Numerical Investigation on Train Vertical Dynamics with Damper Rubber Joint Stiffness

Yuewei Yu, Yunpeng Song, Leilei Zhao and Changcheng Zhou

Abstract—**For the railway vehicle vertical dynamic model under the effect of damper rubber joint stiffness, there is no reliable and fast method to solve its dynamic response at present. In this paper, to improve the efficiency of numerical analysis, by reducing the degree of freedom of the system and transforming the vibration differential equation, a numerical solution method for railway vehicles vertical dynamic response under the effect of damper rubber joint stiffness is established. Through a case analysis, the variation of vertical random vibration response of railway vehicles with the stiffness of damping rubber joints is studied by using random vibration theory. This study provides an effective solution method for the numerical simulation of railway vehicles dynamic response under the effect of the damping rubber joint stiffness, and provides a guidance for the stiffness design of the damper rubber joints for railway vehicles.**

Index Terms—**railway vehicle, modelling and simulation, numerical solution, damper rubber joint stiffness**

I. Introduction

As an important vibration absorbing parts of railway vehicles, the reliability and performance of the damper have a very important impact on railway vehicles operation quality [1]. In the study of railway vehicles dynamics, damper dynamics simulation is an important part, and its simulation accuracy has a very important impact on the simulation accuracy of railway vehicles dynamics. However, in the previous research, when establishing the railway vehicles suspension system model, the damper was mostly regarded as a single damper [2-4], and the elasticity of the damper rubber joints was rarely considered. But in practical application, both ends of the damper are not rigidly connected, they are connected to the vehicle system through rubber joints. The purpose on the one hand is to improve the ability of the vibration isolation, and the noise reduction of the system, on the other hand, is to avoid the impact of vibration in other directions on the components connected on both sides of the damper, thus extending its service life [5-7].

With the increase of railway vehicles running speed, track irregularities excitation frequency is further increased. Meanwhile, the dynamic action between the wheel-set and track increases, which makes the effect of the damper elastic connection stiffness more prominent [8]. Therefore, many scholars began to consider the effect of the damper elastic connection stiffness, and established many railway vehicle dynamic models under the effect of damper elastic connection stiffness [9-14]. However, as the mass of the damper is several times smaller than that of the other components in the vehicle, it can be totally neglected. As a result, the introduction of the damper elastic connection stiffness greatly increases the difficulty of solving the system vibration response. At present, there is no reliable and fast method to solve the vibration response of railway vehicle system with damping rubber joint stiffness. Also, the discussion on the influence of damping rubber joint stiffness on railway vehicles vibration response is not deep enough, and its influence mechanism has not been understood yet.

By reducing the degree of freedom of the system and transforming the vibration differential equation, this paper establishes a numerical solution method for railway vehicles vertical dynamic response under the effect of damper rubber joint stiffness. Through a case analysis, the variation of vertical random vibration response of railway vehicles with the stiffness of damping rubber joints is studied, which is carried out based on the random vibration theory. This study provides an effective solution method for the numerical simulation of railway vehicles dynamic response under the effect of the damping rubber joint stiffness.

II. Train vertical dynamic model

A. Traditional model

In the previous research on the vertical dynamic modeling of traditional railway vehicles, the influence of damper elastic connection stiffness is usually ignored. Generally, the suspension system is simplified to the form of parallel spring-damper [15,16], as shown in Figure 1. Here, M_w is the half mass of the wheel-set; M_t and J_t are the half of the bogie frame mass and its moment of inertia; M_c and J_c are the half of the car body mass and its moment of inertia. K_p and C_p are the vertical stiffness and damping of the primary suspension; K_s and C_s are the vertical stiffness and damping of the secondary suspension. K_H is the equivalent linear contact stiffness between the wheel and rail. L_c and L_t are the half of the fixed distance of the vehicle and the bogie wheelbase. z_{t1}, z_{t2}, and z_c are the vertical displacements of the front bogie frame, the rear bogie frame, and the car body. β_{t1}, β_{t2}, and β_c are the pitching displacements of the front bogie frame, the

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rear bogie frame, and the car body. z_{w1}~z_{w4} are the wheel-set vertical displacements. P_1~P_4 are the interaction forces between the train and track. z_{v1}~z_{v4} are the track irregularities random input. The contact stiffness K_H can be written as [12]

$$K_{\mathrm{H}} = \frac{1.5P_0^{1/3}}{G} \tag{1}$$

where, $P_0=(M_c/4+M_t/2+M_w)g$, and g=9.8 m/s^2; G is the contact constant, its specific values can be founded in reference [14].

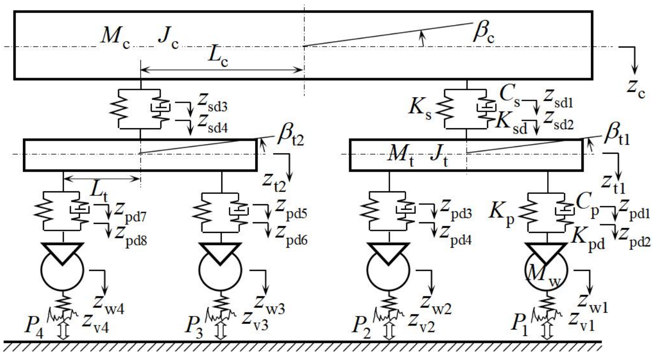


Fig. 1. Traditional railway vehicle vertical dynamic model

Here, according to z_{v1}, z_{v2}~z_{v4} can be obtained by the following formula, that is

$$z_{v2} = z_{v1}(t-\tau_1), \quad z_{v3} = z_{v1}(t-\tau_2), \quad z_{v4} = z_{v1}(t-\tau_3) \tag{2}$$

in which, t represents time; τ denotes time lag, which can be calculated according to vehicle running speed, fixed distance of vehicle and bogie wheelbase, i.e. $\tau_1=2L_t/v$, $\tau_2=2L_c/v$, $\tau_3=2(L_t+L_c)/v$, v is vehicle running speed.

B. Railway vehicle vertical dynamic model with damper rubber joint stiffness

In order to reflect the actual working characteristics of the damper effectively, regarding the damper as a "stiffness-damping-stiffness" series form, the railway vehicle vertical dynamic model under the effect of damper rubber joint stiffness can be established, as shown in Figure 2 [11,12]. In which, the meaning of each parameter in Figure 2 is the same as those of the traditional model shown in Figure 1, and only the stiffness of the damper rubber joint is considered. Here, K_{pd} is the primary damper rubber joint stiffness, K_{sd} is the secondary damper rubber joint stiffness; z_{pd1}~z_{pd8} and z_{sd1}~z_{sd4} are the vertical displacements of the two ends of the damper.

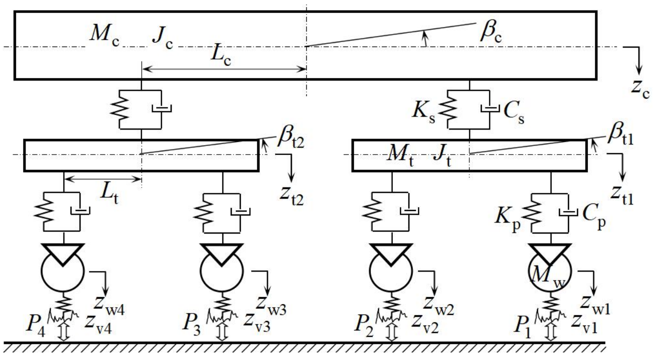


Fig. 2. Railway vehicle vertical dynamic model under the effect of damper rubber joint stiffness

III. Mathematical model of the railway vehicle model under effect of the damper rubber joint stiffness

According to Newton's second law, based on the analysis of the force relationship between each component, the vibration differential equations of the system shown in Figure 2 can be calculated, that is

- *The motion of the car body*

$$\begin{cases} M_c\ddot{z}_c = -K_s(z_c - z_{t1} - L_c\beta_c) - K_{sd}(z_c - z_{sd1} - L_c\beta_c) \\ \quad -K_s(z_c - z_{t2} + L_c\beta_c) - K_{sd}(z_c - z_{sd3} + L_c\beta_c) \\ J_c\ddot{\beta}_c = [K_s(z_c - z_{t1} - L_c\beta_c) + K_{sd}(z_c - z_{sd1} - L_c\beta_c)]L_c \\ \quad -[K_s(z_c - z_{t2} + L_c\beta_c) + K_{sd}(z_c - z_{sd3} + L_c\beta_c)]L_c \end{cases} \tag{3}$$

- *The motion of the bogie frame*

$$\begin{cases} J_t\ddot{\beta}_{t1} = [K_p(z_{t1} - z_{w1} - L_t\beta_{t1}) + K_{pd}(z_{t1} - z_{pd1} - L_t\beta_{t1})]L_t \\ \quad -[K_p(z_{t1} - z_{w2} + L_t\beta_{t1}) + K_{pd}(z_{t1} - z_{pd3} + L_t\beta_{t1})]L_t \\ M_t\ddot{z}_{t1} = -K_s(z_{t1} + L_c\beta_c - z_c) + K_{sd}(z_{t1} - z_{sd2}) \\ \quad -K_p(z_{t1} - z_{w1} - L_t\beta_{t1}) - K_{pd}(z_{t1} - z_{pd1} - L_t\beta_{t1}) \\ \quad -K_p(z_{t1} - z_{w2} + L_t\beta_{t1}) - K_{pd}(z_{t1} - z_{pd3} + L_t\beta_{t1}) \\ J_t\ddot{\beta}_{t2} = [K_p(z_{t2} - L_t\beta_{t2} - z_{w3}) + K_{pd}(z_{t2} - L_t\beta_{t2} - z_{pd5})]L_t \\ \quad -[K_p(z_{t2} + L_t\beta_{t2} - z_{w4}) + K_{pd}(z_{t2} + L_t\beta_{t2} - z_{pd7})]L_t \\ M_t\ddot{z}_{t2} = -K_s(z_{t2} - z_c - L_c\beta_c) - K_{sd}(z_{t2} - z_{sd4}) \\ \quad -K_p(z_{t2} - L_t\beta_{t2} - z_{w3}) - K_{pd}(z_{t2} - L_t\beta_{t2} - z_{pd5}) \\ \quad -K_p(z_{t2} + L_t\beta_{t2} - z_{w4}) + K_{pd}(z_{t2} + L_t\beta_{t2} - z_{pd7}) \end{cases} \tag{4}$$

- *The motion of the wheel-set*

$$\begin{cases} M_w\ddot{z}_{w1} = -K_p(z_{w1} - z_{t1} + L_t\beta_{t1}) - K_{pd}(z_{w1} - z_{pd2}) \\ \quad -K_H(z_{w1} - z_{v1}) \\ M_w\ddot{z}_{w2} = -K_p(z_{w2} - z_{t1} - L_t\beta_{t1}) - K_{pd}(z_{w2} - z_{pd4}) \\ \quad -K_H(z_{w2} - z_{v2}) \\ M_w\ddot{z}_{w3} = -K_p(z_{w3} - z_{t2} + L_t\beta_{t2}) - K_{pd}(z_{w3} - z_{pd6}) \\ \quad -K_H(z_{w3} - z_{v3}) \\ M_w\ddot{z}_{w4} = -K_p(z_{w4} - z_{t2} - L_t\beta_{t2}) - K_{pd}(z_{w4} - z_{pd8}) \\ \quad -K_H(z_{w4} - z_{v4}) \end{cases} \tag{5}$$

- *The motion of the primary vertical damper*

$$\begin{cases} C_p(\dot{z}_{pd1} - \dot{z}_{pd2}) + K_{pd}(z_{pd1} - z_{t1} + L_t\beta_{t1}) = 0 \\ C_p(\dot{z}_{pd2} - \dot{z}_{pd1}) + K_{pd}(z_{pd2} - z_{w1}) = 0 \\ C_p(\dot{z}_{pd3} - \dot{z}_{pd4}) + K_{pd}(z_{pd3} - z_{t1} - L_t\beta_{t1}) = 0 \\ C_p(\dot{z}_{pd4} - \dot{z}_{pd3}) + K_{pd}(z_{pd4} - z_{w2}) = 0 \\ C_p(\dot{z}_{pd5} - \dot{z}_{pd6}) + K_{pd}(z_{pd5} - z_{t2} + L_t\beta_{t2}) = 0 \\ C_p(\dot{z}_{pd6} - \dot{z}_{pd5}) + K_{pd}(z_{pd6} - z_{w3}) = 0 \\ C_p(\dot{z}_{pd7} - \dot{z}_{pd8}) + K_{pd}(z_{pd7} - z_{t2} - L_t\beta_{t2}) = 0 \\ C_p(\dot{z}_{pd8} - \dot{z}_{pd7}) + K_{pd}(z_{pd8} - z_{w4}) = 0 \end{cases} \tag{6}$$

- *The motion of the secondary vertical damper*

$$\begin{cases} C_s(\dot{z}_{sd1} - \dot{z}_{sd2}) + K_{sd}(z_{sd1} - z_c + L_c\beta_c) = 0 \\ C_s(\dot{z}_{sd2} - \dot{z}_{sd1}) + K_{sd}(z_{sd2} - z_{t1}) = 0 \\ C_s(\dot{z}_{sd3} - \dot{z}_{sd4}) + K_{sd}(z_{sd3} - z_c - L_c\beta_c) = 0 \\ C_s(\dot{z}_{sd4} - \dot{z}_{sd3}) + K_{sd}(z_{sd4} - z_{t2}) = 0 \end{cases} \tag{7}$$

IV. Numerical solution of the railway vehicle model under effect of the damper rubber joint stiffness

From equations (3)~(7), it can be seen that, the equation groups contain six second-order differential equations with acceleration variables and twelve first-order differential equations with velocity variables. If it is not transformed, it can not be expressed as a solvable state-space equation. Therefore, for the purpose of solving the problem conveniently, the degree of freedom of the damper should be reduced by variable substitution, that is, let $\Delta z_{s1}=z_{sd1}-z_{sd2}$, $\Delta z_{s2}=z_{sd3}-z_{sd4}$, $\Delta z_{p1}=z_{pd1}-z_{pd2}$, $\Delta z_{p2}=z_{pd3}-z_{pd4}$, $\Delta z_{p3}=z_{pd5}-z_{pd6}$, $\Delta z_{p4}=z_{pd7}-z_{pd8}$, and the following transformation relations can be obtained:

$$\begin{cases} z_{sdi}=\frac{1}{2}(z_c-L_c\beta_c+z_{t1}\pm\Delta z_{s1}) \\ z_{sdj}=\frac{1}{2}(z_c+L_c\beta_c+z_{t2}\pm\Delta z_{s2}) \\ z_{pdi}=\frac{1}{2}(z_{t1}-L_t\beta_{t1}+z_{w1}\pm\Delta z_{p1}) \\ z_{pdj}=\frac{1}{2}(z_{t1}+L_t\beta_{t1}+z_{w2}\pm\Delta z_{p2}) \\ z_{pdp}=\frac{1}{2}(z_{t2}-L_t\beta_{t2}+z_{w3}\pm\Delta z_{p3}) \\ z_{pdq}=\frac{1}{2}(z_{t2}+L_t\beta_{t2}+z_{w4}\pm\Delta z_{p4}) \end{cases} \tag{8}$$

where, i=1,2; j=3,4; p=5,6; q=7,8. Only when i=1, j=3, p=5, q=7, the "±" in the formula takes "+".

Substituting equation (8) into equations (3)~(7), the following matrix form can be obtained:

$$\boldsymbol{M}\ddot{\boldsymbol{X}}+\boldsymbol{K}\boldsymbol{X}=\boldsymbol{P}(t) \tag{9}$$

where, $\boldsymbol{M}$ is a 16×16-order matrix, which is composed of the mass and damping of the vibration system; $\boldsymbol{K}$ is a 16×16-order matrix expressed by the stiffness of the vibration system; $\ddot{\boldsymbol{X}}$ is a 16×1-order matrix expressed by the velocity and acceleration of the vibration system; $\boldsymbol{X}$ is a 16×1-order matrix expressed by the displacements of the vibration system; $\boldsymbol{P}(t)$ is a 16-dimensional vector composed of the excitation load of the vibration system. Here,

$$\ddot{\boldsymbol{X}}=\left[\ddot{z}_c\ \ddot{\beta}_c\ \ddot{z}_{t1}\ \ddot{\beta}_{t1}\ \ddot{z}_{w1}\ \ddot{z}_{w2}\ \ddot{z}_{t2}\ \ddot{\beta}_{t2}\ \ddot{z}_{w3}\ \ddot{z}_{w4}\ \Delta\dot{z}_{s1}\ \Delta\dot{z}_{s2}\ \Delta\dot{z}_{p1}\ \Delta\dot{z}_{p2}\ \Delta\dot{z}_{p3}\ \Delta\dot{z}_{p4}\right]^T_{1\times16};$$

$$\boldsymbol{X}=\left[z_c\ \beta_c\ z_{t1}\ \beta_{t1}\ z_{w1}\ z_{w2}\ z_{t2}\ \beta_{t2}\ z_{w3}\ z_{w4}\ \Delta z_{s1}\ \Delta z_{s2}\ \Delta z_{p1}\ \Delta z_{p2}\ \Delta z_{p3}\ \Delta z_{p4}\right]^T_{1\times16}.$$

According to equation (9), the state vector $\boldsymbol{x}=\left[\dot{z}_c\ \dot{\beta}_c\ \dot{z}_{t1}\ \dot{\beta}_{t1}\ \dot{z}_{w1}\ \dot{z}_{w2}\ \dot{z}_{t2}\ \dot{\beta}_{t2}\ \dot{z}_{w3}\ \dot{z}_{w4}\ \Delta z_{s1}\ \Delta z_{s2}\ \Delta z_{p1}\ \Delta z_{p2}\ \Delta z_{p3}\ \Delta z_{p4}\ z_c\ \beta_c\ z_{t1}\ \beta_{t1}\ z_{w1}\ z_{w2}\ z_{t2}\ \beta_{t2}\ z_{w3}\ z_{w4}\right]^T_{1\times26}$ is defined, and the initial value of each state variable is specified, i.e., $\boldsymbol{x}(t_0)$=0. Therefore, equation (9) can be transformed into the form of state equation, that is

$$\begin{cases}\dot{\boldsymbol{x}}=\boldsymbol{C}\boldsymbol{x}+\boldsymbol{D}\boldsymbol{u} \\ \boldsymbol{y}=\boldsymbol{E}\boldsymbol{x}+\boldsymbol{F}\boldsymbol{u}\end{cases} \tag{10}$$

where, the system matrix $\boldsymbol{C}=\begin{bmatrix}\boldsymbol{O}_{16\times10} & \boldsymbol{H}_{16\times16} \\ \boldsymbol{I}_{10\times10} & \boldsymbol{O}_{10\times16}\end{bmatrix}_{26\times26}$, the input matrix $\boldsymbol{D}=\begin{bmatrix}\boldsymbol{M}^{-1} & \boldsymbol{O}_{16\times10} \\ \boldsymbol{O}_{10\times16} & \boldsymbol{O}_{10\times10}\end{bmatrix}_{26\times26}$, the output matrix $\boldsymbol{E}=\boldsymbol{I}_{26\times26}$, the transfer matrix $\boldsymbol{F}=\boldsymbol{O}_{26\times26}$, the input vector $\boldsymbol{u}=\begin{bmatrix}\boldsymbol{P}(t) \\ \boldsymbol{O}_{10\times1}\end{bmatrix}_{26\times1}$; the output vector $\boldsymbol{y}=\left[\dot{z}_c\ \dot{\beta}_c\ \dot{z}_{t1}\ \dot{\beta}_{t1}\ \dot{z}_{w1}\ \dot{z}_{w2}\ \dot{z}_{t2}\ \dot{\beta}_{t2}\ \dot{z}_{w3}\ \dot{z}_{w4}\ \Delta z_{s1}\ \Delta z_{s2}\ \Delta z_{p1}\ \Delta z_{p2}\ \Delta z_{p3}\ \Delta z_{p4}\ z_c\ \beta_c\ z_{t1}\ \beta_{t1}\ z_{w1}\ z_{w2}\ z_{t2}\ \beta_{t2}\ z_{w3}\ z_{w4}\right]^T_{1\times26}$. Here, $\boldsymbol{O}$ is a zero matrix, $\boldsymbol{I}$ is an unit matrix, $\boldsymbol{H}$ is a matrix obtained by interchanging columns 1-10 and 11-16 in matrix $-\boldsymbol{M}^{-1}\boldsymbol{K}$ as a whole.

According to the fourth order Runge Kutta method, particularly its variable-step size solution method, the state space equation (10) can be solved, then the output results of the railway vehicle model under the action of damping rubber joint stiffness can be obtained.

V. Solving Example

In this section, the vertical vibration response of a given train is simulated using the established numerical solution method. For the given vehicle, its parameters are: the half mass of the wheel-set M_w=700 kg; the half mass and pitching inertia of the bogie frame M_t=600 kg, J_t=700 kg·m^2; the half mass and pitching inertia of the car body M_c=15 200 kg, J_c=1 019 000 kg·m^2; the primary suspension vertical stiffness K_p=875 000 N/m, the primary damper rubber joint stiffness K_{pd}=5 MN/m; the secondary suspension vertical stiffness K_s=412 000 N/m, the secondary damper rubber joint stiffness K_{sd}=5 MN/m; the primary suspension vertical damping C_p=17 000 N·s/m, the secondary suspension vertical damping C_s=54 000 N·s/m; the half of the fixed distance of the vehicle L_c=3.289 m, the half of the fixed distance of the bogie wheelbase L_t=0.797 m; furthermore, the vehicle running speed is 250 km.

A. Input model of the track random irregularity

In the random vibration analysis of railway vehicles, the commonly used track excitation model is the German low interference spectrum, which can well represent the actual characteristics of the rail, its spatial frequency analytic expression is [17]

$$S_v(\Omega)=\frac{A_v\Omega_c^2}{(\Omega^2+\Omega_r^2)(\Omega^2+\Omega_c^2)} \tag{11}$$

where, Ω is the track irregularity spatial frequency; A_v is the rail roughness coefficient, A_v =4.032× 10^{-7} m^2·rad/m; Ω_r and Ω_c are the truncated spatial frequencies, Ω_r=0.020 6 rad/m, Ω_c=0.824 6 rad/m.

According to the time-frequency conversion method [18], the frequency-domain excitation shown in equation (11) can be converted into time-domain model, on this foundation, the vertical vibration response of the train with damper rubber joint stiffness can be solved. Here, the track irregularity time domain signal of each wheel-set is given when the vehicle running speed v=250 km/h, as shown in Figure 3. According to the analysis of Figure 3, we can see that, the changes of the track vertical irregularity at each wheel-set are consistent, and there is only a certain lag in time between the four wheel-sets.

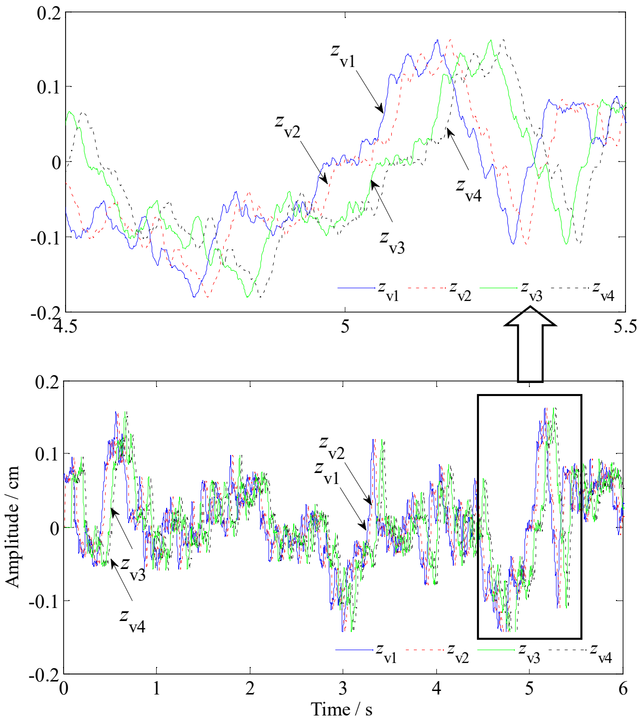
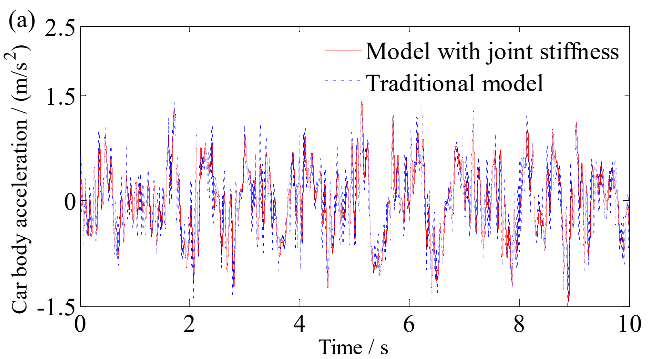
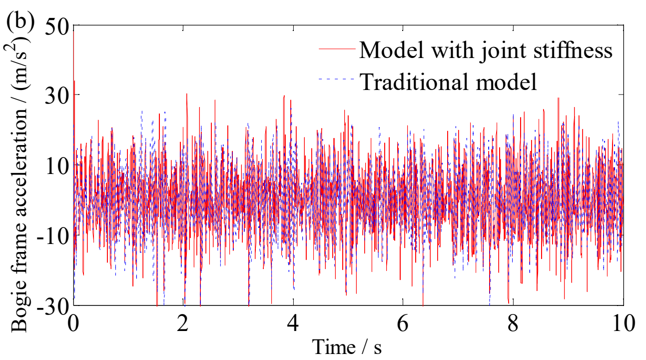


Fig. 3. Track vertical irregularity

B. Vertical random vibration response analysis

Figure 4 shows the variation curves of the vertical vibration acceleration of each component and the vertical stroke of each suspension under the two models obtained by the simulation analysis. Note that, the traditional model is calculated by the Newmark-β method, the model with joint stiffness is calculated by the established method.





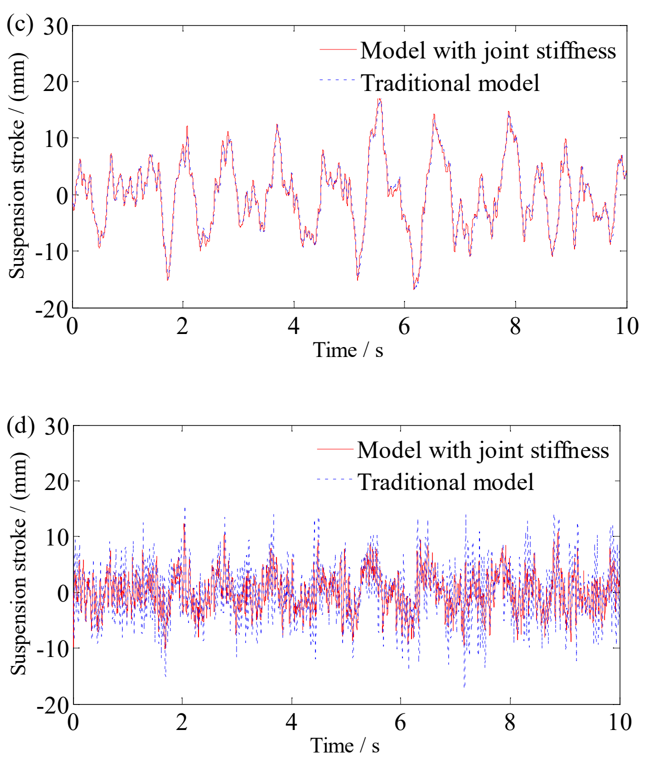


Fig. 4. Analysis results: (a) the car body acceleration; (b) the bogie frame acceleration; (c) the secondary suspension stroke; (d) the primary suspension stroke

As displayed in Figure 4, the car body acceleration and secondary suspension vertical stroke is basically identical under the two models. In addition, the bogie frame vertical acceleration and primary suspension vertical stroke are somewhat different. The reason for this difference is that, the impact of the rubber joint stiffness of the damper on the train is mainly reflected in its high frequency range, has little impact on its low frequency characteristics. The high-frequency vibration response components (affected by damper rubber joint stiffness) and the basic response (refer to the traditional railway vehicle model shown in Figure 1) are mixed together, which makes the difference between the analysis results of the bogie frame larger, while the difference between the analysis results of the car body smaller. This is obviously due to the stiffness of the rubber joint of the damper. It can be concluded that, the dynamic response of railway vehicles is affected by the stiffness of damping rubber joints, which more strongly proves that, the calculation solution method for railway vehicles with considering the effect of damper rubber joint stiffness established is correct.

VI. Influence Analysis of the Damper Rubber Joint Stiffness

For the purpose of finding out the effect of the damper rubber joint stiffness on railway vehicles vertical dynamic performance, the vehicle given in Section V under different running speed and different damper rubber joint stiffness is analyzed. Here, in the analysis, the RMS (i.e., root mean square) values of the car body vertical acceleration, the bogie frame vertical acceleration, the secondary suspension vertical stroke, and the primary suspension vertical stroke were taken as the evaluation criteria. The rubber joint stiffness of the suspension dampers are shown in Table 1.

TABLE I
RUBBER JOINT STIFFNESS VALUES OF THE DAMPER

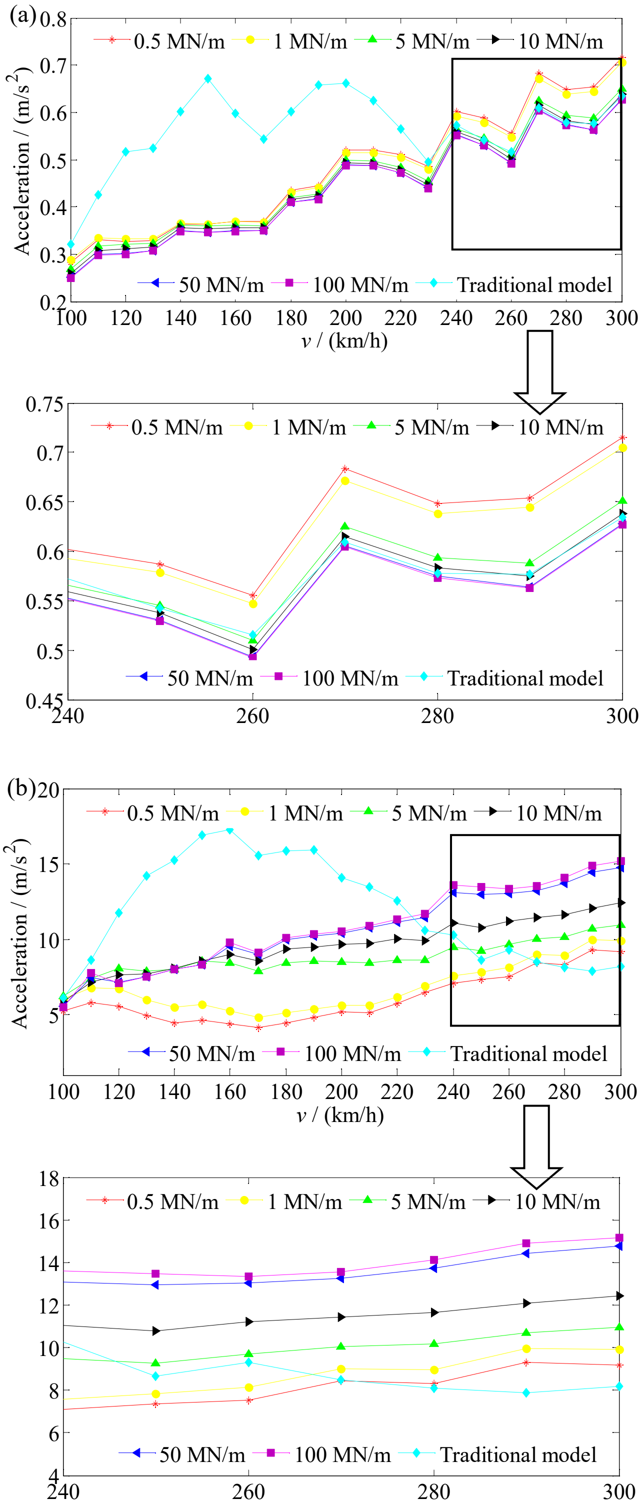
Rubber joint stiffness	Values					
K_{pd}/(MN/m)	0.5	1	5	10	50	100
K_{pd}/(MN/m)	0.5	1	5	10	50	100

A. Influence analysis of the primary vertical damper rubber joint stiffness

When only primary vertical damper rubber joint stiffness changes, and other parameters remain unchanged, the RMS curves of the railway vehicle vertical random response at different running speed are obtained, as shown in Figure 5. In the simulation, the secondary vertical damper rubber joint stiffness K_{sd}=5 MN/m; the vehicle running speed v=100~300 km/h, incremental speed step is 10 km/h; the simulation time is 20 s.

Inspection of Figure 5 shows that:

1) The effect of the primary damper rubber joint stiffness on car body vertical acceleration is mainly reflected in the lower vehicle running speed. When the vehicle is running at a low speed, the car body vertical acceleration with primary vertical damper rubber joint stiffness is less than that of the traditional model, and is basically consistent with the traditional model when the running speed is high, but as long as the joint stiffness exists, the car body vertical acceleration under two models will not be coincidence. In terms of the damper rubber joint stiffness, with the increase of the stiffness of the primary damper rubber joint, the car body acceleration shows a decreasing trend, and the higher the vehicle speed is, the more obvious this effect is. In addition, when the primary vertical damper joint stiffness increases to a certain extent, the car body acceleration is gradually close to that without considering the impact of the damper joint stiffness, but the change is very slow.
2) Compared with the traditional model, the primary vertical damper rubber joint stiffness has a certain influence on the bogie frame acceleration at different vehicle running speed, which is more significant at the lower vehicle running speed. When the vehicle is running at a low speed, the bogie frame vertical acceleration with considering the effect of the primary vertical damper rubber joint stiffness is less than that of the traditional model, but when the vehicle running speed is high, it is greater than that of the traditional model. In terms of the damper rubber joint stiffness, the greater the primary vertical damper joint stiffness is, the greater the bogie frame vertical acceleration will be, and the impact of the primary damper rubber joint stiffness on the vertical acceleration of the bogie frame increases with the increase of vehicle running speed.
3) Whether considering the effect of the primary vertical damper rubber joint stiffness or not, the vertical stroke of the secondary suspension changes little. In terms of the primary vertical damper rubber joint stiffness, the greater the joint stiffness is, the smaller the secondary suspension vertical stroke will be, and the influence of the joint stiffness of the primary vertical damper on the vertical stroke of the secondary suspension increases with the increase of vehicle speed. In addition, when the primary vertical damper joint stiffness increases to a certain extent, the secondary suspension vertical stroke with considering the effect of the primary vertical damper rubber joint stiffness gradually approaches the traditional model, but the change is very slow. As long as the joint stiffness exists, the secondary suspension vertical stroke under the two models can not coincide.



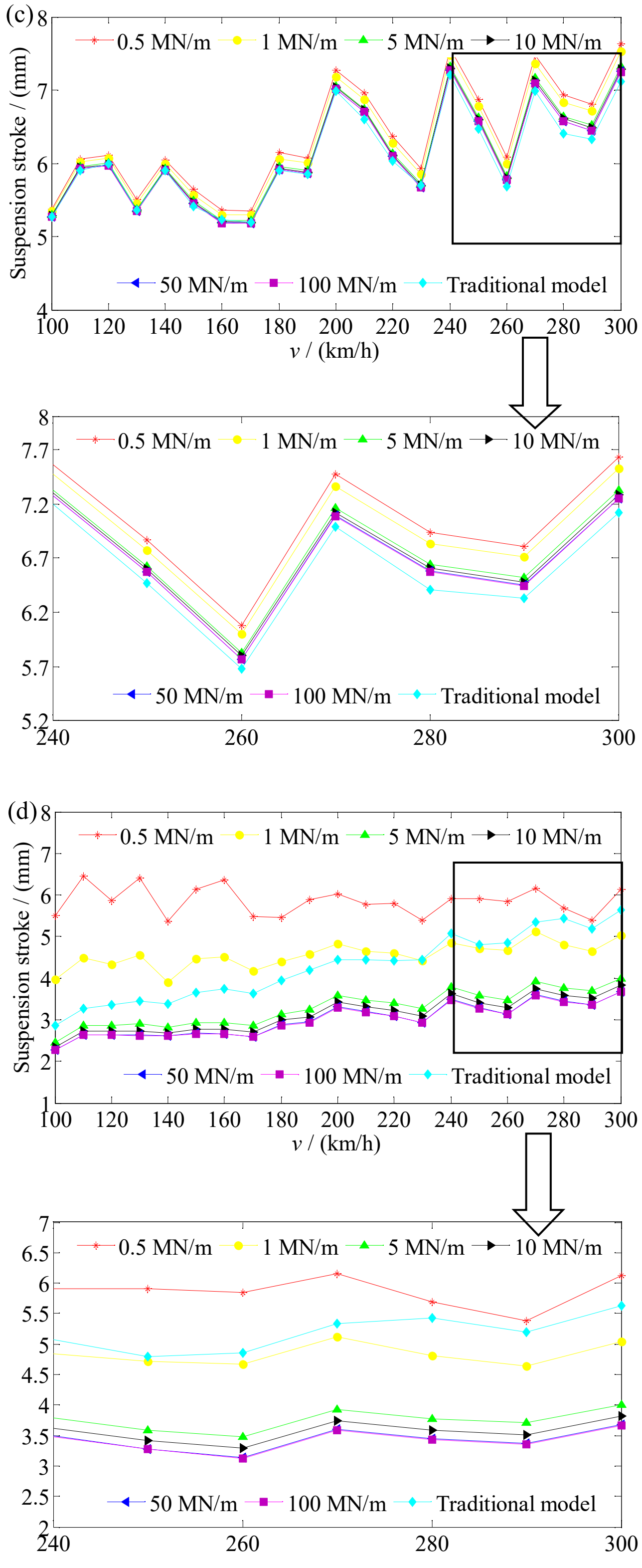


Fig. 5. Effect of the primary vertical damper rubber joint stiffness on the railway vehicle vertical random vibration: (a) the change of the RMS of the car body acceleration; (b) the change of the RMS of the bogie frame acceleration; (c) the change of the RMS of the secondary suspension stroke; (d) the change of the RMS of the primary suspension stroke

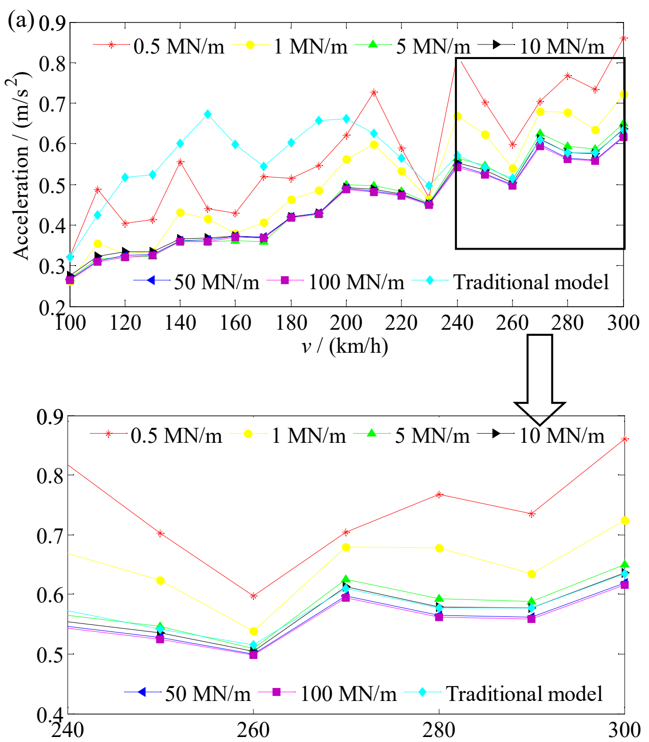
4) Increasing the primary vertical damper rubber joint stiffness, the primary suspension vertical stroke firstly gradually approaches the traditional model, and then gradually moves away from it, and this effect is more obvious and higher with the increase of the vehicle running speed. In terms of the damper rubber joint stiffness, the greater the primary vertical damper joint stiffness is, the smaller the primary suspension vertical stroke will be. The variation law of the vertical stroke of the primary suspension with the rubber joint stiffness of the primary vertical damper is basically the same under the different vehicle running speed.

B. Influence analysis of the secondary vertical damper rubber joint stiffness

When only secondary vertical damper rubber joint stiffness changes, and other parameters remain unchanged, the RMS curves of the railway vehicle vertical random response at different running speed are obtained, as shown in Figure 6 and Figure 7. In the simulation, K_{pd}=5 MN/m,the vehicle running speed v=100~300 km/h, incremental speed step is 10 km/h; the simulation time is 20 s.

Inspection of Figure 6 and Figure 7 shows that:

1) The effect of the secondary damper rubber joint stiffness on car body vertical acceleration is mainly reflected in the lower vehicle running speed. When the vehicle is running at a low speed, the car body vertical acceleration with secondary vertical damper rubber joint stiffness is less than that of the traditional mode, and is basically the same as that of traditional model when the vehicle running speed is high, but as long as the joint stiffness exists, the car body vertical acceleration under two models will not be coincidence. In terms of the damper rubber joint stiffness, with the increase of the stiffness of the secondary damper rubber joint, the car body acceleration shows a decreasing trend, and this effect is basically the same at different vehicle running speed. In addition, when the secondary vertical damper rubber joint stiffness increases to a certain extent, and the vehicle speed is high, the car body vertical acceleration will gradually approach traditional model.



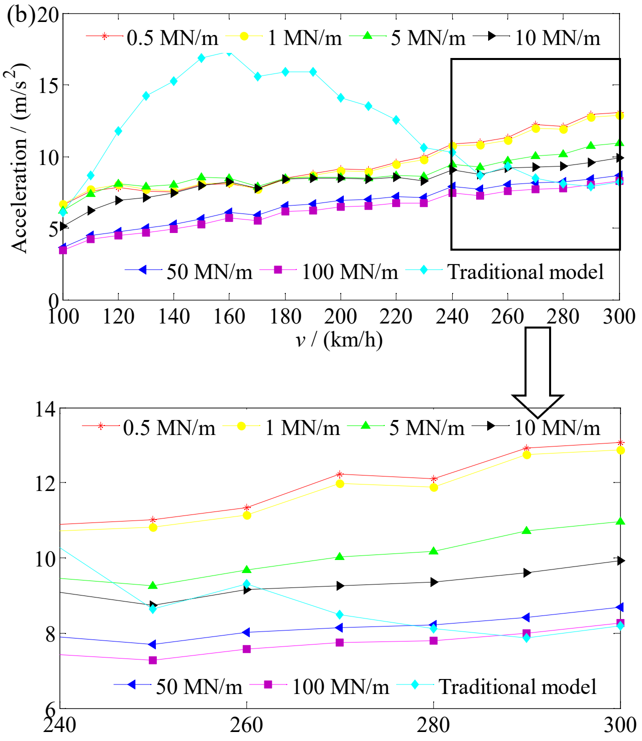


Fig. 6. Effect of the secondary vertical damper rubber joint stiffness on the vertical acceleration: (a) the change of the RMS of the car body acceleration; (b) the change of the RMS of the bogie frame acceleration

2) Compared with the traditional model, the secondary vertical damper rubber joint stiffness has a certain influence on the bogie frame acceleration at different vehicle running speed, which is more significant at the lower vehicle running speed. When the vehicle is running at a low speed, the bogie frame vertical acceleration with considering the effect of secondary vertical damper rubber joint stiffness is less than that of the traditional model, but at the high vehicle running speed, with the increase of the secondary vertical damper joint stiffness, it gradually approaches the traditional model. In terms of the damper rubber joint stiffness, the greater the secondary vertical damper joint stiffness is, the smaller the bogie frame vertical acceleration will be, and the variation law of the bogie frame vertical acceleration with the rubber joint stiffness of the secondary vertical damper is basically the same under the different vehicle running speed.
3) With the increase of the secondary vertical damper rubber joint stiffness, the vertical stroke of the secondary suspension gradually approaches the traditional model, and when the secondary vertical damper joint stiffness increases to a certain extent, the secondary suspension vertical stroke under the two models is basically the same. In terms of the damper rubber joint stiffness, the greater the secondary vertical damper joint stiffness is, the smaller the secondary suspension vertical stroke will be, and the variation law of the vertical stroke of the secondary suspension with the rubber joint stiffness of the secondary vertical damper is basically the same under the different vehicle running speed.

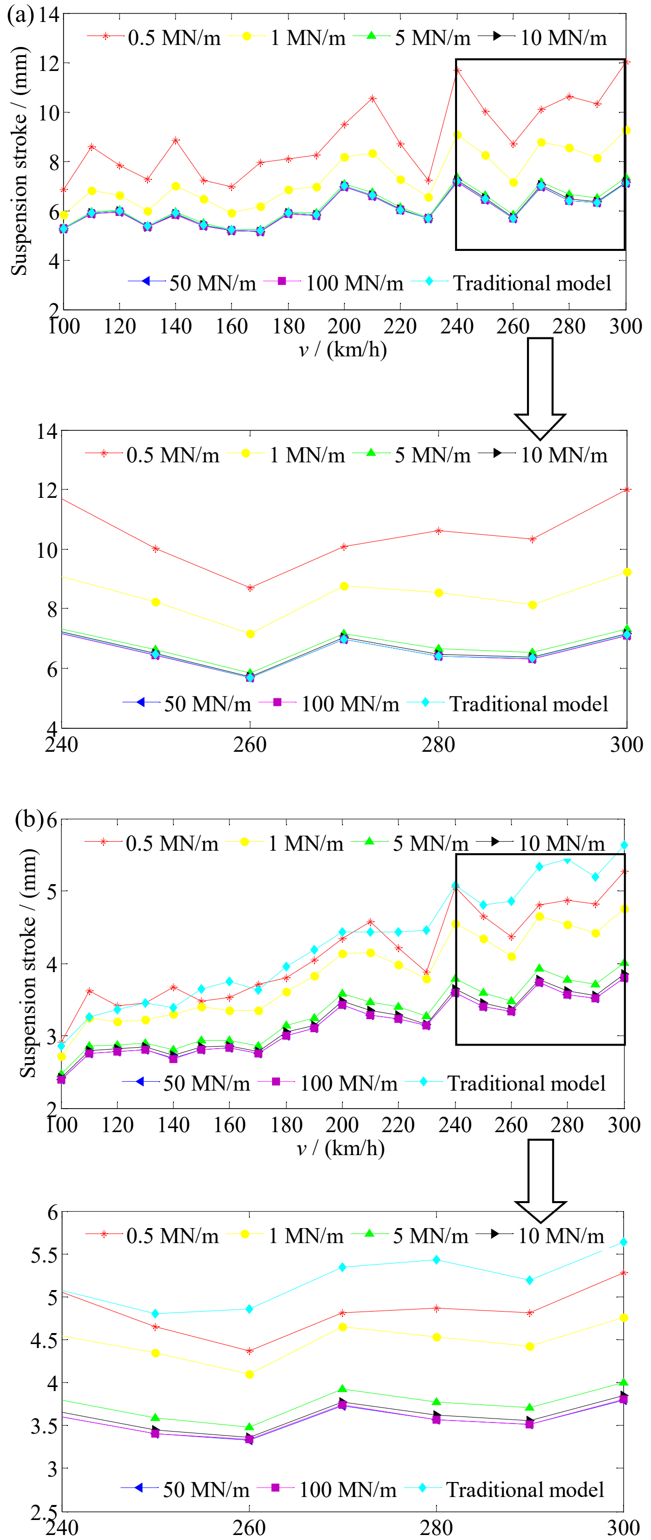


Fig. 7. Effect of the secondary vertical damper rubber joint stiffness on the suspension vertical stroke: (a) the change of the RMS of the secondary suspension stroke; (b) the change of the RMS of the primary suspension stroke

4) The smaller the secondary vertical damper rubber joint stiffness is, the closer the primary suspension vertical stroke under the two models is, and the more consistent the change law is. In terms of the damper rubber joint stiffness, the greater the secondary vertical damper joint stiffness is, the smaller the primary suspension vertical stroke will be. In addition, the variation law of the

vertical stroke of the primary suspension with the rubber joint stiffness of the secondary vertical damper is basically the same under the different vehicle running speed.

VII. CONCLUSIONS

For the railway vehicle vertical dynamic model under the effect of damper rubber joint stiffness, there is no reliable and fast method to solve its dynamic response at present. In this paper, to improve the efficiency of numerical analysis, by reducing the degree of freedom of the system and transforming the vibration differential equation, a numerical solution method for railway vehicles vertical dynamic response under the effect of damper rubber joint stiffness is established. Through a case analysis, the variation of vertical random vibration response of railway vehicles with the stiffness of damping rubber joints is studied by using random vibration theory, some useful conclusions can be obtained:

1) The car body vertical acceleration, the bogie frame vertical acceleration, the secondary suspension vertical stroke, and the primary suspension vertical stroke are significantly affected by the rubber joint stiffness of the primary and secondary vertical dampers.
2) The traditional model will cause great errors in calculating the car body vertical acceleration and the bogie frame vertical acceleration in the low vehicle running speed range. When the damper rubber joint stiffness increases to a certain extent, the results of the vibration response analysis tend to be consistent.
3) By selecting a proper damper rubber joint stiffness, the vibration of the railway vehicle can be alleviated. In addition, the influence of the damper rubber joint stiffness should be fully considered when selecting damping parameters and analyzing railway vehicles dynamics.

This study provides an effective solution method for the numerical simulation of railway vehicles dynamic response under the effect of the damping rubber joint stiffness, and provides a guidance for the stiffness design of the damper rubber joints for railway vehicles.

REFERENCES

[1] B. M. Eickhoff, J. R. Evans and A. J. Minnis, "A review of modelling methods for railway vehicle suspension components," *Vehicle System Dynamics*, vol. 24, no. 6-7, pp. 469-496, 1995.

[2] V. K. Garg and R. V, *Dukkipatim, "Dynamics of vehicle system dynamics*. Academic Press, Toronto, 1984.

[3] H. Tsunashima, Y. Naganuma and T. Kobayashi, "Track geometry estimation from car-body vibration," *Vehicle System Dynamics*, vol. 52, no. supl, pp. 207-219, 2014.

[4] K. Y. Wang, W. M. Zhai and K. K. Lv, "Numerical investigation on wheel-rail dynamic vibration excited by rail spalling in high-speed railway," *Shock and Vibration*, vol. 2016, pp. 1-1, 2016.

[5] G. Z. Yang and F. T. Wang, *Locomotive vehicle hydraulic shock absorber*. China Railway Publishing House, Beijing, 2002.

[6] B. M. Eickhoff, J. R. Evans and A. J. Minnis, "A review of modelling methods for railway vehicle suspension components," *Vehicle System Dynamics*, vol. 24, no. 6-7, pp. 469 – 496, 1995.

[7] H. K. Kim, "Novel service lifetime estimation for an oil-damper rubber bush for a railway vehicle," *Advanced Materials Research*, vol. 739, pp. 437 – 441, 2013.

[8] J. Zeng and P. B. Wu, "Influence of the damper rubber joint stiffness on the critical speed of railway passenger car system," *China Railway Science*, vol. 29, no. 2, pp. 94 – 98, 2008.

[9] J. H. Hao, J. Zeng and P. B. Wu, "Vertical vibration isolation and suspension parameter optimization of railway vehicle," *Journal of Traffic and Transportation Engineering*, vol. 5, no. 4, pp. 10 – 14, 2005.

[10] J. H. Hao, J. Zeng and P. B. Wu, "Optimization of vertical random vibration isolation and suspension parameters of railway passenger car systems," *Journal of the China Railway Society*, vol.28, no. 6, pp. 35 – 40, 2006.

[11] J. J. Meng, Z. Q. Yang and G. H. Pu, "Vertical vibration response analysis of railway vehicle based on pseudo excitation method," *China Railway Science*, vol. 33, no. 2, pp. 89 – 94, 2012.

[12] S. Li, Y. R. Yu and L. Chen, "Global sensitivity analysis on the ride quality of railway vehicle with stochastic suspension parameters," *Journal of the China Railway Society*, vol. 37, no. 8, pp. 29 – 35, 2015.

[13] D. B. Cui, L. Li and X. S. Jin, "Influence of vehicle parameters on critical hunting speed based on Ruzicka model," *Chinese Journal of Mechanical Engineering*, vol. 25, no. 3, pp. 536-542, 2012.

[14] Y. W. Yu, C. C. Zhou, L. L. Zhao, "Vertical coupled vibration mechanism of bogie-body-seat system and joint optimization of suspension parameters for high-speed train," *Journal of Mechanical Engineering*, vol. 54, no. 8, pp. 57-67, 2018.

[15] G. Chen, W. M. Zhai and C. B. Cai, "Comparison on vertical random vibration responses of traditional vehicle model with vehicle/track coupling model," *Journal of the China Railway Society*, vol. 21, no. 5, pp. 70 – 75, 1999.

[16] X. W. Li, J. W. Zhang and T. L. Lu, "Optimization of suspension parameters based on vehicle-track coupled model for a special railway vehicle," *Journal of Shanghai Jiaotong University*, vol. 46, no. 3, pp. 346-351, 2012.

[17] J. F. Mao, Z. W. Yu and Y. J. Xiao, "Random dynamic analysis of a train-bridge coupled system involving random system parameters based on probability density evolution method," *Probabilistic Engineering Mechanics*, vol. 46, pp. 48-61, 2016.

[18] W. M. Zhai, K. Y. Wang and C. B. Cai, "Fundamentals of vehicle – track coupled dynamics," *Vehicle System Dynamics*, vol. 47, no. 11, pp. 1349-1376, 2009.