Analysis on the Fracture Mechanism of Cracked Materials Considering the Influence of Micro-cracks in the Vicinity of the Macro-crack Tip

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***Abstract*—In order to resolve the fracture mechanism of engineering structures with a macro-crack, a multi scale analysis method for crack tip fields with consideration to the influence of micro-cracks has been proposed based on continuum damage mechanics. The micro-cracks induced stress field of cracked materials has been expressed and the plastic area near the macro-crack tip has been represented considering the influence of the micro-cracks. The equivalence principle of stress has been put forward to study the influence of micro-cracks behavior on stress intensity factor and analysis on the fatigue macro-crack propagation has been performed based on the developed equivalence principle. The analysis results illustrated that the shape of the plastic area near the crack tip is irregular in the structure service lifetime caused by the behavior of micro-cracks. The micro-crack behavior along the crack tip exhibits great influence on the macro-crack propagation under cyclic loading.**

***Index Terms*—damage, fracture mechanism, macro-crack, micro-crack, plastic area**

I. INTRODUCTION

Fracture caused by macro-cracks is the common failure form of civil engineering structures, which has attracted much attention from both engineers and scholars. Fracture mechanics is traced back to the event that Griffith successfully explained the phenomenon that the brittle materials fail under the condition where the stress level is far away its yield strength. The definitions of stress intensity factor, energy release rate, *J*-integral and crack tip opening displacement contribute to relate fracture mechanics to the failure problem of engineering structures [1]. Westergaard [2] put forward the stress solution in the vicinity of the crack tip, namely the famous Westergaard solution. However, the Westergaard solution indicates that the stress field exhibits singularity in the crack tip, while the stress field will never be infinite. As a result, the concept of plastic area along the crack tip was introduced to explain this contradiction. Irwin proposed that the crack length should be replaced by effective crack length to study the influence of plastic area on the crack tip fields [1]. Dugdale and Barenblan [3, 4] assumed that the plastic areas with the length of (c-a) were concentrated in the crack tip, namely the profound D-B model. Up to now, there are many studies concentrated on the fracture behaviors of cracked materials and structures [5-8].

Although the fracture mechanics phenomenologically described fracture of engineering structures, but failed to explain the essential mechanism of fracture behavior. Research [9-11] showed that there are a certain number of micro-cracks in the material and the performance degradation of structures is caused by the collective behavior of the micro-cracks. Generally, the micro-cracks propagate much faster than macro-cracks subjected to the same loading. Therefore, scholars tended to pay attention to the influence of micro-cracks on the macro-crack tip field. Kachanov et al. [12] analyzed the mechanics of interaction of a macro-crack with micro-crack arrays. Feng et al. [13] studied the fracture process zone near the crack tip in a brittle damaged material based on D-B model and elastic-damage model with residual strength. Tai al et. [14] studied the damage evolution ahead of a crack tip in high-impact polystyrene specimen. Bouiadjra al et. [15] and Li al et. [16] analyzed the influence of micro-cracks on the plastic zone near the macro-crack tip. Fan al et. [17] developed an equivalent method to deal with the external crack problem with a Dugdale cohesive zone. Zhu al et. [18] analyzed the influence of surface tension on the behavior of adhesive contact based on Maugis–Dugdale model.

It is commonly accepted that the degradation of engineering structures is caused by collective behavior of micro-cracks inner the materials and its damage accumulation process can be described by the micro-crack density [19-21]. The research [22-24] illustrated that the characters of micro structure, such as micro-crack orientation and the grain size, of metallic materials show great influence on the performance of structures. Moreover, research [25] demonstrated that the length of micro-cracks of the material shows a large scatter subjected to cyclic loading.

Up to now, the majority of studies on the propagation of macro-crack tended to ignore the influence of the micro-cracks. Although some research tried to study macro-crack tip fields with consideration to the influence of micro-cracks, the current research was limited to analyze the influence of a micro-crack or several ideal micro-cracks.

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However, the randomness of distribution and discreteness of length of micro-cracks have not been considered. Therefore, although many efforts have been made on the crack tip fields, the mechanism of fracture failure problem has not been resolved.

In order to resolve the fracture failure problem of cracked engineering structures, a multi scale analysis method for the damaged macro-crack tip fields has been proposed based on continuum damage mechanics to study the influence of micro-cracks near the macro-crack tip. The damage induced macro-crack tip fields and the plastic area near the crack tip have been described in which the influence of the micro-cracks was considered. The equivalence principle of stress has been put forward to express the micro-cracks induced stress intensity factor and analyze the propagation of macro crack in damaged materials.

II. REPRESENTATION OF THE DAMAGED CRACK TIP

According to continuum damage mechanics [26], the damage variable D is used to describe the degradation of materials, illustrated as Fig.1:

$$D = \frac{S - S_{\mathrm{eff}}}{S} \tag{1}$$

where S and S_{eff} are the cross-sectional area and effective cross-sectional area of the representative volume element (RVE), respectively.

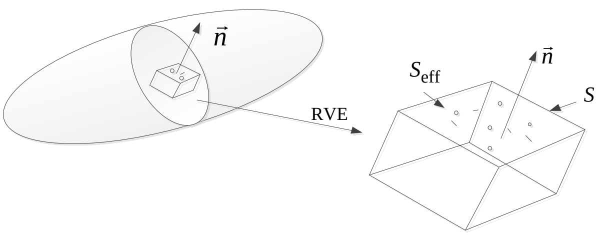


Fig. 1. Schematic diagram for the definition of damage

In this study, it is supposed that the structure of cracked materials with a macro-crack can be divided into two areas to study the influence of micro-cracks, illustrated as Fig. 2. The area 1 is the damaged area with relative low stress level. The area 2 is the plastic area in the vicinity of the crack tip in which the stress level reaches its yield strength.

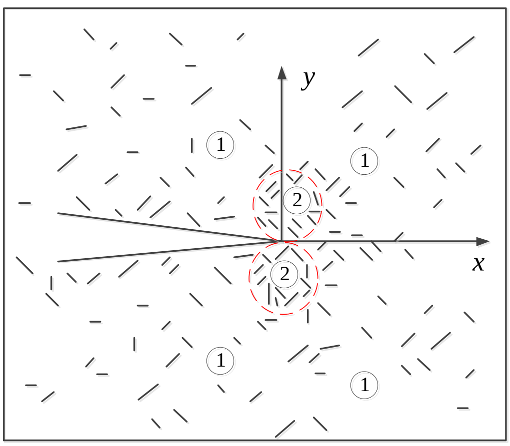


Fig. 2. Structure of the crack tip

The collective behavior of micro cracks can be described by the balance theory for micro-cracks number density developed by Bai et al. [23]:

$$\frac{\partial\left(n(a, N/N_{\mathrm{f}})\right)}{\partial(N/N_{\mathrm{f}})} = \dot{n}_{\mathrm{N}}(a, N/N_{\mathrm{f}}) - \frac{\partial n(a, N/N_{\mathrm{f}})\dot{a}(a, N/N_{\mathrm{f}})}{\partial a} \tag{2}$$

where $n(a, N/N_{\mathrm{f}})$ is number of crack with length a when cycle ratio is N/N_{f}.

In practice, the majority of structural failure is related to fatigue damage which caused by micro-cracks [27-29]. The collective behavior of micro-cracks under cyclic loading can be illustrated as Fig. 3. In the author's previous research [30], a multi scale fatigue damage model had been developed in which the scatter of micro-crack length was analyzed. The fatigue damage model was expressed as:

$$D_{\mathrm{f}} = 1 - \frac{1}{\frac{\pi}{4} f_1 + \frac{D_E^{2D}}{16} f_1^2} \tag{3-1}$$

$$f_1 = \left[H(a)\right]^2 \frac{L_{\mathrm{lf}}\left(e^{A_{\alpha\mathrm{f}} t_1} - 1\right)F(a \le d)}{A_{\alpha\mathrm{f}}} + \frac{L_{\mathrm{lf}}\left(1 - \frac{1}{e^{B_{\beta\mathrm{f}} t_1}}\right)\left[1 - F(a \le d)\right]}{B_{\beta\mathrm{f}}} \tag{3-2}$$

where L_{lf}, $A_{\alpha\mathrm{f}}$, $B_{\alpha\mathrm{f}}$ are the model parameter, $F(x)$ and $H(x)$ are probability distribution function and Weibull distribution expectancy, respectively. $D_{\mathrm{E}}^{2\mathrm{D}}$ is a parameter depended on Poisson's ratio.

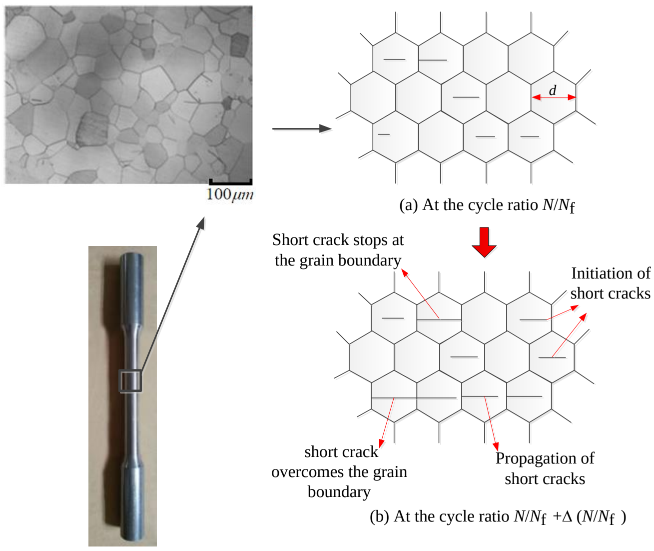


Fig. 3. The collective behaviors of micro-cracks under cyclic loading

III. MICRO-CRACK INDUCED PLASTIC AREA IN THE VICINITY OF THE CRACK TIP

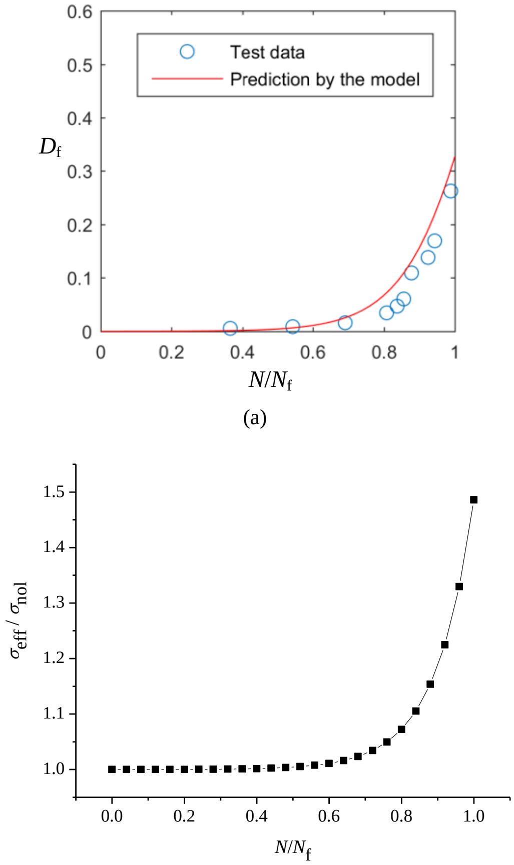
On the basis of damage mechanics, the relationship between nominal stress and effective stress can be described as:

$$\sigma_{\mathrm{eff}} = \frac{\sigma_{\mathrm{nom}}}{1 - D} \tag{4}$$

where σ_{eff} is effective stress, σ_{nom} is nominal stress.

In the author's previous research [30], the fatigue damage evolution curve for alloy IN 100 was obtained, shown as Fig. 4(a). On the basis of the equation (4) and the damage evolution curve, the normalized effective stress field for alloy IN 100 can be represented in Fig. 4(b) in which the influence of micro-cracks was considered.

It can be obtained from Fig. 4 that the normalized effective stress level increases greatly in the later stage of damage evolution process. Therefore, the nominal stress should be replaced by the effective stress to consider the influence of micro-cracks near the crack tip.



(a)

(b)

Fig. 4. The effective stress for alloy IN 100
(a) The damage evolution curve (b) The normalized effective stress

In a polar coordinate system (r, θ) whose origin being at the crack tip, the effective stress should be used to modify the Westergaard solution for stress fields:

$$\begin{cases} \sigma_{xx}(r,\theta)=\dfrac{K_I}{\left[1\text{-}D_{(n,t)}\right]\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\left(1-\sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right) \\ \sigma_{yy}(r,\theta)=\dfrac{K_I}{\left[1\text{-}D_{(n,t)}\right]\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\left(1+\sin\dfrac{\theta}{2}\sin\dfrac{3\theta}{2}\right) \\ \sigma_{xy}(r,\theta)=\dfrac{K_I}{\left[1\text{-}D_{(n,t)}\right]\sqrt{2\pi r}}\cos\dfrac{\theta}{2}\sin\dfrac{\theta}{2}\cos\dfrac{3\theta}{2} \end{cases} \tag{5}$$

where r is the distance from the crack tip, θ is crack angle, σ_{ij} is stress, K_I is stress intensity factor for mode I, $D(n, t)$ is the damage caused by the n micro cracks in the unit area.

According to material mechanics, the Mises yield criterion is expressed as:

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2=2\sigma_s^2 \tag{6}$$

The relationship between principal stresses and nominal stresses is described as:

$$\begin{cases} \sigma_1=\dfrac{\sigma_{xx}+\sigma_{yy}}{2}+\dfrac{\left(\sigma_{xx}-\sigma_{yy}\right)^2}{4}+\tau_{xy}^2 \\ \sigma_2=\dfrac{\sigma_{xx}+\sigma_{yy}}{2}-\dfrac{\left(\sigma_{xx}-\sigma_{yy}\right)^2}{4}+\tau_{xy}^2 \\ \sigma_3=\begin{cases} 0 & \text{for plane stress} \\ \mu(\sigma_1+\sigma_2) & \text{for plane straint} \end{cases} \end{cases} \tag{7}$$

The Westergaard solution implies that there is a singularity, namely $r\rightarrow 0$, $\sigma_{ij}\rightarrow\infty$. In this study, the plastic area based on Mises yield criterion is used to explain the contradiction. Based on the modified Westergaard stress solution and Mises yield criterion, the plastic area in the vicinity of crack tip can be described in which the influence of micro-cracks was considered. The size of the plastic area can be obtained by combining the equations (5), (6) and (7).

The size of the plastic area for plane stress problem gives:

$$r=\frac{1}{2\pi}\frac{K_I^2}{\left[1\text{-}D_{(n,t)}\right]^2\sigma_s^2}\left[\cos^2\frac{\theta}{2}\left(1+3\sin^2\frac{\theta}{2}\right)\right] \tag{8}$$

The size of the plastic area for plane strain gives:

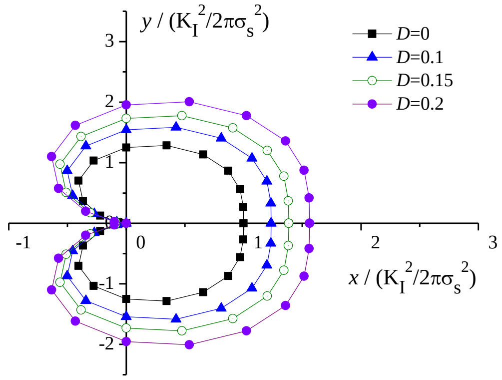
$$r=\frac{1}{2\pi}\frac{K_I^2}{\left[1\text{-}D_{(n,t)}\right]^2\sigma_s^2}\cos^2\frac{\theta}{2}\left[\left(1-2\mu\right)^2+3\sin^2\frac{\theta}{2}\right] \tag{9}$$

Research [30] showed that the critical damage value of metallic material ranges 0.2~0.8. It is assumed that the average damage enables to describe damage evolution process of metallic materials. Analysis on the shape of plastic areas adjacent to the crack tip of metallic material with different average damage level has been performed, shown as Fig. 5. The growth of the point $r_{\theta=0}$ and $r_{\theta=90^\circ}$ characterizes size of the plastic area near the crack tip. The growth of micro-cracks induced plastic areas for both plane stress problem and plane strain problem has been studied, illustrated in Fig. 6.

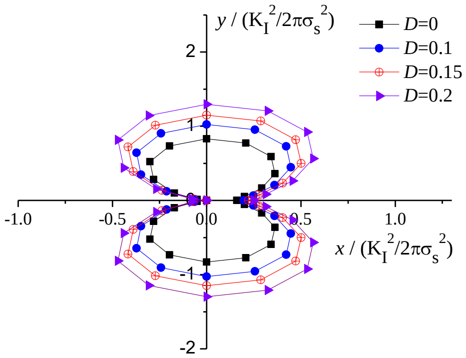
The conclusions can be draw from Fig. 5 and Fig. 6:

(1) The size of plastic areas increases due to the damage accumulation, which illustrated that the collective behavior of micro-cracks leads to significant growth of the plastic area in the vicinity of the crack tip.

(2) The growth rate of $r_{\theta=0}$ and $r_{\theta=90^\circ}$ exhibits unproportional, which implies the shape of plastic areas is irregular in the structure service lifetime, compared with plastic areas with regular shape based on traditional fracture mechanics.



(a)



(b)

Fig. 5. The shape of plastic areas in the vicinity of the crack tip
(a) plane stress problem (b) plane strain problem

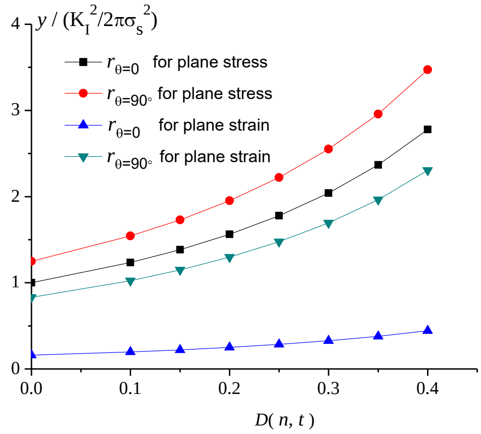


Fig. 6. The growth of plastic areas in the vicinity of the crack tip

IV. THE EQUIVALENCE PRINCIPLE OF STRESS AND THE APPLICATION

A. *The Equivalence Principle of Stress*

As stated above, there are two parts contribute to stress intensity factor K in the crack tip, namely K^{macro} caused by the macro-crack and K^{micro} caused by micro-cracks. Therefore, the K in the crack tip is described as:

$$K = K^{macro} + K^{micro} \tag{10}$$

The influence of micro-cracks on the crack tip field involves amplification effect and shield effect. Research [12] demonstrated that the shield effect can be ignored under the condition where the random of micro-cracks is considered. As a result, only amplification effect caused by micro-cracks should be analyzed in this study.

Fracture analysis on an infinite sheet with a line crack under uniaxial tensile loading has been performed. The stress intensity factor for mode I in the crack tip based on traditional fracture mechanics is expressed:

$$K_I^{macro} = Yp\sqrt{\pi a} \tag{11}$$

where Y is shape parameter, p is nominal applied stress.

However, the traditional fracture mechanics failed to describe the behavior of micro cracks, including the K^{micro} caused by micro-cracks. On the basis of the continuum damage mechanics, the stress of damaged body generated under nominal loading can be equal to the stress of undamaged body generated under effective loading. As a result, the equivalence principle of stress can be developed to calculate the stress intensity factor in the vicinity of the crack tip considering the influence of micro-cracks, illustrated as Fig. 7.

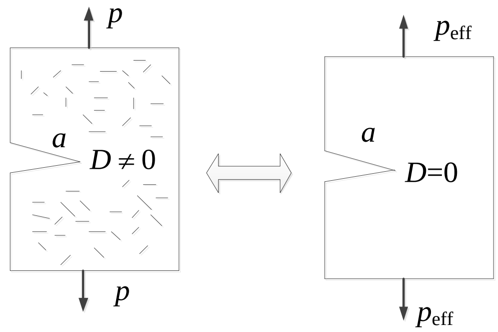


Fig. 7. The equivalence principle of stress

Based on the developed equivalence principle, the relationship between applied loading and the stress filed can be obtained:

$$\sigma_{ij} = \frac{p_{eff}}{S} = \frac{p}{S\left[1 - D_{(n,t)}\right]} \tag{12}$$

As a result, the micro-cracks induced stress intensity factor can be expressed using the developed equivalence principle of stress:

$$K_I^m = K_I^{macro} + K_I^{micro} = Yp_{eff}\sqrt{\pi a} = Y\frac{p}{1 - D_{(n,t)}}\sqrt{\pi a} \tag{13}$$

It is supposed that the applied stress p =10MPa and the initial crack length a_0=3mm. The stress intensity factor of an infinite sheet under different damaged condition has been calculated and the results based on the two different methods are listed in Tab. 1. The normalized stress intensity factor K_I^m/K_I^0 in the damage evolution process is illustrated as Fig. 8.

The conclusions can be draw from Table Ⅰ and Fig. 8:

(1) The relative difference between K_I^0 and K_I^m gradually increases in the process of damage evolution, which implies that attention should be paid on the influence of micro-cracks on the stress intensity factor.

(2) The normalized stress intensity factor is over-high under the condition with high damage level, which revealed the limitation of traditional fracture mechanics to be applied into fatigue fracture analysis for the structures with high micro-crack density.

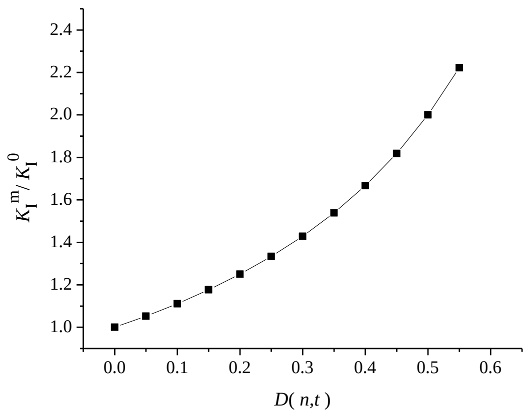


Fig. 8. The curve of normalized stress intensity factor

Table I
The values of stress intensity factor

D / K/MPa · mm$^{1/2}$	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35
K_I^0 using Eq. (11)	30.70	30.70	30.70	30.70	30.70	30.70	30.70	30.70
K_I^m using Eq. (13)	30.70	32.32	34.11	36.12	38.37	40.93	43.86	47.23
Relative differences / %	0	5.26	11.11	17.65	25.00	33.33	42.86	53.85

B. *The Micro-crack Induced Fatigue Crack Propagation*

The majority of failure of engineering structures is related to fatigue fracture caused by crack behavior, including micro-cracks and macro-cracks. Paris law is limited to describe the behavior of macro fatigue crack propagation and it can be expressed as [32]:

$$\frac{da}{dN}=C\left(\Delta K_I^{\text{macro}}\right)^m \quad (14)$$

where C and m are material constant.

In order to analyze the influence of micro-cracks along the crack tip on macro-crack propagation, the Paris law has been modified based on the developed equivalence principle of stress. In the modified model, K_I^{marco} should be replaced by the micro-cracks induced K_I^m:

$$\frac{da}{dN}=C\left(\Delta K_I^m\right)^m \quad (15)$$

It is assumed that C=1.3*10^{-13}, m=2.2 and the applied stress amplitude $\Delta\sigma$=205.5MPa. The damage parameters of $D(n,t)$ in equation (3) were obtained in the previous study [29], and the damage evolution curve of alloy IN 100 is shown as Fig. 4. Fatigue crack propagation analysis for the alloy IN 100 has been performed based on Paris law and modified Paris law, respectively. Comparison has been made between the analysis results based on different crack propagation theories. The results are shown as Fig. 9 and Fig. 10. Note that curve C describes the results with consideration to the influence of micro-cracks and curve NC is for no consideration.

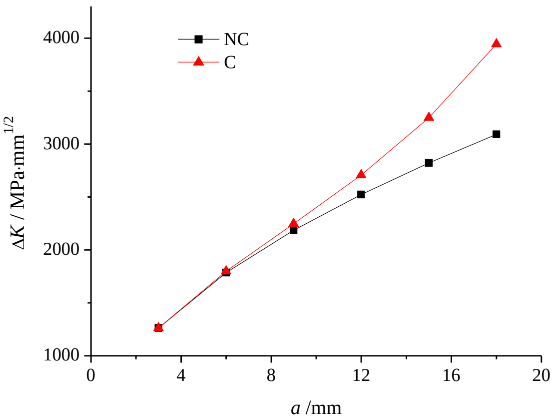


Fig. 9. The curve of stress intensity factor amplitude VS crack length

It can be obtained from Fig. 9 and Fig. 10:

(1) The influence of micro-cracks on ΔK gradually exhibits prominent in the process of the fatigue macro-crack propagation.

(2) The fatigue macro-crack represented by curve C propagates much faster than that for curve NC at later stage of structure lifetime, which implies that micro-cracks near the crack tip enable to remarkably accelerate the propagation of the fatigue macro-crack.

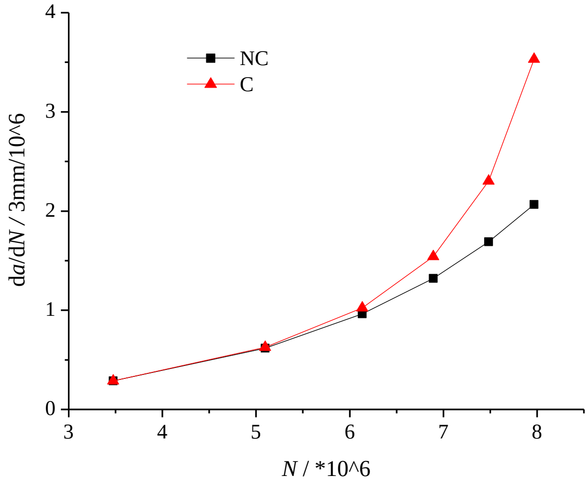


Fig. 10. The curve of fatigue crack propagation

V. CONCLUSIONS

In order to reveal the natural mechanism of fracture failure of engineering structures with a macro crack, a micro-cracks induced analysis method based on continue damage mechanics has been proposed. The structure of cracked materials has been divided into the damaged area and the plastic area to consider the influence of micro-cracks on the crack tip field. The damaged crack tip has been represented and analysis on fatigue crack propagation has been performed in which the influence of micro-cracks near the macro-crack tip was studied. The main works and conclusions are listed as following:

(1) The shape of the plastic area near the macro-crack tip is irregular in the structure service lifetime due to the behavior of micro-cracks, compared with the plastic area with regular shape based on traditional fracture mechanics.

(2) The equivalence principle of stress has been put forward to calculate the micro-crack induced stress intensity factor and the limitation of traditional fracture mechanics to be applied into fatigue analysis for the structures with high micro-crack density has been demonstrated.

(3) The micro-cracks near the macro-crack tip enable to remarkably accelerate the propagation of the fatigue macro-crack.

REFERENCES

[1] C. T. Sun, Z. H. Jin, *Fracture Mechanics*, American: Academic Press, 2012

[2] H. Westergaard, "Bearing pressures and cracks," Journal of Applied Mechanics, vol. 6, pp. A49-A53, 1939.

[3] D. S. Dugdale, “Yielding of steel sheets containing slits,” Journal of the Mechanics and Physics of Solids, vol. 8, no. 2, pp. 100-104, 1960.
[4] G. I. Barenblatt, “The mechanical theory of equilibrium cracks in brittle fracture,” Advances in Applied Mechanics, vol. 7, pp. 55-129, 1962.
[5] N. K. Sharma, D. K. Sehgal, R. K. Pandey, “Orientation dependence of elastic-plastic fracture toughness and micro-fracture mechanism in cortical bone,” Engineering Letters, vol. 19, no. 4, pp. 304-309, 2011.
[6] J. Song, J. T. Zhou, R. Jiang, M. B. Wang, X. Yu, S. L. Jia, F. Y. Chen, “Fatigue crack propagation in BFRP reinforced RC beams based on fracture mechanics and golden ratio,” Engineering Letters, vol. 24, no. 4, pp. 449-454, 2016.
[7] A. M. Zakhezin, Y. G. Pryadko, O. P. Kolosova, S. V. Aliukov, “Diagnostics of fatigue cracks in rotor systems of machines,” Engineering Letters, vol. 28, no. 1, pp. 209-214, 2020.
[8] Rika Rokhana, Eko Mulyanto Yuniarno, I Ketut Eddy Purnama, Kayo Yoshimoto, Hideya Takahashi, and Mauridhi Hery Purnomo, “Accuracy on Bovine Bone Fracture Detection of Two Dimensional B-Mode Ultrasound Images using Polynomial - Intensity Gradient,” IAENG International Journal of Computer Science, vol. 46, no. 2, pp. 217-227, 2019.
[9] K. J. Miller, “The behaviour of short fatigue cracks and their initiation part II –A general summary,” Fatigue & Fracture of Engineering Materials & Structures, vol. 10, no. 2, pp. 93-113, 1987.
[10] D. Angelova, R. Akid, “A note on modelling short fatigue crack behavior,” Fatigue & Fracture of Engineering Materials & Structures, vol. 21, no. 6, pp. 771-779, 1998.
[11] Y. Huang, K. X. Hu, A. Chandra, “A generalized self-consistent mechanics method for microcracked solids,” Journal of Mechanics Physics of Solids, vol. 42, no. 8, pp. 1273-1291, 1994.
[12] M. Kachanov, E. L. E. Montagut, J. P. Laures, “Mechanics of crack-microcrack interactions,” Mechanics of Materials, vol. 10, pp. 59-71, 1990.
[13] X. Q. Feng, S. W. Yu, “Analyses of damage localization at crack tip in a brittle damaged material,” Engineering Fracture Mechanics, vol. 53, no. 2, pp. 169-177, 1996.
[14] W. H. Tai, W. B. Lee, C. Y. Tang, “Damage analysis of crack tip zone in high impact polystyrene,” Engineering Fracture Mechanics, vol. 56, no. 5, pp. 615-621, 1997.
[15] B. B. Bouiadjra, M. Benguediab, M. Elmeguenni, M. Belhouari, B. Serier, M. N. A. Aziz, “Analysis of the effect of micro-crack on the plastic strain ahead of main crack in aluminium alloy 2024 T3,” Computational Materials Science, vol. 42, no. 1, pp. 100-106, 2008.
[16] X. T. Li, X. Li, H. D. Yang, X. Y. Jiang, “Effect of micro-cracks on plastic zone ahead of the macro-crack tip,” Journal of Materials Science, vol. 52, no. 23, pp. 13490-13503, 2017.
[17] F. Jin, D. H. Yue, “An equivalent indentation method for the external crack with a Dugdale cohesive zone,” Journal of Elasticity, vol. 141, no. 1, pp. 31-49, 2020.
[18] X. Y. Zhu, W. Xu, Y. D. Qin, “Effect of surface tension on the behavior of adhesive contact based on Maugis-Dugdale model,” European Journal of Mechanics A-Solids, vol. 81, pp. 1039301-10393022, 2020.
[19] Y. S. Hong, Z. Y. Gu, B. Fang, Y. L. Bai, “Collective evolution characteristics and computer simulation of short fatigue cracks,” Philosophical Magazine A, vol. 75, no. 6, pp. 1517-1531, 1997.
[20] Y. L. Bai, H. Y. Wang, M. F. Xia, F. J. Ke, “Trans-scale mechanics: Looking for the missing links between continuum and micro/nanoscopic reality,” Acta Mechanica Sinica. vol. 24, no. 2, pp. 111-126, 2008.
[21] B. Sun, Y. L. Xu, Z. X. Li, “Multi-scale fatigue model and image-based simulation of collective short cracks evolution process,” Computational Materials Science, vol. 117, pp. 24-32, 2016.
[22] V. P. Bennetta, D. L. McDowell, “Polycrystal orientation distribution effects on microslip in high cycle fatigue,” International Journal of Fatigue, vol. 25, no. 1, pp. 27-39, 2003.
[23] Y. L. Bai, H. Y. Wang, M. F. Xia, F. J. Ke, “Statistical mesomechanics of solid, linking coupled multiple space and time scales,” Advances in Mechanics, vol. 58, no. 6, pp. 286-305, 2006.
[24] R. Z. Wang, C. Bo, X. C. Zhang, S. T. Tu, W. Ji, C. C. Zhang, “The effects of inhomogeneous microstructure and loading waveform on creep-fatigue behaviour in a forged and precipitation hardened nickel-based superalloy,” International Journal of Fatigue, vol. 97, pp. 190-201, 2017.
[25] M. Goto, “Statistical investigation of the behavior of microcraks in carbon steels,” Fatigue & Fracture of Engineering Materials & Structures, vol. 14, no. 8, pp. 833-845, 1991.
[26] J. Lemaitre, *A Course on Damage Mechanics*, Germany: Springer-Verlag Berlin Heidelberg, 1996.
[27] J. L. Chaboche, P. M. Lesne, “A non-linear continuous fatigue damage model,” Fatigue & Fracture of Engineering Materials & Structures, vol. 11, no. 1, pp. 1-17, 1988.
[28] Q. R. Yue, Y. Zheng, X. Chen, X.G. Liu, “Research on fatigue performance of CFRP reinforced steel crane girder,” Composite Structures, vol. 154, pp. 277-285, 2016.
[29] A. Pouya, C. Zhu, C. Arson, “Self-consistent micromechanical approach for damage accommodation in rock-like polycrystalline materials,” International Journal of Damage Mechanics, vol. 28, no. 1, pp. 134-161, 2019.
[30] H .J. Guo, B. Sun, Z. X. Li, “A multi-scale fatigue-creep coupled damage model for steel structures under extreme cyclic loading and temperature,” International Journal of Damage Mechanics, vol. 29, no. 4, pp. 591-609, 2020.
[31] J. Lemaitre, J. L. Chaboche, *Mechanics of solid materials*, England: Cambridge university press, 1994.
[32] P. Paris, F. Erdogan, “A critical analysis of crack propagation laws,” Transactions of the ASME, vol. 85, no. 4, pp. 528-533, 1963.