A Two-Stage Robust Omega Portfolio Optimization with Cardinality Constraints

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***Abstract*—The Omega portfolio is one of the non-Gaussian portfolio models independent of the strict distribution assumption, which has attracted interest from scholars. Furthermore, the robust Omega portfolio is capable of coping with the scenario of data uncertainty. However, the conservatism inherent in robust optimization has not been applauded by the activist investors, prompting us to polish the classical portfolio model. Therefore, this paper aims to develop the less conservative robust Omega portfolio, in which a two-stage portfolio structure is designed. In the first stage, Genetic algorithm, Gurobi solver, and Mosek solver are employed to solve the mixed-integer programming problem to screen out qualified risky assets for the sequel modeling. Then, the robust Omega portfolios are built based on the selected assets in the second stage. The US 30 industry portfolio data set is used for the empirical research, whose results demonstrate the effectiveness of the proposed methodology.**

***Index Terms*—Portfolio selection, Genetic algorithm, Omega ratio, Cardinality constraint**

I. Introduction

As a successful theoretical model, the mean-variance (MV) portfolio[1] has gained much attention from academia and industry. With advances in computer science and engineering, the fundamental assumption of MV seems to be too strict to satisfy the growing investor demand. To remedy the defect, some scholars proposed and developed the Omega portfolio[2], [3], [4], [5]. Compared to the traditional MV model, the Omega portfolio takes into account the state of all observations, rather than characterizing the overall distribution using only two representative parameters, mean and variance.

Theoretically, modeling Omega portfolio requires precise distributional knowledge is required, which poses a challenge for practitioners mired in data ambiguity. Fortunately, the robust Omega portfolio relaxes the precise restriction on the experimental samples, which assumes the data could vary within the range. Considering the worst-case scenario is a common methodology to transform and solve the robust optimization problem[6], [7], [8]. However, this approach is prone to improving the out-of-sample robustness while ignoring the potential asset gain, which would result in a conservative portfolio.

For investors with a risk-seeking preference, the over-conservative portfolio may not be favored, which motivates researchers to develop models balancing risk and return. [7] designed the hybrid robust portfolio, in which both the best-case and the worst-case counterparts are considered. [9] analyzed the cost of robustness theoretically. [8] tried to reduce the conservatism of the worst-case portfolio by introducing the higher moments. Existing studies also point out that some off-the-shelf techniques could provide insights for constructing promising portfolios, such as machine learning[10], [11], [12], deep learning[13], [14], clustering[15], [16], regression[17], [18], [19], [20], heuristic algorithms[21], [16], etc.

Based on the current research progress, this paper contributes to developing the less conservative robust Omega portfolio, in which a two-stage portfolio structure is designed. Intuitively, the first stage is the preselection aiming to select qualified risky assets, and the second stage is to construct the robust Omega portfolio based on the screened assets. A heuristic algorithm, genetic algorithm (GA), and two state-of-art solvers, Gurobi and Mosek, are used to solve the mixed-integer programming problem proposed in the preselection. To verify the effectiveness of the proposed model, the US 30 industry portfolio data set is employed to implement the numerical experiments. In addition, we also test some benchmark models such as $1/N$ strategy, Markowitz portfolio, and the single-stage robust Omega portfolio. Computation results and the corresponding comparative analysis support the superiority of the GA-based two-stage robust Omega portfolio.

The organization of this paper is as follows. Section II introduces the robust Omega portfolio and the corresponding cardinality constraint. Section III discusses the preselection in the first stage, and Section IV shows the work in the second stage, in which the necessary logical relationship between the two stages is explained. Section V implements the numerical experiments and verifies the effectiveness of the propose model. Section VI concludes the paper.

II. Primal portfolio model

Suppose that there are n financial risky assets in the market, whose returns are indicated by a random vector $\tilde{\boldsymbol{r}} = \{\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n\}$. Denote $\boldsymbol{w} \in \mathbb{R}^n$ as the portfolio weight vector held by an investor. According to the definition given by [2], the Omega ratio of the ith risky

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asset is as follows:

$$\Omega_i = \frac{\int_{\tau}^{\infty}[1-F(r_i)]dr_i}{\int_{-\infty}^{\tau}F(r_i)dr_i} = 1 + \frac{\mathbb{E}[(\tilde{r}_i-\tau)]}{\mathbb{E}[(\tau-\tilde{r}_i)^+]} \tag{1}$$

where $F(r_i)$ is the cumulative density function, and $f(r_i)$ is the probability density function. Therefore, the Omega portfolio optimization problem can be addressed as follows:

$$\begin{aligned} \max_{\boldsymbol{w}\in\mathbb{R}^n} \quad & \frac{\boldsymbol{w}'\mathbb{E}[\tilde{\boldsymbol{r}}]-\tau}{\mathbb{E}[(\tau-\boldsymbol{w}\tilde{\boldsymbol{r}})^+]} \\ s.t. \quad & \mathbf{1}'\boldsymbol{w}=1 \\ & \underline{\boldsymbol{w}} \le \boldsymbol{w} \le \overline{\boldsymbol{w}} \end{aligned} \tag{2}$$

Kapsos et al. [4], [5] provided the sample-based linear programming (LP) form of (2), in which they assume m possible scenarios and the probability of the jth scenario is $p_j = 1/m$. Define $\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_m$ be the m-rounds sampling assets returns, the problem (2) can be transformed into the following linear fractional optimization:

$$\begin{aligned} \max_{\boldsymbol{w}\in\mathbb{R}^n, \boldsymbol{u}\in\mathbb{R}^m} \quad & \frac{\boldsymbol{w}'\bar{\boldsymbol{r}}-\tau}{(1/m)\mathbf{1}'\boldsymbol{u}} \\ s.t. \quad & u_j \ge \tau - \boldsymbol{w}'\boldsymbol{r}_j \\ & u_j \ge 0 \\ & \mathbf{1}'\boldsymbol{w}=1 \\ & \boldsymbol{w} \in [\underline{\boldsymbol{w}}, \overline{\boldsymbol{w}}] \\ & \boldsymbol{w}'\bar{\boldsymbol{r}} \ge \tau \end{aligned} \tag{3}$$

Obviously, the Omega portfolio requires to be fully aware of the distribution of the achievable assets. However, access to this information is costly and time-consuming, and practitioners have to resort to the method of sample estimation to fulfill the task. To alleviate the influence bought by potential error-prone estimation, the robust variant of the Omega portfolio is developed according to the results in [22], [4]. To be specific, we consider the following polytopic uncertainty where the distribution of $\tilde{\boldsymbol{r}}$ is constrained by the pre-specified mixture distributions:

$$p(\tilde{\boldsymbol{r}}) \in \mathcal{P} \triangleq \left\{ \sum_{i=1}^{k} \lambda_i p_i(\tilde{\boldsymbol{r}}), \lambda_i \ge 0, \sum \lambda_i = 1 \right\} \tag{4}$$

The worst-case approach is feasible to deal with the robust portfolio problem[6]. Accordingly, the worst-case Omega portfolio can be derived:

$$\begin{aligned} \max_{\theta,\boldsymbol{x},\boldsymbol{q},z} \min_{\boldsymbol{\pi}} \quad & \theta \\ s.t. \quad & (\boldsymbol{R}\boldsymbol{x})'\boldsymbol{\pi} - \tau z \ge \theta \\ & \boldsymbol{\pi}'\boldsymbol{q} = 1 \\ & \boldsymbol{q} \ge \tau z \mathbf{1} - \boldsymbol{R}\boldsymbol{x} \\ & \boldsymbol{q} \ge \mathbf{0} \\ & z = \mathbf{1}'\boldsymbol{w} \\ & z\underline{\boldsymbol{w}} \le \boldsymbol{x} \le z\overline{\boldsymbol{w}} \\ & z \ge 0, \theta \ge 0 \end{aligned} \tag{5}$$

where the rescaled vector $\boldsymbol{x} = z\boldsymbol{w}$, $\boldsymbol{\pi}$ is the $S \times 1$ vector representing the sampling scenario, $\boldsymbol{R} \in \mathbb{R}^{S\times n}$ is the discrete distribution. Empirical studies point out that holding a sparse portfolio is beneficial to individual investors, which could be implemented by the cardinality constraint:

$$\mathcal{C}_K = \{\boldsymbol{w} \in \mathbb{R}^n, \boldsymbol{y} \in \{0,1\}^n : \mathbf{1}'\boldsymbol{w} = 1; \mathbf{1}'\boldsymbol{y} = K; \boldsymbol{w} \le \boldsymbol{y}\} \tag{6}$$

where y_i is the binary variable indicating whether an asset has been selected.

As a result, the Omega portfolio constrained by $\mathcal{C}_K$ is a mixed-integer programming (MIP) problem, which has been recognized as an NP-hard problem. In additional to the computational difficulties, conservatism is also the main problem in the worst-case portfolio optimization, which is not preferred by the risk-seeking investors. To overcome the potential conservatism, we divide the proposed Omega portfolio into the two stages. In the first stage, i.e., preselection, we implement a cardinality constrained portfolio to select K assets with satisfactory performance, in which the profit-pursuing rule is designed. Based on the picked risky assets in the prior stage, the robust Omega portfolio is constructed in the second stage.

III. First stage: Preselection

The classical mean-variance model paves the way for the portfolio selection problem. Hence, we consider building the following Markowitz portfolio with cardinality constraints to select promising assets for the second phase:

$$\begin{aligned} \min \quad & \boldsymbol{w}'\Sigma\boldsymbol{w} \\ s.t. \quad & \boldsymbol{w}'\mathbb{E}[\tilde{\boldsymbol{r}}] \ge \mu \\ & \mathbf{1}'\boldsymbol{w} = 1 \\ & \mathbf{1}'\boldsymbol{y} = K \\ & \boldsymbol{w} \le \boldsymbol{y} \\ & \boldsymbol{w} \ge 0, \boldsymbol{y} \in \{0,1\}^n \end{aligned} \tag{7}$$

where μ is the target return of the portfolio. Due to the objective function is quadratic, and y_i is the binary variable, the problem (7) is hard to solve by some well-implemented algorithms such as simplex and the interior-point method. To deal with these complicated optimization problems, scholars have developed lots of exact and heuristic algorithms. For instance, branch & bound[23] and cutting planes[24] are two mainstreams in solving the MIP problem, which have been implemented in some commercial solvers like Gurobi[25], CPLEX[26], and Mosek[27]. Randomized algorithms also play some indispensable roles in solving large-scale integer programming. Compared with the exact algorithm, the solution of the randomized algorithm may be suboptimal, but the heuristic algorithm has some advantages in solving efficiency. Therefore, the heuristic solution provided by the randomized algorithm could be used as the warm-start in some optimization toolkits.

Accordingly, GA, Gurobi 9.5.0, and Mosek 9.3 are the three selected solvers for the first stage problem (7), and the corresponding results are delivered to the second stage for constructing the robust Omega portfolio. Considering the stochastic nature of GA, we fixed the random seed in programming for reproducing.

IV. Second stage: Robust Omega portfolio

Based on the risky assets selected from the first stage, we build the robust Omega portfolio (5) in this stage. By adjusting the target return μ in problem (7), the constituents of this phase's portfolio may be changed accordingly. Intuitively, we assume that using aggressive μ would somewhat reduce the conservatism of the worst-case Omega portfolio. However, the actual effectiveness of the strategy depends on the outcome of the preselection stage, as well as the performance of the solver [28].

The threshold τ, in addition to the risky assets selected, is also a key parameter influencing the robust Omega portfolio performance. In this paper, we use the rule-of-thumb to determine the threshold as in existing research [29], [5]. Note that the main goal of this work is to construct the less conservative robust Omega portfolio, this work tries to do this by separating the integer constraint $\mathcal{C}_K$ and establishing the independent problem (7). To be fair, customizing the dynamic adjusting strategy for the threshold would also contribute to improving the portfolio performance. But this topic is out of the scope of this study, and will be discussed in the forthcoming full-length paper.

Actually, the equality constraint could be violated in the process of solving, thus considering the relaxed formulation of $\mathcal{C}_K$ is feasible in programming:

$$\mathcal{C}'_K = \{\boldsymbol{w} \in \mathbb{R}^n, \boldsymbol{y} \in \{0,1\}^n : \mathbf{1}'\boldsymbol{w} \leq 1; \mathbf{1}'\boldsymbol{y} \leq K; \boldsymbol{w} \leq \boldsymbol{y}\} \tag{8}$$

Accordingly, the relaxed formulation of the problem (7) in the first stage and the problem (5) in the second stage respectively could be obtained in the same vein.

V. Numerical experiments

A. Data set

The US 30 industry data set from the website of Kenneth. R. French is employed in our numerical experiments to demonstrate the effectiveness of the proposed strategy. Table I presents the descriptive statistics of the experimental data set ranging from 2012 to 2021. It can be observed that the overall distribution of the industry data is stable, which is good for avoiding accidental errors in our experiments. K-S tests are also carried out in the empirical research, and the statistical results strongly reject the normality hypothesis, which in turn is suitable for some non-Gaussian models such as the Omega portfolio.

The first 500 observations are fed into the proposed portfolio model for preselection. Then the $601 \sim 1200$ samples are used for the robust Omega portfolio modeling, from which the optimized weight vectors can be obtained. Whilst, the remaining data will be used for out-of-sample experiments. To comprehensively evaluate the performance of the two-stage portfolio model, the MV portfolio, $1/N$ strategy[30], and the single-stage robust Omega portfolio are also gauged for comparative analysis.

All of the numerical experiments were carried out on the Apple M1 Max computer with the Mac OS Monterey operating system, 64G LPDDR5 memory, and a 2T SSD hard disk. The Python 3.8 platform was used to accomplish the task of data processing, and the Matlab R2021a was employed to program the mathematical models.

B. Results: the first stage

It is efficient and flexible for an individual investor to manage a portfolio consisting of about 10 risky assets[7]. Therefore, we test $K = 9, 10, 11, 12$ respectively in the constraint $\mathcal{C}_K$ at the first stage. As a common-used evolutionary algorithm in financial applications[16], [31], GA employs the operators such as selection, crossover, and mutation to explore the solution space, and drives the population to converge to the best fitness chromosome, which is denoted as the weight of an individual asset within the portfolio. Fig. 1 illustrates the principle of GA briefly. In this numerical experiment, the size of the population is 150, the maximal generation is 500, and the number of the elite is 10.

The fitness curves of GA for the problem (7) with different K are shown in Fig. 2. It can be found that all of the evolution curves converge within 120 generations. Table II$\sim$III show the results of preselection solved by GA, Gurobi, and Mosek, respectively. Apparently, the three solvers have different characteristics in dealing with the gradually changing constraints. GA shows evident randomness and high sensitivity to the constraint condition, in that it filters distinct subsets of the risky assets with different values of K. Gurobi is the most conservative solver and insensitive to the $\mathcal{C}_K$. Contrary to GA, Gurobi has the consistent assets subset as the first stage. Mosek neutralizes the characteristics of the two solvers, which ensuring that there is no great difference between the solution sets while keeping the constraint active.

C. Results: the second stage

The corresponding robust Omega portfolio models are built and investigated in this stage. To avoid the issue of data snooping, we select the data with an interval from the training set as the testing set for evaluating the portfolio performance.

Fig. 3 presents the out-of-sample performance of the proposed two-stage robust Omega portfolio based on GA (GA-Ro-Omega) as well as the benchmark models. Due to the two-stage robust Omega portfolio based on Gurobi (Gro-Ro-Omega) and Mosek (Msk-Ro-Omega) do not show significant superiority over the single-stage robust Omega portfolio (Ro-Omega), we have omitted their cumulative return curves for greater clarify the figure. Roughly speaking, GA-Ro-Omega outperforms other models in terms of cumulative return, which preliminary confirms the effectiveness of the GA-based preselection. However, more detailed comparative analysis should be done for deepening the conclusion.

Table IV presents the comparative analysis from the three perspectives: ROI, STD, MDD. GA-Ro-Omega has outstanding performance on both profitability and robustness. For instance, when $K = 11$, GA-Ro-Omega yields the highest ROI of 1.5752, followed by $1/N$, 1.0936; GA-Ro-Omega has the optimal MDD of 0.2867, MV follows with 0.3094. Although GA-Ro-Omega shows clear advantages in seeking returns and controlling extreme risks, it also assumes more volatility than other models, which can be inferred from the STD.

TABLE I
DESCRIPTIVE STATISTICS OF THE DATA SET.

Industry	**Mean** (%)	**Stdev.** (%)	25% **Quantile** (%)	50% **Quantile** (%)	75% **Quantile** (%)	**Range**
Food	0.0424	0.9150	-0.3900	0.0600	0.5200	0.1623
Beer	0.0580	1.0306	-0.4200	0.0700	0.5500	0.2274
Smoke	0.0436	1.2302	-0.5600	0.0700	0.6800	0.2149
Games	0.0800	1.5269	-0.6800	0.1200	0.9000	0.2304
Books	0.0431	1.3340	-0.6400	0.0700	0.7800	0.2144
Hshld	0.05160	0.9727	-0.3800	0.0700	0.5300	0.1910
Clths	0.0685	1.4214	-0.6200	0.0900	0.8200	0.2747
Hlth	0.0663	1.0671	-0.4400	0.0900	0.6300	0.1661
Chems	0.0575	1.3064	-0.5700	0.0900	0.7500	0.2409
Txtls	0.0692	1.9462	-0.7100	0.0900	0.9300	0.4032
Cnstr	0.0754	1.5251	-0.6400	0.1000	0.8700	0.3149
Steel	0.0530	1.9205	-1.0000	0.0500	1.0300	0.2659
FabPr	0.0701	1.4181	-0.6300	0.0900	0.7800	0.2560
ElcEq	0.0611	1.4438	-0.6200	0.0800	0.8000	0.2767
Autos	0.1114	1.8416	-0.7100	0.0900	0.9300	0.2974
Carry	0.0611	1.5829	-0.5900	0.0700	0.7400	0.3111
Mines	0.0383	1.6870	-0.9000	0.0400	1.0000	0.2580
Coal	-0.0309	3.0904	-1.6000	-0.0300	1.4700	0.3835
Oil	0.0215	1.7846	-0.7700	0.0100	0.8200	0.3602
Util	0.0426	1.0860	-0.4500	0.0800	0.5700	0.2343
Telcm	0.0492	1.0153	-0.4300	0.0700	0.5700	0.1817
Servs	0.0862	1.2472	-0.4500	0.1100	0.7100	0.2387
BusEq	0.0933	1.3161	-0.4900	0.1300	0.7400	0.2357
Paper	0.0490	1.0883	-0.4500	0.0900	0.5900	0.1939
Trans	0.0630	1.2982	-0.5400	0.1000	0.7400	0.2418
Whlsl	0.0595	1.1602	-0.4800	0.0900	0.6500	0.2291
Rtail	0.0762	1.0548	-0.4200	0.1200	0.6300	0.1705
Meals	0.0643	1.1638	-0.4200	0.0900	0.6300	0.3117
Fin	0.0732	1.3394	-0.5000	0.1000	0.7200	0.2781
Other	0.04998	1.0587	-0.4400	0.0700	0.5500	0.1978

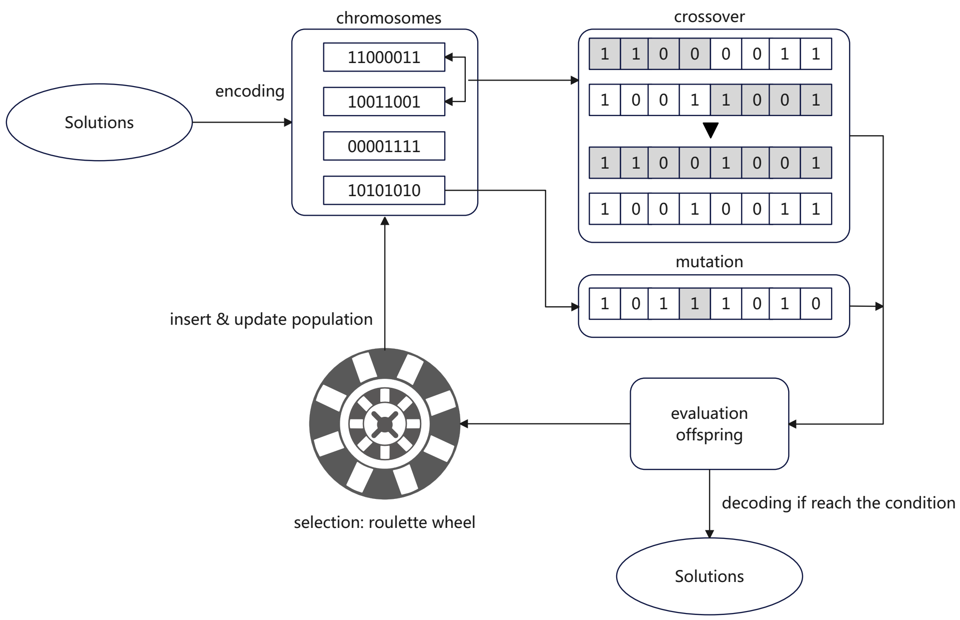

Fig. 1. Flowchart of GA.

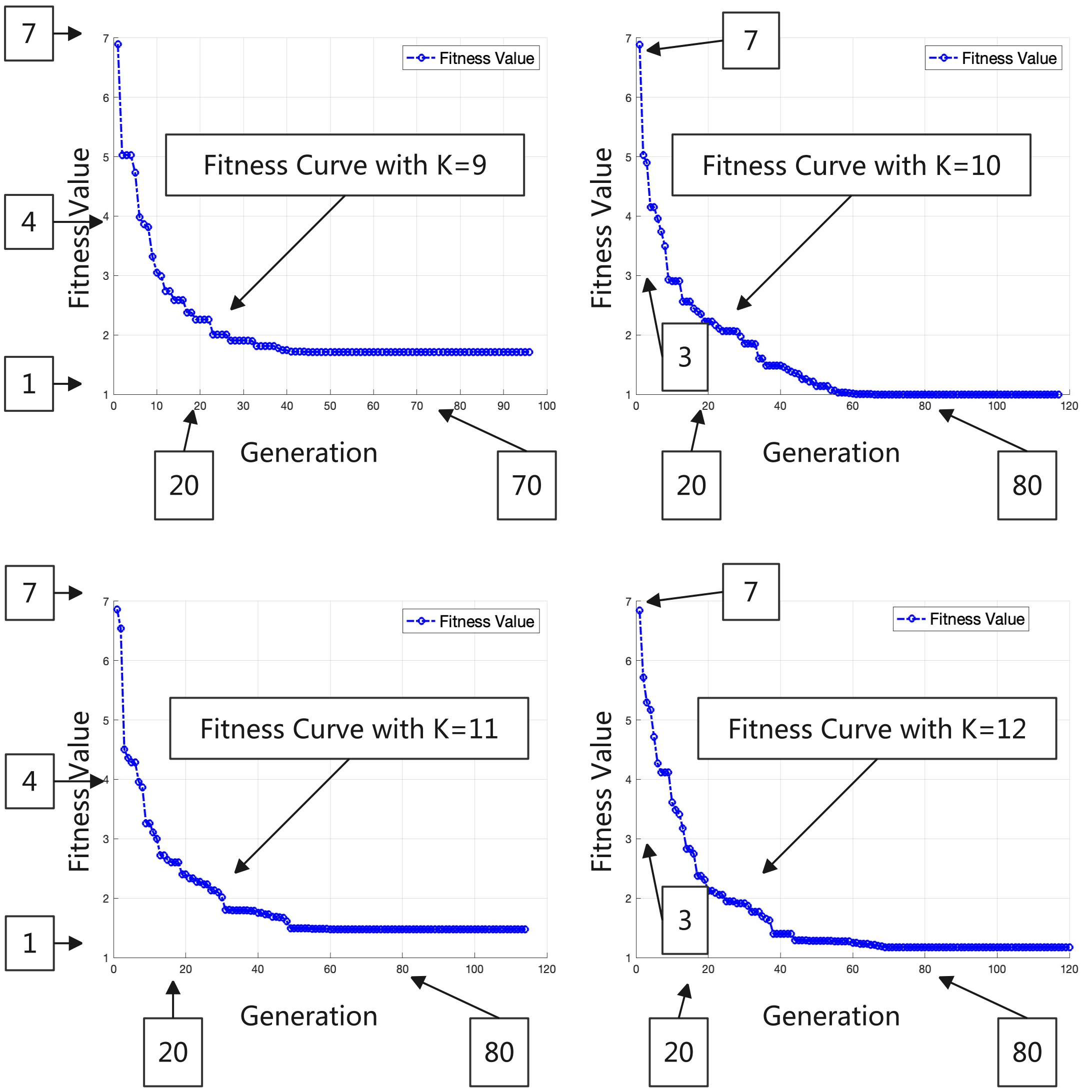

Fig. 2. Fitness curve of GA.

TABLE II
SELECTED ASSETS IN THE FIRST STAGE.

Solver	K=9		K=10	
	Assets selected	Selected subset	Assets selected	Selected subset
GA	5	$\{4, 5, 10, 21, 27\}$	5	$\{18, 22, 27, 28, 30\}$
Gurobi	7	$\{2, 3, 8, 20, 23, 27, 28\}$	7	$\{2, 3, 8, 20, 23, 27, 28\}$
Mosek	9	$\{1, 2, 3, 8, 20, 23, 27, 28, 29\}$	10	$\{1, 2, 3, 6, 8, 20, 23, 27, 28, 29\}$

TABLE III
SELECTED ASSETS IN THE FIRST STAGE.

Solver	K=11		K=12	
	Assets selected	Selected subset	Assets selected	Selected subset
GA	7	$\{8, 18, 19, 22, 24, 27, 29\}$	5	$\{6, 8, 11, 20, 22\}$
Gurobi	7	$\{2, 3, 8, 20, 23, 27, 28\}$	7	$\{2, 3, 8, 20, 23, 27, 28\}$
Mosek	11	$\{1, 2, 3, 6, 8, 11, 20, 23, 27, 28, 29\}$	12	$\{1, 2, 3, 6, 8, 10, 11, 20, 23, 27, 28, 29\}$

TABLE IV
PERFORMANCE OF THE TESTED PORTFOLIO MODELS.

Proposed model	K=9			K=10			K=11			K=12		
	ROI	STD	MDD	ROI	STD	MDD	ROI	STD	MDD	ROI	STD	MDD
GA-Ro-Omega	0.8055	0.2453	0.4886	2.1363	0.2018	0.2730	1.5752	0.1937	0.2867	1.5590	0.1928	0.2858
Gro-Ro-Omega	0.4068	0.1904	0.3170	0.4068	0.1904	0.3170	0.4068	0.1904	0.3170	0.4068	0.1904	0.3170
Msk-Ro-Omega	0.4046	0.1879	0.3158	0.4040	0.1895	0.3156	0.4033	0.1892	0.3150	0.4009	0.1884	0.3138

Benchmarks	ROI	STD	MDD
$1/N$	1.0936	0.2006	0.3835
MV	0.7417	0.1642	0.3094
Ro-Omega	0.3906	0.1871	0.3118

Acronyms: ROI: Return of investment; STD: Annualized standard deviation; MDD: Max drawdown.

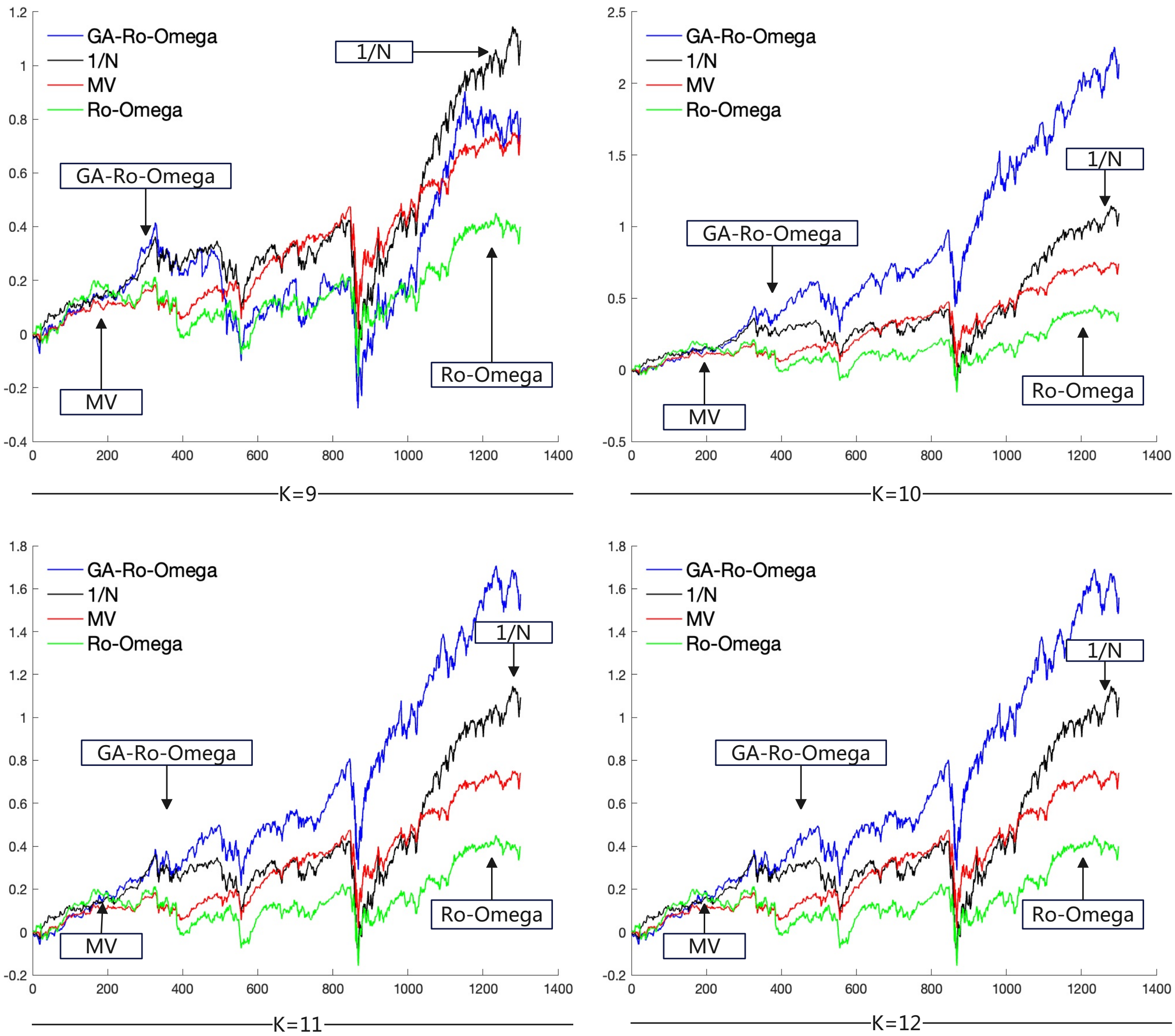

Fig. 3. Cumulative returns of the portfolio models.

The effectiveness of the preselection depends on the solver involved in the first stage. Due to both Gurobi and Mosek trying to solve the MIP problem by resorting to the exact algorithm, they show similar performance on the numerical experiments. However, neither Gro-Ro-Omega nor Msk-Ro-Omega shows significant performance improvement for Ro-Omega. Essentially, the problem (7) is a minimum-variance portfolio essentially, the exact solution given by Gurobi or Mosek exhausts the information of the training samples, while may not be generic enough to deal with the out-of-sample observations. One possible approach to tackle the issue is to set an aggressive target return in the first stage, but the inherent shortcomings of the MV model may obstruct the decision makers from achieving their goals. Instead of tuning the parameters in problem (7), the inherent randomness of GA-Ro-Omega improves the out-of-sample generalization of the proposed portfolio. To summarize, the experimental results still verify the validity of the specified preselection, especially using the GA solver in the first stage.

VI. Conclusions & Discussions

In this study, we mainly investigate the issue of overcoming the potential conservatism of the robust Omega portfolio. As a result, we design and develop the two-stage robust Omega portfolio, where a MIP-based preselection is specified to screen out the risky assets for the sequel stage modeling. Both the heuristic algorithm and exact method are tested in the first stage to select the qualified assets subset. The corresponding robust Omega portfolios (GA-Ro-Omega, Gro-Ro-Omega, Msk-Ro-Omega) are constructed in the second stage. Computational results support the efficiency of the proposed models, which also demonstrate the effectiveness of GA. Even though the suboptimal solutions provided by some heuristic algorithms are not quite exact, they can still play a crucial role in elevating the out-of-sample portfolio performance.

As demonstrated in the existing studies, the standard formulation of the Omega ratio optimization portfolio tends to be idealistic in the circumstance of lacking precise return distribution information, especially the estimation error would do harm to the efficiency and effectiveness of the portfolio model. Although the robust variant considering different types of uncertainty is beneficial to constructing realistic portfolio models, the ensuing conservatism becomes a novel controversial topic. This paper tries to overcome the inherent conservatism of the robust Omega portfolio model to some extent by the mean of the random algorithm, GA. Some other random algorithms such as particles swarm optimization (PSO), neural network (NN) are also worthwhile to improve the performance of the robust Omega portfolio model.

Future work will revolve around developing intelligent portfolio models, in which more heuristic algorithms, machine learning models, and deep learning models are involved. In addition, optimizing the existing robust Omega portfolio model from a theoretical point of view will be one of the priorities. In this vein, designing and developing a dynamic framework for the robust Omega portfolio is a feasible research direction.

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