Comparison of Single Server Queuing Performance Using Fuzzy Queuing Model and Intuitionistic Fuzzy Queuing Model with Finite Capacity

S. Aarthi, and M. Shanmugasundari

***Abstract*—We describe a systematically confined single-server queuing model using triangular (TFN) and triangular intuitionistic fuzzy numbers (TIFN). The core objective of this study is to investigate the performance of a single-server queuing model with finite capacity in terms of fuzzy queuing theory and intuitionistic fuzzy queuing theory. The entering rate and departure rate are characterized as fuzzy and intuitionistic fuzzy, with fuzzy number computation implemented. The fuzzy queuing theory model's evaluation metrics are supplied as a range of values, but the intuitionistic fuzzy queuing theory model provides a plethora of values. An approach is conducted to evaluate the process metrics utilizing a defined methodology in which the fuzzy values are taken as-is without being processed into crisp values. The mathematical predecessor is wrapped around each type of fuzzy number to validate the miniature's attainability.**

***Index terms*—finite capacity, intuitionistic fuzzy number, queuing theory, single server, triangular fuzzy number.**

I. INTRODUCTION

THE statistical investigation of waiting lines, or queues, is characterized as queuing theory. This methodology provides a framework for assessing the resources necessary to offer a service leveraging a conceptual framework. Whenever the present demand for a service surpasses the contemporary ability to supply that service, the emergence of long lines is a frequent occurrence. A queue forms when a consumer is forced to wait because the number of buyers outweighs the supplier. Queuing theory is used to require careful assessment among buyers. In a plethora of different scenarios, fuzzy queuing systems are more pragmatic than traditional queuing systems. In this article, we employ triangular and triangular intuitionistic fuzzy numbers to manage uncertain parameters in the queuing model FM/FM/1 with constrained potential and FCFS control.

How to look at fuzzy numbers is a crucial hurdle in conceptualizing the fuzzy set assertions since fuzzy numbers do not frame a conventional, structured approach like genuine numbers.

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For positioning fuzzy numbers, various methodologies have been developed where researchers transform the fuzzy values into crisp. When a fuzzy problem formulation is retrofitted into a classical one, the essence of the actual problem is distorted. As a result, we are planning to retain the fuzziness till the end. Hence, we provide a strategy for solving the limited capacity single server fuzzy queuing model in both fuzzy and intuitionistic fuzzy environments without jeopardizing its core. This approach contributes more than previous methods in that it is compact, customized, and topical. According to the analysis, the fuzzy queuing model's performance measurements are within the ballpark of the intuitionistic fuzzy queuing model's estimated evaluation criteria. If you have a single-server fuzzy queuing model, you can use this method to look at the membership function of execution proportions.

Many researchers have investigated queues in fuzzy domains as part of their fuzzy logic study. Li et al. [3] explored statistical results for two typical fuzzy queues using Zadeh's extension theory in 1989. Later, in 1992, Negi et al. [4] exploited alpha-cuts and computation offloading to evaluate fuzzy queues. The use of a triangular fuzzified to assess anthropic reasoning abilities was proposed by Voskoglou et al. [1]. The trapezoidal fuzzy logic model for the assessment programs was researched by Subbotin et al. [2]. Zimmermann [5] examined fuzzy and linear programming with a variety of objectives. Yager [6] gives an alternate interpretation of the fuzzy set extension principle. In 1999, Kao et al. [7] applied the membership functions for fuzzy queues and offered a comprehensive way to estimate the participating components of the $FM/FM/1$, $F/F/1$, $F/M/1$, and $M/F/1$ lines, where F signifies fuzziness and FM specifies exponential time. Similarly, Jau Chuan Ke et al. [8] used the retrial line model's aspects as well as the depiction of a fuzzy aspect of the admittance and monitoring rate to compute the framework's membership capacity. Especially because Kao et al. [7] used fuzzy line models in their derivation. Meanwhile, the exhibition proportions of the line were handled using a nonlinear parametric methodology. Several academics have explained combined IFS with concurrent intuitionistic fuzzy sets. R. Sethi et al. [11] used a systematic strategy to retrieve stable queue distributions, establish numerous evaluation metrics, and undertake empirical tests to quantify the system's behavior indices as various control variables are altered. F. Ferdowsi [12] suggested an intuitionistic fuzzy measure to deal with unpredictability, in which he deployed a trustworthiness metric to turn the fuzzy model into a crisp one. Arpita Kabiraj et al. [13] applied intuitionistic approaches to a linear programming problem to solve a fuzzy linear programming

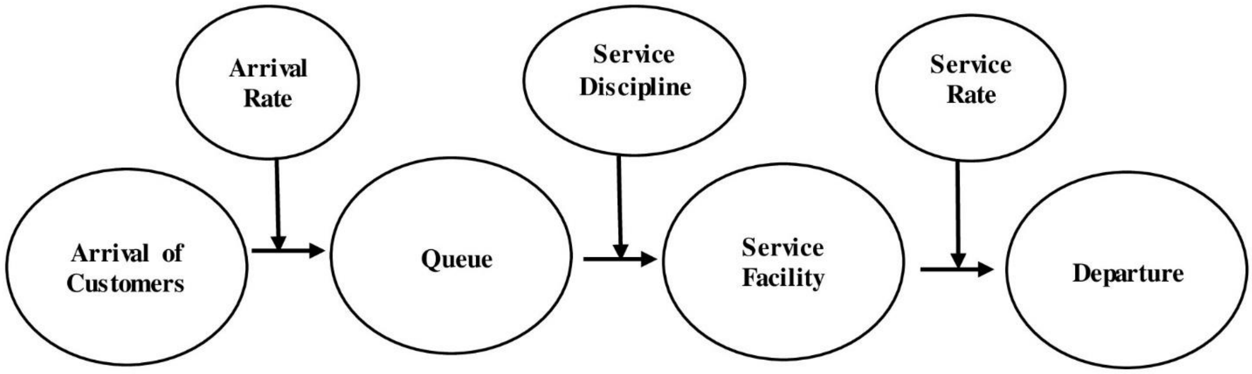

Fig. 1. The fundamental characteristics of a queuing system

difficulties. In their work, S. Hanumantha Rao et al. [14] recommended a single semi-Markov queue system with restricted capacity, encouraging or discouraging arrivals, and an adjusted client reneging policy. A. Tamilarasi [15] investigated the queuing model using trapezoidal intuitionistic fuzzy numbers. The fuzzy cost function's membership degree was devised by S. Hanumantha Rao et al. [16] to generate reliable estimates for several essential metrics of a customized two different service dedicated server Markovian gating queues with server starts and breakdown across N-policy. G. Chen et al. [17] looked at optimal and equilibria procedures in $M/M/1$ fuzzy queues, using all fuzzy integers as control variables. For input parameters, S. Narayana Moorthy et al. [18] employed intuitionistic fuzzy numbers, and the main methodology is predicated on Atanassov's extension principle and α - cut method. Using the DSW method, Noor Hidayah Mohd Zaki et al. [19] analyzed the queuing and the fuzzy queuing model. Muddasir Ahmad et al. [21] solved a solo server finite capacity queuing model with an encouraging arrival rate where the model's economic analysis is present and the cost model is being developed. Usha Prameela et al. [20] used the alpha cut method and DSW algorithm to find out the execution indicators of the single server fuzzy queuing model with finite capacity. Liyuan Zhang et al. [24] used case-based reasoning to help make emergency decisions with the help of triangular fuzzy numbers. Using fuzzy soft set algorithms, Kanwara Waraha et al.. [25] took an approach to the flood alerting prediction process. In this paper, we have presented an approach to resolving issues that is swift, advantageous, and customizable. The core principle of this approach is to preserve the fuzziness till the end, then use the fuzzy numbers in the queuing performance calculations. Achieving the intended effect with ease.

The layout of this article is as regards: Remnant 1 gives a summary, Remnant 2 explains some basic definitions, Remnant 3 discusses the processable, Remnant 4 depicts the computational formula, Remnant 5 explains a few basic theorems, Remnant 6 clarifies the fuzzy queuing system, Remnant 7 explains the performance measures, Remnant 8 tends to give illustrative simulations, Remnant 9 grants discussion of the findings, Remnant 10 shows the constraints, and Remnant 11 wraps up the content.

II. PRELIMINARIES

The motive of this division is to give some basic definitions, annotations, and outcomes that are used in our further calculations.

Definition 1. [22] A fuzzy set $\tilde{A}$ is defined on R, the set of real numbers is called a **fuzzy number** if its membership function $\mu_{\tilde{A}}: R \to [0,1]$ has the following conditions:

(a) $\tilde{A}$ is convex, which means that there exists $x_1, x_2 \in R$ and $\lambda \in [0,1]$, such that $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$

(b) $\tilde{A}$ is normal, which means that there exists an $x \in R$ such that $\mu_{\tilde{A}}(x) = \tilde{1}$

(c) $\tilde{A}$ is piecewise continuous.

Definition 2. [22] A fuzzy number $\tilde{A}$ is defined on R, the set of real numbers is said to be a **triangular fuzzy number (TFN)** if its membership function $\mu_{\tilde{A}}: R \to [0,1]$ which satisfies the following conditions:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-\tilde{a}_1}{\tilde{a}_2-\tilde{a}_1} \text{ for } \tilde{a}_1 \leq x \leq \tilde{a}_2 \\ 1 \text{ for } x = \tilde{a}_2 \\ \frac{\tilde{a}_3-x}{\tilde{a}_3-\tilde{a}_2} \text{ for } \tilde{a}_2 \leq x \leq \tilde{a}_3 \\ 0 \text{ otherwise} \end{cases}$$

Figure 2 shows a schematic representation of the TFN.

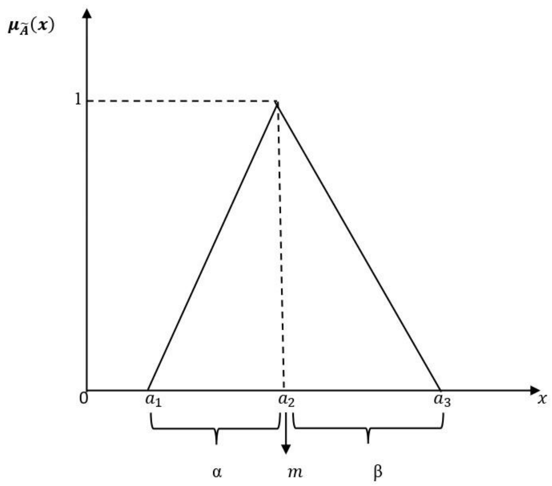

Fig. 2. TFN

Definition 3. Let the two triangular fuzzy numbers be $\tilde{P} \approx (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3) = (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1)$ and $\tilde{Q} \approx (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2)$ and then the **arithmetic operations on TFN** be given as follows:

(A)Addition

$$\tilde{P} + \tilde{Q} \approx \left(\tilde{m}_1 + \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}\right) \quad (1)$$

(B)Subtraction

$$\tilde{P} - \tilde{Q} \approx \left(\tilde{m}_1 - \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}\right) \quad (2)$$

(C) Multiplication

$$\tilde{P}.\tilde{Q} \approx \left(\tilde{m}_1.\tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}\right) \quad (3)$$

(D) Division

$$\frac{\tilde{P}}{\tilde{Q}} \approx \left(\frac{\tilde{m}_1}{\tilde{m}_2}, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}\right) \quad (4)$$

(E) Scalar Multiplication

$$k\tilde{P} \approx \begin{cases} (ka_2, \alpha_1, \beta_1), k \geq 0 \\ (-ka_2, \alpha_1, \beta_1), k < 0 \end{cases} \quad (5)$$

Definition 4. For every triangular fuzzy number $\tilde{P} \approx (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3) \in F(R)$ **ranking function** $\Re: F(R) \to R$ is defined by graded mean as

$$\Re(\tilde{P}) = \frac{(\tilde{a}_1 + 4\tilde{a}_2 + \tilde{a}_3)}{6}$$

For any two TFN $\tilde{P} \approx (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3)$ and $\tilde{Q} \approx (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3)$ we have the following correlations,

$$(a)\tilde{P} \succ \tilde{Q} \Leftrightarrow \Re(\tilde{P}) > \Re(\tilde{Q})$$
$$(b)\tilde{P} \prec \tilde{Q} \Leftrightarrow \Re(\tilde{P}) < \Re(\tilde{Q})$$
$$(c)\tilde{P} \approx \tilde{Q} \Leftrightarrow \Re(\tilde{P}) = \Re(\tilde{Q})$$
$$(d)\tilde{P} - \tilde{Q} \approx 0 \Leftrightarrow \Re(\tilde{P}) - \Re(\tilde{Q}) = 0$$

A TFN $\tilde{P} \approx (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3) \in F(R)$ is known to be **positive** if $\Re(\tilde{P}) > 0$ and defined by $\tilde{P} \succ 0$

Definition 5. [23] Let a non–empty set be X. An **Intuitionistic fuzzy set (IFS)** $\tilde{A}'$ is defined as $\tilde{A}' = \{(x, \mu_{\tilde{A}'}(x), \gamma_{\tilde{A}'}(x)/x \in X)\}$, where $\mu_{\tilde{A}'}: X \to [0,1]$ and $\gamma_{\tilde{A}'}: X \to [0,1]$ denotes the degree of membership and degree of non–membership functions respectively, where $x \in X$, for every $x \in X, 0 \leq \mu_{\tilde{A}'}(x) + \gamma_{\tilde{A}'}(x) \leq 1$

Definition 6. [23] An intuitionistic fuzzy set described on R, the real numbers are said to be an **Intuitionistic fuzzy number (IFN)** if its membership function $\mu_{\tilde{A}'}: R \to [0,1]$ and its non–membership function $\gamma_{\tilde{A}'}: R \to [0,1]$ should be agreeable to the following conditions:

i) $\tilde{A}'$ is normal, which means that there exists an $x \in R$, such that $\mu_{\tilde{A}'}(x) = 1, \gamma_{\tilde{A}'}(x) = 0$

ii) $\tilde{A}'$ is convex for the membership functions $\mu_{\tilde{A}'}$, which means that there exists $x_1, x_2 \in R$ and $\lambda \in [0,1]$ such that $\mu_{\tilde{A}'}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\tilde{A}'}(x_1), \mu_{\tilde{A}'}(x_2)\}$

iii) $\tilde{A}'$ is concave for the non–membership function $\gamma_{\tilde{A}'}$, which means that there exists $x_1, x_2 \in R$ and $\lambda \in [0,1]$ such that $\gamma_{\tilde{A}'}(\lambda x_1 + (1-\lambda)x_2) \leq max\{\gamma_{\tilde{A}'}(x_1), \gamma_{\tilde{A}'}(x_2)\}$

Definition 7. [23] A fuzzy number $\tilde{A}'$ on R is said to be a **triangular intuitionistic fuzzy number (TIFN)** if its membership function $\mu_{\tilde{A}'}: R \to [0,1]$ and non-membership function $\gamma_{\tilde{A}'}: R \to [0,1]$ has the following conditions:

$$\mu_{\tilde{A}'}(x) = \begin{cases} \frac{x - \tilde{a}_1}{\tilde{a}_2 - \tilde{a}_1} \text{ for } \tilde{a}_1 \leq x \leq \tilde{a}_2 \\ 1 \text{ for } x = \tilde{a}_2 \\ \frac{\tilde{a}_3 - x}{\tilde{a}_3 - \tilde{a}_2} \text{ for } \tilde{a}_2 \leq x \leq \tilde{a}_3 \\ 0 \text{ otherwise} \end{cases}$$

and

$$\gamma_{\tilde{A}'}(x) = \begin{cases} 1 \quad \text{for } x < \tilde{a}'_1, x > \tilde{a}'_3 \\ \frac{\tilde{a}_2 - x}{\tilde{a}_2 - \tilde{a}'_1} \text{ for } \tilde{a}'_1 \leq x \leq \tilde{a}_2 \\ 0 \text{ for } x = \tilde{a}_2 \\ \frac{x - \tilde{a}_2}{\tilde{a}_3 - \tilde{a}_2} \text{ for } \tilde{a}_2 \leq x \leq \tilde{a}'_3 \end{cases}$$

and is given by $\tilde{A}' = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3; \tilde{a}'_1, \tilde{a}_2, \tilde{a}'_3)$ where $\tilde{a}'_1 \leq \tilde{a}_1 \leq \tilde{a}_2 \leq \tilde{a}_3 \leq \tilde{a}'_3$.

Cases: Let $\tilde{A}' = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3; \tilde{a}'_1, \tilde{a}_2, \tilde{a}'_3)$ be a TIFN then the following cases arise.

Case:1 If $\tilde{a}'_1 = \tilde{a}_1, \tilde{a}'_3 = \tilde{a}_3$ then $\tilde{A}'$ represent a triangular fuzzy number.

Case:2 If $\tilde{a}'_1 = \tilde{a}_1 = \tilde{a}_2 = \tilde{a}'_3 = \tilde{a}_3 = \tilde{m}$ then $\tilde{A}'$ represent a real number $\tilde{m}$. The parametric form of TIFN $\tilde{A}'$ is represented as $\tilde{A}' = (\tilde{\alpha}, \tilde{m}, \tilde{\beta}; \tilde{\alpha}', \tilde{m}, \tilde{\beta}')$ where $\tilde{\alpha}, \tilde{\alpha}' \& \tilde{\beta}, \tilde{\beta}'$ represents the left spread and right spread of membership functions and non–membership functions respectively.

The triangular intuitionistic fuzzy number is illustrated in Figure 3.

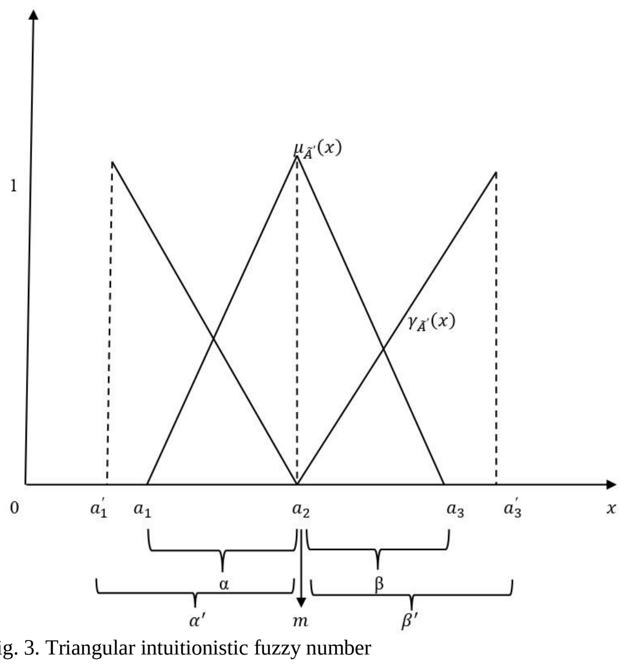

Fig. 3. Triangular intuitionistic fuzzy number

Definition 8. [23] TIFN $\tilde{A}' \in F(R)$, (where $F(R)$ is the set of all TIFN) can also be represented as a pair $\tilde{A}' = (\tilde{a}, \tilde{\tilde{a}}; \tilde{a}', \tilde{\tilde{a}}')$ of functions $\tilde{a}(\tilde{r}'), \tilde{\tilde{a}}(\tilde{r}'), \tilde{a}'(\tilde{r}') \& \tilde{\tilde{a}}'(\tilde{r}')$ for $0 \leq \tilde{r}' \leq 1$ which satisfies the following requirements:

i) $\tilde{a}(\tilde{r}') \& \tilde{\tilde{a}}'(\tilde{r}')$ is a bounded monotonic increasing left continuous function for membership and non–membership functions respectively.

ii) $\tilde{\tilde{a}}(\tilde{r}^{'})\&\tilde{a}^{'}(\tilde{r}^{'})$ is a bounded monotonic decreasing left continuous function for membership and non–membership functions respectively.

iii) $\tilde{a}(\tilde{r}^{'}) \le \tilde{\tilde{a}}(\tilde{r}^{'}), 0 \le \tilde{r}^{'} \le 1$

iv) $\tilde{a}^{'}(\tilde{r}^{'}) \le \tilde{\tilde{a}}^{'}(\tilde{r}^{'}), 0 \le \tilde{r}^{'} \le 1$

Definition 9. The extension of fuzzy arithmetic operations of Ming Ma et al. [22] to the set of triangular intuitionistic fuzzy numbers based upon both location indices and functions of fuzziness indices. The location indices number is taken in the regular arithmetic while the functions of fuzziness indices are assumed to follow the lattice rule, which is the least upper bound in the lattice $\tilde{I}^{'}$.

For any two arbitrary TIFN $\tilde{P}^{'} \approx (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3; \tilde{a}_1^{'}, \tilde{a}_2^{'}, \tilde{a}_3^{'}) \approx (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1^{'}, \tilde{\beta}_1^{'})$, $\tilde{Q}^{'} \approx (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3; \tilde{b}_1^{'}, \tilde{b}_2^{'}, \tilde{b}_3^{'}) \approx (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2^{'}, \tilde{\beta}_2^{'})$ and $* \in \{+, -, \times, \div\}$, then the **arithmetic operations on TIFN** are defined by

$$\tilde{P}^{'} * \tilde{Q}^{'} = \begin{pmatrix} \tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1 \vee \tilde{\alpha}_2, \tilde{\beta}_1 \vee \tilde{\beta}_2; \\ \tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1^{'} \vee \tilde{\alpha}_2^{'}, \tilde{\beta}_1^{'} \vee \tilde{\beta}_2^{'} \end{pmatrix}$$

We define:

(A) Addition

$$\tilde{P}^{'} + \tilde{Q}^{'} = \begin{pmatrix} \tilde{m}_1 + \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}; \\ \tilde{m}_1 + \tilde{m}_2, max\{\tilde{\alpha}_1^{'}, \tilde{\alpha}_2^{'}\}, max\{\tilde{\beta}_1^{'}, \tilde{\beta}_2^{'}\} \end{pmatrix} \quad (6)$$

(B) Subtraction

$$\tilde{P}^{'} - \tilde{Q}^{'} = \begin{pmatrix} \tilde{m}_1 - \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}; \\ \tilde{m}_1 - \tilde{m}_2, max\{\tilde{\alpha}_1^{'}, \tilde{\alpha}_2^{'}\}, max\{\tilde{\beta}_1^{'}, \tilde{\beta}_2^{'}\} \end{pmatrix} \quad (7)$$

(C) Multiplication

$$\tilde{P}^{'} \times \tilde{Q}^{'} = \begin{pmatrix} \tilde{m}_1 \times \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}; \\ \tilde{m}_1 \times \tilde{m}_2, max\{\tilde{\alpha}_1^{'}, \tilde{\alpha}_2^{'}\}, max\{\tilde{\beta}_1^{'}, \tilde{\beta}_2^{'}\} \end{pmatrix} \quad (8)$$

(D) Division

$$\tilde{P}^{'} \div \tilde{Q}^{'} = \begin{pmatrix} \tilde{m}_1 \div \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}; \\ \tilde{m}_1 \div \tilde{m}_2, max\{\tilde{\alpha}_1^{'}, \tilde{\alpha}_2^{'}\}, max\{\tilde{\beta}_1^{'}, \tilde{\beta}_2^{'}\} \end{pmatrix} \quad (9)$$

(E) Scalar Multiplication

$$k\tilde{A}' = \begin{cases} (ka_2, \alpha_1, \beta_1; ka_2, \alpha_1', \beta_1'), \text{ for } k \ge 0 \\ (-ka_2, \alpha_1, \beta_1; -ka_2, \alpha_1', \beta_1'), \text{ for } k < 0 \end{cases} \quad (10)$$

Definition 10. Consider an arbitrary TIFN $\tilde{A}' = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3; \tilde{a}_1', \tilde{a}_2, \tilde{a}_3') = (\tilde{m}, \tilde{\alpha}, \tilde{\beta}; \tilde{m}, \tilde{\alpha}', \tilde{\beta}')$ and the magnitude of TIFN $\tilde{A}'$ is given by

$$mag(\tilde{A}') = \frac{1}{2}\int_0^1 (\tilde{a} + \tilde{\tilde{a}} + 2\tilde{m} + \tilde{a'} + \tilde{\tilde{a}'})f(r)dr$$

$$mag(\tilde{A}') = \frac{1}{2}\int (\beta + \beta' + 6\tilde{m} - \alpha - \alpha')f(r)dr$$

In real-life scenarios, decision-makers select the value of $\tilde{f}(\tilde{r}')$ based on their circumstances. Here for our ease, we choose $\tilde{f}(\tilde{r}') = \tilde{r}'^2$

$$\therefore\ mag(\tilde{A}') = \left(\frac{\beta + \beta' + 6\tilde{m} - \alpha - \alpha'}{6}\right)$$
$$= \left(\frac{\tilde{a} + \tilde{\tilde{a}} + 2\tilde{m} + \tilde{\alpha}' + \tilde{\tilde{a}}'}{6}\right)$$

For any two TIFN $\tilde{P}' \approx (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1'), \tilde{Q}' \approx (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$ in $F(R)$, we define

$(a) \tilde{P}^{'} \ge \tilde{Q}^{'} \Leftrightarrow mag(\tilde{P}^{'}) \ge mag(\tilde{Q}^{'})$

$(b) \tilde{P}^{'} \le \tilde{Q}^{'} \Leftrightarrow mag(\tilde{P}^{'}) \le mag(\tilde{Q}^{'})$

$(c) \tilde{P}^{'} \approx \tilde{Q}^{'} \Leftrightarrow mag(\tilde{P}^{'}) = mag(\tilde{Q}^{'})$

III. MODEL DESCRIPTION

We cite a single server restricted limit with first-come, first-served (FCFS) discipline queuing model $(FM/FM/1)$: $(N/FCFS)$, in which the inter-entrance period and the service time follow Poisson and exponential diffusion distributions with fuzzy parameters $\tilde{\lambda}$ and $\tilde{\mu}$ respectively. Both TFN and TIFN are being used to compute the arrival and service rates. The system's utmost limit is set to a certain range. The main purpose is to determine evaluation criteria using both fuzzy and intuitionistic fuzzy numbers, and models are contrasted based on the average number of consumers in the queue and system, as well as their sojourn time in the queue and system. The problems are solved by sustaining the fuzziness values until the end, i.e., without switching them to crisp. As a consequence, it is more appropriate for specific circumstances.

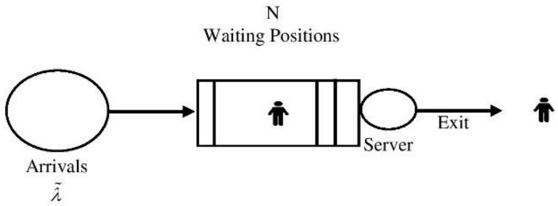

Fig. 4. A formal outline of the model

IV. HYPOTHESES AND SYNTAXES

A. Hypotheses

i) The admissions to the queuing system start happening one by one, according to a Poisson process with a mean rate $\tilde{\lambda}$.

ii) With a parameter $\tilde{\lambda}$, the inter-arrival times are unilaterally, symmetrically and exponentially distributed.

iii) There is only one server, and each customer is served individually.

iv) With a parameter $\tilde{\mu}$, service times are dispersed unilaterally, predictably, and exponentially.

v) The system's capacity is finite, say N.

vi) Services are provided in the order in which they arrived, i.e., First-Come, First-Served.

vii) The arrival rate and service rate are taken as TFN and TIFN.

B. Syntaxes

Here we are using the following notations:

$\tilde{\lambda}, \tilde{\lambda}^{'} \rightarrow$ The mean No. of consumers who arrive in a predetermined period of time.

$\tilde{\mu}, \tilde{\mu}^{'} \rightarrow$ The mean No. of consumers being serviced per unit of time.

$\tilde{\rho} \rightarrow$Traffic intensity

$\tilde{N}_q, \tilde{N}_q^{'} \rightarrow$ The mean No. of consumers in the line.
$\tilde{N}_s, \tilde{N}_s^{'} \rightarrow$The mean No. of consumers in the system.
$\tilde{T}_q, \tilde{T}_q^{'} \rightarrow$ The mean sojourn time of the consumers in the queue.
$\tilde{T}_s, \tilde{T}_s^{'} \rightarrow$ The mean sojourn time of the consumers in the system.
$\tilde{P}, \tilde{P}^{'} \rightarrow$ Interarrival rate.
$\tilde{Q}, \tilde{Q}^{'} \rightarrow$ Service rate
$N \rightarrow$The capacity of the system
$X \rightarrow$ Set of the arrival time
$Y \rightarrow$Set of the service time

V. A SINGLE-SERVER EXPONENTIAL QUEUING SYSTEM HAVING FINITE CAPACITY

In the prior model, we presupposed that the number of customers who could be logged into the system concurrently had no upper bound. However, in practice, the system always has a finite capacity of N, meaning that there can never be more than N concurrent users. This means that a customer can't use the system if he/she shows up and finds that there are already N people there.

The restricting probability that there are n customers in the system is signified by $\tilde{P}_n$, where $0 \leq n \leq N$.

Consider the scenario where the system's capacity is N times its maximum. In any case, the number of arrivals won't be greater than N. The model's physical interpretation is as follows:

1. The system only contains n units.
2. If the queue length is too lengthy, the arriving customers will permanently seek their service elsewhere ($\leq N$).

Theorem 5.1

In the $(FM/FM/1):(N/FCFS)$ queuing model, at the initial state, when there are no customers in the state, prove that the rate is $\tilde{\lambda}^{'}\tilde{P}_0^{'} = \tilde{\mu}^{'}\tilde{P}_1^{'}$.

Proof. Let,

$$\tilde{\lambda}_n^{'} = \begin{cases} \tilde{\lambda}^{'}, n = 0,1,2,3,\ldots N-1 \\ 0, n \geq N \end{cases}$$
$$\tilde{\mu}_n^{'} = \tilde{\mu}^{'} \text{for } n = 1,2,3,\ldots$$

For $n = 0$,

The possibility that somehow there won't be any entities in the system at $(\tilde{t}^{'} + \Delta\tilde{t}^{'})$ will be equal to the sum of the next two separate probabilities:

i) $\tilde{P}_0^{\prime}(\tilde{t}^{\prime})(1 - \tilde{\lambda}^{\prime}\Delta\tilde{t}^{\prime})$ is the statistical likelihood that there would be no entity in the system at $\tilde{t}^{'}$ and no arrival at $\Delta\tilde{t}^{'}$

ii) $\tilde{P}_1^{\prime}(\tilde{t}^{\prime}).\tilde{\mu}^{\prime}\Delta\tilde{t}^{\prime}.(1 - \tilde{\lambda}^{\prime}\Delta\tilde{t}^{\prime}) \cong \tilde{P}_1^{\prime}(\tilde{t}^{\prime})\tilde{\mu}^{\prime}\Delta\tilde{t}^{\prime} + o(\Delta\tilde{t}^{\prime})$ is the statistical likelihood that one unit is present in the system at $\tilde{t}^{\prime}$, one unit is serviced at $\Delta\tilde{t}^{\prime}$, and no arrival at $\Delta\tilde{t}^{\prime}$.

As a result, the probability of $n = 0$ is $\tilde{P}_0(\tilde{t}^{\prime} + \Delta\tilde{t}^{\prime}) = \tilde{P}_0(\tilde{t}^{\prime})(1 - \tilde{\lambda}^{\prime}\Delta\tilde{t}^{\prime}) + \tilde{P}_1(\tilde{t}^{\prime})\tilde{\mu}^{\prime}\Delta\tilde{t}^{\prime} + o(\Delta\tilde{t}^{\prime})$ (11)

Now dividing the aforementioned equation by $\Delta\tilde{t}^{\prime}$ and assuming the limit as $\Delta\tilde{t}^{\prime} \rightarrow 0$, hence it transforms into:

$$\tilde{P}_0^{\prime}(\tilde{t}^{\prime}) = -\tilde{\lambda}^{\prime}\tilde{P}_0^{\prime}(\tilde{t}^{\prime}) + \tilde{\mu}^{\prime}\tilde{P}_1^{\prime}(\tilde{t}^{\prime}) \quad (12)$$

In the scenario of a steady state where $\Delta\tilde{t}^{\prime} \rightarrow \infty$, $\tilde{P}_n^{\prime}(\tilde{t}^{\prime}) \rightarrow \tilde{P}_n^{\prime}$ (independent of $\tilde{t}^{'}$) and consequently $\tilde{P}_n^{\prime}(\tilde{t}^{\prime}) \rightarrow 0$. Eventually, the steady-state difference equations of the system are provided by

$0 = -\tilde{\lambda}^{\prime}\tilde{P}_0^{\prime} + \tilde{\mu}^{\prime}\tilde{P}_1^{\prime}$ (From 12)

Furthermore, $\tilde{\lambda}^{\prime}\tilde{P}_0^{\prime} = \tilde{\mu}^{\prime}\tilde{P}_1^{\prime}; n = 0$ (13)

Theorem 5.2

In the $(FM/FM/1):(N/FCFS)$ queuing model, at the state $1 \leq n \leq N-1$, where $0 \leq n \leq N$ proves that the rate is $(\tilde{\lambda}^{\prime} + \tilde{\mu}^{\prime})\tilde{P}_n = \tilde{\lambda}^{\prime}\tilde{P}_{n-1} + \tilde{\mu}^{\prime}\tilde{P}_{n+1}$.

Proof. Let,

$$\tilde{\lambda}_n^{'} = \begin{cases} \tilde{\lambda}^{'}, n = 0,1,2,3,\ldots N-1 \\ 0, n \geq N \end{cases}$$
$$\tilde{\mu}_n^{'} = \tilde{\mu}^{'} \text{for } n = 1,2,3,\ldots$$

For $n = 1,2,3,\ldots N-1$,

When independent conditions are incorporated together, the probability will be shown as follows:

i) At any time $\tilde{t}^{\prime}$, there are n entities in the system are $\tilde{P}_n^{\prime}(\tilde{t}^{\prime})$. However, at $\Delta\tilde{t}^{\prime}$, there are no arrivals and no services respectively, $(1 - \tilde{\lambda}^{\prime}\Delta\tilde{t}^{\prime})$ and $(1 - \tilde{\mu}^{\prime}\Delta\tilde{t}^{\prime})$. As a consequence, the probability is provided as
$$\Rightarrow \tilde{P}_n^{\prime}(\tilde{t}^{\prime})(1 - \tilde{\lambda}^{\prime}\Delta\tilde{t}^{\prime})(1 - \tilde{\mu}^{\prime}\Delta\tilde{t}^{\prime})$$
$$\Rightarrow \tilde{P}_n^{\prime}(\tilde{t}^{\prime})\left(1 - \Delta\tilde{t}^{\prime}(\tilde{\lambda}^{\prime} + \tilde{\mu}^{\prime})\right) + o_1(\Delta\tilde{t}^{\prime}) \quad (14)$$

ii) At any time $\tilde{t}^{\prime}$, there are $(n-1)$ entities in the system are $\tilde{P}_{n-1}^{\prime}(\tilde{t}^{\prime})$; there is one entrance at $\Delta\tilde{t}^{\prime}$ is $\tilde{\lambda}^{\prime}\Delta\tilde{t}^{\prime}$ and no assistance at $\Delta\tilde{t}^{\prime}$is $(1 - \tilde{\mu}^{\prime}\Delta\tilde{t}^{\prime})$. As a result, the probability is given as
$$\Rightarrow \tilde{P}_{n-1}^{\prime}(\tilde{t}^{\prime})\tilde{\lambda}^{\prime}\Delta\tilde{t}^{\prime}(1 - \tilde{\mu}^{\prime}\Delta\tilde{t}^{\prime})$$
$$\Rightarrow \tilde{\lambda}^{\prime}\Delta\tilde{t}^{\prime}\tilde{P}_{n-1}^{\prime}(\tilde{t}^{\prime}) + o_2(\Delta\tilde{t}^{\prime}) \quad (15)$$

iii) At any time $\tilde{t}^{\prime}$, there are $(n+1)$ units in the system are $\tilde{P}_{n+1}^{\prime}(\tilde{t}^{\prime})$; there is no inflow at $\Delta\tilde{t}^{\prime}$ is $(1 - \tilde{\lambda}^{\prime}\Delta\tilde{t}^{\prime})$ and one service at $\Delta\tilde{t}^{\prime}$ is $\tilde{\mu}^{\prime}\Delta\tilde{t}^{\prime}$. As an outcome, the probability is given as
$$\Rightarrow \tilde{P}_{n+1}^{\prime}(\tilde{t}^{\prime})(1 - \tilde{\lambda}^{\prime}\Delta\tilde{t}^{\prime})\tilde{\mu}^{\prime}\Delta\tilde{t}^{\prime}$$
$$\Rightarrow \tilde{\mu}^{\prime}\Delta\tilde{t}^{\prime}\tilde{P}_{n+1}^{\prime}(\tilde{t}^{\prime}) + o_3(\Delta\tilde{t}^{\prime}) \quad (16)$$

By adding (14), (15) & (16), we obtain

$$\tilde{P}_n^{\prime}(\tilde{t}^{\prime} + \Delta\tilde{t}^{\prime}) = \tilde{P}_n^{\prime}(\tilde{t}^{\prime})\left[1 - (\tilde{\lambda}^{\prime} + \tilde{\mu}^{\prime})\Delta\tilde{t}^{\prime}\right] + \tilde{\lambda}^{\prime}\Delta\tilde{t}^{\prime}\tilde{P}_{n-1}^{\prime}(\tilde{t}^{\prime}) + \tilde{\mu}^{\prime}\Delta\tilde{t}^{\prime}\tilde{P}_{n+1}^{\prime}(\tilde{t}^{\prime}) + o(\Delta\tilde{t}^{\prime}); n = 1,2,3,\ldots N-1 \quad (17)$$

The above equation now becomes $\tilde{P}_n^{\prime}(\tilde{t}^{\prime})$ by dividing it by $\Delta\tilde{t}^{\prime}$ and presuming that $\Delta\tilde{t}^{\prime} \rightarrow 0$ is the limit hence it transforms into:

$$\tilde{P}_n^{\prime}(\tilde{t}^{\prime}) = -(\tilde{\lambda}^{\prime} + \tilde{\mu}^{\prime})\tilde{P}_n^{\prime}(\tilde{t}^{\prime}) + \tilde{\lambda}^{\prime}\tilde{P}_{n-1}^{\prime}(\tilde{t}^{\prime}) + \tilde{\mu}^{\prime}\tilde{P}_{n+1}^{\prime}(\tilde{t}^{\prime}) \quad (18)$$

In the circumstance of a steady state where $\Delta\tilde{t}^{\prime} \rightarrow \infty$, $\tilde{P}_n^{\prime}(\tilde{t}^{\prime}) \rightarrow \tilde{P}_n^{\prime}$ (independent of $\tilde{t}^{\prime}$) and hence $\tilde{P}_n^{\prime}(\tilde{t}^{\prime}) \rightarrow 0$. So, the system of steady-state difference equations is given by

$0 = -(\tilde{\lambda}^{\prime} + \tilde{\mu}^{\prime})\tilde{P}_n^{\prime} + \tilde{\lambda}^{\prime}\tilde{P}_{n-1}^{\prime} + \tilde{\mu}^{\prime}\tilde{P}_{n+1}^{\prime}$ (From 18)

Hence,

$$(\tilde{\lambda}^{\prime} + \tilde{\mu}^{\prime})\tilde{P}_n^{\prime} = \tilde{\lambda}^{\prime}\tilde{P}_{n-1}^{\prime} + \tilde{\mu}^{\prime}\tilde{P}_{n+1}^{\prime}; n = 1,2,3,\ldots N-1 \quad (19)$$

Theorem 5.3

In the $(FM/FM/1):(N/FCFS)$ queuing model, at the state N, where $0 \leq n \leq N$ proves that the rate is $\tilde{\mu}^{\prime}\tilde{P}_N = \tilde{\lambda}^{\prime}\tilde{P}_{N-1}$.

Proof. Let,

$$\tilde{\lambda}_n^{'} = \begin{cases} \tilde{\lambda}^{'}, n = 0,1,2,3,\ldots N-1 \\ 0, n \geq N \end{cases}$$
$$\tilde{\mu}_n^{'} = \tilde{\mu}^{'} \text{for } n = 1,2,3,\ldots$$

For $n \geq N$,

Using equation (17),

When the value of $n = N$, then $\tilde{P}_{N+1}^{\prime}(\tilde{t}^{\prime}) = 0; \tilde{\lambda}^{\prime} = 0$.

As a consequence, the probability value becomes,

$$\tilde{P}_N^{\prime}(\tilde{t}^{\prime} + \Delta\tilde{t}^{\prime}) = \tilde{P}_N^{\prime}(\tilde{t}^{\prime})[1 - \tilde{\mu}^{\prime}\Delta\tilde{t}^{\prime}] + \tilde{P}_{N-1}^{\prime}(\tilde{t}^{\prime})\tilde{\lambda}^{\prime}\Delta\tilde{t}^{\prime} + o(\Delta\tilde{t}^{\prime}) \quad (20)$$

Now dividing the aforementioned equation by $\Delta\tilde{t}^{'}$ and assuming the limit as $\Delta\tilde{t}^{\prime} \rightarrow 0$, hence it transforms into:

$$\tilde{P}_N^{\prime}(\tilde{t}^{\prime}) = -\tilde{\mu}^{\prime}\tilde{P}_N^{\prime}(\tilde{t}^{\prime}) + \tilde{\lambda}^{\prime}\tilde{P}_{N-1}^{\prime}(\tilde{t}^{\prime}); n = N \quad (21)$$

In the situation of a steady state where $\Delta\tilde{t}' \to \infty$, $\tilde{P}_n'(\tilde{t}') \to \tilde{P}_n'$ (independent of $\tilde{t}'$) and hence $\tilde{P}_n'(\tilde{t}') \to 0$. Hence, the system of steady-state difference equations is given by
$0 = -\tilde{\mu}'\tilde{P}_N' + \tilde{\lambda}'\tilde{P}_{N-1}'$ (From 21)
Hence, $\tilde{\lambda}'\tilde{P}_{N-1}' = \tilde{\mu}'\tilde{P}_N'; n = N$ (22)

Theorem 5.4
In the $(FM/FM/1):(N/FCFS)$ queuing system, the steady-state probability $\tilde{P}_n'$ is given by

$$\tilde{P}_n' = \frac{\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n \left(1-\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)\right)}{\left(1-\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^{N+1}\right)}, 0 < n < N$$

Proof
The rate at which the process enters and leaves various states is given as follows:
At the initial state, when there are no customers in the state the rate becomes $\tilde{\lambda}'\tilde{P}_0' = \tilde{\mu}'\tilde{P}_1'$. At the state $1 \le n \le N-1$, the rate is given as $(\tilde{\lambda}' + \tilde{\mu}')\tilde{P}_n' = \tilde{\lambda}'\tilde{P}_{n-1}' + \tilde{\mu}'\tilde{P}_{n+1}'$. At the state N, the rate value is $\tilde{\mu}'\tilde{P}_N' = \tilde{\lambda}'\tilde{P}_{N-1}'$.
We can solve the system of differential equations as follows,
Hence, initially $\tilde{P}_0' = \tilde{P}_0'$
From (13), $\tilde{P}_1' = \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)\tilde{P}_0'$
From (19), $(\tilde{\lambda}' + \tilde{\mu}')\tilde{P}_n' = \tilde{\lambda}'\tilde{P}_{n-1}' + \tilde{\mu}'\tilde{P}_{n+1}'$
Substitute $n = 1$ and the value of $\tilde{P}_1'$ in the above equation, it becomes

$$(\tilde{\lambda}' + \tilde{\mu}')\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)\tilde{P}_0' = \tilde{\lambda}'\tilde{P}_0' + \tilde{\mu}'\tilde{P}_2'$$

$$\frac{(\tilde{\lambda}')^2}{\tilde{\mu}'}\tilde{P}_0' + \tilde{\lambda}'\tilde{P}_0' = \tilde{\lambda}'\tilde{P}_0' + \tilde{\mu}'\tilde{P}_2'$$

$$\tilde{P}_2' = \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^2\tilde{P}_0'$$

Similarly, $\tilde{P}_3' = \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^3\tilde{P}_0'$

$$\vdots$$

$$\tilde{P}_n' = \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^n\tilde{P}_0'; n < N$$

$$\tilde{P}_N' = \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^N\tilde{P}_0'; n = N$$

$$\tilde{P}_{N+1}' = 0; n > N$$

By the total probability, we have $\sum_{n=0}^{N}\tilde{P}_n' = 1$
$\Rightarrow \tilde{P}_0' + \tilde{P}_1' + \tilde{P}_2' + \tilde{P}_3' + \ldots + \tilde{P}_N' = 1$

$$\tilde{P}_0' + \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)\tilde{P}_0' + \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^2\tilde{P}_0' + \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^3\tilde{P}_0' + \ldots + \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^N\tilde{P}_0' = 1$$

$$\tilde{P}_0'\left[1 + \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right) + \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^2 + \left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^3 + \ldots\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^N\right] = 1$$

$$\tilde{P}_0'\left[\frac{1-\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^{N+1}}{1-\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)}\right] = 1$$

(or)

$$\tilde{P}_0' = \left[\frac{1-\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)}{1-\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^{N+1}}\right]; \tilde{\lambda}' \ne \tilde{\mu}'$$

$$\tilde{P}_0' = \left[\frac{1}{N+1}\right]; \tilde{\lambda}' = \tilde{\mu}'$$

$$\therefore \tilde{P}_n' = \left[\frac{1-\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)}{1-\left(\frac{\tilde{\lambda}'}{\tilde{\mu}'}\right)^{N+1}}\right]\tilde{\rho}'^n; n = 0,1,2,3,\ldots N$$

VI. (FM/FM/1): (N/FCFS) QUEUES

We assume a single-server fuzzy queuing system with finite capacity. The inter-arrival rate $\tilde{P}$ and the service rate $\tilde{Q}$ are nearly comprehended and depicted by a fuzzy set,

$$\tilde{P} = \{p, \mu_{\tilde{P}}(p)/p \in X\}$$
$$\tilde{Q} = \{q, \mu_{\tilde{Q}}(q)/q \in Y\}$$

In this, X is the inter-entrance period configuration and Y is the service time configuration. $\mu_{\tilde{P}}(p)$ is the inter-entrance time's membership function and $\mu_{\tilde{Q}}(q)$ is the enlistment capacity of the service time. In addition to that, consider a single server intuitionistic fuzzy queuing system with finite capacity. The inter-arrival rate $\tilde{P}'$ and the service rate $\tilde{Q}'$ are nearly comprehended and depicted by an intuitionistic fuzzy set,

$$\tilde{P}' = \{p, \mu_{\tilde{P}'}(p), \gamma_{\tilde{P}'}(p)/p \in X\}$$
$$\tilde{Q}' = \{q, \mu_{\tilde{Q}'}(q), \gamma_{\tilde{Q}'}(q)/q \in Y\}$$

In this, X is the inter-entrance duration customization and Y is the service time customization. $\mu_{\tilde{P}'}(p)\&\gamma_{\tilde{P}'}(p)$ is the membership and non-membership functions respectively of the inter-arrival time. $\mu_{\tilde{Q}'}(q)\&\gamma_{\tilde{Q}'}(q)$ are the membership and non-membership functions respectively of the service time.

VII. SINGLE SERVER FUZZY QUEUING MODEL WITH FINITE CAPACITY

Let $\tilde{\lambda}$ and $\tilde{\lambda}'$ be the fuzzy and intuitionistic fuzzy arrival rates respectively. Let $\tilde{\mu}$ and $\tilde{\mu}'$ be the fuzzy and intuitionistic fuzzy service rates respectively. At the steady state, the FCFS discipline is upheld, but the capacity is limited to a certain extent.
The following are the fabrication characteristics of the above model:
(a) Number of consumers predicted in the system

$$\tilde{N}_s = \tilde{\rho}\frac{[1-(N+1)\tilde{\rho}^N + N(\tilde{\rho}^{N+1})]}{(1-\tilde{\rho})(1-\tilde{\rho}^{N+1})} \quad (23)$$

(b) Estimated number of consumers standing in line

$$\tilde{N}_q = \tilde{\rho}^2\frac{[1-(N)\tilde{\rho}^{N-1} + (N-1)(\tilde{\rho}^N)]}{(1-\tilde{\rho})(1-\tilde{\rho}^{N+1})} \quad (24)$$

(c) The expected volume of time a customer invests in the system

$$\tilde{T}_s = \tilde{\rho}\frac{[1-(N+1)\tilde{\rho}^N + N(\tilde{\rho}^{N+1})]}{\tilde{\lambda}(1-\tilde{\rho})(1-\tilde{\rho}^{N+1})} \quad (25)$$

(d) The average length of time a customer stood in line

$$\tilde{T}_q = \tilde{\rho}^2\frac{[1-(N)\tilde{\rho}^{N-1} + (N-1)(\tilde{\rho}^N)]}{\tilde{\lambda}(1-\tilde{\rho})(1-\tilde{\rho}^{N+1})} \quad (26)$$

VIII. SOLO SERVER FUZZY QUEUING MODEL WITH LIMITED CAPABILITY

Interpret the entry rate and the departure rate as both TFNs and TIFNs symbolized by $\tilde{\lambda}$, $\tilde{\lambda}'$ and $\tilde{\mu}$,$\tilde{\mu}'$ respectively. We postulate the system's greatest limit, i.e., $N = 2$.

A. Single server fuzzy queuing model with finite capacity

Let $\tilde{\lambda} = (3,4,5)$ is the arrival rate and $\tilde{\mu} = (13,14,15)$ is the service rate of the queuing model.

Determine the TFN in the form of $(\tilde{m}, \tilde{\alpha}, \tilde{\beta})$ as $\tilde{\lambda} = (4,1,1)$ and $\tilde{\mu} = (14,1,1)$.

To determine the values of a No. of consumers and their sojourn time in the queue as well as a system using suitable formulas among (23), (24), (25), & (26). It is necessary to use the appropriate arithmetic operations described in (1), (2), (3), (4), and (5) for add, sub, multiply, and divide, respectively.

For instance, the value of $\widetilde{N}_s$ is calculated as follows:

$$\widetilde{N}_s = \frac{(0.2857,1,1)[1-3(0.2857,1,1)^2 + 2(0.2857,1,1)^3]}{(1-(0.2857,1,1))(1-(0.2857,1,1)^3)}$$

$$\widetilde{N}_s = \frac{(0.2857,1,1)[1-(0.2448,1,1)+(0.0466,1,1)]}{(0.7143,1,1)(0.9767,1,1)}$$

$$\widetilde{N}_s = \frac{(0.2857,1,1)(0.8018,1,1)}{(0.6976,1,1)}$$

$$\widetilde{N}_s = \frac{(0.2290,1,1)}{(0.6976,1,1)}$$

$$\widetilde{N}_s = (0.3282,1,1)$$

$$\widetilde{N}_s = (-0.6718,0.3282,1.3282)$$

Similarly, calculate the remaining parameters and the metrics of performance are calculated and tabulated in Table I.

TABLE I
PERFORMANCE MEASURES USING TRIANGULAR FUZZY NUMBERS

	Quantifiable metrics using TFN
$\widetilde{N}_q$	$(-0.9404,0.0596,1.0596)$
$\widetilde{N}_s$	$(-0.6718,0.3282,1.3282)$
$\widetilde{T}_q$	$(-0.9851,0.0149,1.0149)$
$\widetilde{T}_s$	$(-0.918,0.082,1.082)$

The following figures depict the visualizations of Table I.

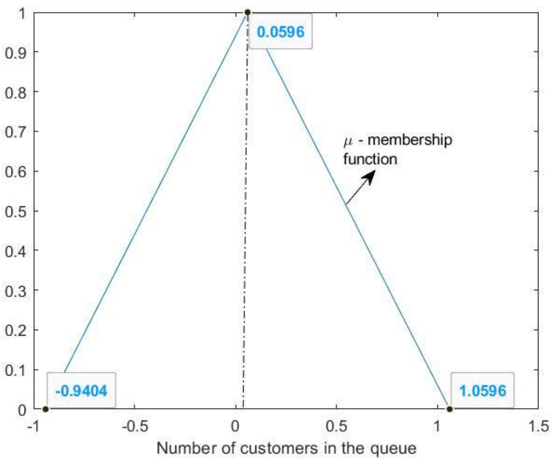

Fig. 5. The value of a No. of consumers in the queue $\tilde{N}_q$

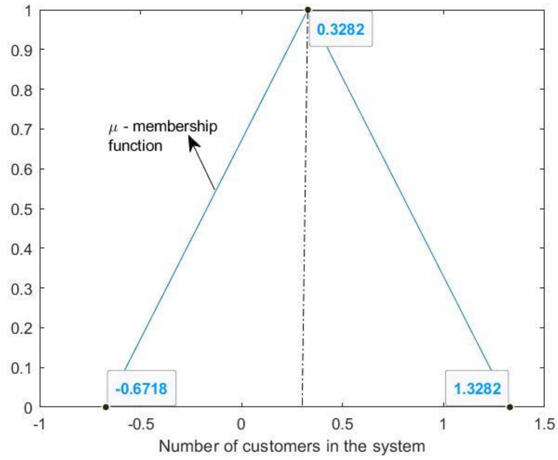

Fig. 6. The value of a No. of consumers in the system $\tilde{N}_s$

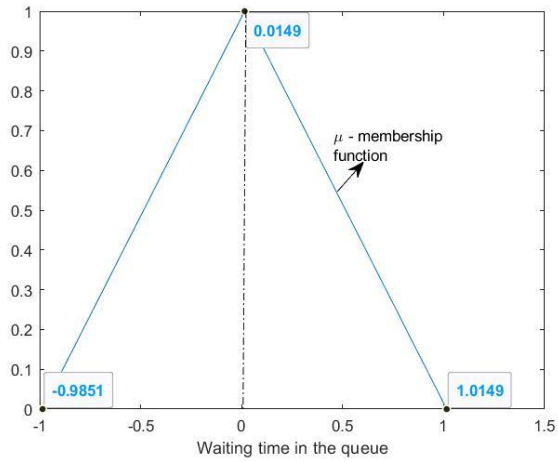

Fig. 7. The value of the sojourn time of consumers in the queue $\tilde{T}_q$

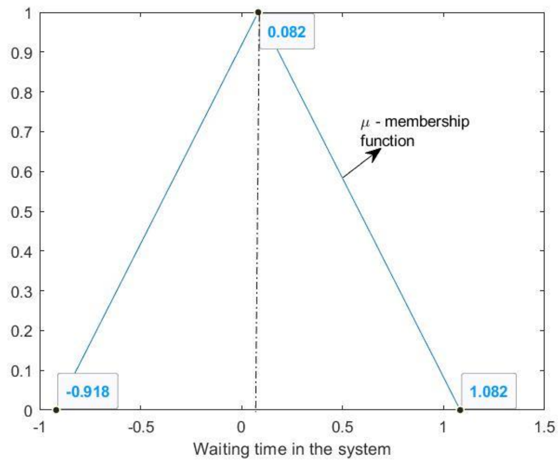

Fig. 8. The value of the sojourn time of consumers in the system $\tilde{T}_s$

B. Single server intuitionistic fuzzy queuing model with finite capacity

Let $\tilde{\lambda}' = (3.5,4,4.5; 3,4,5)$ is the arrival rate and $\tilde{\mu}' = (13.5,14,14.5; 13,14,15)$ is the service rate of the queuing model.

Determine the TIFN in the form of $(\tilde{m},\tilde{\alpha},\tilde{\beta};\tilde{m},\tilde{\alpha}',\tilde{\beta}')$ as $\tilde{\lambda}'=(4,0.5,0.5;4,1,1)$ and $\tilde{\mu}'=(14,0.5,0.5;14,1,1)$.

To determine the values of a No. of consumers and their sojourn time in the queue as well as a system using suitable formulas among (23), (24), (25), & (26). It is necessary to use the appropriate arithmetic operations described in (6), (7), (8), (9), and (10) for addition, subtraction, multiplication, and division, respectively.

For instance, the value of $\tilde{N}_s$ is calculated as follows:

$$\tilde{N}_s=\frac{(0.2857,0.5,0.5;0.2857,1,1)[1-3(0.2857,0.5,0.5;0.2857,1,1)^2+2(0.2857,0.5,0.5;0.2857,1,1)^3]}{\big(1-(0.2857,0.5,0.5;0.2857,1,1)\big)\big(1-(0.2857,0.5,0.5;0.2857,1,1)^3\big)}$$

$$\tilde{N}_s=\frac{(0.2857,0.5,0.5;0.2857,1,1)(0.8018,0.5,0.5;0.8018,1,1)}{(07143,0.5,0.5;0.7143,1,1)(0.9767,0.5,0.5;0.9767,1,1)}$$

$$\tilde{N}_s=\frac{(0.2290,0.5,0.5;0.2290,1,1)}{(0.6976,0.5,0.5;0.6976,1,1)}$$

$$\tilde{N}_s=(-0.1718,0.3282,0.8282;-0.6718,0.3282,1.3282)$$

Similarly, calculate the remaining parameters and the metrics of performance are calculated and tabulated in Table II.

TABLE II
PERFORMANCE MEASURES USING INTUITIONISTIC TRIANGULAR FUZZY NUMBERS

	Quantifiable metrics using TIFN
$\tilde{N}'_q$	$(-0.4404,0.0596,0.5596;-0.9404,0.0596,1.0596)$
$\tilde{N}'_s$	$(-0.1718,0.3282,0.8282;-0.6718,0.3282,1.3282)$
$\tilde{T}'_q$	$(-0.4851,0.0149,0.5149;-0.9851,0.0149,1.0149)$
$\tilde{T}'_s$	$(-0.418,0.0820,0.582;-0.918,0.0820,1.0820)$

The following figures depict the visualizations of Table II.

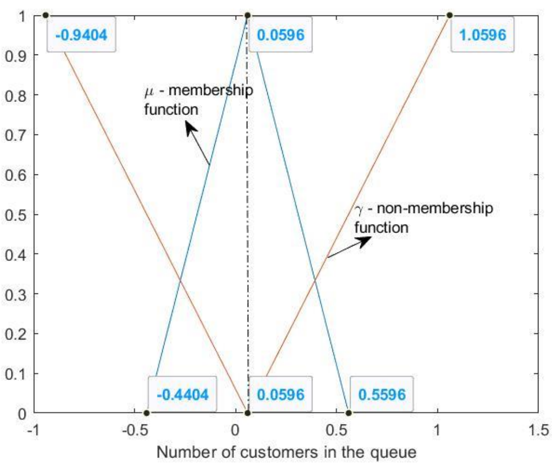

Fig. 9. The membership (μ) and the non-membership (γ) functions of the No. of consumers in the queue $\tilde{N}'_q$

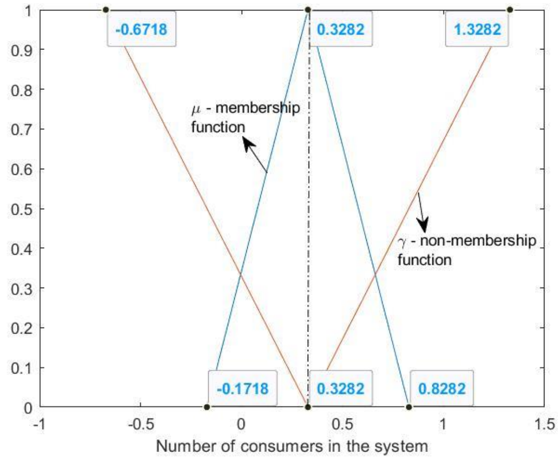

Fig. 10. The membership (μ) and the non-membership (γ) functions of the No. of consumers in the system $\tilde{N}'_s$

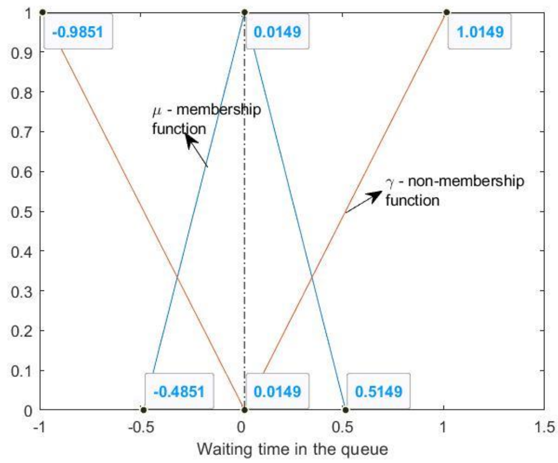

Fig. 11. The membership (μ) and the non-membership (γ) functions of the sojourn time of consumers in the queue $\tilde{T}_q{}'$

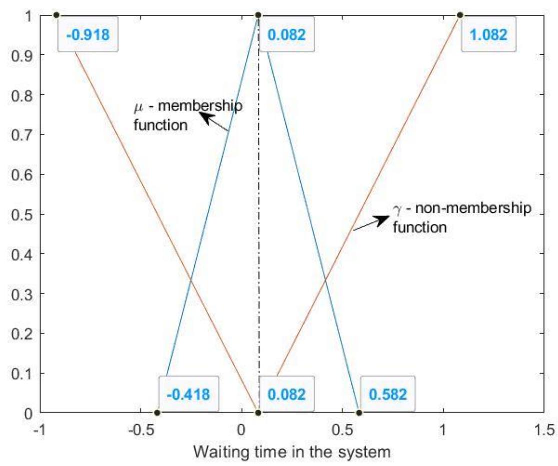

Fig. 12. The membership (μ) and the non-membership (γ) functions of the sojourn time of consumers in the system $\tilde{T}'_s$

IX. RESULTS AND DISCUSSIONS

Here, we've utilized the fuzzy set and intuitionistic fuzzy set notions to tackle the issue. Compared to the identical method utilizing fuzzy sets, we achieved better results using intuitionistic fuzzy sets. The performance of the intuitionistic fuzzy classifier is highly dependent on the recognition capacity of a smaller class. It is directly related to the fact that Atanassov's [9] intuitionistic fuzzy sets, which are a generalization of fuzzy sets, require more parameters, which makes the resulting models more reliable (both for memberships and non-memberships). In this work, we compare fuzzy and intuitionistic fuzzy queuing models and discuss queuing conundrums in practical settings. And their evaluations are tabulated as follows:

Intuitionistic fuzzy optimization approaches offer a useful and effective tool for modelling and optimizing the unstructured system. Intuitionistic fuzzy modelling enhances the validity of poorly organized systems by providing a complete awareness of the complexity of the decision parameters. One of the most helpful aspects of intuitionistic fuzzy set theory is its capacity to generate ambiguous and hazy goals and restrictions in problem-solving situations. The intuitionistic fuzzy decision methodology's objective is to make the best choice while adhering to a set of constraints and attaining a certain set of goals. The intuitionistic fuzzy decision-building method is utilized when the decision-maker does not place equal weight on the aims, purposes and limitations.

Tables I and II provides the results, which show different assessments for a multitude of membership functions (TFN and TIFN).

i) The mean value of $\widetilde{N}_q = 0.0596$ and the left and right stretched values are -0.9404 and 1.0596 respectively emphasizing that the queue length of consumers is closely between -0.9404 and 1.0596. Its most assured value is 0.0596.

ii) The mean value of $\widetilde{N}_s = 0.3282$ and the left and right stretched values are -0.6718 and 1.3282 respectively emphasizing that the system length of consumers is closely between -0.6718 and 1.3282. Its most assured value is 0.3282.

iii) The mean value of $\widetilde{T}_q = 0.0149$ and the left and right stretched values are -0.9851 and 1.0149 respectively emphasizing that the sojourn time of consumers in the queue is closely between -0.9851 and 1.0149. Its most assured value is 0.0149.

iv) The mean value of $\widetilde{T}_s = 0.082$ and the left and right stretched values are -0.918 and 1.082 respectively emphasizing that the sojourn time of consumers in the system is closely between -0.918 and 1.082. Its most assured value is 0.082.

v) The mean value of $\widetilde{N}_q' = 0.0596$ and the left and right fuzziness of membership (μ) functions are -0.4404 and 0.5596 respectively and the left and right fuzziness of non-membership (γ) functions are -0.9404 and 1.0596 respectively. Its most assured value is 0.0596.

vi) The mean value of $\widetilde{N}_s' = 0.3282$ and the left and right fuzziness of membership (μ) functions are -0.1718 and 0.8282 respectively and the left and right fuzziness of non-membership (γ) functions are -0.6718 and 1.3282 respectively. Its most assured value is 0.3282.

vii) The mean value of $\widetilde{T}_q' = 0.0149$ and the left and right fuzziness of membership (μ) functions are -0.4851 and 0.5149 respectively and the left and right fuzziness of non-membership (γ) functions are -0.9851 and 1.0149 respectively. Its most assured value is 0.0149.

viii) The mean value of $\widetilde{T}_s' = 0.082$ and the left and right fuzziness of membership (μ) functions are -0.418 and 0.582 respectively and the left and right fuzziness of non-membership (γ) functions are -0.918 and 1.082 respectively. Its most assured value is 0.082.

Table III indicates that the performance metrics for the intuitionistic fuzzy queuing model and the fuzzy queuing theory model are interoperable. For the fuzzy queuing theory model, the values of $\widetilde{N}_q$, $\widetilde{N}_s$, $\widetilde{T}_q$, and $\widetilde{T}_s$ are within the range of values of the intuitionistic fuzzy queuing theory model. As a result, the outcomes of performance measurements for the intuitionistic fuzzy queuing model and the fuzzy queuing theory model exemplify that both models are comparable. While intuitionistic fuzzy queuing encompasses a wide range of values, fuzzy queuing offers a range of values. Since the obtained value of the intuitionistic fuzzy queuing model falls within the range of performance measures, Subsequently, it demonstrates that the outcome is coherent.

Using the TFN in this study's intuitionistic fuzzy environment, we adjust for stabilization and deprivation levels to ensure that the total of both virtues never outstrips one. For this kind of fuzzified integer, we implemented different non-normal arithmetic methods. The envisioned configurations are short and to the point because they were developed using traditional algorithmic mathematics. This campaign is easy and simple to use in practical systems. The TIFN is then measured to the nearest interval number. The foremost strength of this strategy is that it facilitates us to adapt a multi-section heuristic hastily to solve a compelled unbridled optimization framework with TIFN correlations. In focusing on customer equity, fundraising, administration, and earth sciences, which will be the focus of our future research, the contemporary strategies and proposals are consigned to be pertinent to various sorts of updated decision-making bollards. TIFN encompasses a wider variety of options than TFN, even though their average correlation is fairly comparable. Although the fuzzy set theory is used to deal with uncertainty in decision-making situations, it only accounts for membership extent and lacks a model for hesitance. The distinctive quality of intuitionistic fuzzy sets is the cognizance of the confirmation and deprivation levels of each asset. It consequently becomes more factual, pertinent, and implementable.

X. THE AMENDED MODEL'S BOUNDARIES

The envisaged model has some detriments. One of them is the prospect that the waiting environment will be constricted. The reasonable assumption is that the birth rate may be regarded as dependent on the state. Multi-channel queuing problems occur recurrently in sectors and providers. After a customer has been identified and a provider has been supplied, the customer may need to acquire another provider

TABLE III
COMPARISON PERFORMANCE MEASURES BETWEEN FUZZY QUEUING THEORY MODEL AND INTUITIONISTIC FUZZY QUEUING THEORY MODEL

Performance Measures	Mean	Fuzzy Queuing Model	Intuitionistic Fuzzy Queuing Model
No. of customers in the queue	0.0596	$(-0.9404, 0.0596, 1.0596)$	$(-0.4404, 0.0596, 0.5596; -0.9404, 0.0596, 1.0596)$
No. of customers in the system	0.3282	$(-0.6718, 0.3282, 1.3282)$	$(-0.1718, 0.3282, 0.8282; -0.6718, 0.3282, 1.3282)$
Waiting time in the queue	0.0149	$(-0.9851, 0.0149, 1.0149)$	$(-0.4851, 0.0149, 0.5149; -0.9851, 0.0149, 1.0149)$
Waiting time in the system	0.082	$(-0.918, 0.082, 1.082)$	$(-0.418, 0.0820, 0.582; -0.918, 0.0820, 1.0820)$

from another facility, which will necessitate them to queue again. In such instances, assessing the problem becomes even more complicated.

XI. CONCLUSION

When attempting to deal with putative deployment in single server queuing models with finite capacity, IFS is shown to be a more valuable tool than the fuzzy set theory in this manuscript. Amidst altering the composition of the queues from fuzzy to crisp, we assessed the system using benchmark grading rubrics such as the projected length of the customer line and the system for both classifications of arrivals. Besides that, fuzzy values and intuitionistic fuzzy values are used to quantify the postulated sojourn time of consumers in the line and across the system. Another rationale for using the suggested technique indicator is that it provides more than one solution to morals in the queuing system, utilizing multiple kinds of membership functions (TFN and TIFN) while sustaining a precise calculation within the shuttered crisp interval.

Using various fuzzy numbers (TFN and TIFN), we surmise that fuzzy identity has been stimulated to a restricted extent in miniature. The characterizations of the time between entries and service time are nebulous. It should be acknowledged that by increasing the number of variables, the success of the queuing model can be improved. The proposed model can help endeavors, distributors and retail outlets determine the ideal lethal injection proportions of the queuing model without a doubt.

The fuzzy and intuitionistic fuzzy queue with finite capacity is explained in greater detail, and the prediction model is used to arrive at scientific conclusions. The TFN and TIFN mathematical manifestations are used to evaluate the proposed queuing system's accuracy and completeness. The recommended methodology's versatility is revealed by a numerical model. Because the intuitionistic fuzzy theory is more flexible and scalable, the intuitionistic fuzzy queuing model is significantly more productive and advantageous in reviewing and evaluating the dimensions of queuing models. As a result, intuitionistic fuzzy queuing, according to this investigation, is one of the healthiest modes of computing performance standards because the evidence gathered from the application is easier to recognize and better comprehend.

The deterministic parameters for the fuzzy numbers can be combined to increase the scope of this article. Neutrosophic sets are yet another potential area of study for the future. The authors are collaborating on more sophisticated concepts for user contexts, like scenarios where multiple servers process a customer simultaneously across multiple serving streams or phases.

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