Separation Axioms in Soft L-topological Spaces

Sandhya S Pai, Baiju Thankachan*

***Abstract*—The soft set is a mapping from a parameter to the power set of the universe. Molodtsov developed the notion of soft sets as a tool for modeling with unknown variables. Soft L-topological spaces are defined over a soft lattice L with a specified set of parameters P by the same authors and the continuity of soft L-topological space mappings has also been investigated. This paper introduces the soft L-T_i-space $(i = 0; 1; 2; 2\frac{1}{2}; 3; 3\frac{1}{2})$ on a soft L-topological space and explores some of their properties. It also discusses the soft L-invariant properties based on soft L-T_i-space $(i = 0; 1; 2; 2\frac{1}{2}; 3; 3\frac{1}{2})$ namely soft L-hereditary property and soft L-topological property.**

***Index Terms*—Soft L-set, soft L-topology, soft L-T_i-space $(i = 0; 1; 2; 2\frac{1}{2}; 3; 3\frac{1}{2})$, invariant properties**

I. INTRODUCTION

IN 1999 D. Molodtsov [1], [2] introduced the concept of a soft set and started to develop basics of the corresponding theory as a new approach for modeling uncertainties. The aim of this notion was to make a certain discretization of such fundamental mathematical concepts with effectively continuous nature and to provide a new tool for the mathematical analysis in real life problems. To achieve this aim a certain parameterization of a given set X was proposed resulting in the concept of a soft structure over the set X. Later, Maji et.al [3], [4] investigated Molodtsov's soft sets [1], [2] and provided definitions based on the equality of two soft sets, the subset and superset of a soft set, the complement of a soft set, the null soft set, and the absolute soft set, along with examples and basic properties. The algebraic structure of set theory dealing with uncertainties is also studied by many authors [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18]. The concept of soft set is extended to soft lattices and soft fuzzy sets by Li F [19] in the year 2010. Shabir and Naz [20] introduced the concept of soft topological spaces in 2011 and obtained some of its basic properties. In 2016, Cigdem Gunduz Aras, Ayse Sonmez, and Huseyin Cakalli [22] introduced soft continuous mappings and studied some of its properties. Hazra, Majumdar, and Samanta [23] defined continuity of soft mappings in 2012, and many other authors [24], [25] have also studied the same. Moreover, in 2015, Yang et.al[26] first explored the concept of soft continuous mapping between two soft topological spaces. Tantawy et.al[27] introduced separation axioms T_i $(i = 0; 1; 2; 3; 4; 5)$ in a soft L-topological space and explored some of their properties. These axioms are soft topological features under specific soft mapping. Soft separation axioms are studied by [28] and Soft Urysohn space was studied by Ramkumar et.al[29].

Using the notion of soft set initiated by Molodtsov [1], [2], the authors extended this idea to the field of soft lattices and obtained the topological properties of soft lattices [21]. Soft Lattice topological spaces (Soft L-topological spaces or Soft L-space) are introduced by the same authors [21] over an initial universe X with a predefined set of parameters P. The authors have defined some basic properties of soft L-topological spaces and also introduced soft L-open and soft L-closed sets [21]. The soft L-closure of a soft lattice, which is a generalization of the closure of a set, is also defined in the same work. The idea of parameters is critical when dealing with the collection of parameterized topologies on the initial universe. One can build a topological space for each parameter, which increases the importance of parameter involvement. On the initial universe, the authors have proved that a soft L-topological space generates a parameterized family of topologies [21]. But the converse need not be true. It means that, if we are given some topologies for each parameter, it is not possible to construct a soft L-topological space. Soft L-continuous mappings are defined by the same authors in [30] with a fixed set of parameters across an initial universe. Some algebraic properties of soft L-mappings such as injection, surjection, bijection, and composition of soft L-mappings are discussed, and studied the continuity mapping under soft L-topological spaces. Soft open and soft closed L-mappings, and soft L-homeomorphism are introduced and some interesting results are obtained. Furthermore, the same authors [31] investigated the concept of soft L-continuous mapping between two soft L-topological spaces and cartesian product in soft L-topological spaces.

The separation axioms are merely axioms in the sense that they may be added to the definition of topological space as supplementary axioms to achieve a more constrained definition of what a topological space is. In the present paper, we propose soft L-separation axioms i.e., soft L-T_i-spaces $(i = 0; 1; 2; 2\frac{1}{2}; 3; 3\frac{1}{2})$ for soft L-topological spaces and have discussed some of the basic results. The notions of soft L-normal and soft L-regular spaces are explored in this paper. Based on the above-mentioned soft L-separation axioms, we also investigate invariant properties in soft L-topological spaces such as soft L-hereditary property and soft L-topological property.

II. PRELIMINARIES AND BASIC DEFINITIONS

We refer to L as a complete lattice throughout this study, and our assumption is that L is consistent. A unary operation $\prime: L \longrightarrow L$ is a quasi complementation if it is an involution (i.e., $\alpha^{''} = \alpha$ for all $\alpha \in L$) that inverts the ordering (i.e., $\alpha \leq \beta \Longrightarrow \beta' \leq \alpha'$).

Definition 1: [2] "Let X be an initial universe set and P be a set of parameters. The power set of X is denoted as

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Dr. Sandhya S. Pai is an Assistant Professor of Mathematics Department, Manipal Institute of Technology, Manipal Academy of Higher Education Manipal-576104, Karnataka, India. (Email: sandhya.pai@manipal.edu).

Dr. Baiju Thankachan is an Associate Professor (Senior scale) of Mathematics Department, Manipal Institute of Technology, Manipal Academy of Higher Education Manipal-576104, Karnataka, India. (Email: baiju.t@manipal.edu). *Corresponding author

$\wp(X)$ and $A \subset P$. Then a pair (F, A) is said to be a soft set over X, where the mapping F is given by $F : A \to \wp(X)$. i.e., a soft set over X is regarded as a parameterized family of subsets of the universe X. For $a \in A$, the set of approximate elements of the soft set (F, A) denoted by $F(a)$."

Definition 2: [19] "Consider $M = (f, X, L)$, where L is a complete lattice, $f : X \longrightarrow \wp(L)$ is a mapping, X is a universe set, then M is called the soft lattice denoted by f_P^L. ie., for every $x \in X$, f_P^L is a soft lattice over L, if $f(x)$ is a sub lattice of L."

Definition 3: [21] "The relative complement of a soft lattice f_P^L is denoted by $(f_P^L)'$ and is defined as $(f_P^L)' = (f'^L_P)$ where $f' : P \longrightarrow \wp(L)$ is a mapping given by $f'(p) = L - f(p)$ for all $p \in P$."

Definition 4: [21] "Consider X as an initial universe set and P as the non-empty set of parameters.
Let τ be the set of complete, uniquely complemented soft lattices over L, then τ is said to be a soft lattice topology on L if;
$(i)\phi, L$ belongs to τ.
(ii) The arbitrary union of soft lattices in τ belongs to τ.
(iii) The finite intersection of soft lattices in τ belongs to τ.
Then (L, τ, P) is called a soft lattice topological space (soft topological lattice space or soft L -space) over L."

Example 1: [21] Let $L = \{0, 1, l_1, l_2, l_3\}$ be the lattice where l_1, l_2, l_3 represents the students of class 12, $P = \{p_1, p_2\}$ be the parameter in which p_1: brilliant and p_2: average.
Let us consider a collection $\tau = \{\phi, L, f_{1P}^L, f_{2P}^L, f_{3P}^L, f_{4P}^L\}$, where $f_{1P}^L, f_{2P}^L, f_{3P}^L, f_{4P}^L$ are soft lattices over L in which f_1, f_2, f_3, f_4 represents subjects like Mathematics, Physics, Chemistry, Computer science respectively. It is defined as follows,
$f_1(p_1) = \{l_2\}, f_1(p_2) = \{l_1\}$,
$f_2(p_1) = \{l_2, l_3\}, f_2(p_2) = \{l_1, l_2\}$,
$f_3(p_1) = \{l_1, l_2\}, f_3(p_2) = \{L\}$,
$f_4(p_1) = \{l_1\}, f_4(p_2) = \{l_1, l_3\}$
Therefore τ is a soft lattice topology.

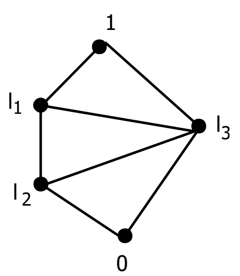

Fig. 1. Example for Complete Soft Lattice

Hence (L, τ, P) is a soft L-topological space.
Further, $\tau_{p_1} = \{\phi, L, \{l_1\}, \{l_2\}, \{l_2, l_3\}, \{l_1, l_2\}\}$
and $\tau_{p_2} = \{\phi, L, \{l_1\}, \{l_1, l_3\}, \{l_1, l_2\}\}$ are topologies on L. Hence these collections based on each parameter gives a soft lattice topology on L.

Definition 5: [21] "Consider (L, τ, P) as a soft lattice topological space over L, then the members of τ are called soft L-open sets in L."

Definition 6: [21] "Let (L, τ, P) be a soft lattice topological space over L. A soft lattice f_P^L over L is said to be a soft L-closed set in L, if its relative complement $(f_P^L)'$ belongs to τ."

Definition 7: [21] " Consider L be a lattice, P be the set of parameters and $\tau = \{\phi, L\}$. Then τ is called the soft indiscrete lattice topology on L and (L, τ, P) is said to be a soft indiscrete lattice topological space over L."

Definition 8: [21] "Consider L be a lattice, P be the set of parameters and let τ be the collection of all soft lattices which can be defined over L. Then τ is called the soft discrete lattice topology on L and (L, τ, P) is said to be a soft discrete lattice topological space over L."

Definition 9: [21] " Consider (L, τ, P) as a soft lattice topological space over L and f_P^L be a soft lattice over L. Then the soft lattice closure of f_P^L, denoted by $\overline{f}_P^L$, is the intersection of all soft L-closed super sets of f_P^L."

Definition 10: [21] "Let (L, τ, P) be a soft lattice topological space over L and f_P^L be a soft lattice over L. Then we associate with $\underline{f_P^L}$, a soft lattice L, denoted by $\overline{f}_P^L$ and defined as $\overline{f}(p) = \overline{f(p)}$, where $\overline{f(p)}$ is the soft L-closure of $f(p)$ in τ_p for each $p \in P$."

Definition 11: [21] "Consider (L, τ, P) as a soft lattice topological space over L, g_P^L be a soft lattice over L and $x \in L$. Then x is said to be a soft L-interior point of g_P^L if there exists a soft L-open set f_P^L such that $x \in f_P^L \subset g_P^L$. It is denoted by $(f_P^L)^o$."

Definition 12: [21] "Let (L, τ, P) be a soft lattice topological space over L, g_P^L be a soft lattice over L and $x \in L$. Then g_P^L is said to be a soft lattice neighbourhood of x if there exists a soft L-open set f_P^L such that $x \in f_P^L \subset g_P^L$."

Definition 13: [21] "Let (L, τ, P) be a soft lattice topological space over L and Y be a non-empty subset of L. Then $\tau_Y = \{Y f_P^L \mid f_P^L \in \tau\}$ is said to be the soft relative lattice topology on Y and (Y, τ_Y, P) is called a soft L-subspace of f_P^L."

Proposition 1: [21] "Let (L, τ, P) be a soft L-space. Then the set $\tau_p = \{f(p) | f_P^L \in \tau\}$ for all $p \in P$ gives a topology on L."

Proposition 2: [21] "Let (L, τ, P) be a soft lattice topological space over L, g_P^L be a soft lattice over L and $x \in L$. If x is a soft L-interior point of g_P^L, then x is an interior point of $g(p)$ in (L, τ_p), for each $p \in P$."

Proposition 3: [21] "Let (L, τ, P) be a soft lattice topological space over L. Then
(1) each $x \in L$ has a soft lattice neighbourhood.
(2) if f_P^L and g_P^L are soft lattice neighbourhoods of some $x \in L$, then $f_P^L \cap g_P^L$ is also a soft lattice neighbourhood of x.
(3) if f_P^L is a soft lattice neighbourhoods of $x \in L$ and $f_P^L \tilde{\subset} g_P^L$, then g_P^L is a soft lattice neighbourhoods of $x \in L$."

Proposition 4: [21] "Let (L, τ, P) be a soft lattice topological space over L. For any soft L-open set f_P^L over L, f_P^L is a soft L-neighbourhood of each point of $\cap_{p \in P} f(p)$."

Proposition 5: [21] "Let (L, τ, P) be a soft lattice topological space over L and Y be a non-empty subset of L. Then (Y, τ_{pY}) is a subspace of (L, τ_p, P) for each $p \in P$."

Proposition 6: [21] "Let (L, τ_Y, P) be a soft L-subspace of a soft L-topological space (L, τ, P) and f_P^L be a soft L-open set in Y. If $Y \in \tau$, then $f_P^L \in \tau$."

Theorem 7: [21] "Let (L, τ_Y, P) be a soft L-subspace of a soft L-topological space (L, τ, P) and f_P^L be a soft L-open set in Y. Then

$(i) f_P^L$ is soft L-open in Y iff $f_P^L = \widetilde{Y} \cap g_P^L$ for some $g_P^L \in \tau$.
$(ii) f_P^L$ is soft L-closed in Y iff $f_P^L = \widetilde{Y} \cap g_P^L$ for some soft L-closed set g_P^L in L."

Definition 14: [30] "Consider f_P^L as a soft lattice over L. The soft lattice f_P^L is called a soft L-point, denoted by (l_p, P), for the element $p \in P$, if $f(p) = \{l\}$ and $f(p') = \phi$ for all $p' \in P - \{l\}$."

Definition 15: [30] "Let (L_1, τ_1, P) and (L_2, τ_2, P) be two soft lattice topological spaces. The mapping f_g is called a soft L-mapping from L_1 to L_2 denoted by $f_g: (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$, where $f: L_1 \longrightarrow L_2$ and $g: P \longrightarrow P$ are two mappings. For each soft L-neighbourhood g_P^L of $(f(l)_p, P)$, if there exist a soft L-neighbourhood f_P^L of (l_p, P) such that $f_g(f_P^L \subset g_P^L)$, then f_g is said to be soft L-continuous mapping at (l_p, P).
If f_g is soft L-continuous mapping for all (l_p, P), then f_g is called soft L-continuous mapping."

Definition 16: [30] "Let (L_1, τ_1, P) and (L_2, τ_2, P) be two soft lattice topological spaces, $f_g: (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ be a mapping. Then
(a) If the image $f_g(f_P^L)$ of each soft L-open set f_P^L over L_1 is a soft L-open set in L_2, then f_g is said to be a soft L-open mapping.
(b) If the image $f_g(h_P^L)$ of each soft L-closed set h_P^L over L_1 is a soft L-closed set in L_2, then f_g is said to be a soft L-closed mapping."

Definition 17: [30] "Consider (L_1, τ_1, P) and (L_2, τ_2, P) as two soft lattice topological spaces, $f_g: (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ be a mapping. If f_g is a bijection, soft L-continuous and f_g^{-1} is a soft L-continuous mapping, then f_g is said to be soft L-homeomorphism from L_1 to L_2.
When a soft homeomorphism f_g exists between L_1 and L_2, we say that L_1 is soft L-homeomorphic to L_2."

Theorem 8: [30] "Let (L_1, τ_1, P) and (L_2, τ_2, P) be two soft lattice topological spaces,
$f_g: (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ be a bijection mapping. Then the following conditions are equivalent:
(1) f_g is a homeomorphism on soft L-topological space,
(2) f_g is a continuous and closed mapping on soft L-topological space,
(3) f_g is a continuous and open mapping on soft L-topological space."

Theorem 9: [31] "We know that (L_1, τ_1, P) and (L_2, τ_2, P) are two soft lattice topological spaces, $f_g: (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ be a mapping. Then the following conditions are equivalent:
(1) $f_g: (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ is a soft L-continuous mapping.
(2) For each soft L-open set G_P^L over L_2, $f_g^{-1}(g_P^L)$ is a soft L-open set over L_1.
(3) For each soft L-closed set H_P^L over L_2, $f_g^{-1}(h_P^L)$ is a soft L-closed set over L_1.
(4) For each soft L-set F_P^L over L_1, $f_g(\overline{f_P^L}) \subset \overline{f_g(f_P^L)}$.
(5) For each soft L-set G_P^L over L_2, $\overline{f_g^{-1}(g_P^L)} \subset f_g(\overline{g_P^L})$.
(6) For each soft L-set g_P^L over L_2, $f_g^{-1}((g_P^L)^o) \subset (f_g^{-1}(g_P^L))^o$."

Theorem 10: [31] "consider (L_1, τ_1, P) and (L_2, τ_2, P) be two soft lattice topological spaces, $f_g: (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ be a mapping. Then f_g is soft L-continuous if and only if $f_g(\overline{f_P^L}) \subset \overline{f_g((f_P^L))}$.
A bijective soft L-continuous mapping f_g is a soft L-homeomorphism if and only if
$\overline{f_g(f_P^L(p))} = f_g(\overline{f_P^L})(p) \forall p \in P$."

III. Soft lattice separation axioms

Definition 18: Let (L, τ, P) be a soft lattice topological spaces over L and $l_1, l_2 \in L$ such that $l_1 \neq l_2$. If there exist soft L-open sets f_P^L and g_P^L such that $l_1 \in f_P^L$ and $l_2 \notin f_P^L$ or $l_2 \in g_P^L$ and $l_1 \notin g_P^L$, then (L, τ, P) is called a soft L-T_0-space.

Definition 19: Let (L, τ, P) be a soft lattice topological spaces over L and $l_1, l_2 \in L$ such that $l_1 \neq l_2$. If there exist soft L-open sets f_P^L and g_P^L such that $l_1 \in f_P^L$ and $l_2 \notin f_P^L$ and $l_2 \in g_P^L$ and $l_1 \notin g_P^L$, then (L, τ, P) is called a soft L-T_1-space.

Definition 20: Let $l \in L$, then l_P^L denotes the soft L-set over L for which $l(p) = \{l\}$ for all $p \in P$.

Lemma 1: Let f_P^L be a soft L-set and $l_1 \in L$. Then
(1) $l_1 \in f_P^L$ if and only if $l_{1P}^L \subset f_P^L$.
(2) if $l_{1P}^L \cap f_P^L = \phi$, then $l_1 \notin f_P^L$.

Proof: The proof is obvious. ■

Theorem 11: Let (L, τ, P) be a soft L-topological space over L and $l_1 \in L$. If (L, τ, P) is a soft L-T_1-space, then for each soft L-open set f_P^L with $l_1 \in f_P^L$,
(1) $l_{1P}^L \subset \cap f_P^L$.
(2) for all $l_{2P}^L \neq l_{1P}^L$, $l_{2P}^L \notin \cap f_P^L$.

Proof: (1) Since $l_1 \in \cap f_P^L$, by lemma 1, it is obvious that $l_{1P}^L \subset \cap f_P^L$.
(2) Let $l_{1P}^L \neq l_{2P}^L$ for $l_{1P}^L, l_{2P}^L \in L$, then there exist a soft L-open set g_P^L such that $l_{1P}^L \in g_P^L$ and $l_{2P}^L \notin g_P^L$. So $l_{2P}^L \notin g(p)$ for some $p \in P$ and we have $l_{2P}^L \notin \cap g(p)$ for all $p \in P$. This implies $l_{2P}^L \notin \cap f_P^L$. ■

Generally, the converse of (2) of lemma 1 is not true and the same is illustrated by the following example.

Example 2: Let $L = \{l_1, l_2\}, P = \{p_1, p_2, p_3\}$ and let f_P^L be a soft L-set as shown below:
$f(p_1) = \{l_1\}, f(p_2) = \{l_1\}, f(p_3) = \{l_1, l_2\}$.
Then $\{l_2\} \in f_P^L$. But $l_{2P}^L \in f_P^L = \phi$ because of $l_2(p_3) \cap f(p_3) \neq \phi$ for $p_3 \in P$.
As shown in the example below, the equality in (1) of theorem 11 may not hold.

Example 3: Let $L = \{l_1, l_2\}, P = \{p_1, p_2, p_3, p_4\}$ and $\tau = \{\phi, L, f_{1P}^L, f_{2P}^L, f_{3P}^L, f_{4P}^L, f_{5P}^L\}$ is a soft L-topological space over L, where $f_{1P}^L, f_{2P}^L, f_{3P}^L, f_{4P}^L, f_{5P}^L$ are soft lattices over L, defined as follows,
$f_1(p_1) = \{l_1, l_2\}, f_1(p_2) = \{l_1\}, f_1(p_3) = \{l_1\}, f_1(p_4) = \{l_1\}$,
$f_2(p_1) = \{l_1, l_2\}, f_2(p_2) = \{l_1, l_2\}, f_2(p_3) = \{l_1\}, f_2(p_4) = \{l_1\}$
$f_3(p_1) = \{l_2\}, f_3(p_2) = \{l_2\}, f_3(p_3) = \{l_2\}, f_2(p_4) = \{l_1, l_2\}$,
$f_4(p_1) = \{l_2\}, f_4(p_2) = \phi, f_4(p_3) = \phi, f_4(p_4) = \{l_1\}$,
$f_5(p_1) = \{l_2\}, f_5(p_2) = \{l_2\}, f_5(p_3) = \phi, f_5(p_4) = \{l_1\}$.
For $l_1, l_2 \in L$, since $l_1 \neq l_2$, we have two soft L-sets f_{1P}^L, f_{2P}^L such that $l_1 \in f_{1P}^L$ and $l_2 \notin f_{1P}^L$ and $l_2 \in f_{2P}^L$ and $l_1 \notin f_{3P}^L$. Hence the soft L-topological space (L, τ, P) is a soft L-T_1-space.
But for all soft L-open sets $l_1 \in f_{1P}^L$ and $l_1 \in f_{2P}^L$, $f_{1P}^L \cap f_{2P}^L = f_{1P}^L \neq l_{1P}^L$.

Theorem 12: Let (L, τ, P) be a soft lattice topological spaces over L. If l_{1P}^L is a soft L-closed set in τ for each $l_1 \in L$, then (L, τ, P) is a soft L-T_1-space over L.

Proof: If l_{1P}^L is a soft L-closed set in τ, then $(l_{1P}^L)'$ is a soft L-open set in τ for each $l_1 \in L$. Let $l_1, l_2 \in L$ such that $l_1 \neq l_2$. For $l_1 \in L$, $(l_{1P}^L)'$ is a soft L-open set in τ such that $l_2 \in (l_{1P}^L)'$ and $l_1 \notin (l_{1P}^L)'$.
Similarly, $l_2 \in L$, $(l_{2P}^L)'$ is a soft L-open set in τ such that $l_1 \in (l_{1P}^L)'$ and $l_2 \notin (l_{1P}^L)'$.
Thus (L, τ, P) is a soft L-T_1-space over L. ∎

Remark 1: The example below demonstrates that the converse of theorem 12 is false.

Example 4: Suppose $L = \{l_1, l_2\}, P = \{p_1, p_2\}$ and $\tau = \{\phi, L, f_{1P}^L, f_{2P}^L, f_{3P}^L\}$ is a soft L-topological space over L, where $f_{1P}^L, f_{2P}^L, f_{3P}^L$ are soft lattices over L, defined as follows,
$f_1(p_1) = L, f_1(p_2) = \{l_2\}$,
$f_2(p_1) = \{l_1\}, f_2(p_2) = L$,
$f_3(p_1) = \{l_1\}, f_3(p_2) = \{l_2\}$,
Thus (L, τ, P) be a soft lattice topological spaces over L.
Also $\tau_{p_1} = \{\phi, L, \{l_1\}\}$ and $\tau_{p_2} = \{\phi, L, \{l_2\}\}$.
Neither (L, τ_{p_1}) and (L, τ_{p_2}) is a soft L-T_1-space but $l_1, l_2 \in L$ such that $l_1 \neq l_2$, also $l_2 \in f_{1P}^L$ but $l_1 \notin f_{1P}^L$ and $l_1 \in f_{2P}^L$ but $l_2 \notin f_{2P}^L$.
Thus (L, τ, P) is a soft L-T_1-space over L.
For l_{1P}^L, l_{2P}^L over L, where
$l_1(p_1) = \{l_1\}, l_1(p_2) = \{l_1\}$,
$l_2(p_1) = \{l_2\}, l_2(p_2) = \{l_2\}$,
The relative complement $(l_{1P}^L)', (l_{2P}^L)' \in \tau$ over L which is defined by
$l_1'(p_1) = \{l_2\}, l_1'(p_2) = \{l_2\}$,
$l_2'(p_1) = \{l_1\}, l_2'(p_2) = \{l_1\}$,
Neither $(l_{1P}^L)'$ nor $(l_{2P}^L)' \in \tau$.
Hence converse is not true.
The prerequisites for resolving this challenge are outlined in the following two propositions.

Proposition 13: Let (L, τ, P) be a soft lattice topological spaces over L and $l_1, l_2 \in L$ such that $l_1 \neq l_2$. If there exist soft L-open sets f_P^L and g_P^L such that $l_1 \in f_P^L$ and $l_2 \in (f_P^L)'$ or $l_2 \in g_P^L$ and $l_1 \in (g_P^L)'$, then (L, τ, P) is called a soft L-T_0-space.

Proof: Let $l_1, l_2 \in L$ such that $l_1 \neq l_2$ and f_P^L and g_P^L are soft L-open sets over L such that $l_1 \in f_P^L$ and $l_2 \in (f_P^L)'$ or $l_2 \in g_P^L$ and $l_1 \in (g_P^L)'$.
If $l_2 \in (f_P^L)'$, then $l_2 \in (f(p))'$ for each $p \in P$.
This implies $l_2 \notin f(p)$ for each $p \in P$.
Therefore $l_2 \notin f_P^L$.
Similarly, we can show that if $l_1 \in (g_P^L)'$, then $l_1 \notin g_P^L$.
Hence (L, τ, P) is called a soft L-T_0-space. ∎

Proposition 14: Consider (L, τ, P) as a soft lattice topological spaces over L. If (L, τ, P) is called a soft L-T_0-space over L, then (L, τ_p) is a soft L-T_0-space for each parameter $p \in P$.

Proof: For any $p \in P$, (L, τ_p) is a soft topological spaces and $l_1 \in f_P^L$ and $l_2 \in (f_P^L)'$ or $l_2 \in g_P^L$ and $l_1 \in (g_P^L)'$ so that $l_1 \in f(p)$ and $l_2 \notin f(p)$ or $l_2 \in g(p)$ and $l_1 \notin g(p)$.
Hence (L, τ_p) is a soft L-T_0-space for each parameter $p \in P$. ∎

A result similar to proposition 13 can be obtained for soft L-T_1-space.

Proposition 15: Consider (L, τ, P) as a soft lattice topological spaces over L and $l_1, l_2 \in L$ such that $l_1 \neq l_2$. If there exist soft L-open sets f_P^L and g_P^L such that $l_1 \in f_P^L$ and $l_2 \in (f_P^L)'$ or $l_2 \in g_P^L$ and $l_1 \in (g_P^L)'$, then (L, τ, P) is called a soft L-T_1-space.
A result similar to proposition 14 can be obtained for soft L-T_1-space.

Proposition 16: We consider (L, τ, P) to be a soft lattice topological spaces over L. If (L, τ, P) is called a soft L-T_1-space over L, then (L, τ_p) is a soft L-T_1-space for each parameter $p \in P$.

Definition 21: Let (L, τ, P) be a soft lattice topological spaces over L and $l_1, l_2 \in L$ such that $l_1 \neq l_2$. If there exist soft L-open sets f_P^L and g_P^L such that $l_1 \in f_P^L$ and $l_2 \in g_P^L$ and $f_P^L \cap g_P^L = \phi$, then (L, τ, P) is called a soft L-T_2-space.

Proposition 17: We consider (L, τ, P) to be a soft lattice topological spaces over L. If (L, τ, P) is called a soft L-T_2-space or soft L-Hausdorff space over L, then (L, τ_p) is a soft L-T_2-space for each parameter $p \in P$.

Proof: Suppose (L, τ, P) is said to be a soft L-T_2-space over L.
For any $p \in P$, $\tau_p = \{f(p) \mid f_P^L \in \tau\}$.
Let $l_1, l_2 \in L$ such that $l_1 \neq l_2$, there exist soft L-open sets f_P^L and g_P^L such that $l_1 \in f_P^L$ and $l_2 \in g_P^L$ and $f_P^L \cap g_P^L = \phi$.
which implies $l_1 \in f(p)$ and $l_2 \in g(p)$ and $f(p) \cap g(p) = \phi$.
Hence (L, τ_p) is a soft L-T_2-space for each parameter $p \in P$. ∎

Remark 2: (1) Soft L-T_1-space $\Longrightarrow$ Soft L-T_0-space.
(2) Soft L-T_2-space $\Longrightarrow$ Soft L-T_1-space.

Proof: Let (L, τ, P) be a soft lattice topological spaces over L and $l_1, l_2 \in L$ such that $l_1 \neq l_2$.
(1) If (L, τ, P) is a soft L-T_1-space, then there exist soft L-open sets f_P^L and g_P^L such that $l_1 \in f_P^L$ and $l_2 \notin f_P^L$ and $l_2 \in g_P^L$ and $l_1 \notin g_P^L$.
Obviously, we have $l_1 \in f_P^L$ and $l_2 \notin f_P^L$ or $l_2 \in g_P^L$ and $l_1 \notin g_P^L$. Thus (L, τ, P) is called a soft L-T_0-space.
(2) If (L, τ, P) is a soft L-T_1-space, then there exist soft L-open sets f_P^L and g_P^L such that $f_P^L \cap g_P^L = \phi$.
Since $f_P^L \cap g_P^L = \phi$, so $l_1 \notin g_P^L$ and $l_2 \notin f_P^L$. Hence (L, τ, P) is called a soft L-T_1-space. ∎
The example below demonstrates that the converse of the remark 2 is false.

Example 5: Suppose $L = \{l_1, l_2\}, P = \{p_1, p_2\}$ and $\tau = \{\phi, L, f_{1P}^L, f_{2P}^L, f_{3P}^L\}$ is a soft L-topological space over L, where $f_{1P}^L, f_{2P}^L, f_{3P}^L$ are soft lattices over L, defined as follows,
$f_1(p_1) = L, f_1(p_2) = \{l_2\}$,
$f_2(p_1) = \{l_1\}, f_2(p_2) = L$,
$f_3(p_1) = \{l_1\}, f_2(p_2) = \{l_2\}$,
Thus (L, τ, P) be a soft lattice topological spaces over L.
Also (L, τ, P) is called a soft L-T_1-space over L but not a soft L-T_2-space because $l_1, l_2 \in L$ and there do not exist soft L-open sets f_P^L and g_P^L such that $l_1 \in f_P^L$ and $l_2 \in g_P^L$ and $f_P^L \cap g_P^L = \phi$.
Now consider the following soft L-topology on L, $\tau = \{\phi, L, f_{1P}^L\}$, where $f_1(p_1) = L, f_1(p_2) = \{l_2\}$,
Thus (L, τ, P) be a soft lattice topological spaces over L.
Also (L, τ, P) is called a soft L-T_0-space over L but not a soft L-T_1-space because $l_1, l_2 \in L$ and there do not exist soft L-open sets f_P^L and g_P^L such that $l_1 \in f_P^L$ and $l_2 \notin f_P^L$ and $l_2 \in g_P^L$ and $l_1 \notin g_P^L$.

Theorem 18: Let (L, τ, P) be a soft L-topological space over L and $l_1 \in L$. If (L, τ, P) is a soft L-T_2-space, then $l_{1P}^L = \cap f_P^L$ for each soft L-open set f_P^L with $l_1 \in L$.

Proof: Suppose there exist $l_2 \in L$ such that $l_1 \neq l_2$ and

$l_2 \in \cap f_P^L$ for some $p \in P$.
Since (L, τ, P) is a soft L-T_2-space, there exist soft L-open set g_P^L and h_P^L such that $l_1 \in g_P^L$ and $l_2 \in h_P^L$ and $g_P^L \cap h_P^L = \phi$ and so $g_P^L \cap l_{1P}^L = \phi$ and $g(p) \cap l_2(p) = \phi$.
This is a contradiction to $l_2 \in \cap f_P^L$ for some $p \in P$. ■

Definition 22: Let (L, τ, P) be a soft lattice topological spaces over L. Then (L, τ, P) is a soft L-$T_{2\frac{1}{2}}$-space or soft L-Urysohn space if for $l_1, l_2 \in L$ such that $l_1 \neq l_2$, there exist two soft L-open sets f_P^L and g_P^L such that $l_1 \in f_P^L$ and $l_2 \in g_P^L$ and $\overline{f_P^L} \cap \overline{g_P^L} = \phi$.

Theorem 19: A soft L-$T_{2\frac{1}{2}}$-space is a soft L-T_2-space.

Proof: The proof is straight forward. ■

Definition 23: Let (L, τ, P) be a soft lattice topological spaces over L, g_P^L be a soft L-closed set in L and $l_1 \in L$ such that $l_2 \notin g_P^L$. If there exist soft L-open sets f_{1P}^L and f_{2P}^L such that $l_1 \in f_{1P}^L$, $g_P^L \subset f_{2P}^L$ and $f_{1P}^L \cap f_{2P}^L = \phi$, then (L, τ, P) is called a soft L-regular space.

Definition 24: Let (L, τ, P) be a soft lattice topological spaces over L. Then (L, τ, P) is said to be a soft L-T_3-space if it soft L-regular and soft L-T_1-space.

Lemma 2: Let (L, τ, P) be a soft L-topological space over L and let g_P^L be a soft L-closed set in L and $l_1 \in L$ such that $l_1 \notin g_P^L$. If (L, τ, P) is a soft L-regular space, then there exist soft L-open set f_P^L such that $f_P^L \cap g_P^L = \phi$.

Proof: If g_P^L be a soft L-closed set in L and $l_1 \in L$ such that $l_1 \notin g_P^L$, then by the definition of soft L-regularity, there exist soft L-open set f_P^L such that $f_P^L \cap g_P^L = \phi$. ■

Theorem 20: Let (L, τ, P) be a soft L-topological space over L and $l_1 \in L$. If (L, τ, P) is a soft L-regular space, then
(1) For a soft L-closed set g_P^L, $l_1 \notin g_P^L$ if and only if $l_P^L \cap g_P^L = \phi$.
(2) For a soft L-open set f_P^L, $l_1 \notin f_P^L$ if and only if $l_P^L \cap f_P^L = \phi$.

Proof: (1) Let $l_1 \notin g_P^L$. Then by lemma 2, then there exist soft L-open set f_P^L such that $f_P^L \cap g_P^L = \phi$.
Since $l_P^L \subset f_P^L$, we have $l_P^L \cap g_P^L = \phi$.
(2) Let $l_1 \notin f_P^L$. Then there are two cases.
(i) $l_1 \in f(p_1)$ for all $p_1 \in P$.
(ii) $l_1 \in f(p_1)$ and $l_1 \in f(p_2)$ for some $p_1, p_2 \in P$.
In case (i), it is obvious that $l_P^L \cap g_P^L = \phi$.
In the other case, $l_1 \in f'(p_1)$ and $l_1 \notin f'(p_2)$ for some $p_1, p_2 \in P$ and so $(f_P^L)'$ is a soft L-closed set such that $l_1 \notin (f_P^L)'$ by (1), $l_P^L \cap f_P^L = \phi$.
So $l_P^L \subset f_P^L$ but this contradicts $l_1 \in f(p_1)$ for all $p_1 \in P$.
This implies $l_P^L \cap f_P^L = \phi$.
The converse is obvious. ■

Theorem 21: Let (L, τ, P) be a soft L-topological space over L and $l_1 \in L$. Then the following are equivalent:
(1) (L, τ, P) is a soft L-regular space.
(2) For each soft L-closed set g_P^L such that $l_P^L \cap g_P^L = \phi$, there exist soft L-open sets f_{1P}^L and f_{2P}^L such that $l_P^L \subset f_P^L$, $g_P^L \subset f_{2P}^L$ and $f_{1P}^L \cap f_{2P}^L = \phi$.

Proof: Proof follows from theorem 20 and lemma 1. ■

Theorem 22: Let (L, τ, P) be a soft L-topological space over L and $l_1 \in L$. If (L, τ, P) is a soft L-regular space, then
(1) For a soft L-open set f_P^L, $l_1 \in f_P^L$ if and only if $l_1 \in f(p_1)$ for all $p_1 \in P$.
(2) For a soft L-open set f_P^L, $f_P^L = \cup\{l_1 \in f_P^L : l_1 \in f(p_1)$ for all $p_1 \in P\}$.
(3) For each $p_1, p_2 \in P$, $\tau_{p_1} = \tau_{p_2}$.

Proof: (1) Assume that for some $p_1 \in P$, $l_1 \in f(p_1)$ and $l_1 \notin f_P^L$. Then by theorem 20, $l_P^L \cap f_P^L = \phi$.
By the assumption, this is a contradiction and so $l_1 \in f_P^L$.
The converse is obvious.
(2) The proof follows from (1) and $l_1 \in f_P^L$ iff $l_P^L \subset f_P^L$.
(3) follows from (2). ■

Theorem 23: Let (L, τ, P) be a soft L-topological space over L and $l_1 \in L$. If (L, τ, P) is a soft L-regular space, then the following are equivalent:
(1) (L, τ, P) is a soft L-T_1-space.
(2) For $l_1, l_2 \in L$ such that $l_1 \neq l_2$, there exist soft L-open sets f_P^L and g_P^L such that $l_{1P}^L \subset f_P^L$ and $l_{2P}^L \cap f_P^L = \phi$ and $l_{2P}^L \subset g_P^L$ and $l_{1P}^L \cap g_P^L = \phi$.

Proof: It is obvious that $l_1 \in f_P^L$ iff $l_{1P}^L \subset f_P^L$ and by theorem 20, $l_1 \notin f_P^L$ iff $l_{1P}^L \cap f_P^L = \phi$.
Hence they are equivalent. ■

Theorem 24: Let (L, τ, P) be a soft L-topological space over L. If (L, τ, P) is a soft L-T_3-space, then for $l_1 \in L$, l_{1P}^L is a soft L-closed set.

Proof: To prove: $(l_{1P}^L)'$ is a soft L-open set.
For each $l_2 \in X - \{l\}$, since (L, τ, P) is a soft L-regular space and soft L-T_1-space by theorem 23, there exist a soft L-open set $f_{l_2P}^L$ such that $l_{2P}^L \subset f_{l_2P}^L$ and $l_{1P}^L \subset f_{l_2P}^L = \phi$.
Therefore, $\cup_{l_2 \in X-\{l\}} f_{l_2P}^L \subset (l_{1P}^L)'$.
In addition, let $\cup_{l_2 \in X-\{l\}} f_{l_2P}^L = h_P^L$, where $h(p) = \cup_{l_2 \in X-\{l\}} f_{l_2}(p)$ for all $p \in P$.
From the definition 3 and definition 15, we have $l'(p) = X - \{l\}$ for each $p \in P$.
Now for each $l_2 \in X - \{l\}$ and for each $p \in P$, $l'(p) = X - \{l\} = \cup_{l_2 \in X-\{l\}} \{l_2\} = \cup_{l_2 \in X-\{l\}} l_2(p) \subset \cup_{l_2 \in X-\{l\}} f_{l_2}(p) = h(p)$.
By definition 3, $l'(p) = X - \{l\} = \cup_{l_2 \in X-\{l\}} f_{l_2P}^L$.
Since $f_{l_2P}^L$ is a soft L-open set for each $l_2 \in X - \{l\}$.
Hence l_{1P}^L is a soft L-closed set. ■

Theorem 25: A soft L-T_3-space is a soft L-T_2-space.

Proof: Let (L, τ, P) be any soft L-T_3-space.
For $l_1, l_2 \in L$ such that $l_1 \neq l_2$, by theorem 24, l_{2P}^L is a soft L-closed set and $l_1 \notin l_{2P}^L$. From the definition of soft L-regularity, there exist soft L-open sets f_P^L and g_P^L such that $l_{1P}^L \in f_P^L$ and $l_{2P}^L \in g_P^L \subset g_P^L$ and $l_P^L \cap g_P^L = \phi$.
Hence (L, τ, P) is a soft L-T_2-space. ■

Theorem 26: If (L, τ, P) is a soft L-T_3-space, then (L, τ_p) is a soft L-T_3-space for some parameter $p \in P$.

Proof: The proof is obvious using theorem 25 and proposition 15. ■

Theorem 27: Let (L, τ, P) be a soft L-topological space over L. If (L, τ, P) is a soft L-regular space and if $l_1 \in L$ is a soft L-closed set for each $l_1 \in L$, then (L, τ, P) is a soft L-T_3-space.

Proof: Since $l_1 \in L$ is a soft L-closed set for each $l_1 \in L$, by theorem 12, (L, τ, P) is a soft L-T_1-space.
Even it is also soft L-regular space by theorem 22 and definition 23.
Hence (L, τ, P) is a soft L-T_3-space. ■

Definition 25: Let (L, τ, P) be a soft lattice topological spaces over L, then (L, τ, P) is called a soft L-completely regular space if every soft L-closed subset f_P^L and any given soft L-point $l_P^L \notin f_P^L$, then there is a soft L-continuous function $f_g \colon (L, \tau, P) \longrightarrow (L, \tau, P)$ such that $f(l) = \phi$ and $f(f_P^L) = L$.

Otherwise, we say l and f_P^L can be seperated by a soft L-continuous function.

Definition 26: A soft L-topological space (L, τ, P) is said to be soft L-$T_{3\frac{1}{2}}$-space if it is a soft L-completely regular space and a soft L-T_1-space.

Theorem 28: A soft L-$T_{3\frac{1}{2}}$-space is a soft L-T_3-space.

Proof: The proof is straight forward. ■

A. Soft L-Hereditary Property

Proposition 29: Consider (L, τ, P) as a soft lattice topological spaces over L and let Y be a non-empty subset of L. If (L, τ, P) is a soft L-T_0-space, then (Y, τ_Y, P) is a soft L-T_0-space.

Proof: Let $l_1, l_2 \in L$ such that $l_1 \neq l_2$ and f_P^L and g_P^L are soft L-open sets over L such that $l_1 \in f_P^L$ and $l_2 \notin f_P^L$ or $l_2 \in g_P^L$ and $l_1 \notin g_P^L$.
Now $l_1 \in Y \Rightarrow l_1 \in f_P^L$.
Hence $l_1 \in Y \cap f_P^L = Yf_P^L$, where $f_P^L \in \tau$
Consider $l_2 \notin f_P^L \Rightarrow l_2 \notin f(p)$ for some $p \in P$. Then $l_2 \notin Y \cap f(p) = Y(p) \cap f(p)$.
Therefore $l_2 \notin Y \cap f_P^L = Yf_P^L$.
Similarly, it can be proved that $l_2 \in g_P^L$ and $l_1 \notin g_P^L$, then $l_2 \in Yg_P^L$ and $l_1 \notin Yg_P^L$.
Thus (Y, τ_Y, P) is a soft L-T_0-space. ■
A result similar to proposition 29 can be obtained for soft L-T_1-space.

Proposition 30: Let (L, τ, P) be a soft lattice topological spaces over L and Y be a non-empty subset of L. If (L, τ, P) is a soft L-T_1-space, then (Y, τ_Y, P) is a soft L-T_1-space.

Proposition 31: Consider (L, τ, P) to be a soft lattice topological spaces over L and Y be a non-empty subset of L. If (L, τ, P) is a soft L-T_2-space, then (Y, τ_Y, P) is a soft L-T_2-space.

Proof: Let $l_1, l_2 \in L$ such that $l_1 \neq l_2$, there exist soft L-open sets f_P^L and g_P^L such that $l_1 \in f_P^L$ and $l_2 \in g_P^L$ and $f_P^L \cap g_P^L = \phi$.
So for each parameter $p \in P$, $l_1 \in f(p)$ and $l_2 \in g(p)$ and $f(p) \cap g(p) = \phi$.
which implies $l_1 \in Y \cap f(p)$ and $l_2 \in Y \cap g(p)$ and $f(p) \cap g(p) = \phi$.
Hence $l_1 \in Yf_P^L$ and $l_2 \in Yg_P^L$ and $Yf_P^L \cap Yg_P^L = \phi$ where $Yf_P^L, Yg_P^L \in \tau_Y$.
Thus (Y, τ_Y, P) is a soft L-T_2-space. ■
A result similar to proposition 31 can be obtained for soft L-$T_{2\frac{1}{2}}$-space.

Proposition 32: Consider (L, τ, P) to be a soft lattice topological spaces over L and Y be a non-empty subset of L. If (L, τ, P) is a soft L-$T_{2\frac{1}{2}}$-space, then (Y, τ_Y, P) is a soft L-$T_{2\frac{1}{2}}$-space.

Theorem 33: Let (L, τ, P) be a soft L-regular space and (Y, τ_Y, P) is a soft L-subspace of (L, τ, P) such that $\tau_Y = \{Y_P^L \mid f_P^L \in \tau\}$ is soft L-relative topology on Y. Then (Y, τ_Y, P) is a soft L-regular space.

Proof: (Y, τ_Y, P) is a soft L-subspace of soft L-regular space (L, τ, P). Let $y \in Y$ and g_P^L be soft L-closed set in Y s.t. $y \notin g_P^L$. Now $\overline{g_P^L} \cap Y = g_P^L$. Clearly, $y \notin \overline{g_P^L}$. Thus $\overline{g_P^L}$ is soft L-closed set in Y.
Since (L, τ, P) is a soft L-regular space, then $\exists$ soft L-open sets f_{1P}^L and f_{2P}^L s.t. $y \in f_{1P}^L$, $\overline{g_P^L} \subset f_{2P}^L$ and $f_{1P}^L \cap f_{2P}^L = \phi$.
Then $Y \cap f_{1P}^L$, $Y \cap f_{2P}^L$ are two distinct soft L-open sets in Y s.t. $y \in Y \cap f_{1P}^L$ and $g_P^L \subseteq Y \cap f_{2P}^L$.
Hence the proof. ■

Proposition 34: Consider (L, τ, P) as a soft lattice topological spaces over L and Y be a non-empty subset of L. If (L, τ, P) is a soft L-T_3-space, then (Y, τ_Y, P) is a soft L-T_3-space.

Proof: By proposition 30, (Y, τ_Y, P) is a soft L-T_1-space.
Let $l_2 \in f_P^L$. then $l_2 \notin (Yf_P^L \cap Yg_P^L)$, where $f_P^L = (Yf_P^L \cap Yg_P^L)$ for some soft L-closed set in L by theorem 12.
But $l_2 \in f_P^L$, so $l_2 \notin f_P^L$.
As (L, τ, P) is a soft L-T_3-space, so that there exist soft L-open sets g_{1P}^L and g_{2P}^L in L such that $l_2 \in g_{1P}^L$, $g_{1P}^L \subset g_{2P}^L$ and $g_{1P}^L \cap g_{2P}^L = \phi$.
Now if we take $f_{1P}^L = Y_P^L \cap g_{1P}^L$ and $f_{2P}^L = Y_P^L \cap g_{2P}^L$, then f_{1P}^L and f_{2P}^L belongs to τ_Y such that $l_2 \in f_{1P}^L$ and $f_P^L \subset Y_P^L \cap g_{2P}^L = f_{2P}^L$ and $f_{1P}^L \cap f_{2P}^L \subset g_{1P}^L \cap g_{2P}^L = \phi$,
That is $f_{1P}^L \cap f_{2P}^L = \phi$.
Thus (Y, τ_Y, P) is a soft L-T_3-space. ■
A result similar to proposition 34 can be obtained for soft L-$T_{3\frac{1}{2}}$-space.

Proposition 35: Consider (L, τ, P) as a soft lattice topological spaces over L and Y be a non-empty subset of L. If (L, τ, P) is a soft L-$T_{3\frac{1}{2}}$-space, then (Y, τ_Y, P) is a soft L-$T_{3\frac{1}{2}}$-space.

B. Soft L-Topological Property

Theorem 36: The property of being soft L-T_i-space $(i = 0; 1; 2; 2\frac{1}{2})$ is a topological property.

Proof: We prove the theorem for $i = 2$ and the remaining cases can be proved in the same manner.
Let $f_g: (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ be a soft L-continuous mapping such that
(i) f_g is one-to-one, onto and soft L-open mapping.
(ii) (L_1, τ_1, P) is a soft L-T_2-space.
To prove: (L_2, τ_2, P) is a soft L-T_2-space.
Let $l_1, l_2 \in L_2$ such that $l_1 = l_2$.
Since f_g is one-to-one, onto mapping, then there exist two soft L-open sets f_P^L and g_P^L in L_1 such that $f_g(f_P^L) = l_1, f_g(g_P^L) = l_2$ and $f_P^L \neq g_P^L$.
But (L_1, τ_1, P) is a soft L-T_2-space, then there exist $f_P^L, g_P^L \in \tau_1$ such that $f_P^L \subset f_P^L, g_P^L \subset g_P^L$ and $f_P^L \cap g_P^L = \phi_{L_1}$.
This implies $f_g(f_P^L) = l_1 \in f_g(f_P^L), f_g(g_P^L) = l_2 \in f_g(g_P^L)$ and $f_g(f_P^L \cap g_P^L) = f_g(f_P^L) \cap f_g(g_P^L) = f_g(\phi_{L_1}) = \phi_{L_2}$.
Since $f_P^L, g_P^L \in \tau$ and f_g is a soft L-open mapping, then $f_g(f_P^L), f_g(g_P^L) \in \tau_2$.[from the definition of soft L-open mapping].
Now there exist $f_g(f_P^L), f_g(g_P^L) \in \tau_2$ such that $l_1 \in f_g(F_P^L)$, $l_2 \in f_g(g_P^L)$ and $f_g(f_P^L) \cap f_g(g_P^L) = \phi$.
Hence (L_2, τ_2, P) is a soft L-T_2-space. ■

Theorem 37: The property of being soft L-T_i-space $(i = 3; 3\frac{1}{2})$ is a soft L-topological property or it is preserved under a soft L-homeomorphism mapping.

Proof: We prove that the theorem is true for i = 3; the remaining case is analogous. Since the property of being soft L-T_1-space is a soft L-topological property, we need to show that the property of soft L-regularity is a soft L-topological property.
Let $f_g: (L_1, \tau_1, P) \longrightarrow (L_2, \tau_2, P)$ be a soft L-continuous

mapping such that
(1) f_g is a homeomorphism from L_1 to L_2.
(2) (L_1, τ_1, P) is a soft L-regular space.
Let g_P^L be a τ_2-closed soft L-subset of L_2 and let $l_2 \in L_2$ such that $l_2 \notin g_P^L$.
Since f_P^L is an onto-mapping, there exist a soft L-open set $f_P^L \in L_1$ such that $f_g(f_P^L) = l_2$.
Since f_P^L is soft L-continuous mapping and g_P^L is a τ_2-closed soft L-subset of L_2, we have $f_g^{-1}(g_P^L)$ is a τ_1-closed soft L-subset of L_1. (from theorem 9).
Since $f_g(f_P^L) = l_2 \notin g_P^L$, we have $f_g^{-1}(f_g(g_P^L)) = f_P^L \notin f_g^{-1}(g_P^L)$.
Now $f_g^{-1}(g_P^L)$ is a τ_1-closed soft L-subset of L_1, $f_P^L \in L_1$ such that $f_P^L \notin f_g^{-1}(g_P^L)$.
But (L_1, τ_1, P) is a soft L-regular space, there exist $f_P^L, g_P^L \in \tau_1$ such that $f_P^L \in f_P^L$, $f_g^{-1}(g_P^L) \subset g_P^L$ and $f_P^L \cap g_P^L = \phi_{L_1}$ and therefore
$f_g(f_P^L) = l_2 \in f_g(f_P^L)$, $f_g(f_g^{-1}(g_P^L)) = g_P^L \subset f_g(g_P^L)$ and $f_g(f_P^L \cap g_P^L) = f_g(f_P^L) \cap f_g(g_P^L) = f_g(\phi_{L_1}) = \phi_{L_2}$.
Since f_g^{-1} is a soft L-continuous mapping i.e., f_g is an soft L-open mapping (by theorem 8).
Now $f_P^L, g_P^L \in \tau_1$ and f_g is an soft L-open mapping, then $f_g(f_P^L)$ and $f_g(g_P^L) \in \tau_2$ (from definition 15 of soft L-open mapping).
So finally, $f_g(f_P^L)$ and $f_g(g_P^L) \in \tau_2$ such that $l_2 \in f_g(f_P^L)$, $g_P^L \subset f_g(g_P^L)$ and $f_g(f_P^L) \cap f_g(g_P^L) = \phi_{L_2}$.
Hence (L_2, τ_2, P) is a soft L-regular space. ■

IV. Conclusion

Soft set theory is a generalized method for solving problems with uncertainty. The authors extended the concept of soft set using soft lattice in a soft L-topology context. This paper dealt with the soft L-separation axioms in soft L-topological spaces and obtained some basic results based on properties such as soft L-hereditary property and soft L-topological property.

These soft L-separation axioms are important for the development of soft L-topology theory in order to tackle complex issues with uncertainties in economics, engineering, medicine, the environment, and general man-machine systems of many sorts. These discoveries contribute to the expansion of the soft L-topology tool set.

This paper has paved way for a new beginning of a structure, which helped us to study new ideas that is useful in expanding theoretical research.

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