted as a probable fatal scenario. Take notice also that in the twelfth month the indicator \( \Omega_{II}(12) \simeq 1.3 \), which could be interpreted as the return of the couple to the honeymoon stage, is an indicator that the cycle of violence could be restarting.

Upon the basis of the data calculated by the mathematical model, the care center could make a decision about immediate actions that should be taken to support the IPV victim. However, it is the expert who plays the key role in interpreting the probable violence scenarios and the stage for the specific case (see section III-A). Once the phase of arrival of the victim at the care center has elapsed, the expert must diagnose the probability of the type of loss of control the perpetrator has incurred in (see Figures 1, 5 and Table III).

To illustrate this point, a scenario is selected where the victim, according to the expert, is in the tension stage (see Figures 1–4, while the loss of control of her intimate partner...
changes month after month (see Figure 1, scenario $\sigma(k)$)).

According to these tables and Graphs III, the scenario began to be really dangerous from the fourth month onwards, when the violence indicator $\Omega_{12}(k) > \Omega_{12}(0)$, exceeded the initial condition of violence towards the victim. At this stage, the health expert must provide help to the victim (see Figure 5, scenario $\Omega_{12}(k)$).

The model generated by equations (7)-(9) is very general because it does not include some factors that could influence the state of loss of control by the perpetrator, such as alcoholism and drug addiction, while the victim is assumed to be completely submissive.

Furthermore, the modeled loss of control by the perpetrator is assumed to be cumulative, increasingly dangerous and harmful, while the model disregards likely the control of the perpetrator over his own violent impulses. Nonetheless these limitations, the model can serve as a useful tool for care centers and above all for victims because both their current risk and probable violence scenarios can be observed and measured, so care centers could provide timely help to break the cycle of domestic violence.

After all of the simulations, it can be observed that most of the scenarios do not end up in fatal outcomes at the end of the twelve-month cycle (see Figures 4 and 8 and Table III). This conclusion is not a generalization.

As a final remark, we hope that this work will be useful for care centers and researchers who study the complex phenomenon of domestic violence that affects all the societies in the whole world.

### V. FORTHCOMING WORK

A clinical validation will be conducted by experts concerning the levels of violence inflicted against IPV victims and compared with our own modeled scenarios to test the reliability of the model. The clinical validation will also serve to observe the reactions by the victims when noticing the probable scenarios of violence that they may suffer in the absence of help or professional attention from experts in domestic violence issues.

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