Prescribed-Time Stabilization of Double Integrator Systems with Application to Wheeled Mobile Robot

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***Abstract*—In this paper, we study the prescribed-time stabilization (PTS) problem of a class of stable double integrals System. A scale-free design scheme of state feedback is proposed to ensure the time stability of the closed loop system by appropriately introducing a time-varying function into the virtual (real) controller. Finally, a wheeled mobile robot is taken as an example to verify the effectiveness of this method.**

***Index Terms*—double integrator systems, time-varying function, non-scaling design, prescribed-time stabilization, wheeled mobile robot**

I. Introduction

STABILIZATION and control design of linear/nonlinear systems has recently received extensive attention due to its theoretical and practical significance. Sports from the perspective of convergence rate, The current results are divided into finite-time stabilization (FTS) and infinite-time stabilization. By comparison, the former is more attractive because of its good features of convergence and disturbance rejection [1-6]. However, the existing FTS results suffer from the shortcoming that the convergence time relies on the system initial conditions.

To deal with this, the fixed-time stability is proposed in [7], by requiring the upper boundedness of system convergence time irrespective of initial condition. Soon afterwards, the research on fixed-time stabilization (FixTS) of nonlinear system has become a popular topic. [8-15]. It must be said that in the existing resutls, the upper boundedness of system convergence time even is bounded, but it is hard to adjust according the different needs [16].

However, quite a few engineering applications require that the system has a prespecifiable convergence time. This impels that the prescribed-time stability has been proposed to research on the problem of system stabilization [17-22], where the upper boundedness of system convergence time is chosen by user. Especially, the scaling technique, i.e.,scaling the states by a function that grows unboundedly to the terminal time, was given in [17] to solve the PTS of Brunovsky systems. However, this technique may result in the computationally singular appearing in the designed controller.

To address the above problem, this paper studies the PTS problem of stable double integrator systems. The main contributions are twofold: (i) We introduce a witched scaling function in which the switching rule is dependent on time and system states to overcome the computationally singular problem of the scaling-based controller of [17]. (ii) For double integrator systems, a non-scaling design is given. Different from the scaling-based design of [17], where the time-varying function scales the states in all the transformations, we employs the given switched function scaling the virtual (scaling) controllers to achieve the PTS. In this way, the computation burden is reduced and thus the proposed method leads to a simpler controller.

The rest of this paper is organised as follows. Section II elaborates the problem to be investigated. Section III gives the design and analysis. Section IV where an application of the presented scheme to a wheeled mobile robot is provided. Finally, some concluding remarks are given in Section VI.

II. Problem Formulation and Preliminaries

A. Problem formulation

Consider a stable double integrator as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -ax_1 - bx_2 + u, \end{aligned} \tag{1}$$

where $x = (x_1, x_2)^T \in \mathbb{R}^2$, $u \in \mathbb{R}$ are the system state and input respectively.

Assumption 1: The linear plant (1) is globally stabilizable from u, i.e., $a, b \geq 0$.

The control object is to design a state feedback controller via non-scaling technique to stabilize system (1) within any prescribed time $T > 0$.

Remark 1: It is clear that there is a smooth function $\varphi_2 \geq 0$ and a constant $\tau \in (0,1)$ such that $|f_2(\cdot)| \leq \varphi_2(\cdot)\sum_{j=1}^{i}|x_j|^{\frac{\lambda_i-\tau}{\lambda_j}}$, where $\lambda_i = 1-(i-1)\tau > 0$, i=1,2,3.

B. Preliminaries

Consider the nonlinear system

$$\dot{x} = f(t,x), \quad x(0) = x_0 \in \mathbb{R}^n, \tag{2}$$

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where $f(\cdot)$ is a (discontinuous) nonlinear vector field satisfying $f(t,0)=0$.

The following results can found in [1,5,10].

Definition 1: The origin of system (2) is fixed-time stable if it is asymptotically stable and the settling-time function $T(x_0)$ exists and is bounded, that is, $x(t,x_0)=0$, $\forall t\geq T(x_0)$ and $\exists\ T_{\max}>0$, s.t. $T(x_0)\leq T_{\max}$, $\forall x_0\in\mathbb{R}^n$.

Definition 2: The origin of system (2) is prescribed-time stable if it is fixed-time stable and for any prescribed time $T_c>0$, a tunable parameter $\theta\in\mathbb{R}$ exists such that $T(x_0)\leq T_c$, $\forall x_0\in\mathbb{R}$.

Lemma 1: For system (2), there exists a C^1, positive definite function $V(x)$ and real numbers $c>0$, $0<\alpha<1$ such that

$$\dot V(x)\leq -cV^{\alpha}(x).$$

Then, the origin of system (2) is finite-time stable with the setting time $T(x_0)$ satisfying

$$T(x_0)\leq\frac{V^{1-\alpha}(0)}{c(1-\alpha)}.$$

Lemma 2: For $\zeta_1\in\mathbb{R}$, $\zeta_2\in\mathbb{R}$, and a constant $m\geq 1$, one has (i)$|\zeta_1+\zeta_2|^m\leq 2^{m-1}|\zeta_1^m+\zeta_2^m|$; (ii)$(|\zeta_1|+|\zeta_2|)^{1/m}\leq|\zeta_1|^{1/m}+|\zeta_2|^{1/m}\leq 2^{(m-1)/m}(|\zeta_1|+|\zeta_2|)^{1/m}$.

Lemma 3: If c,d are positive constant and $\gamma(\zeta_1,\zeta_2)>0$ are real-valued function, then

$$|\zeta_1|^c|\zeta_2|^d\leq\frac{c}{c+d}\gamma(\zeta_1,\zeta_2)|\zeta_1|^{c+d}+\frac{d}{c+d}\gamma^{-\frac{c}{d}}(\zeta_1,\zeta_2)|\zeta_2|^{c+d}.$$

Lemma 4: For $\zeta_1\in\mathbb{R}$, $\zeta_2\in\mathbb{R}$ and constant $0<m\leq 1$ and $a>0$, one has

$$|\lceil\zeta_1\rceil^{am}-\lceil\zeta_2\rceil^{am}|\leq 2^{1-m}|\lceil\zeta_1\rceil^{a}-\lceil\zeta_2\rceil^{a}|^m.$$

III. Prescribed-Time Stabilization

A. Scaling function

For the control object of this paper, we introduce the following switched scaling function:

$$F=\begin{cases}\Gamma, & x\in\{\mathbb{R}^2-\Xi\}\ \&\ t<T_1,\\ 1, & \text{otherwise},\end{cases}\tag{3}$$

where Ξ is a small closed neighborhood of origin and

$$\Gamma=\frac{T_1}{T_1-t},\tag{4}$$

with $0<T_1<T$.

B. Controller design

The controller is developed by the recursive idea.

Step 1: Take

$$V_1=\frac{1}{2}x_1^2,\tag{5}$$

as the Lyapunov function for this step. Then the derivative of V_1 arrives

$$\dot V_1\leq x_1(x_2-x_2^*)+x_1x_2^*,\tag{6}$$

where x_2^* is the virtual controller of x_2.

Select

$$\begin{aligned}x_2^* =&\ -F\left(1+c+\varphi_1\right)\lceil x_1\rceil^{\lambda_2}\\ :=&\ -F\beta_1\lceil x_1\rceil^{\lambda_2},\end{aligned}\tag{7}$$

where c is a positive constant. Then, by substituting (7) into (6), one has

$$\dot V_1\leq -(1+c)F|x_1|^{2-\tau}+x_1\left(x_2-x_2^*\right).\tag{8}$$

Step 2: Define

$$z_2=\lceil x_2\rceil^{\frac{1}{\lambda_2}}-\lceil x_2^*\rceil^{\frac{1}{\lambda_2}},\tag{9}$$

and take the Lyapunov function $V_2=V_1+W_2$ with

$$W_2=\int_{x_2^*}^{x_2}\left\lceil\lceil s\rceil^{\frac{1}{\lambda_2}}-\lceil x_2^*\rceil^{\frac{1}{\lambda_2}}\right\rceil^{2-\lambda_2}ds.\tag{10}$$

From

$$\begin{cases}\dfrac{\partial W_2}{\partial x_2}=\lceil z_2\rceil^{2-\lambda_2},\\ \dfrac{\partial W_2}{\partial\vartheta}=-(2-\lambda_2)\dfrac{\partial\left(\lceil x_2^*\rceil^{\frac{1}{\lambda_2}}\right)}{\partial\vartheta}\\ \qquad\times\displaystyle\int_{x_2^*}^{x_2}\left|\lceil s\rceil^{\frac{1}{\lambda_2}}-\lceil x_2^*\rceil^{\frac{1}{\lambda_2}}\right|^{1-\lambda_2}ds,\end{cases}\tag{11}$$

where $\vartheta=t$ or $\vartheta=x_1$, a direct calculation gives

$$\begin{aligned}\dot V_2\leq&-(1+c)F|x_1|^{2-\tau}+x_1\left(x_2-x_2^*\right)+\lceil z_2\rceil^{2-\lambda_2}u\\ &+\lceil z_2\rceil^{2-\lambda_2}f_2+\frac{\partial W_2}{\partial x_1}(x_2+f_1)+\frac{\partial W_2}{\partial t}.\end{aligned}\tag{12}$$

We gives the estimates of some terms of (12) in the following inequalities and their proofs are listed in Appendix.

$$x_1\left(x_2-x_2^*\right)\leq\frac{1}{4}|x_1|^{2-\tau}+|z_2|^{2-\tau}\varrho_{21},\tag{13}$$

$$\lceil z_2\rceil^{2-\lambda_2}f_2\leq\frac{1}{4}|x_1|^{2-\tau}+F^{\frac{2-\tau}{1+\tau}}|z_2|^{2-\tau}\varrho_{22},\tag{14}$$

$$\frac{\partial W_2}{\partial x_1}(x_2+f_1)\leq\frac{1}{4}|x_1|^{2-\tau}+F^{\frac{(2-\tau)^2}{(1-\tau)^2}}|z_2|^{2-\tau}\varrho_{23},\tag{15}$$

$$\frac{\partial W_2}{\partial t}\leq\frac{1}{4}|x_1|^{2-\tau}+F^{\frac{(2-\tau)^2}{(1-\tau)^2}}|z_2|^{2-\tau}\varrho_{24},\tag{16}$$

where ϱ_{21}, ϱ_{22}, ϱ_{23} and ϱ_{24} are nonnegative smooth functions.

Substituting (13), (14), (15) and (16) into (12) yields

$$\begin{aligned}\dot V_2\leq&-cF|x_1|^{2-\tau}+\lceil z_2\rceil^{2-\lambda_2}u\\ &+|z_2|^{2-\tau}\left(\varrho_{21}+F^{\frac{2-\tau}{1+\tau}}\varrho_{22}+F^{\frac{2-\tau}{1-\tau}}\varrho_{23}+F^{\frac{(2-\tau)^2}{(1-\tau)^2}}\varrho_{24}\right).\end{aligned}\tag{17}$$

Therefore, one can design the actual controller

$$u=-F^{\frac{(2-\tau)^2}{(1-\tau)^2}}\lceil z_2\rceil^{\lambda_2-\tau}\beta_2,\tag{18}$$

with

$$\beta_2=\left(c+\varrho_{21}+\varrho_{22}+\varrho_{23}+\varrho_{24}\right),\tag{19}$$

is a smooth function, which together with the fact that $F\geq 1$ for all $t\geq 0$ is such that

$$\dot V_2\leq -cF|x_1|^{2-\tau}-cF|z_2|^{2-\tau}.\tag{20}$$

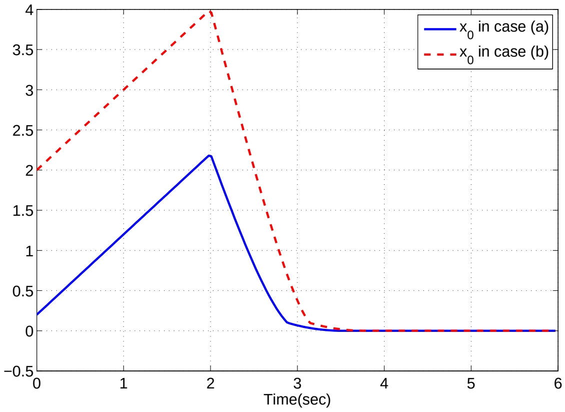

Fig. 1. System state x_0.

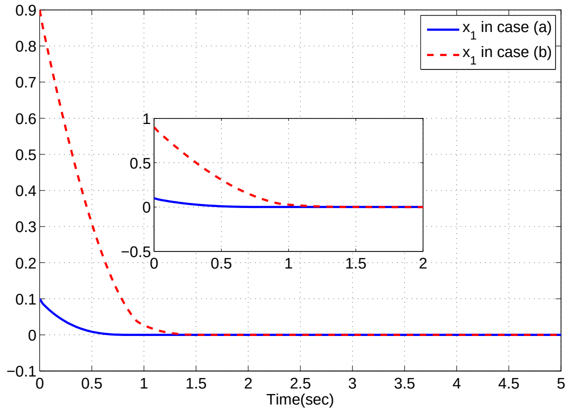

Fig. 2. System state x_1.

C. Stability analysis

We give the main result in this paper.

Theorem 1: For system (1) under Assumption 1, the controller (18) with properly choosing the design parameters renders the origin is prescribed-time stable within any given settling time T.

Proof: The main proof is divided into three steps.

Step 1: From Lemma 4, it is obtained that

$$\begin{aligned} W_2 &= \int_{x_2^*}^{x_2} \left[\lceil s \rfloor^{\frac{1}{\lambda_2}} - \lceil x_2^* \rfloor^{\frac{1}{\lambda_2}} \right]^{2-\lambda_2} ds. \\ &\leq |z_2|^{2-\lambda_2} |x_2 - x_2^*| \\ &\leq 2^{1-\lambda_2} |z_2|^2. \end{aligned} \tag{21}$$

As a result, we have the following estimation.

$$\begin{aligned} V_2^{\frac{2-\tau}{2}} &= (V_1 + W_2)^{\frac{2-\tau}{2}} \\ &\leq \left(\frac{1}{2}|x_1|^2 + 2^{1-\lambda_2}|z_2|^2 \right)^{\frac{2-\tau}{2}} \\ &\leq |x_1|^{2-\tau} + |z_2|^{2-\tau}. \end{aligned} \tag{22}$$

which together with (20) renders

$$\begin{aligned} \dot{V}_2 &\leq -c\digamma |x_1|^{2-\tau} - c\digamma |z_2|^{2-\tau} \\ &\leq -c\digamma V_2^{\frac{2-\tau}{2}}. \end{aligned} \tag{23}$$

When $\digamma = \Gamma$, (23) means Ξ is prescribed-time attractive and the convergence time satisfies

$$T_a \leq T_1 \left(1 - exp \left(-\frac{2V_2^{\frac{\tau}{2}}(0)}{c\tau} \right) \right) < T_1. \tag{24}$$

Step 2 : When $\digamma = 1$, set $M = \max_{\zeta \in \Xi} V_2(\zeta)$. Then from (23), we know that the CLS is locally finite-time stable in the domain Ξ with

$$T_l \leq \frac{2V_2^{\frac{\tau}{2}}(0)}{c\tau} \leq \frac{2M^{\frac{\tau}{2}}}{c\tau}. \tag{25}$$

Therefore, by choosing $c \geq (2M^{\frac{\tau}{2}})/(\tau T - \tau T_1)$, one has $T_l \leq T - T_1$.

Step 3: (23) indicates that system (1) under controller (18) is Lyapunov asymptotically convergent (stable) in both operational domains. By the properties of existence and continuation of the solutions, the whole system is Lyapunov asymptotically stable. Therefore, the origin of the CLS is prescribed-time stable within $T_p < T_a + T_l < T$.

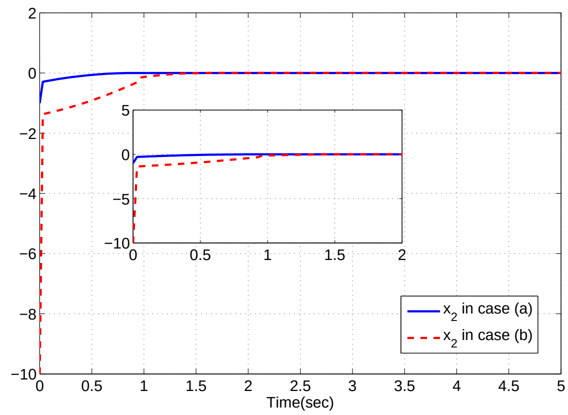

Fig. 3. System state x_2.

IV. An Application Example

Consider a tricycle-type wheeled mobile robot whose kinematic equations are represented by

$$\begin{aligned} \dot{x}_0 &= u_0, \\ \dot{x}_1 &= u_0 x_1, \\ \dot{x}_2 &= u_1 - u_0 x_1. \end{aligned} \tag{26}$$

For the x_0-subsystem, if we take

$$u_0 = 1, \tag{27}$$

then the x-subsystem becomes

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= u_1 - x_1, \end{aligned} \tag{28}$$

Note that this system is a special case of system (1) with $f_2 = -x_1$. Hence, the proposed control design can be employed. For the x-subsystem, the prescribed-time controller u_1 is designed as (18) with $\tau = 1/3$, $c = 0.5$, $T_1 = 1.8$, $\Xi = \{x : x_1^2 + x_2^2 \leq 0.01\}$ to drive the states to zero within the prescribed time $T_s = 2s$.

Then when $t \geq T_s$, switch u_0 to

$$u_0 = -\bar{\digamma} \lceil x_0 \rfloor^{\frac{1}{2}} \tag{29}$$

where

$$\bar{\digamma} = \begin{cases} \dfrac{T_2 - T_s}{T_2 - T_s - t}, & x_0 \in \{\mathbb{R} - \Xi_0\} \ \& \ T_s \leq t < T_2, \\ 1, & t \geq T_2, \end{cases} \tag{30}$$

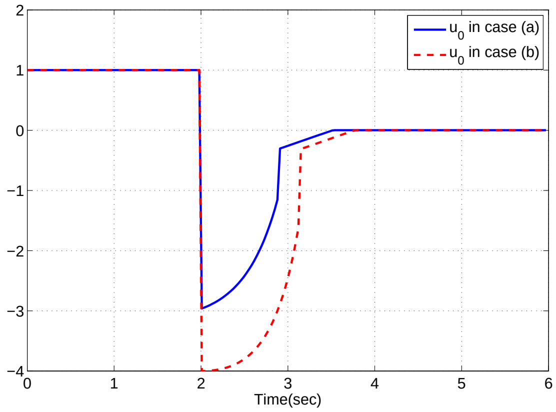

Fig. 4. Control input u_0.

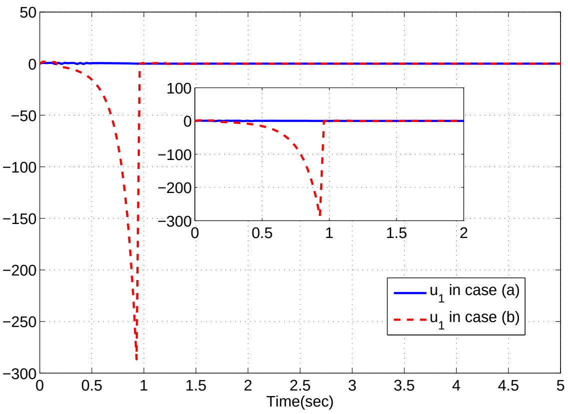

Fig. 5. Control input u_1.

with $\Xi_0 = \{x_0^2 \le 0.01\}$ which regulates the state x_0 to zero within a prescribed time $T_2 = 4s$.

For different initial conditions: (a) $(x_0(0), x_1(0), x_2(0))=$ $(0.2, 0.1, -1)$ and (b) $(x_0(0), x_1(0), x_2(0))=$ $(2, 0.9, -10)$, Figs.1–5 are given to depict the responses of the CLS, which confirm the effectiveness of the proposed scheme.

V. Conclusion

In this paper, a switched, non-scaling design has been developed for the PTS of double integrator systems. The suitable switching mechanism renders the proposed control scheme ensuring the prescribed-time stability of the origin of the CLS. At the same time it solves the computationally singular problem effectively and leads to a simpler controller. Extension of the result to more general systems is one of our future research topics.

Appendix

Based on the fact $\digamma \ge 1$ for all $t \ge 0$, we given the proofs of (13), (14), (15) and (16).

Proof of (13): Based on the definitions of z_2 and x_2^* and Lemma 2.3, one has

$$\begin{aligned} |x_2 - x_2^*| &\le 2^{1-\lambda_2}\left|\lceil x_2 \rfloor^{\frac{1}{\lambda_2}} - \lceil x_2^* \rfloor^{\frac{1}{\lambda_2}}\right|^{\lambda_2} \\ &= 2^{1-\lambda_2}|z_2|^{\lambda_2}. \end{aligned} \tag{31}$$

Thus, from Lemma 4, it is obtained that

$$\lceil x_1 \rfloor^{2-\lambda_1}(x_2 - x_2^*) \le \frac{1}{4}|x_1|^{2-\tau} + |z_2|^{2-\tau}\varrho_{21}, \tag{32}$$

where $\varrho_{21} \ge 0$ is a smooth function.

Proof of (14): Firstly, from Remark 1 and Lemma 2, one gets

$$\begin{aligned} |f_2| &\le \varphi_2\left(|x_1|^{\frac{\lambda_2-\tau}{\lambda_1}} + |x_2|^{\frac{\lambda_2-\tau}{\lambda_2}}\right) \\ &\le \varphi_2\left(|x_1|^{\lambda_2-\tau} + |z_2|^{\lambda_2-\tau} + \digamma\beta_1|x_1|^{\lambda_2-\tau}\right). \end{aligned} \tag{33}$$

Using (33) and Lemma 3 yields

$$\lceil z_2 \rfloor^{2-\lambda_2} f_2 \le \frac{1}{4}|x_1|^{2-\tau} + \digamma^{\frac{2-\tau}{1+\tau}}|z_2|^{2-\tau}\varrho_{22}, \tag{34}$$

where $\varrho_{22} \ge 0$ is a smooth function.

Proof of (15) and (16): Notice that

$$\begin{aligned} \int_{x_2^*}^{x_2}\left|\lceil s \rfloor^{\frac{1}{\lambda_2}} - \lceil x_2^* \rfloor^{\frac{1}{\lambda_2}}\right|^{1-\lambda_2} ds &\le \frac{2\rho-\lambda_2}{\rho}|z_2|^{\frac{\rho-\lambda_2}{\rho}}|x_2 - x_2^*| \\ &\le 2^{1-\lambda_2}|z_2|, \end{aligned} \tag{35}$$

$$\begin{aligned} \left|\frac{\partial\left(\lceil x_2^* \rfloor^{\frac{1}{\lambda_2}}\right)}{\partial x_1}\right| &\le \digamma^{\frac{1}{\lambda_2}}\left|\frac{\partial \beta_1^{\frac{1}{\lambda_2}}}{\partial x_1}\right||x_1| + \digamma^{\frac{1}{\lambda_2}}\beta_1^{\frac{1}{\lambda_2}} \\ &\le \digamma^{\frac{1}{\lambda_2}}\gamma_{21}, \end{aligned} \tag{36}$$

$$\begin{aligned} \left|\frac{\partial\left(\lceil x_2^* \rfloor^{\frac{1}{\lambda_2}}\right)}{\partial t}\right| &\le \frac{2}{T_1\lambda_2}\digamma^{\frac{1+\lambda_2}{\lambda_2}}\beta_1^{\frac{1}{\lambda_2}}|x_1| \\ &\le \digamma^{\frac{1+\lambda_2}{\lambda_2}}|x_1|\gamma_{22}, \end{aligned} \tag{37}$$

where γ_{21} and γ_{22} are some nonnegative smooth functions.

Then, from (35), (36), (37) and Lemma 3, one arrives

$$\frac{\partial W_2}{\partial x_1}(x_2 + f_1) \le \frac{1}{4}|x_1|^{2-\tau} + \digamma^{\frac{2-\tau}{1-\tau}}|z_2|^{2-\tau}\varrho_{23}, \tag{38}$$

$$\frac{\partial W_2}{\partial t} \le \frac{1}{4}|x_1|^{2-\tau} + \digamma^{\frac{(2-\tau)^2}{(1-\tau)^2}}|z_2|^{2-\tau}\varrho_{24}, \tag{39}$$

where $\varrho_{23} \ge 0$ and $\varrho_{24} \ge 0$ are smooth functions.

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