A Risk Assessment Model for Airborne Infection in a Ventilated Room using the Adaptive Runge-Kutta Method with Cubic Spline Interpolation

Wasu Timpitak, and Nopparat Pochai

***Abstract*—Bacteria or viruses that are spread by tiny respiratory droplets are known as airborne infections. These infectious vehicles can move along air currents, stay in the air, or stick to surfaces before being inhaled by another person. Airborne transmission may happen across long ranges and time periods. Increased infection rates or clusters of airborne infections are linked to a lack of ventilation or low ventilation rates. While normal people remain in the same room as infectors, this research will utilize a mathematical model for estimating the concentration of exhaled air in a space with an outlet ventilation system, as well as the risk of infection. As a result, the exhaled air concentration and infection risk are affected by the actual concentration level, the number of users, and the rate of ventilation. The adaptive Runge-Kutta technique and the standard fourth-order Runge-Kutta technique are used to estimate the model solution. Because the number of individuals who stay in the space varies over time, the Lagrange interpolating polynomial and cubic splines interpolation are employed to represent the number of individuals in the space. A good agreement solution is obtained using the adaptive Runge-Kutta method with cubic spline interpolation. The proposed strategy represents the balance in the air quality management process between the number of individuals allowed to stay in the space and the performance of the air ventilation system. For the optimal outcomes, the proposed technique was capable of converting field data from a the number of individuals using cubic splines and adaptive RK methods. The model can also be utilized as a part of an internet of things (IoT) system to develop new approaches to controlling infection-free zones. We demonstrate that the proposed strategy works in real-world scenarios.**

***Index Terms*—airborne, infectious, diseases, ventilation system.**

I. Introduction

BACTERIA or viruses that are spread by tiny respiratory droplets are known as airborne infections. These infectious vehicles can move along air currents, stay in the air, or stick to surfaces before being inhaled by another person. Airborne transmission may happen across long ranges and time periods. Increased infection rates or clusters of airborne infections are linked to a lack of ventilation or low ventilation rates. TB, COVID-19, MERS, and SARS are all dangerous infectious illnesses that transmit through the air or aerosol in a multitude of ways, including coughing, spitting, sneezing, speaking, or through wounds.

In [1], they proposed the risk of inhalation of indoor airborne infection by using the probability transmission dynamic modeling method. Three examples were estimated using Wells-Riley mathematical models: (1) CO2 exposure concentrations in indoor settings based on epidemiological data, (2) baseline reproduction numbers, and (3) local air variability. The risk of infection in susceptible populations under a variety of scenarios of exposure. Improved indoor ventilation has been demonstrated to minimize the risk of infection in studies. In [2], they proposed the Wells-Riley mathematical model for predicting the risk of influenza infection during train transportation. The marginal incidence of infection rises as the number of people using public transportation rises. Improving ventilation is an efficient method of preventing influenza infection. In [3], they proposed an assessment of the feasibility of using natural ventilation to control infection in naturally ventilated hospital wards in Hong Kong. A high rate of natural ventilation can reduce cross-infection of airborne diseases. In [4], they proposed to develop multilevel IAQ control strategies to reduce the risk of infection in buildings and transport areas. A multi-level IAQ control technique is evaluated for indoor environments, including long-term care facilities such as schools, universities, retail stores, hospitals, and transport areas. Assessing the effectiveness of IAQ control strategies can be used to help address the current challenges of COVID-19. In [5], [6], they proposed that as the exhaled air concentration in a room rises in the presence of infectors, the probability of vulnerable individuals contracting infectious diseases transmitted by the air, this is because contaminated people's exhaled air also contains contagious airborne particles inside the nuclei with droplets that can stay airborne for extended periods and infect a susceptible person when inhaled. In [7], they propose a mathematical model to estimate the risk of transmissible diseases in the air. The calculations revealed that the probability of infection increased when the numeral of ill people and airborne viral infections increased.

In [8], they proposed a numerical model that can be used to describe the dynamic dispersion of airborne infectious diseases in an outpatient room. In [9], they proposed developing a model for superspreading episodes of infectious diseases based on the SARS epidemic. In [10], they proposed the random forest method to fuse the WRF model, using the atmospheric pollutant concentration and fundamental meteo-

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N. Pochai is an Assistant Professor of Department of Mathematics, Faculty of Science, King Mongkut's Institute of Technology Ladkrabang, Bangkok, 10520, Thailand (corresponding author to provide phone: 662329-8400; fax: 662-329-8400; e-mail: nop_math@yahoo.com).

W. Timpitak is a PhD candidate of Mathematics Department, Faculty of Science, King Mongkut's Institute of Technology Ladkrabang, Bangkok, 10520, Thailand (e-mail: zienws@gmail.com).

rological parameters training model, and add the atmospheric thermal stability factor as an extra element to model and forecast the municipal PM2.5 concentration. While normal people do remain in the same room as infectors, this research will utilize a mathematical model for estimating the concentration of exhaled air in a space with an outlet ventilation system, as well as the risk of infection.

II. GOVERNING EQUATION

CO_2 concentration in the air of approximately 400 ppm in a area, but when people enter it, exhaled air concentration begins to rise, depending on the rate of ventilation each person, the length of the room, and the proportion of persons in the area [11],[12], and [13].

We suppose that an indoor area, such as a room with a volume of V, begins the day with an environment CO_2 concentration of C_E roughly 400 ppm and is inhabited by the number of individuals(n). Given the presence of infectors, the exhale air concentration that may include airborne contagious particles may tend to rise in the room, determined by the rate of ventilation (Q) and n. We simply assume that persons in the room make a significant contribution to the production of CO_2, which serves as an exhaled air marker. The general equation of the accumulation rate exhaled air concentration in a room with C_E, is equivalent to the exhaled air rate generated by inhabitants plus the rate of C_E, minus Q removes exhaled air:

$$V\frac{dC}{dt} = npC_a + QC_E - QC, \tag{1}$$

where C is the exhale air concentration indoor(ppm), C_a is a fraction of the CO_2 contained in inbreathed air and p is the rate of respiration in the room for each person(L/s). t is the duration time and T is the stationery simulation time. Initial condition $C(0) = C_0$ where C_0 is the latent CO_2 concentration.

In this paper, we consider airborne infections generated by inhabitants, and the value of Q is assumed by Q_{out}, then this value is named the outlet ventilation rate. In a simple scenario, the number of people is varied and depends on the time are assumed by $n(t)$. The general equation of the accumulation rate exhaled air concentration in a room with C_E in Eq.(1) can be written as:

$$V\frac{dC}{dt} = n(t)pC_a - Q_{out}C, \tag{2}$$

for all $0 \leq t \leq T$.

A. The percentage of exhaled air in an unstable state

In Eq.(2), calculation of the percentage of exhaled air in a room with an outlet ventilation system under unsteady state conditions, we get

$$f(t) = \frac{C(t)}{C_a}. \tag{3}$$

B. The airborne infection particles concentration

In [14], [15], they proposed when a susceptible individual inhales the infected particle, a limited amount of contaminated particles may reach the location of the respiratory ailment. This is because the infected particle has varied sizes and deposition percentages in different parts of the respiratory system. In determining the risk of airborne infection the accumulation proportion of airborne infection particles in the airways must also be considered.

Given $(\beta - \mu)$ is the rate of survival of airborne infection particles generated by the infector that reaches its target the infected area of the person who is vulnerable to infection at a threshold value (particles per second) as illustrated in Figure 1,

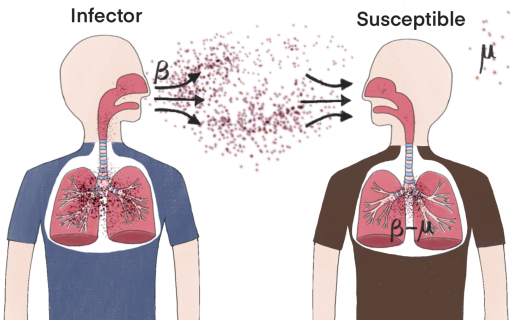

Fig. 1. Movement of airborne infectious particles.

where β is the infector's production rate of total released airborne infection particles and μ is the rate of infected particles death in the air caused by the infector that cannot be embedded in the alveoli layer.

The infection-causing concentration of airborne infection particle, $N(t)$, is expressed as [7]:

$$N(t) = \frac{If(t)(\beta - \mu)}{np}, \tag{4}$$

where I is the number of people infected inside the room. Substituting Eq.(3) into Eq.(4), we obtain the concentration of infectious airborne particles under unstable conditions,

$$N(t) = \frac{IC(t)(\beta - \mu)}{npC_a}. \tag{5}$$

C. The number of airborne infection particle

Not that all infected particles can reach the alveolar cavity and deposit there; let θ be the proportion of airborne infection particles that penetrate and deposit at the host's location of the infected area. As a result, the number of airborne infection particles(λ), inhaled by an individual susceptible and resulting in infection is expressed as [7],

$$\lambda = pt\theta N, \tag{6}$$

where the time consumed (t) in the room got to the moment of infection in the room and $(0 < \theta < 1)$.

D. The probability of airborne infectors

In [7],[6] and [16], they proposed that TB transmission is governed by a Poisson distribution, the probability of airborne infectors is expressed as:

$$P(t) = 1 - e^{-\lambda(t)}, \tag{7}$$

where the probability (P) of susceptible individuals with airborne infectors risk.

III. Numerical technique

There will be no continuous approximation to the solution $C(t)$; instead, approximations to C will be constructed at various values as in interval $[0, T]$, known as mush points.

Once the estimated solution at the points is determined, an approximation can be used to find the approximate solution at other points in the interval. We first make the stipulation that the mesh points are distributed equally over the interval $[0, T]$.

This condition is achieved by selecting a positive integer N and the mesh points $t_i = a+ih$, for each $i = 0, 1, 2, ..., N$. The general distance between the points $h = (T - 0)/N = t_{i+1} - t_i$ is called the step size.

A. The classical fourth-order Runge-Kutta method

The classical fourth-order Runge-Kutta method is expressed as [17],

$$C \cong C_i, \tag{8}$$

$$C_{i+1} = C_i + \frac{1}{6}(k_1 + 2k_2 + 2k3 + k_4)h, \tag{9}$$

$$k_1 = f(t_i, C_i), \tag{10}$$

$$k_2 = f(t_i + \frac{1}{2}h, C_i + \frac{1}{2}k_1 h), \tag{11}$$

$$k_3 = f(t_i + \frac{1}{2}h, C_i + \frac{1}{2}k_2 h), \tag{12}$$

$$k_4 = f(t_i + h, C_i + k_3 h), \tag{13}$$

from Eq.(2), we get the classical fourth-order RK method

$$\frac{dC}{dt} = f(t_i, C_i), \tag{14}$$

$$f(t_i, C_i) = \frac{1}{V}(n(t)pC_a - Q_{out}C_i). \tag{15}$$

B. Adaptive Runge-Kutta method

This technique uses a Runge-Kutta method with local truncation error of order five. The Adaptive Runge-Kutta method is expressed as, [17],

$$\tilde{C}_{i+1} = C_i + \frac{16}{135}k_1 + \frac{6656}{12825}k_3 + \frac{28561}{56430}k_4 - \frac{9}{50}k_5 + \frac{2}{55}k_6, \tag{16}$$

to calculate the local error in a four-order Runge-Kutta technique given by

$$C_{i+1} = C_i + \frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4140}k_4 - \frac{1}{5}k_5, \tag{17}$$

where the coefficient equations are

$$C \cong C_i, \tag{18}$$

$$k_1 = hf(t_i, C_i), \tag{19}$$

$$k_2 = hf(t_i + \frac{h}{4}, C_i + \frac{1}{4}k_1), \tag{20}$$

$$k_3 = hf(t_i + \frac{3h}{8}, C_i + \frac{3}{32}k_1 + \frac{9}{32}k_2), \tag{21}$$

$$k_4 = hf(t_i + \frac{12h}{13}, C_i + \frac{1932}{2197}k_1 - \frac{7200}{2197}k_2 + \frac{7296}{2197}k_3), \tag{22}$$

$$k_5 = hf(t_i + h, C_i + \frac{439}{216}k_1 - 8k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4), \tag{23}$$

$$k_6 = hf(t_i + \frac{h}{2}, C_i - \frac{8}{27}k_1 + 2k_2 - \frac{3544}{2565}k_3 + \frac{1859}{4104}k_4 - \frac{11}{40}k_5), \tag{24}$$

from Eq.(2), we get the Adaptive Runge-Kutta method method

$$\frac{dC}{dt} = f(t_i, C_i), \tag{25}$$

$$f(t_i, C_i) = \frac{1}{V}(n(t)pC_a - Q_{out}C_i). \tag{26}$$

C. Lagrange interpolating polynomial

Suppose we formulate a linear interpolating polynomial as the weighted average of the two values that we are connecting by a straight line [18]:

$$f(x) = L_1 f(x_1) + L_2 f(x_2), \tag{27}$$

where the L_1 and L_2 are the weighting coefficients. It is logical that the first weighting coefficient is the straight line that is equal to 1 at x_1 and 0 at x_2:

$$L_1 = \frac{x - x_2}{x_1 - x_2}. \tag{28}$$

Similarly, the second coefficient is the straight line that is equal to 1 at x_2 and 0 at x_1:

$$L_2 = \frac{x - x_1}{x_2 - x_1}. \tag{29}$$

Substituting these coefficients into Eq. (27),

$$f_1(x) = \frac{x - x_2}{x_1 - x_2}f(x_1) + \frac{x - x_1}{x_2 - x_1}f(x_2), \tag{30}$$

where the nomernclature $f_1(x)$ designates that this is a first-order polynomial. Eq. (30) is referred to as the linear lagrange interpolating polynomial. Such a second-order Lagrange interpolating polynomial can be written as

$$f_2(x) = \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)}f(x_1) + \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)}f(x_2) + \frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)}f(x_3). \tag{31}$$

Notice how the first term is equal to $f(x_1)$ at x_1 and is equal to zero at x_2 and x_3. The other terms work in a similar fashion. Both the first-order and second-order versions as well as higher-order Lagrange polynomials can be represented concisely as,

$$f_{n-1}(x) = \sum_{i=1}^{n} L_i(x)f(x_i). \tag{32}$$

D. Cubic splines interpolation

As the preceding example demonstrates, a spline defined on an interval that is divided into n subintervals will require determining $4n$ constants. To generate the cubic spline interpolation for a given function f, the definition's conditions are applied to cubic polynomials [17],

$$S_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3, \tag{33}$$

for each $j = 0, 1, \ldots, n-1$. Since $S_j(x_j) = a_j = f(x_j)$, condition (c) can be applied to obtain,

$$a_{j+1} = S_{j+1}(x_{j+1}) = a_j + b_j(x_{j+1} - x_j)$$

$$+c_j(x_{j+1} - x_j)^2 + d_j(x_{j+1} - x_j), \tag{34}$$

for $j = 0, 1, \ldots, n-2$.

As the terms $x_{j+1} - x_j$ appear several times in this progression, it is more comfortable to utilize the simplified notation.

$$h_j = x_{j+1} + x_j,$$

for each $j = 0, 1, \ldots, n-1$. If we also define $a_n = f(x_n)$, then the equation

$$a_{j+1} = a_j + b_j h_j + c_j h_j^2 + d_j h_j^3, \tag{35}$$

holds for each $j = 0, 1, \ldots, n-1$. Similarly, define $b_n = S'(x_n)$ and observe that

$$S'_j(x) = b_j + 2c_j(x - x_j) + 3d_j(x - x_j)^2.$$

Implies $S'_j(x) = b_j$, for each $j = 0, 1, \ldots, n-1$. Using condition (d), we get

$$b_{j+1} = b_j + 2c_j h_j + 3d_j h_j^2, \tag{36}$$

for each $j = 0, 1, \ldots, n-1$. The additional connection here between coefficients of S_j is derived by defining $c_n = S''(x_n)/2$ and applying condition (e). Then, for each $j = 0, 1, \ldots, n-1$,

$$c_{j+1} = c_j + 3d_j h_j. \tag{37}$$

Solving for d_j in Eq.(37) and substituting this value into Eqs.(35) and (36) gives, for each $j = 0, 1, \ldots, n-1$, the new equations

$$a_{j+1} = a_j + b_j h_j + \frac{{h_j}^2}{3}(2c_j + c_{j+1}), \tag{38}$$

and

$$b_{j+1} = b_j + h_j(c_j + c_{j+1}). \tag{39}$$

The concluding coefficient relation is achieved by first calculating the relevant equation in the formula of equation (38), for b_j,

$$b_{j+1} = \frac{1}{h_j}(a_{j+1} - a_j) - \frac{h_j}{3}(2c_j + c_{j+1}), \tag{40}$$

and then, with a decrease in the index, for b_{j-1}. This gives

$$b_{j-1} = \frac{1}{h_{j-1}}(a_j - a_{j-1}) - \frac{h_{j-1}}{3}(2c_{j-1} + c_j).$$

Substituting these values into the equation derived from Eq.(39), with the index reduced by one, gives the linear system of equations,

$$h_{j-1}c_{j-1} + 2(h_{j-1} + h_j)c_j + h_j c_{j+1} = \frac{3}{h_j}(a_{j+1} - a_j)$$

$$-\frac{3}{h_{j-1}}(a_j - a_{j-1}) - \frac{3}{h_{j-1}}(a_j - a_{j-1}), \tag{41}$$

for each $j = 0, 1, \ldots, n-1$.

This system involves only the $\{c_j\}_{j=0}^{n}$ as unknowns. The values of $\{h_j\}_{j=0}^{n-1}$ and $\{a_j\}_{j=0}^{n}$ are given, respectively, by the spacing of the nodes $\{x_j\}_{j=0}^{n}$ and the values of f at the nodes. So once the values of $\{c_j\}_{j=0}^{n}$ are determined, it is a straightforward affair to find the reminder of the constant $\{b_j\}_{j=0}^{n-1}$ from Eq.(40) and $\{d_j\}_{j=0}^{n-1}$ from Eq.(37). Then we can construct the cubic polynomials $\{S_j(x)\}_{j=0}^{n-1}$.

IV. Numerical experiments and results

Assuming that the respiration rate assumed by $p = 0.12$ (L/s) and a fraction of the Covid-19 concentration contained inbreathed air $C_a = 0.04$. By employing the classical fourth-order Runge-Kutta method Eqs.8-15 and the adaptive fourth-order Runge-Kutta method Eqs.16-26.

The number of people in the room is represented using the Lagrange interpolating polynomial and the cubic splines interpolation since the number of people who stay in the room varies over time.

A. Simulation 1: an ideal carbon dioxide concentration measurement.

Table I, lists the model's physical parameters. $C_0 = 0.01$ is the ambient carbon dioxide concentration (ppm). The analytical solution for this case can be obtained by [7] such as,

$$C(t) = C_E + \frac{npC_a}{Q}[1 - e^{-Qt/V}]. \tag{42}$$

Table II presents the maximum error of the fourth-order Runge-Kutta solution and the adaptive fourth-order Runge-Kutta solution with the analytical solution. As seen in Figure 2, the adaptive fourth-order Runge-Kutta solution and the fourth-order Runge-Kutta solution are compared to the analytical solution.

TABLE I
PHYSICAL PARAMETERS.

$n(t)$	C_E	V	Q
50	0.004	75	8

TABLE II
THE MAXIMUM ERROR OF THE RK4 SOLUTION AND THE ADRK4 SOLUTION WITH THE ANALYTICAL SOLUTION.

$\triangle t$	Maximum error RK4	Maximum error ADRK4
0.100	9.6097×10^{-13}	1.1102×10^{-15}
0.050	5.9789×10^{-14}	1.3184×10^{-16}
0.025	3.7192×10^{-15}	1.3878×10^{-16}

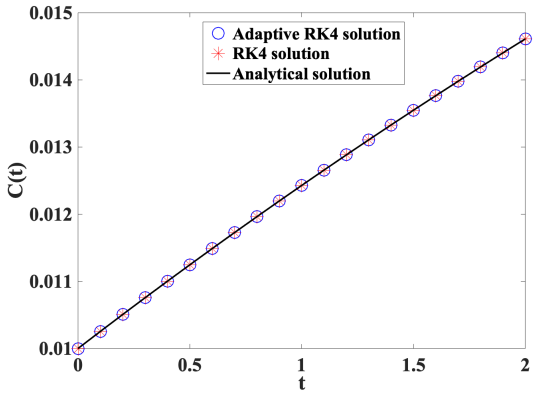

Fig. 2. The approximated air exhaled indoor concentration in a room $T = 180$.

B. Simulation 2 : The concentration measurement of exhaled air with $n(t)$ is function.

The parameters are assumed in Table III. A number of people are assumed by $n(t) = 45 + 5\sin(\pi t)$ is illustrated in Figure 3. Figure 4 shows the approximated solution.

TABLE III
PARAMETERS

C_E	V	Q_{out}	C_0
0.004	75	4	0.01

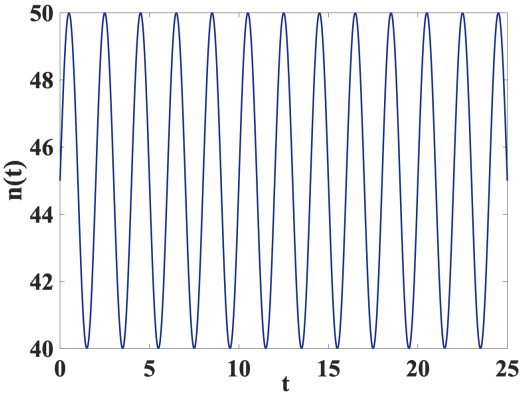

Fig. 3. A number of people in a room $0 \leq t \leq 25$.

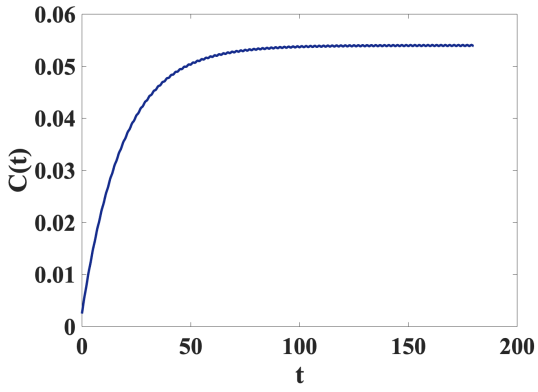

Fig. 4. The approximated air exhaled indoor concentration in a room with $n(t)$ is function $\triangle t = 0.025$ $T = 180$.

C. Simulation 3 : The concentration measurement of exhaled air with cubic splines interpolation of function $n(t)$.

The parameters are assumed in Table IV. A number of people are assumed by $n(t) = 45 + 5\sin(\pi t)$. As seen in Figure 5, the cubic spline interpolation is compared to the function of $n(t)$. Figure 6 shows the approximated solution.

TABLE IV
PARAMETER

C_E	V	Q_{out}	C_0
0.004	75	4	0.01

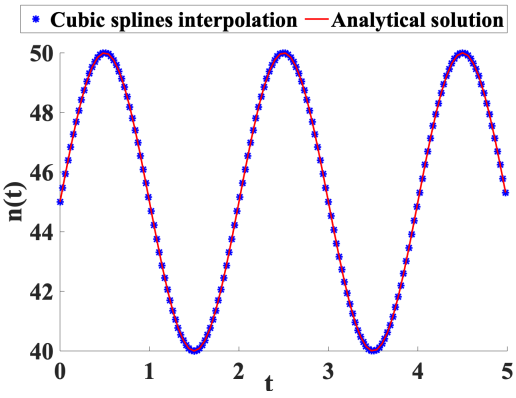

Fig. 5. The cubic splines interpolation is compared to the function of $n(t)$

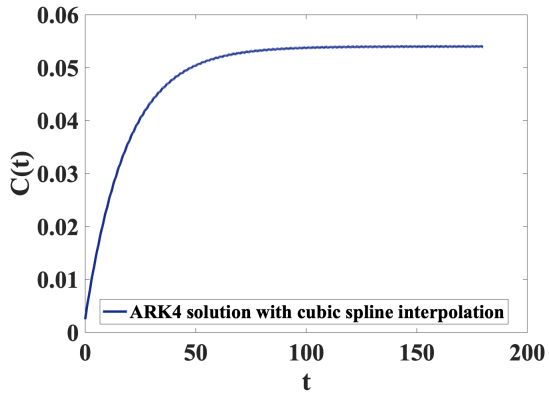

Fig. 6. The approximated air exhaled indoor concentration in a room with cubic splines interpolation of function $n(t)$ $\triangle t = 0.025$ $T = 180$.

D. Simulation 4 : The concentration measurement of exhaled air with lagrange interpolation of function $n(t)$.

The parameters are assumed in Table V. A number of people are assumed by $n(t) = 45 + 5\sin(\pi t)$. As seen in Figure 7, the lagrange interpolation is compared to the function of $n(t)$. Figure 8 shows the approximated solution.

TABLE V
PARAMETER

C_E	V	Q_{out}	C_0
0.004	75	4	0.01

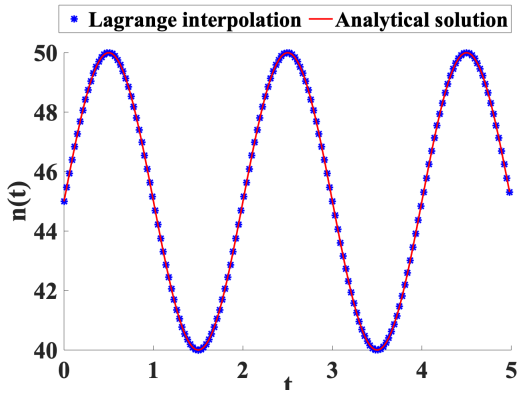

Fig. 7. The lagrange interpolation is compared to the function of $n(t)$

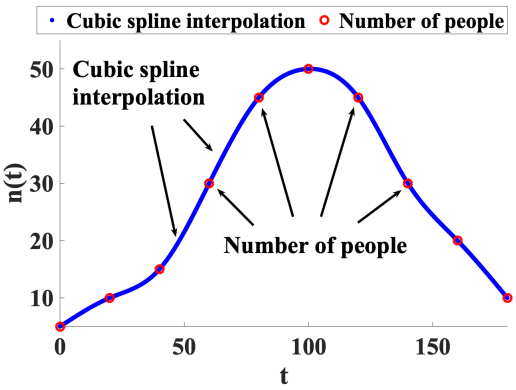

Fig. 9. The cubic spline interpolation is compared to the number of people

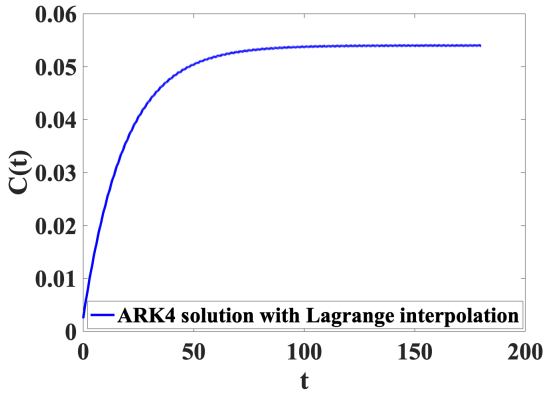

Fig. 8. The approximated air exhaled indoor concentration in a room with lagrange interpolation of function $n(t)$ $\triangle t = 0.025$ $T = 180$.

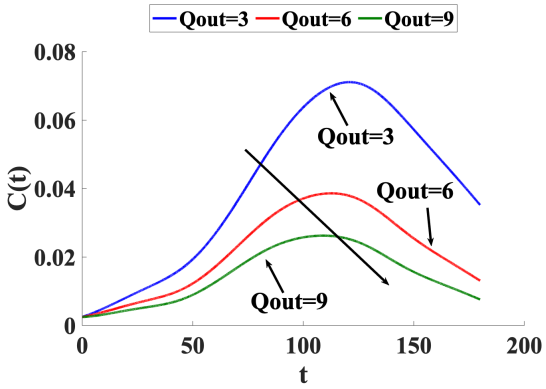

Fig. 10. The approximated air exhaled indoor concentration in a room with cubic spline interpolation $\triangle t = 0.025$ $T = 180$.

TABLE VI
THE ROOT MEAN SQUARE ERROR OF THE CUBIC SPLINES INTERPOLATION AND THE LAGRANGE INTERPOLATION ARE COMPARED TO THE FUNCTION OF $n(t)$

RMSE of the lagrange polynomial	RMSE of the cubic splines
3.8748×10^{-5}	1.0157×10^{-7}

E. Simulation 5 : The risk of normal peoples who staying in a room with infectors.

The initial condition is assumed by $C_0 = 0.01$ and the environmental carbon dioxide concentration(CE) is 0.004. The size of the room is 75 (m^3) and ventilation fan levels are assumed in three cases by $0.18, 0.36$, and 0.54 (m^3/min). As assumed in Table VII, the number of people changes over time.

As seen in Figure 9, the cubic splines interpolation is compared to the number of people. Figure 10 shows the approximated air exhaled indoor concentration in a room with cubic spline interpolation. Table VIII presents the approximated air exhaled indoor concentration in a room when ventilation fan levels are assumed in three cases.

TABLE VII
A NUMBER OF PEOPLE $n(t)$

t	0	20	40	60	80	100	120	140	160	180
$n(t)$	5	10	15	30	45	50	45	30	20	10

TABLE VIII
THE APPROXIMATED AIR EXHALED INDOOR CONCENTRATION IN A ROOM

t	C		
	$Q_{out} = 3$	$Q_{out} = 6$	$Q_{out} = 9$
20	0.0082	0.0058	0.0044
40	0.0145	0.0092	0.0067
60	0.0267	0.0171	0.0126
80	0.0464	0.0290	0.0209
100	0.0637	0.0371	0.0258
120	0.0711	0.0380	0.0254
140	0.0642	0.0305	0.0192
160	0.0500	0.0211	0.0128
180	0.0352	0.0131	0.0076

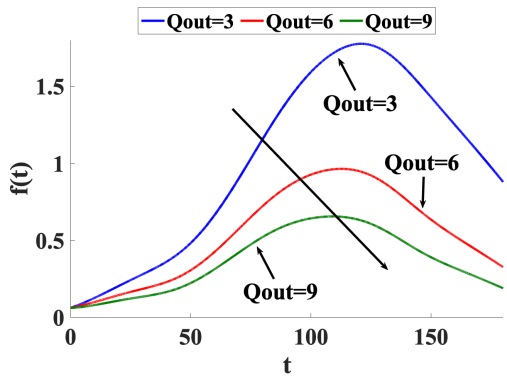

Fig. 11. The percentage of exhaled air

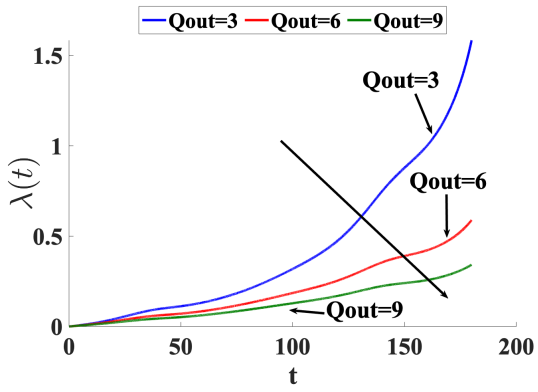

Fig. 12. The number of airborne infection particles

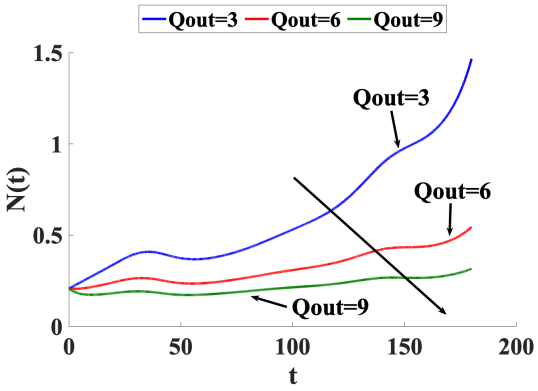

Fig. 13. The airborne infection particles concentration

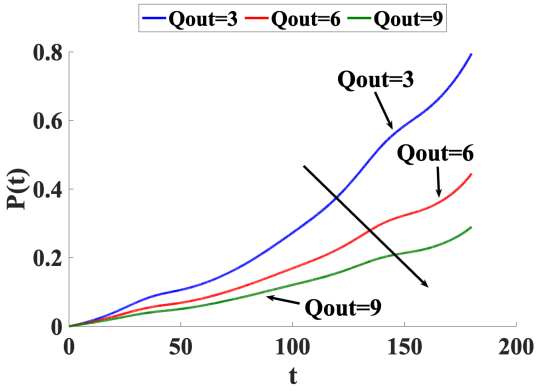

Fig. 14. The probability of airborne infector

V. Discussion

Simulation 1 compares the analytical solution with the classical fourth-order Runge-Kutta method and the Adaptive Runge-Kutta method. In Table II, the maximum error of the classical fourth-order Runge-Kutta method is more than the Adaptive Runge-Kutta method. Figure 2 shows the comparison of both approximation techniques.

Simulation 2, in reality, people were constantly entering and leaving our room, implying that $n(t)$ is a function. Figure 3 shows the number of individuals in the room is between 40 and 50. The approximated air exhaled indoor concentration in a room with $n(t)$ is function as illustrated in Figure 4.

Simulation 3, the cubic spline interpolation is used to interpolate the function $n(t)$. Figure 5 shows the comparison of the cubic spline interpolation and the exact solution. Figure 6 shows the approximated air exhaled indoor concentration in a room with cubic spline interpolation of the function $n(t)$.

Simulation 4, the Lagrange interpolation is used to interpolate the function $n(t)$. Figure 7 shows the comparison of the Lagrange interpolation and the exact solution. Figure 8 shows the approximated air exhaled indoor concentration in a room with the Lagrange interpolation of the function $n(t)$. Table VI shows the root mean square error of both interpolation techniques.

Simulation 5, the number of people as illustrated in Table VII and interpolated by the cubic spline interpolation. Figure 9 shows the comparison of the cubic spline interpolation and the number of people. Figure 10 shows the approximated air exhaled indoor concentration has reduced when outlet ventilation has increased. Figures 11-14 shows the percentage of exhaled air, the number of airborne infection particles, the airborne infection particles concentration, and the probability of airborne infector.

VI. Conclusion

This study will use a mathematical model to predict the concentration of exhaled air in a space with an outlet ventilation system and the risk of infections when healthy people remain in the same room as infected people. As a result, the actual concentration level, the number of users, and the ventilation rate all impact the exhaled air concentration and infection risk. The adaptive Runge-Kutta approach and the classic fourth-order Runge-Kutta method are all used to estimate the model solution. The number of people in the room is represented using the Lagrange interpolating polynomial and the cubic splines interpolation since the number of individuals who stay in the room varies over time. The adaptive Runge-Kutta technique with cubic splines interpolation turns out to be a good agreement solution. The proposed strategy represents the balance in the air quality management process between the number of individuals allowed to stay in the space and the performance of the air ventilation system. For the optimal outcomes, the proposed technique was capable of converting field data from a number of people using cubic splines and adaptive RK methods. The model can also be utilized as a part of an internet of things (IoT) system to develop new approaches to controlling infection-free zones. Due to the proposed numerical enhancement of the adaptive Runge-Kutta technique with cubic spline interpolation, we show that the suggested strategy is effective in real-world scenarios.

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N. Pochai is a researcher of Centre of Excellence in Mathematics, MHESI, Bangkok 10400, Thailand.

W. Timpitak is an assistant researcher of Centre of Excellence in Mathematics, MHESI, Bangkok 10400, Thailand.